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An Analysis of the Common Core State Standards for Mathematics and the Content of Math 095: Essentials of Algebra at the University of Wisconsin-Milwaukee

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AN ANALYSIS OF THE COMMON CORE STATE STANDARDS FOR MATHEMATICS AND THE CONTENT OF MATH 095: ESSENTIALS OF ALGEBRA AT THE UNIVERSITY OF WISCONSIN-MILWAUKEE

by

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ABSTRACT

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In this analysis we present the content in Math 095: Essentials of Algebra at the University of Wisconsin-Milwaukee that is aligned to the Common Core State Standards for Mathematics. We find that the content in Math 095 is aligned to a small subset of the high school Number and Quantity, Algebra, and Function standards. We present a representative sample of homework and assessment items from the traditional lecture format of Math 095 and compare them to assessment items released by the Smarter Balanced Assessment Consortium and Illustrative Mathematics. We then discuss content from the Common Core State Standards for Mathematics that is absent from Math 095 or overlaps with Math 105: Intermediate Algebra. We conclude that overall the content from Math 095 meets the standards that require students to solve, graph, factor, rewrite expressions, and apply operations to expressions; however, we find that standards expecting students to understand concepts, model with mathematics, and explain reasoning are not met in the course content. We conclude by presenting recommendations based on our findings.

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Chapter 1: Common Core State Standards

1.1 Motivation

The Common Core State Standards (herein also referred to as "the standards" or as "CCSSM" when specifically referring to the mathematics standards) are a set of kindergarten through high school standards developed by the National Governors Association Center for Best Practices and the Council of Chief State School Officers. The official Common Core website as of April 2013 gives the mission of the standards [7]:

> The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy.

The standards for English language arts and mathematics were officially released in June 2010, and as of April 2013 they have been adopted by forty five states, the District of Columbia, four territories, and the Department of Defense Education Activity according to the initiative website [7]. Alaska, Nebraska, Puerto Rico, Texas, and Virginia have opted not to adopt either the mathematics or English language arts standards, while Minnesota has adopted only the English language arts standards [7]. States adopting the Common Core standards are required to adopt all of the standards in the two subject areas; however, states may add up to fifteen percent of their own state-specific standards as well [6].

The Common Core standards were designed for several reasons. Policymakers realized that the different proficiency standards across the states were resulting in uneven achievement. For students who move from one state to another, differences in state standards can create gaps or redundancy in grade level content. Further, if there is a need to assess academic performance across states, it is more efficiently accomplished if there are common assessments, which in turn necessitate common content and practices across the grade levels.

As is implied in the mission statement of the Common Core initiative, another important factor in the development of the standards is the goal of improving the U.S. economy by enhancing the labor force. According to a 2012 study performed by the Organization for Economic Cooperation and Development (the OECD, of which the United States is a member), "more than half of the GDP growth in OECD countries over the past decade is related to labor income growth among tertiary-educated individuals" and "employers pay almost twice as much for a 45-54 year-old worker with tertiary education, than for someone without an upper secondary education" [15]. The OECD also has conducted research on the international rankings of its members in terms of mathematics and reading literacy. The Program for International Student Assessment (PISA) measures the performance of 15-year-olds every three years, and the most recent report available is from 2009. PISA found the following key statistics related to mathematical proficiency [9]:

- "Among the 33 other OECD countries, 17 countries had higher average scores than the United States, 5 had lower average scores, and 11 had average scores not measurably different from the U.S. average."
- On a proficiency scale of one through six, with six being advanced, 27 percent of U.S. students score at or above proficiency level four—which was lower than the

> overall 32 percent average in the OECD countries—and 23 percent of U.S. students scores below level two, "what OECD calls a 'baseline level of mathematics proficiency […] at which students begin to demonstrate the kind of literacy skills that enable them to actively use mathematics.' "

The performance of American students in mathematics and English language arts has provided a motivation for the formulation of the Common Core standards.

Finally, in speaking specifically about the mathematics standards, traditional math content and curricula focus on breadth rather than depth. In a video produced by the Hunt Institute discussing the development of the mathematics standards, Jason Zimba, one of the central team writers of the Common Core, states, "the goal wasn't simply for [the states to] all to begin doing the same thing; the goal was for all of them to raise their game and do math education better." He adds that the goal was "to build on what the best states were doing" [11]. Jason describes how the standards were founded on what high-performing countries teach and the true demands of college and careers. To address the conflict between the breadth and depth of content coverage, the standards for mathematics were written with a concentration on focus and coherence rather than on quantity of content [11]. The standards are organized such that teachers have an explicit understanding of the content that students are coming into their course ready to do, the content they should be teaching in their grade, and what students need to be ready to do in the subsequent grade level.

1.2 Authors of the Mathematics Standards

Jason Zimba is one of three members of the central team of writers for the Common Core mathematics standards. His background consists of studies in astrophysics and mathematics, culminating with a Ph.D. in mathematical physics from UC-Berkeley, and he was previously a professor of Physics and Mathematics at Bennington College. Another prominent writer on the central team is William (Bill) McCallum. He is Head of the Department of Mathematics at the University of Arizona and holds a Ph.D. in mathematics from Harvard University. Philip (Phil) Daro is the third member and directs the partnership of University of California, Stanford, and others with the San Francisco Unified School District for SERP (Strategic Education Research Partnership) in San Francisco. All three writers have extensive experience serving on mathematics councils, contributing to advisory boards, and with research related to mathematics education.

While these three writers served as the central team for writing the official standards document, Bill McCallum indicates in the Hunt Institute video that there were many prominent contributors from the field of mathematics education: educators, mathematicians, teachers, policymakers, and so on [11]. Various mathematical organizations under the umbrella of the Conference Board of Mathematical Sciences comprised a work team of about 60 people within these constituencies. These organizations include the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), the Association of Mathematics Teacher Educators (AMTE), and the American Mathematical Society (AMS) and the Mathematical Association of America (MAA) [11]. After several rounds of drafting, editing, and revisions, the Common Core State Standards for Mathematics were released in June 2010 as were the standards for English language arts.

1.3 Structure and Nature of the Standards

The authors' ultimate goal was to make the standards clear and challenging with a higher level of cognitive demand, with students mastering conceptual understanding, procedural awareness and fluency, and problem solving skills. To that end, the standards are comprised of two types: practice and content standards. The Standards for Mathematical Practice are general mathematical habits and mindsets that should be instilled in students across their mathematical careers. According to the standards, they are as follows [14]:

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

These practices go hand in hand with the Standards for Mathematical Content. The content standards are explicitly laid out from kindergarten through eighth grade. Within each grade level, the standards are organized by domains—general groups of related standards—and subgroups of the domains called clusters. There are seven domains that span across multiple grade levels [14]:

- Counting and cardinality (kindergarten)
- Operations and algebraic thinking (kindergarten through grade five)

- Number and operations in base ten (kindergarten through grade five)
- Number and operations—fractions (grades three through five)
- Measurement and data (kindergarten through grade five and high school)
- Geometry (kindergarten through grade eight)
- Ratios and proportional relationships (grades six and seven)
- The number system (grades six through eight)
- Expressions and equations (grades six through eight)
- Functions (grade eight)
- Statistics and probability (grades six through eight)

Once students enter high school, the content is not delineated by grade level but rather by broader conceptual categories: number and quantity, algebra, functions, modeling, geometry, and statistics and probability.

The mathematical experience across the grades is meant to be a blend of both content and practice standards. As stated in the Common Core standards, the content standards are meant to combine both procedure and understanding, and many standards beginning with the word "understand" provide an opportunity to connect the content to the practices [14]. The standards further indicate that the standards beginning with "understand" are intended to be "weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics" [14]. In particular, the high school modeling standards are *both* practice and content standards. Modeling is intertwined throughout the curriculum, and content standards that are closely related to or augmented by modeling standards are specially marked with an asterisk.

1.4 Assessment Consortia and Illustrative Mathematics

Shortly after various states adopted the Common Core State Standards in June 2010, two assessment consortia formed. The U.S. Department of Education indicates that under the Race to the Top Assessment program, the Partnership for the Assessment of Readiness for College and Careers (PARCC) and the SMARTER Balanced Assessment Consortium (Smarter Balanced) were awarded grants on September 29, 2010 [17]. It also indicates that just over $185 million was awarded to PARCC and just over $175 million was awarded to Smarter Balanced [17]. As of April 2013, the Smarter Balanced consortium consists of twenty-four states plus the U.S. Virgin Islands as an affiliate member, and all but three of these states are governing members, meaning they have a vote in policy decisions [18]. Similarly, twenty-two states and the U.S. Virgin Islands are members of PARCC according to its website as of April 2013, and all but three of their states are governing board states [16]. Both consortia are charged with developing state-of-the-art assessments to support the CCSSM. It is noteworthy that there are alternate standards as well as alternative assessment consortia designed to maximize college and career readiness for students with disabilities, but the specifics of these will be omitted in this analysis.

The assessments developed by PARCC and Smarter Balanced will be implemented during the 2014-2015 academic year. The Center for K-12 Assessment & Performance Management at ETS (Electronic Testing Service) has released multiple publications regarding the components of the assessments from each consortia. There are multiple similarities and differences between the two consortia. For both PARCC and Smarter Balanced, the assessments will be computer-based and will be administered in grades three through eight and in high school. Each consortia is developing formative and summative assessment items, and the summative assessment items are comprised of two components: performance-based task items and end-of-year items. The performance-based items strive to assess mathematics standards that have traditionally been difficult to assess and are organized around real-world problems and complex tasks [6]. The end-of-year items will focus on the most important content in the mathematics standards. Both consortia will utilize a variety of item types: constructive responses, selected responses, technology-enhanced items (such as those involving graphing tools), and complex performance tasks [6]. The assessments from PARCC and Smarter Balanced will be administered during the final weeks of the school year, and the consortia are developing multiple interim assessments, resources, and tools to be used in CCSSM states.

While the content of the summative assessments and other resources are very similar, there are a few key differences between the consortia. The primary difference lies in whether each consortium will use adaptive technology. The PARCC assessments are fixed-form delivery assessments, where students will be given one of several similarly chosen sets of items and tasks. On the other hand, Smarter Balanced test takers will see an individually tailored set of items and tasks, which is determined by a student's performance [6]. The ETS report indicates, "Although not part of the current design and to be decided only after further study, PARCC will investigate the possible addition of an adaptive add-on to the [end-of-year] component which would be given only to students at the extremes of the performance spectrum in order to provide supplemental performance information" [6]. Another difference includes the frequency at which other developmental assessments are offered by the consortia: PARCC will offer diagnostic and mid-year assessments while Smarter Balanced will offer computer adaptive interim assessments.

Another prominent resource for assessment guidance and sample items is Illustrative Mathematics, an initiative of the Institute for Mathematics and Education funded by the Bill and Melinda Gates Foundation. According to its website, Illustrative Mathematics "provide[s] guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students experience in a faithful implementation of the Common Core State Standards, and by publishing other tools that support implementation of the standards" [12]. Among other content and grade level advisors, the three authors of the CCSSM serve as senior advisors on the team. The items released by Illustrative Mathematics are in close alignment with the Common Core standards and are also developed and reviewed by both mathematical and classroom experts [12].

1.5 Wisconsin

Wisconsin adopted the Common Core State Standards for English language arts and mathematics on June 2, 2010. Wisconsin is a governing member of Smarter Balanced and will be fully implementing the Smarter Balanced assessments starting in the 2014-2015 academic school year. According to the Wisconsin Department of Public Instruction, the implementation of the Common Core State Standards will consist of three general phases: Phase I (2010-2011) focusing on understanding the standards and identifying anticipated changes necessary based on the adoption of the standards; Phase II (2011-2012) focusing on local curriculum development aligned to the Common Core; and Phase III (2012-2015) focusing on instructional support and assessment implementation [19]. In general, the state plans to partner with Cooperative Educational Service Agencies (CESAs), professional organizations, local educational agencies (LEAs), institutes of higher education, and multistate partnerships in order to implement the Common Core standards [19]. The specific mathematics partners listed as of April 2013 include the Wisconsin Mathematics Council, the Wisconsin Association for Supervision and Curriculum Development (WASCD), and the CESAs [20]. As students progress into higher education, the impact of the Common Core in Wisconsin will not end with the K-12 community.

Chapter 2: Math 095

2.1 Course Formats at the University of Wisconsin-Milwaukee

Students attending the University of Wisconsin-Milwaukee (herein referred to as UWM) begin their academic careers with various mathematics placement levels. Those who are deemed not adequately prepared for Math 103: Contemporary Applications of Mathematics, Math 105: Intermediate Algebra, or Math 175: Mathematical Explorations for Elementary Teachers are required to pass Math 095: Essentials of Algebra with a C or better as a prerequisite. There are several different formats for the Math 095 course. They include a traditional classroom setting with a lecture; a computer lab setting using a computer adaptive software program, ALEKS; or an online course which also utilizes ALEKS. Given that most of the students completing the Math 095 course at UWM take it in the traditional classroom setting, the analysis to follow will concentrate on the content presented in this format.

Over the years, the Math 095 course has undergone changes in its textbook, instructors, course coordinator, course components, grading structures, and other elements. This analysis will concentrate on the structure of the Math 095 traditional setting course as of the Fall 2012 semester when full course materials were available for review. Many course components across the various sections of Math 095 are determined in advance by the course coordinator and are mostly uniform. There are multiple areas of assessment in the course: three basic skills tests, two formal content quizzes, three midterm examinations, and a cumulative final examination. In addition, there is daily written homework assigned, drawing on both the textbook *Beginning Algebra* (eighth edition, authors Baratto and Bergman, publisher McGraw-Hill) [5] and the computer program MathZone. There are weekly homework quizzes consisting of problems from both the textbook and from MathZone. The basic skills tests, formal quizzes, and the final examination are common across all sections; the midterm examinations and homework quizzes are created by the individual instructors, with instructors being granted access to test generators and banks of assessment questions. The content of the course centers on Chapters Two through Nine from the textbook, and we now concentrate on the content studied throughout the course.

2.2 Overview of Topics Studied

Math 095 at UWM presents an overview of some of the most essential and fundamental procedural components of algebra. The course begins with a study of linear equations and inequalities in one variable and then builds the skill of translating statements into the symbolic language of algebraic expressions. Further, students are asked to graph solutions of linear inequalities in one variable. The course continues with a study of polynomials, beginning with properties of exponents and then performing the operations of addition, subtraction, multiplication and division on polynomials, mostly up to degree four.

By mid-semester, students then extend properties of polynomials to factoring by the following methods: factoring the greatest common factor; factoring by grouping; factoring trinomials of the form $ax^2 + bx + c$ by guess and check and by rewriting the middle term to apply factoring by grouping; and factoring differences of perfect squares. Students then proceed to solve polynomial equations by factoring into irreducible polynomials. Building off their knowledge of polynomials, students then focus on rational expressions, including simplifying rational expressions and applying the operations of addition, subtraction, multiplication, and division to them. This leads to solving equations with rational expressions involving polynomials, mostly up to degree two.

After a survey of polynomials and rational expressions, students return to the case of linear equations in one or two variables. Students graph these equations on a coordinate plane, determine the slope and intercepts of these graphs, and find equations of parallel and perpendicular lines. Students extend linear equations to linear inequalities in two variables and graph these inequalities as half-planes. They conclude their study of linear equations by finding solutions of systems of linear equations using various methods: by graphing, substitution, and adding a multiple of one equation to another.

Finally, students revisit exponents and use radical notation to represent the square root of an expression. They rewrite expressions involving radicals, only addressing the case of a square root. Students solve simple radical equations in one variable and briefly address applications of the Pythagorean Theorem.

Throughout the semester, application/word problems are mixed into the course content and they reinforce the geometric concepts of area and perimeter of rectangles and triangles. Many word problems are also "number" problems whereby students find one or two numbers satisfying given conditions, resulting in students solving primarily linear and polynomial equations of degree two. Students also apply the process of solving systems of linear equations to find quantity/cost problems (e.g. how many tickets of two types were sold given the cost of the two tickets and the total dollar amount sold).

CHAPTER 3: ANALYSIS

3.1 Methods and Assumptions

In order to analyze the content of Math 095 in conjunction with CCSSM, we will explore the homework and assessment items for students throughout the course to obtain a full picture of what students are expected to master. Assessment items including basic skills exam items, quiz items, and exam items officially measure student mastery of objectives. For the reader's convenience, the assessment items from Exams One, Two, Three, and the Final Examination are in Appendices B, C, D, and E, respectively. Homework questions also indirectly contribute to the assessment of student learning. Aggregately, these items are an indicator of the skills and practices that are expected of students and provide an objective gauge of the depth of their content knowledge. In this analysis all homework and assessment items were gathered and individually aligned with the most appropriate CCSSM standard per the language of the standards. From this matching process, there are approximately twenty CCSSM standards that appear to be addressed to some degree in the content of Math 095. These standards fall within the CCSSM high school conceptual categories of Number and Quantity, Algebra, and Functions.

This chapter represents a summary of the content of Math 095 as viewed through the lens of the standards. We organize the standards first according to conceptual category. Then for each conceptual category, we present an overview of mathematical content from relevant domains in kindergarten through grade eight to demonstrate prerequisite knowledge leading up to the high school standards. We then present the CCSSM standards in the conceptual category to which the Math 095 content is aligned. A statement of the CCSSM standard and representative homework and assessment items from Math 095 will be given for each standard. We then compare the Math 095 items to sample assessment items as proposed by the Smarter Balanced assessment consortium and from Illustrative Mathematics with the goal of gauging the extent to which the Math 095 content fulfills the objectives of the corresponding CCSSM standard.

Once the content present in Math 095 is discussed in conjunction with the standards, we then provide a table (see Appendix A) with a full list of all standards in the Number and Quantity, Algebra, and Function conceptual categories as well as a summary of what is included in Math 095 and in Math 105 for general comparison. This will allow the reader to see content that is mostly absent from or overlapping in these mathematics courses at UWM.

Throughout this analysis, it is assumed that content (homework assigned, quizzes, and examinations) is *mostly* uniform across the various sections of Math 095. Delivery of the content and time spent on each topic will likely vary across sections, so we choose to analyze the uniform content of the course. As previously stated, the materials for this analysis are based on the course structure as of the Fall 2012 semester. The textbook *Beginning Algebra* (eighth edition, authors Baratto and Bergman, publisher McGraw-Hill) [5] was used in the Fall 2012 semester, but a new textbook *Beginning Algebra* (first edition, authors Hendricks and Chow, publisher McGraw-Hill) [10] has been instituted for the Spring 2013 semester. It is assumed that the change of textbook has not drastically altered the course content. Any conclusions drawn from this thesis should take the textbook change into consideration.

We have also assumed that assessment items are the best indicators of the rigor of course content. As previously stated, homework is not considered an assessment item for the purpose of this analysis; however, we present homework problems as they provide insight into the types of practice problems completed by students in support of their mastery of the course objectives. We compare homework and assessment items to the Smarter Balanced and Illustrative Mathematics items under the assumptions that Wisconsin K-12 schools will be adopting the Smarter Balanced assessment and that the Illustrative Mathematics assessment items will be somewhat comparable to the Smarter Balanced style and rigor given the CCSSM authors' involvement with Illustrative Mathematics. Finally, we assume that there are various interpretations of the standards. Therefore, we attempt to use the specific language of each standard and the sample assessment items in order to establish a baseline comparison for the content of Math 095.

3.2 Standards Addressed in Math 095: The Number and Quantity Conceptual Category

3.2.1 Background/Prerequisite Knowledge

Per the standards, by the time students enter high school, they should have developed and revised their concept of "number". In the early grades, students are expected to build the non-negative rational numbers from the whole numbers through the introduction to and construction of fractions. They use the base-ten decimal system to represent whole numbers and are expected to understand the construction of the rational numbers. During the middle grades, students should continue to expand their understanding of "number" to include negative rational numbers, culminating in grade eight where students should "complete" the set of real numbers with irrational numbers. As stated in the standards [14]:

> With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay

> the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Thus, the high school Number and Quantity standards are built on the assumption that students have an established sense of number and its various representations as well as the four aforementioned operations applied to numbers. We now present the Number and Quantity high school standard addressed in Math 095.

3.2.2 Standard N-RN 2

Standard N-RN 2 indicates students should be able to "rewrite expressions involving radicals and rational exponents using the properties of exponents" [14]. In Math 095, students simplify expressions with rational exponents in Sections 3.1 and 3.2 of the textbook [5]. They are asked to solve problems such as the following:

- Write an explanation of why $(x^3)(x^4)$ is not x^{12}. [5] (Section 3.1)
- Simplify. $(a^3b^6)^5$ [1] (Exam 1)
- Simplify. Write your answer with only positive exponents. $\frac{(b^5)^{-6}}{b(b^5)^{-5}}$ [4] (Final)

Students in Math 095 later study rewriting expressions involving radicals in Sections 9.2, 9.3, and 9.4 from the textbook [5]. While topics involving exponents are studied very early in the semester, radical expressions are one of the last topics of the semester due to the textbook's separation of the two concepts. Students are expected to be able to complete the following assessment items:

- Find the area and perimeter of a square with a side of length $\sqrt{3}$. One of these measures, the area, is a rational number, and the other, the perimeter, is an irrational number. Explain how this happened. Will the area always be a rational number? Explain. [5] (Section 9.2)

- Simplify. $\sqrt{48} - \sqrt{3}$ [4] (Final)

Based on sample items released by Smarter Balanced, students would be asked to complete something similar to the following problem to assess mastery of standard N-RN 2 [13]:

For each item determine whether the equation is true or false:

$$\sqrt{32} = 2^{\frac{5}{2}}$$

$$4^{\frac{1}{2}} = \sqrt[4]{64}$$

$$(\sqrt{64})^{\frac{1}{3}} = 8^{\frac{1}{6}}$$

Based on the Math 095 items asking students to rewrite expressions involving integer exponents, it is clear that students are expected to apply the additive and multiplicative properties of integer exponents. They also practice with exponents in rational expressions so that they can simplify/rewrite negative exponents. However, the Math 095 items do not require students to work with rational (non-integer) exponents. When examining the items involving radical expressions, it appears the extent to which students work with radicals in the course is limited to square roots. That is, the cube root or other rational roots are not addressed. It also appears that there is no connection made between rational exponents and their equivalent radical form in Math 095.

Since the course does not require students to apply properties of rational exponents and radicals other than integer exponents and the square root, students would not be able to complete the sample item successfully. This limited perspective may skew students' progress in understanding that numbers are more varied than just integers. It also would inhibit students from strategizing ways to see and visualize real numbers in different yet equivalent forms. At the same time, since Math 095 students work with negative (integer) exponents and rational expressions, they may be advanced in this aspect of the standard.

3.3 Standards Addressed in Math 095: The Algebra Conceptual Category

3.3.1 Background/Prerequisite Knowledge

Students begin to prepare for high school algebra in grades six through eight. The Common Core standards for sixth grade indicate that students should be able to understand the concept of variables and use them in mathematical expressions. More explicitly, sixth graders “write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems” [14]. They rewrite expressions in equivalent forms and apply the idea of maintaining equality on both sides of an equation to solve equations in one step. Sixth graders also analyze information given in tables and use equations to describe how quantities are related.

By seventh grade, students solidify their understanding of ratios and proportionality, solving a “wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease” [14]. Students also graph proportional relationships and “understand the unit rate informally as a measure of the steepness of the related line, called the slope” [14]. By the time they enter eighth grade, they should have familiarity with proportional relationships. According to the standards, eighth graders “use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems” [14], recognizing that the proportional relationships they studied in seventh grade are special linear equations. They build the connection that the slope of a line represents the rate of change and describes the relationship between a change between the input or x-coordinate and the output or y-coordinate. Additionally, eighth grade students use models with real-world contexts to express and interpret relationships between two quantities. Lastly, eighth grade students solve linear equations in one variable and systems of equations in two variables, using their understanding of slope of a line to solve problems.

The high school Algebra standards generally focus on three areas: expressions, equations and inequalities. Students are expected to interpret and rewrite expressions and to understand the meaning of a solution to an equation or inequality. In the high school curriculum, students ideally blend their understanding of expressions, equations, and inequalities by applying their knowledge to functions and modeling of real-world situations.

3.3.2 Standards A-SSE 3 and A-SSE 3a

An essential foundation for the development of algebra is to be able to identify and rewrite expressions in equivalent forms. Standard A-SSE 3 states students should "choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression," and substandard A-SSE 3a expects students to "factor a quadratic expression to reveal the zeros of the function it defines" [14]. In Math 095, choosing and producing an equivalent form of an expression is heavily emphasized in Chapter Four of the text, where students are asked to factor polynomials (usually of at most degree four) in Sections 4.1, 4.4, and 4.5 [5]. There do appear to be any homework or assessment items assigned that provide context for factorization, and the items do not specifically ask students to apply or make the idea of factorization purposeful at this point in the course. However, students later go on to use factorizations to find solutions to polynomial equations. While function notation is not used in Math 095 and most questions do not use the language of "zeros of a function", factoring quadratic equations is also highly emphasized in Sections 4.2 through 4.5 in the textbook. Sample assessment items are as follows:

- Factor completely. $6b^4 - 18b^3 - 60b^2$ [4] (Final)
- Factor completely. $9x^2 - 64$ [2] (Exam 2)
- Factor completely. $40x^3 + 15x^2 - 8x - 3$ [2] (Exam 2)
- Find a positive value for k so that the polynomial can be factored: $x^2 + kx + 8$ [5] (Section 4.3)

Below is a sample assessment item released by Smarter Balanced [13]:

Select the two equations with equivalent zeros:

$$y = x^2 + 14$$

$$y = x^2 + 9x + 14$$

$$y = (x - \frac{9}{2})^2 - \frac{25}{4}$$

$$y = (x + 7)(x + 2)$$

$$y = (\frac{1}{2}x + 7)(2x + 2)$$

While the Smarter Balanced item has rational coefficients, in Math 095 all polynomials have integer coefficients. The Smarter Balanced item presents a polynomial expression that cannot be factored over the real numbers, and students are also expected to be able to recognize completing the square in the third equation. In Math 095, students are asked to factor polynomial expressions given in a prescribed format: ones in which a greatest common factor monomial can be factored out, polynomials that can be rewritten as quadratic in a new variable, ones which can be regrouped, and differences of squares.

There are no homework or assessment items that cannot be factored or encourages students to explore why they cannot be factored over the real numbers. Further, there are not many polynomials of degree higher than two. The problems presented in Math 095 do not indicate that students are encouraged to think about uniqueness of factoring, different representations of the same expression, or when polynomials cannot be factored over the real numbers. Therefore, overall it appears that students practice rewriting polynomials as an equivalent factored expression; however, there does not appear to be an emphasis on explaining properties represented by the equivalent expression. Thus only a portion of the standard is learned and assessed in Math 095. It is noteworthy that factoring is also heavily studied in Math 105, and in this course students are presented with a larger variety of problems.

3.3.3 Standards A-APR 3 and A-REI 4b

Standard A-APR 3 requires students to "identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial" [14]. Very closely related to this standard is standard A-REI 4b: "Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation" (p. 65). Section 6.4 from the textbook [5] addresses solving quadratic equations by factoring and various applications as a result. The section focuses on solutions to polynomial equations and does not address the function defined by the polynomial nor the graphing of the function. It also does not address any methods for solving quadratic equations as listed in Standard A-APR3 other than factoring. Below are representative assessment questions asked in Math 095 related to these two standards:

- Solve the quadratic equation. $3x^2 + 28x = -49$ [2] (Exam 2)

- The sum of an integer and its square is 56. Find the integer. [4] (Final)

Here again the quadratic equations are those with integer coefficients. Note also that the solution to the word problem given above is an integer solution and could be easily guessed.

Smarter Balanced has not released a sample item specific to these standards, but Illustrative Mathematics has posted the following illustration for standard A-REI 4b [12]:

> Suppose that a particular model of car has a braking distance that can be computed as follows. When the car is traveling at v miles per hour, its braking distance is given (in feet) by $d = 2.2v + \frac{v^2}{20}$.
>
> 1. What is the braking distance, in feet, of a car of this model going 30 mph? 60 mph? 90 mph?
> 2. Suppose that a specific car of this model took 500 feet to brake. Use your computations in part (1) to make a predication about how fast it was going when the brakes were applied.
> 3. Use a graph of the distance equation to determine more precisely how fast it was going when the brakes were applied, and check your answer using the quadratic formula.

This sample assessment item expects students to use a graph of the equation in order to solve the problem and then use the quadratic equation as a *tool* to check the estimated and graphical answers as opposed to just using the formula as a way to find the solution out of context. It is clear that the Math 095 materials only address part of each of these standards in the sense that they solve only quadratic equations (not other polynomials of higher degree) using only one method (factoring a trinomial, not using the quadratic formula). Even though quadratic expressions are factored frequently in Math 095, they are not graphed until Math 105. Furthermore, Math 095 does not present content in function notation—but rather presents them as equations only—and the extent of graphing equations is limited to linear equations, as discussed later in this analysis.

3.3.4 Standard A-APR 1

The Common Core State Standards expect that by the end of high school students will recognize other systems that are analogous to the integers and rational numbers, that is, polynomials and rational expressions, respectively. Standard A-APR 1 states that students will be able to "understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials" [14]. Various homework and assessment items and the content covered in Sections 3.3 and 3.4 of the textbook [5] indicate that students in Math 095 are expected to add, subtract, and multiply polynomials:

- Subtract $6w^2 + w - 5$ from $9w^2 + 6w + 6$. [4] (Final)
- A triangle has sides $2x - 5$, $3x + 1$, and $4x + 2$. Find the polynomial that represents its perimeter. Simplify your final answer. [4] (Final)
- Multiply. $(-4x - 2)^2$ [1] (Exam 1)
- Given a square of side length $(a + b)$ with each side split into 2 pieces, one of length a and the other of length b. Students are asked to give the length, width, and area of each sub-rectangle in the figure. [5] (Section 3.4)

Smarter Balanced released a multi-part assessment item with regard to this standard. The details of the three parts are detailed below [13]:

- Part A: A town council plans to build a public parking lot. Figure 1 represents the proposed shape of the parking lot. Write an expression for the area, in square feet, of this proposed parking lot. Explain the reasoning you used to find the expression.

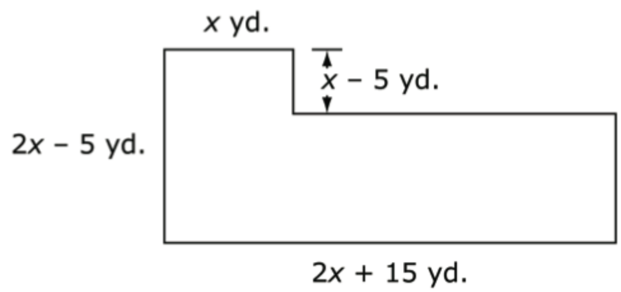


Figure 1: Assessment item A-APR1 Part A

- Part B: The town council has plans to double the area of the parking lot in a few years. They create two plans to do this. The first plan increases the length of the base of the parking lot by p yards, as shown in Figure 2 below. Write an expression in terms of x to represent the value of p, in feet. Explain the reasoning you used to find the value of p.

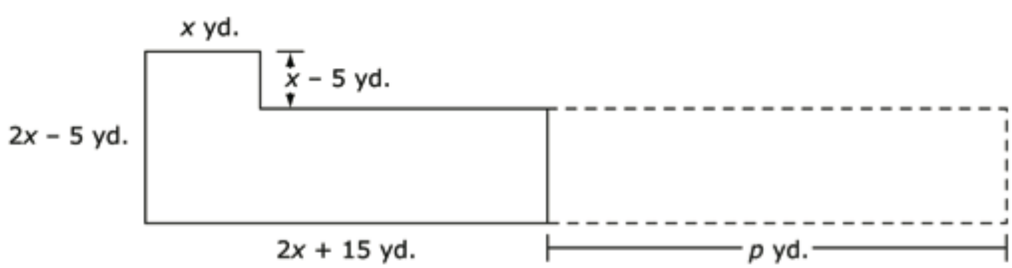


Figure 2: Assessment item A-APR 1 Part B

- Part C: The town council's second plan to double the area changes the shape of the parking lot to a rectangle, as shown in Figure 3. Can the value of z be represented as a polynomial with integer coefficients? Justify your reasoning.

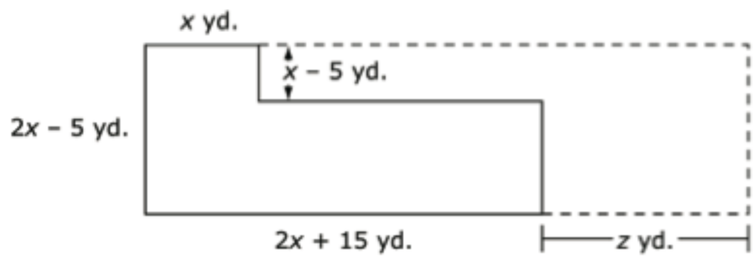


Figure 3: Assessment item A-APR 1 Part C

This Smarter Balanced assessment item not only assesses whether the student understands that one can add, subtract, and multiply polynomials in the same manner that these operations are applied to integers, but takes it a step further in having students explain their understanding of the reasoning behind each part of the problem and also extends into the modeling standards. Part C requires that the student understand that since polynomials are analogous to the integers, then factorization works in the same way as well. The Smarter Balanced assessment item is not comparable to an assessment item in Math 095 given its length. The Math 095 assessment items demonstrate a strong development of the arithmetic operations on polynomials, but students are not asked assessment questions to demonstrate their understanding that the polynomials are analogous to the integers, which represents the heart of the standard. While the Smarter Balanced assessment item applies the operations on polynomials in a real-world modeling context, the Math 095 homework and assessment items for this standard only extend their modeling to perimeter problems aside from leading the students to a visual/geometric representation of why $(a+b)^2 = a^2 + 2ab + b^2$. The extent to which the standard is accomplished in Math 095 mostly depends on the instructor's development of the analogy between the integers and polynomials. Either way, there is certainly no discussion about concepts such as unique factorization or the fundamental theorem of arithmetic.

3.3.5 Standard A-APR 6

It is a natural progression from adding, subtracting, and multiplying polynomials to applying the division algorithm for polynomials. Standard A-APR 6 indicates students should be able to "rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system" [14]. The Math 095 textbook does not address why we can apply the division algorithm to polynomials (or even the division algorithm for integers). Sections 3.5 and 5.1 present rational expressions in which the students are asked to simplify the expression by dividing out common factors or to apply long division [5]. A representative sample of homework and assessment items from Math 095 is as follows:

- Divide. $\frac{20x^2+63x+55}{4x+7}$ [1] (Exam 1)
- Write in simplest form. $\frac{x^2-2x-24}{x^2-36}$ [2] (Exam 2)
- Given a rectangle with width $3x + 2$ and area $6x^2 + 19x + 10$, what is the length of the rectangle? [5] (Section 5.1)

Smarter Balanced has not released sample items for this standard, but Illustrative Mathematics has posted the following assessment item [12]:

> The U.S. Department of Energy keeps track of fuel efficiency for all vehicles sold in the United States. Each car has two fuel economy numbers, one measuring efficiency for city driving and one for highway driving. For example, a 2012 Volkswagen Jetta gets 29.0 miles per gallon (mpg) in the city and 39.0 mpg on the highway.

Many banks have "green car loans" where the interest rate is lowered for loans on cars with high combined fuel economy. This number is not the average of the city and highway economy values. Rather, the combined fuel economy (as defined by the federal Corporate Average Fuel Economy standard) for x mpg in the city and y mpg on the highway, is computed as combined fuel economy $= \frac{1}{\frac{1}{2}(\frac{1}{x}+\frac{1}{y})} = \frac{2xy}{x+y}$.

1. What is the combined fuel economy for the 2012 Volkswagen Jetta? Give your answer to three significant digits.
2. For most conventional cars, the highway fuel economy is 10 mpg higher than the city fuel economy. If we set the city fuel economy to be x mpg for such a car, what is the combined fuel economy in terms of x? Write your answer as a single rational function, $a(x)/b(x)$.
3. Rewrite your answer from (2) in the form of $q(x) + \frac{r(x)}{b(x)}$ where $q(x)$, $r(x)$, and $b(x)$ are polynomials and the degree of $r(x)$ is less than the degree of $b(x)$.
4. Use your answer in (3) to conclude that if the city fuel economy, x, is large, then the combined fuel economy is approximately $x + 5$.

Regardless of the rigor of the assessment items in Math 095, students are not asked to specifically identify the polynomials representing the quotient or the remainder after polynomial division. Further, there is no link between integers and polynomials demonstrated with homework or assessment items in Math 095. There also is no emphasis on the "size" of the polynomial according to degree. The sample assessment item from Illustrative Mathematics demonstrates that the Common Core standards expect that students understand the modeling of real-world contexts and then use properties of integers and polynomials along with algebraic properties to show important information about the real-world context. There are no application questions related to this standard in Math 095, and it is unclear if students are expected to be able to solve polynomial division problems by inspection. Computer algebra systems are also not utilized in Math 095. Thus, it is clear that several components of Standard A-APR 6 are absent from the Math 095 course and that Math 095 deviates significantly from the standard.

3.3.6 Standard A-APR 7

Just as the rational numbers are constructed from the integers, in algebra courses rational expressions are analogously constructed from polynomials. The Common Core State Standards have identified an understanding of this concept as a "+" standard, meaning that the content of the standard is beyond what would be expected of all students at the high school level in order to be college and career ready. This standard states that students will "understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions" [14].

In Math 095, the application of the above four operations to rational expressions is addressed in Sections 5.2 through 5.5 from the textbook [5]. A sample of the problems students are expected to do is as follows:

- Multiply. Write your answer in simplest form. $\frac{5x-45}{x^2+9x} \cdot \frac{4x}{9-x}$ [2] (Exam 2)
- Find the perimeter of a given rectangle with length $\frac{2x}{x+3}$ and width $\frac{6}{x+3}$. [5] (Section 5.3)

- Add. Express your answer in simplest form. $\frac{2x}{x^2+15x+54}+\frac{4}{x+6}$ [2] (Exam 2)

- Simplify the complex fraction. $\frac{\frac{x^2}{y^2}-1}{\frac{x}{y}+1}$ [5] (Section 5.5)

These homework and assessment items demonstrate that students should be able to add, subtract, multiply, and divide rational expressions in Math 095, but they do not indicate that students are expected to understand the connection between the rational numbers and rational expressions. The understanding of the analogy is essential in a student's assignment of value to the four operations, and a lack of encouraging the understanding reduces the study of rational expressions to the application of procedures and algorithms. Perhaps Math 095 instructors informally assess the students' mastery of the underlying understanding of the analogy in class, but this likely depends on the instructor. Neither Smarter Balanced nor Illustrative Mathematics has released any sample items for this standard for a full comparison.

3.3.7 Standard A-CED 1

In the Creating Equations domain in the Algebra conceptual category of the high school standards, students are expected to construct equations and inequalities. Standard A-CED 1 states that students will "create equations and inequalities in one variable and use them to solve problems," and it further specifies that examples include "equations arising from linear and quadratic functions, and simple rational and exponential functions" [14]. Students in Math 095 are exposed to some of the content of this standard—namely, linear equations and inequalities in one variable and equations involving rational expressions—in various parts of the course. In Section 5.7 from the textbook [5], students create and solve simple equations involving rational expressions:

- One number is 3 times another. If the sum of their reciprocals is $\frac{1}{6}$, find the two numbers. [2] (Exam 2)
- A car uses 7 gallons of gasoline on a trip of 168 miles. At that rate, how much gasoline will a trip of 312 miles require? [4] (Final)

Earlier in the semester, students will have worked at a more fundamental level of creating linear equations in one variable in Sections 2.4 and 2.5 of the textbook [5]. Most of the problems students are asked to complete are brain teasers involving integers/rational numbers or problems involving essential geometry concepts. Sample assessment items include:

- If one-half a number is subtracted from five-sixths of the number, the difference is 6. Find the number. [2] (Exam 2)
- The length of a rectangle is 1 in. more than twice its width. If the perimeter of the rectangle is 74 in., find the dimensions of the rectangle. [5] (Section 2.5)
- Find two consecutive integers such that the sum of 3 times the first integer and 2 times the second integer is 22. [1] (Exam 1)

In Section 2.6 of the textbook [5], students then move on to constructing linear inequalities. Most of the section entails solving given inequalities and graphing them, but there was one item in which students were expected to construct an inequality as well. For example:

> An arithmetic student needs an average of 70 or more to receive credit for the course. She scored 79, 75, 81 on the first three exams. Write a simplified inequality representing the score she must get on the last test to receive credit for the course. [1] (Exam 1)

Smarter Balanced has not released an assessment item for this particular standard, but Illustrative Mathematics has released several examples, two of which are below [12]:

- A checking account is set up with an initial balance of \$4800, and \$400 is removed from the account each month for rent (no other transactions occur on the account).
    1. Write an equation whose solution is the number of months, m, it takes for the account balance to reach \$2000.
    2. Make a plot of the balance after m months for $m = 1, 3, 5, 7, 9, 11$ and indicate on the plot the solution to your equation in part (1).
- (Part 1) Below is a quadrilateral $ABCD$. Show, by dividing A□CD into triangles, that the sum of the interior angles is 360°: $m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360$

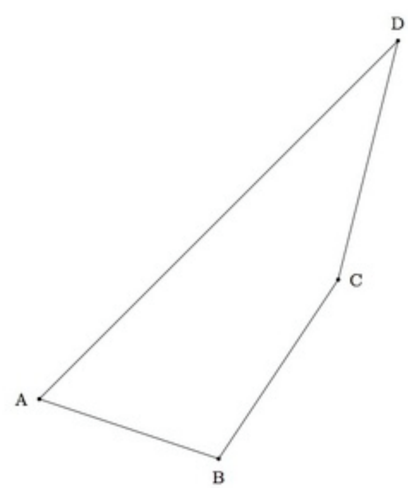


Figure 4: Assessment item A-CED 1 Part 1

(Part 2) Below is a pentagon $ABCDE$. Show that the sum of the interior angles is 540°: $m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) + m(\angle E) = 540$

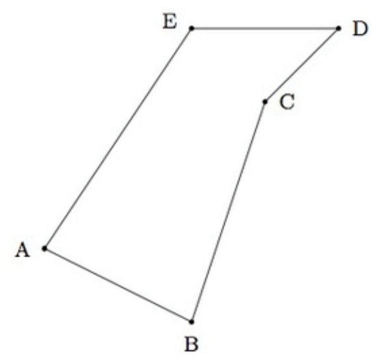


Figure 5: Assessment item A-CED 1 Part 2

> (Part 3) Suppose P is a polygon with $n \geq 3$ sides and assume that all interior angles of P measure less than 180 degrees. Show that the sum of the measures of the interior angles of P is $(n-2)\times 180$ degrees. Check that this formula gives the correct value for equilateral triangles and squares.

Other sample assessment items released by Illustrative Mathematics involve rational equations, quadratic equations, and more complex modeling equations.

It is evident that Math 095 does not address the creation of quadratic or exponential functions as stated in the standard and that function notation is not used. While Math 095 has students create linear equations in one variable, the two Illustrative Mathematics examples presented above delve deeper and extend much further than the homework and assessment items in Math 095. Students are exposed to topics that may be new for them, and they must call on higher levels of reasoning to construct unfamiliar equations based on logic and patterns. Comparatively, it appears in the Math 095 homework and assessment items that the equations created by students are small derivations from the sample items listed above, thus narrowing the students' ability to construct equations in untried real-world contexts. Therefore, while simple linear equations are created in Math 095, it does not appear that the content of the course fulfills the purpose of Standard A-CED 1.

3.3.8 Standard A-CED 2

The next Common Core standard progresses from equations and inequalities in one variable to equations in two or more variables. Standard A-CED 2 indicates students should be able to "create equations in two or more variables to represent relationships between quantities [and] graph equations on coordinate axes with labels and scales" [14]. The two important components of this standard are the creation of algebraic equations and the representation of the same information in graphical form. The first half of the standard is discussed in Section 7.3 of the textbook [5]. Students are usually asked to write a linear equation given pieces of information relating the two variables, such as the slope or the $x-$ or $y-$intercept. Sample assessment items are as follows:

- Write the equation of the line passing through $(-4, 27)$ and $(1, 2)$. Write your results in slope-intercept form, if possible. [4] (Final)
- A temperature of $10°\text{C}$ corresponds to a temperature of $50°\text{F}$. Also, $40°\text{C}$ corresponds to $104°\text{F}$. Find the linear equation relating F and C. [5] (Section 7.3)

The graphing portion of the standard is addressed in Section 6.3 of the textbook [5]. Note that on the examinations, students are already given a coordinate plane with pre-determined labels and scales, and in every case the horizontal axis is represented by the variable x and the vertical axis is represented by the variable y.

- Graph $y = \frac{1}{3}x$. [5] (Section 6.3)
- Graph $2x - 5y = 10$. [4] (Final)

- Write the equation of the line with slope $-\frac{1}{2}$ and y-intercept $(0, 3)$. Then graph the line. [4] (Final)

Smarter Balanced has released the following sample assessment item for Standard A-CED 2 [13]:

David compares the sizes and costs of photo books offered at an online store. Figure 6 below shows the cost for each size photo book:

Book Size	Base Price	Cost for each additional page
7-in. by 9-in.	\$20	\$1.00
8-in. by 11-in.	\$25	\$1.00
12-in. by 12-in.	\$45	\$1.50

Figure 6: Assessment item A-CED 2

The base price reflects the cost for the first 20 pages of the photo book.

1. Write an equation to represent the relationship between the cost, y, in dollars, and the number of pages, x, for each book size. Assume that x is at least 20 pages.
2. What is the cost of a 12-in. by 12-in. book with 28 pages?
3. How many pages are in an 8-in. by 11-in. book that costs \$49?

This assessment item not only has students create (multiple) equations to model the relationship between the cost of the book and the number of pages, but it also has students demonstrate they can apply the equation and see the relationship between the two variables as a linear relationship without explicitly stating so.

The extent of the variety of relationships expected by Standard A-CED 2 is not clear, but it is likely not limited to the case of linear relationships between two variables.

In Math 095, the only relationship *modeled* between two variables is a linear relationship, which could lead students to a false impression that relationships between variables are exclusively linear. While some quadratic and other polynomial equations are given algebraically to students (as opposed to in a qualitative format from which the students construct equations), students do not create their own equations aside from linear equations in one or two variables. Furthermore, students do not even build up the idea of linearity in its own right; rather, the problems specifically indicate that a relationship is linear. For example, in the Celsius-Fahrenheit example (from Section 7.3), instead of giving students a descriptive representation of the relationship between F and C or providing students with a large number of points to graph and visually identify a linear relationship, the problem is reduced to giving the students two (integer-valued) ordered pairs and telling students the relationship is linear. This is a missed opportunity as Standard A-CED 2 is identified as a standard that would be ideal for coupling with the modeling standards.

3.3.9 Standard A-CED 3

One of the aforementioned assessment items from Section 2.6 of the textbook [5] *may* align with part of Standard A-CED 3, which states that students should "represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context" [14]. As shown in the analysis of Standard A-CED 1, on Exam One Math 095 students were asked to find the minimum grade needed on the final examination in order to pass the class[1]. For comparison, two examples from Illustrative Mathematics are presented below [12]:

- Bernardo and Silvia play the following game. An integer between 0 and 999, inclusive, is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. What is the smallest initial number that results in a win for Bernardo?
- The only coins that Alexis has are dimes and quarters. Her coins have a total value of $5.80. She has a total of 40 coins. Which of the following systems of equations can be used to find the number of dimes, d, and the number of quarters, q, Alexis has? Explain your choice.

  a. $\begin{cases} d + q = 5.80 \\ 40d + 40q = 5.80 \end{cases}$

  b. $\begin{cases} d + q = 40 \\ 0.25d + 0.10q = 5.80 \end{cases}$

  c. $\begin{cases} d + q = 5.80 \\ 0.10d + 0.25q = 40 \end{cases}$

  d. $\begin{cases} d + q = 40 \\ 0.10d + 0.25q = 5.80 \end{cases}$

The Math 095 question regarding constructing a constraint for the minimum grade needed appears to be the only one of its type in the course. It demonstrates that students are expected at best to know how to construct a basic linear constraint. However, this

[1] Exam 1 question 5 states [Exam 1]: “An arithmetic student needs an average of 70 or more to receive credit for the course. She scored 79, 75, 81 on the first three exams. Write a simplified inequality representing the score she must get on the last test to receive credit for the course.”

item does not involve extensive modeling or systems of equations/inequalities and may even be solved with a simple linear equation. The first Illustrative Mathematics item demonstrates that students should be able to explicitly recognize the constraint in a given context (benchmarks that constitute a win and a loss in various rounds of the game) and build the inequality to represent the rather complicated constraint. The second item has students model constraints using *systems* of linear equations in two variables and further requires that students defend their choice by explaining their answer. Systems of equations are addressed later in the course materials; however, students are not asked to create or model constraints from them. Overall, it appears that the Math 095 content barely scratches the surface of Standard A-CED 3.

3.3.10 Standard A-CED 4

An important extension of the standards associated with creating equations is the ability to solve for a different variable in a formula. Standard A-CED 4 indicates students should be able to "rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations" [14]. In examining all homework and assessment items asked of students, there were two (and only two) items addressing this standard in Math 095:

- Solve for y. $x + 5y = 25$ [1] (Exam 1)
- Solve for the variable R: $v = \sqrt{2gR}$. [5] (Section 9.5)

Smarter Balanced has not released sample assessment items for comparison, but Illustrative Mathematics has posted the following example [12]:

> Use inverse operations to solve the equations for the unknown variable, or for the designated variable if there is more than one. If there is more than one operation

to 'undo', be sure to think carefully about the order in which you do them. For equations with multiple variables, it may help to first solve a version of the problem with numerical values substituted in.

$5 = a - 3$

$A - B = C$ (solve for A)

$6 = -2x$

$IR = V$ (solve for R)

$\frac{x}{5} = 3$

$W = \frac{A}{L}$ (solve for A)

$7x + 3 = 10$

$ax + c = R$ (solve for x)

$13 = 15 - 4x$

$2h = w - 3p$ (solve for p)

$F = \frac{GMm}{r^2}$ (solve for G)

The instructions for the Illustrative Mathematics items demonstrate that students are expected to reason through the rearrangement of formulas, and the items contain a mix of problems containing one variable and multiple variables. By placing these two different types together we should be able to assess whether or not students understand that variables simply represent numbers and that rearranging formulas is the same as solving an equation in one variable.

It appears that Math 095 tries to touch on standard A-CED 4 within the course content. However, students are not exposed to much practice with rearranging formulas.

It is possible that the only purpose of the Math 095 assessment item in which students solve a linear equation for the variable y is to see if students can convert an equation into slope-intercept form. Furthermore, the only other formula presented was in an isolated homework problem. Since there are only two assessment items related to solving a multi-variable equation for one variable, this standard is not heavily emphasized in Math 095.

3.3.11 Standard A-REI 2

The Common Core standards expect students to understand that solving equations is a process of reasoning. Standard A-REI 2 states students should "solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise" [14]. Students practice solving rational equations in one variable in Sections 5.6 and 5.7 of the textbook [5]. Representative homework and assessment items from Math 095 are as follows:

- Solve and check. [1] (Section 5.6)
  - $\frac{2}{x+3}+\frac{1}{2}=\frac{x+6}{x+3}$ (Note: $x=-2$ is the only solution and no extraneous solutions arise)
  - $\frac{3}{x+3}+\frac{25}{x^2+x-6}=\frac{5}{x-2}$ (Note: $x=2$ is an extraneous solution)
- Solve for x. $\frac{x-5}{4}=\frac{5}{2}$ [2] (Exam 2)
- What values for x, if any, must be excluded in the following algebraic function? $\frac{x+2}{x^2-9x+14}$ [2] (Exam 2)
- A speed of 60 mi/h corresponds to 88 ft/s. If a light plane's speed is 150 mi/h, what is its speed in feet per second? [5] (Section 5.7)

Solving radical equations in one variable is addressed in Sections 9.5 and 9.6 of the textbook [5]. Section Five is focused on how to solve radical equations, and Section Six applies the Pythagorean Theorem for the sake of solving radical equations:

- Solve the following equations. Check your solution. [5] (Section 9.5)
    - $2\sqrt{3z+2}-1=5$ (Note: only real solution is $z=3$ and no extraneous solutions arise in the process)
    - $\sqrt{3x-2}+2=0$ (Note: no real solution, which can be determined by inspection or by identifying an extraneous solution if students carry out squaring both sides)
    - $\sqrt{t+9}+3=t$ (Note: only real solution is $t=7$ and $t=0$ is an extraneous solution)
- Solve. $\sqrt{y+4}=5$ [4] (Final)
- The length of one leg of a right triangle is 3 in. more than the other. If the length of the hypotenuse is 15 in., what are the lengths of the two legs? [1] (Section 9.6)

Smarter Balanced has not released an assessment item for Standard A-REI 2, but Illustrative Mathematics has released two different items: one involving rational equations and one involving radical equations. They are given below [12]:

- Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.
    a. How many games would Chase have to win in a row in order to have a 75% winning record?

b. How many games would Chase have to win in a row in order to have a 90% winning record?

c. Is Chase able to reach a 100% winning record? Explain why or why not.

d. Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?

- (a) Solve the following two equations by isolating the radical on one side and squaring both sides. Be sure to check your solutions.

$$\sqrt{2x+1}-5=-2$$

$$\sqrt{2x+1}+5=2$$

(b) If we raise both sides of an equation to a power, we sometimes obtain an equation which has more solutions than the original one. (Sometimes the extra solutions are called extraneous solutions.) Which of the following equations result in extraneous solutions when you raise both sides to the indicated power? Explain.

$\sqrt{x}=5$, square both sides

$\sqrt{x}=-5$, square both sides

$\sqrt[3]{x}=5$, cube both sides

$\sqrt[3]{x}=-5$, cube both sides

(c) Create a square root equation similar to the one in part (a) that has an extraneous solution. Show the algebraic steps you would follow to look for a solution, and indicate where the extraneous solution arises.

It appears that the Math 095 content encourages students to master solving rational and radical equations in one variable. The coefficients and constants represented in the rational equations are mostly integers, and all radical equations involve the square root (as opposed to any other root such as the cube root). Students are exposed to a variety of homework problems resulting in the existence of solutions, extraneous solutions, and no solution; however, extraneous solutions do not arise in items on the formal assessments in Math 095. The assessment items indicate that students are not necessarily required to explain *why* extraneous solutions arise, but rather to verify by substitution *if* a solution is extraneous.

The rational equation assessment item created by Illustrative Mathematics appears to emphasize both the modeling/creation of the rational equation and finding its solution, which may surpass the expectations of Standard A-REI 2. The multi-part Illustrative Mathematics item involving radical equations not only asks students to solve the equation but also encourages students to generalize when extraneous solutions arise in radical equations. It even goes further in asking students to construct a square root equation that results in an extraneous solution and explain the algebraic reasoning behind why the extraneous solution arises in the first place. This assessment item seems to capture the simultaneous importance of both procedurally solving the equations and explaining why applying certain operations may lead to mathematically non-equivalent statements, thus potentially resulting in extraneous solutions. The level of understanding and reasoning required for the latter is much more demanding and does not appear to be emphasized in Math 095.

3.3.12 Standard A-REI 3

Aside from rational and radical equations, several sections of the Math 095 textbook focus on solving linear equations and inequalities in one variable. The extent to which students are expected to *create* linear equations and inequalities has been discussed in connection with Standards A-CED 1 and A-CED 2. Sections 2.3, 2.4, 2.5, and 2.6 of the textbook [5] focus solely on the *methods of solving* equations and inequalities. Standard A-REI 3 states that students should be able to "solve linear equations and inequalities in one variable, including equations with coefficients represented by letters" [14]. Sample items involving solving linear equations in one variable are as follows:

- Solve for x and show a check. $-4(x+4)+7x=8x+24$ [1] (Exam 1)
- Solve for x and show a check. $\frac{7}{4}x-4=\frac{9}{4}x-7$ [1] (Exam 1)
- Create an equation of the form $ax+b=c$ that has 2 as a solution. [5] (Section 2.3)
- A common mistake is shown. The equation is $6x-(x+3)=\ 5+2x$. The first step in solving is: $6x-x+3=5+2x$. Write a clear explanation of what error has been made and what could be done to avoid the mistake. [5] (Section 2.5)

In Section 2.6 of the textbook [5] students solve and graph linear inequalities in one variable:

- Solve and graph the solution set. $5x+7\geq 4x+11$ [1] (Exam 1)
- Solve and graph. $\frac{2x}{3}<6$ [5] (Section 2.6)

Neither Smarter Balanced nor Illustrative Mathematics has released an assessment item to gauge the rigor of Standard A-REI 3. Given the relative simplicity of the linear equations from Math 095, it may be more appropriate to align this portion of the content from the course with Standard 8.EE.7b from the Expressions and Equations domain in eighth grade. This standard indicates that students are expected to "solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms" [14]. It falls under the umbrella of Standard 8.EE.7, which expects students to "solve linear equations in one variable" [14]. Smarter Balanced has released the following assessment item for the eighth grade standard [13]:

> Consider the equation $3(2x + 5) = ax + b$.
>
> - Part A: Find one value for a and one value for b so that there is exactly one value of x that makes the equation true. Explain your reasoning.
> - Part B: Find one value for a and one value for b so that there are infinitely many values of x that make the equation true. Explain your reasoning.

Similarly, it may be more applicable to align the linear inequalities studied in Math 095 to Standard 7.EE.4b from seventh grade, which states that students should be able to "solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem." The following item was released by Smarter Balanced for Standard 7.EE.4b [13]:

> David wants to buy 2 pineapples and some bananas. The price of 1 pineapple is \$2.99. The price of bananas is \$0.67 per pound. David wants to spend less than \$10.00. Write an inequality that represents the number of pounds of bananas, b, David can buy. On the number line [given to the students with integers from -2

to 8 already marked], draw a graph that represents the number of pounds of bananas David can buy.

In Math 095, it appears that students have many opportunities to demonstrate mastery of solving linear equations and inequalities in one variable through their homework and assessment items. However, the Math 095 assessment items do not seem to challenge students to solve equations with variable coefficients as only one question contains a variable coefficient; in fact, most coefficients are limited to integers. It is clear from the Smarter Balanced assessment item that eighth graders are expected to be comfortable with variable coefficients in linear equations and to use them to determine when a linear equation has one solution versus infinitely many solutions.

The seventh grade linear inequality standard appears to possibly be more comprehensive than the homework and assessment items from Math 095 in that the Smarter Balanced assessment item further required students to construct the inequality that modeled a real-world situation. It is difficult to assess whether the Math 095 content aligns more with the seventh grade linear equality standard and the eighth grade linear equation standard or to Standard A-REI 3 in high school. Perhaps as the assessment consortia and groups such as Illustrative Mathematics generate assessment items for Standard A-REI 3 it will be clearer to judge the rigor expected in high school.

3.3.13 Standard A-REI 6

In algebra, students progress from creating equations in two or more variables to solving systems of equations. Standard A-REI 6 expects students to be able to "solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables" [14]. Students in Math 095 spend several classes building their understanding of systems of equations. First, students graph linear equations and interpret the point(s) of intersection as the solution(s) in Section 8.1 of the text [5]. Sample items from Math 105 include the following:

- A manufacturer has two machines that produce door handles. On Monday, machine A operates for 10 hours and machine B operates for 7 hours, and 290 door handles are produced. On Tuesday, machine A operates for 6 hours and machine B operates for 12 hours, and 330 door handles are produced. Use the system $10x + 7y = 290$ and $6x + 12y = 330$, where x is the number of handles produced by machine A in an hour, and y is the number of handles produced by machine B in an hour. Solve the system graphically. [5] (Section 8.1)
- Solve the system by graphing. $3x + y = 6$ and $3x - y = 0$ [3] (Exam 3) and [4] (Final)

Students then move on to solving equations exactly by adding a multiple of one equation to another and by substitution in Sections 8.2 and 8.3 of the text [5], respectively. Sample items requiring students to demonstrate mastery of adding a multiple of one equation to another are:

- Solve the system by addition: $\begin{cases} 3x + 2y = -7 \\ 6x - y = -1 \end{cases}$ [3] (Exam 3)
- A coffee merchant has coffee beans that sell for \$9 per pound and \$12 per pound. The two types are to be mixed to create 100 pounds of a mixture that will sell for \$11.25 per pound. How much of each type of bean should be used in the mixture? [5] (Section 8.2)

In Section 8.3 students practice using the method of substitution:

- Solve the system by substitution: $\begin{cases} -3x - y = -2 \\ y = -3x + 4 \end{cases}$ [3] (Exam 3)

- The length of a rectangle is 8 in. more than twice its width. If the perimeter of the rectangle is 28 in., find the width of the rectangle. [3] (Exam 3)
- The difference of two numbers is 80. The larger is 8 less than 5 times the smaller. What are the two numbers? [3] (Exam 3)

For comparison purposes, we present the sample assessment item released by Smarter Balanced [13]:

> A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special. On Thursday, the restaurant collected $467 selling 21 vegetarian specials and 40 chicken specials. On Friday, the restaurant collected $484 selling 28 vegetarian specials and 36 chicken specials. What is the cost of each lunch special?

The sample item builds off of and connects with Standard A-CED 2 in that it requires students to first construct equations to model the two constraints in order to solve the system of equations exactly. Most homework and assessment items in Math 095 only ask students to solve the system rather than creating the system as well, but overall the problems appear to address all portions of the standard: students solve the systems both graphically and algebraically using various methods. The course also poses a variety of application questions, from questions requiring students to properly translate sentences to algebraic equations to simple geometric applications. The reader may wish to compare to

Standard 8.EE.8 from eighth grade, as it is very similar to the language of the high school Standard A-REI 6[2].

3.3.14 Standard A-REI 10

In Chapter Six of the textbook, students practice graphing linear equations in two variables, and as shown above in Chapter Eight students graph simultaneous equations to determine solutions to systems of linear equations. Through these learning opportunities, students ideally build the understanding that points whose coordinates satisfy an equation comprise the graph of the equation. To that end, the Common Core specifies in Standard A-REI 10 that students should be able to "understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)" [14].

The content from Section 6.1 of the textbook [5] is approximately aligned with this standard, and in terms of assessment items students in Math 095 are expected to do the following:

- Find four solutions for the equation. $3x + 5y = 15$ [3] (Exam 3)
- Determine which of the ordered pairs $(-3, 1)$, $(2, 0)$, $(-13, -3)$, $(-18, 4)$ are solutions for the equation $x + 5y = 2$. [3] (Exam 3)

---

[2] Standard 8.EE.8 states that students should be able to "analyze and solve pairs of simultaneous linear equations." The sub-standards are [14]:

8.EE.8a: "Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graph, because points of intersection satisfy both equations simultaneously."

8.EE.8b: "Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection."

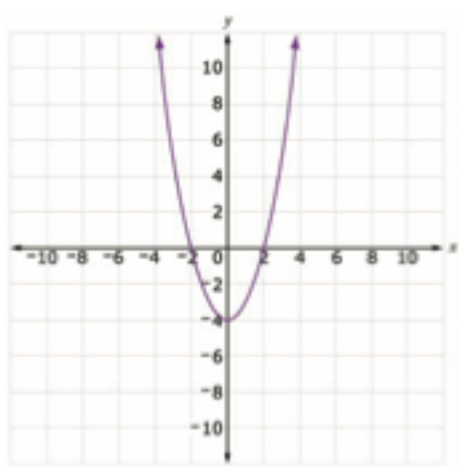
8.EE.8c: "Solve real-world and mathematical problems leading to two linear equations in two variables."

- Complete the ordered pairs so that each is a solution for the equation $y = 4x + 12$. (, 0), ($\frac{1}{4}$,), (0,), ($-\frac{1}{4}$,) [3] (Exam 3)
- You now have had practice solving equations with one variable and equations with two variables. Compare equations with one variable to equations with two variables. How are they alike? How are they different? [5] (Section 6.1)

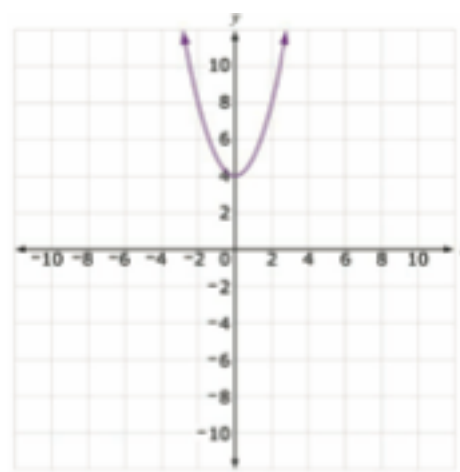
For comparison, Smarter Balanced released the following assessment item for Standard A-REI 10 [13]:

Which graph could represent the solution set of $y = \sqrt{x - 4}$ from Figure 7 below?

A.



C.



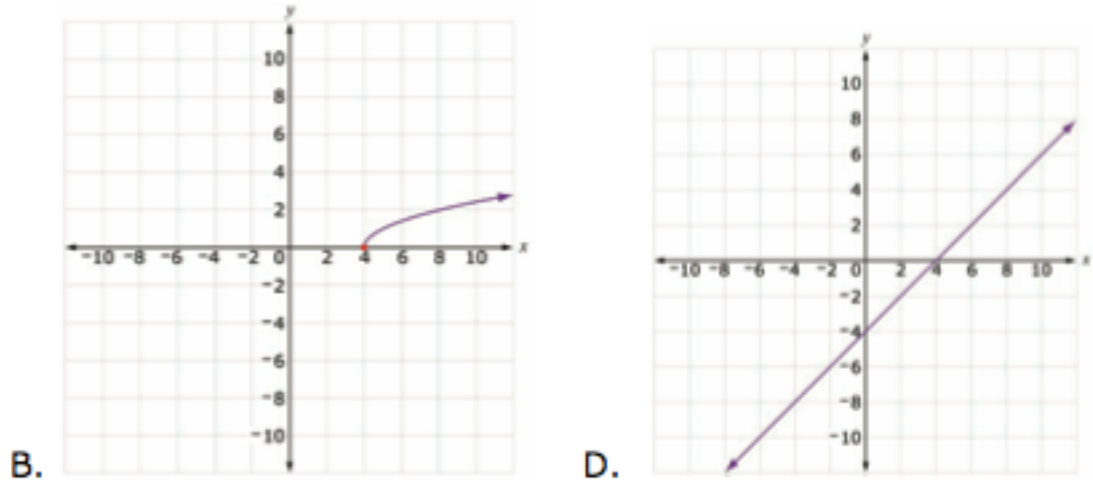


Figure 7: Assessment item A-REI 10

The above Smarter Balanced assessment item illustrates that students should be able to identify to graphs of common families of equations in two variables (namely, from this example, quadratic, radical, and linear equations). In Math 095, students are exposed to linear equations and their graphs, and if students master the homework and assessment items listed above, we can conclude that students understand that linear graphs are a visual representation of the infinite set of solutions to a linear equation. However, there is no indication that graphs of nonlinear equations are explored in Math 095. Without practice with graphs of nonlinear equations it is unclear the extent to which students can master this standard. If only linear equations in two variables are addressed in Math 095, students may not be to evaluate the Smarter Balanced assessment item and analyze options A, B, or C if they did not already build a foundation in recognizing that the graphs may contain solutions to the given equation.

3.3.15 Standard A-REI 12

Students in Math 095 are not only expected to be able to graph linear equations, but also linear inequalities in two variables. Standard A-REI 12 states students should "graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes" [14]. In Math 095, the focus is limited to solely graphing a single linear inequality in two variables as opposed to a system of linear equalities, and this topic is explored in Section 7.4 of the textbook [5]. Representative assessment items from Math 095 are presented below:

- Graph the inequality. $x - y \geq 2$ [3] (Exam 3)
- Graph the inequality. $2x + 3y > 6$ [4] (Final)

For comparison purposes, below is a sample assessment item released by Illustrative Mathematics [12]:

> Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 1200 pounds of people and gear for safety reasons. Assume on average an adult weighs 150 pounds and a child weighs 75 pounds. Also assume each group will require 200 pounds of gear plus 10 pounds of gear per person.
>
> a. Write an inequality that illustrates the weight limit for a group of adults and children on the fishing boat and a second inequality that represents the total number of passengers in the fishing boat. Graph the solution set to the inequalities.
>
> b. Several groups of people wish to rent a boat. Group 1 has 4 adults and 2 children. Group 2 has 3 adults and 5 children. Group 3 has 8 adults. Which of the groups, if any, can safely rent a boat? What other combinations of adults and children are possible?

This Illustrative Mathematics assessment item demonstrates that the Common Core expects students to be able to model systems of linear inequalities in two variables, graph the solution set, and use the graph to determine feasible solutions to the system. In Math 095, students do not examine systems of linear inequalities but rather graph a single linear inequality in the coordinate plane, and it appears that the inequalities are already given rather than constructed by the students. However, students may be able to extrapolate what a system of linear inequalities implies. In Math 095, students study systems of linear *equations* in a subsequent chapter, so perhaps after linking the meaning of systems of linear equations with the concept of linear inequalities then students could piece together the entirety of Standard A-REI 12. Systems of linear inequalities are studied further in Math 105 as well.

3.4 Standards Addressed in Math 095: The Function Conceptual Category

3.4.1 Background/Prerequisite Knowledge

In their mathematical career, students build their understanding of algebraic expressions and later use algebraic expressions as one of various ways to describe functions. Students can also describe functions by means of a graph, among other ways. The use of algebraic expressions and their underlying structure to form functions is paramount for modeling real-world relationships and phenomena. The concepts of growth and rates of change are especially important for decision-making models in fields of business, biology, and so on. To that end, the Common Core aims for students to connect their previous experiences involving expressions, equations, graphs, and modeling to functions.

According to the grade level content contained in the standards, students generally study expressions and equations from sixth through eighth grade, and they begin to work with functions in eighth grade. The summary of eighth grade content indicates the following [14]:

> Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

The high school Functions standards move from interpreting to building functions and then to working with common families of functions: linear, quadratic, exponential, and trigonometric functions.

Math 095 has traditionally been a course that does not address functions. Students are not exposed to the definition or concept of functions nor do they use function notation despite the fact that there are assessment items using the word "function." Students are exposed to a comprehensive treatment of functions in Math 105. Math 095 intentionally skips Section 7.5 of the textbook, which is the only section in the textbook that tries to connect expressions, equations, and graphs to functions [5]. However, students work indirectly with representations of functions in the sense that they study the graphs of linear equations in two variables as well as work with the equation form of linear functions. Students calculate the slope of linear equations and are asked to find and/or interpret the intercepts. Thus, the idea behind functions is not *absent* in the course. There are three general topics related to linear functions in Math 095: finding and comparing the slopes of linear equations, graphing linear equations, and rewriting linear equations in different forms in order to graph them and identify the slope and y-intercept.

3.4.2 Standard F-IF 6

Students in Math 095 work exclusively with rates of change in the form of slopes of linear equations (mostly in two variables). The Common Core would expect that students "calculate and interpret the average rate of change of a function (represented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph" [14]. Section 6.4 from the textbook [5] discusses finding the slope of a line. Representative assessment items are below:

- Find the slope of the graphed line (where students are given the graph). [3] (Exam 3)
- Find the slope of the line through the points $(0, -4)$ and $(0, -2)$. [3] (Exam 3)
- Consider the equation $y = 2x + 3$. [5] (Section 6.4)
  a. Complete the tables values (for x-coordinates 5, 6, 7, 8, 9)
  b. As the x-coordinate changes by 1, by how much do the corresponding y-coordinates change
  c. Is your answer to part (b.) the same if you move from point B to C? From C to D? From D to E?
  d. Describe the "growth rate" of the line using these observations. Complete the statement: When the x-value grows by 1 unit, the y-value __________.
- Complete the statement: "The difference between undefined slope and zero slope is _________" [5] (Section 6.4)

Section 7.2 of the text [5] has students compare the slopes of linear equations in order to determine if lines are parallel, perpendicular, or neither. Below are sample assessment items from Math 095:

- Without graphing, determine whether the following lines are parallel, perpendicular, or neither. Show work for all calculations for full credit: L_1 with equation $x - 3y = 12$; L_2 with equation $3x + y = 3$. [3] (Exam 3)
- Are the pairs of lines parallel, perpendicular, or neither? L_1 through $(8, 5)$ and $(3, -2)$; L_2 through $(-2, 4)$ and $(4, -1)$. [5] (Section 7.2)

In comparison, Smarter Balanced released the following sample assessment item for Standard F-IF 6 [13]:

> The value of an antique has increased exponentially, as shown in Figure 8. Based on the graph, estimate to the nearest $50 the average rate of change in value of the antique for the following time intervals: from 0 to 20 years and from 20 to 40 years.

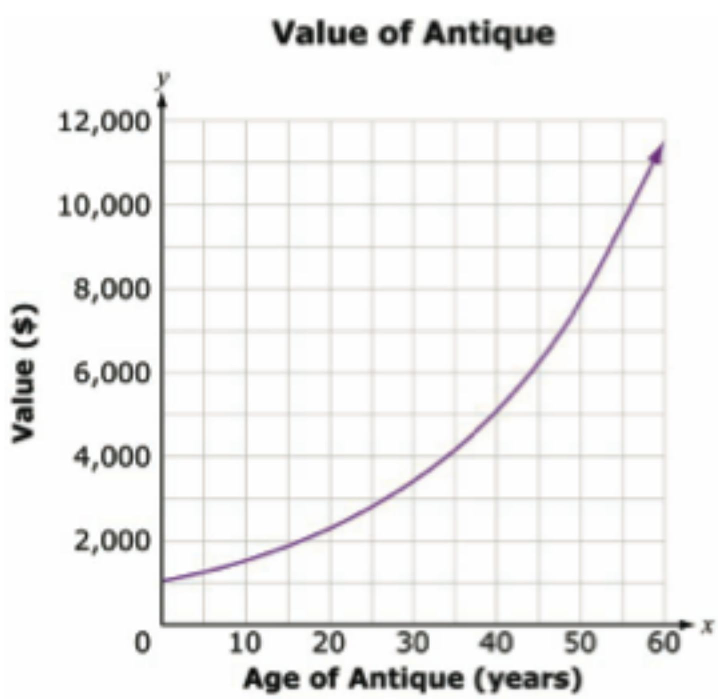


Figure 8: Assessment item F-IF 7

The Math 095 homework and assessment items encourage students to think about the slope as a growth rate and provide a variety of opportunities for calculating the slope (in a table format, given a graph, and using two coordinates). The multi-part question from Section 6.4 tries to have students calculate rates of change, although there are no questions in this section asked of students where they may be able to connect the rate of change to a real-world application. For Standard F-IF 7, there are three key differences in the items asked of Math 095 students and the rigor of the Smarter Balanced assessment item. First, the Smarter Balanced function is nonlinear, as opposed to the linear graph given on Exam Three. In a nonlinear function, the rate of change varies according to the specified interval and in Math 095 students are not exposed to nonlinear examples (or even, for example, *approximately* linear data). Rather, the nonlinear examples are reserved for Math 105. Students may not even connect that slope is a rate of change if it has not been adequately developed with the students. Another difference is that the Smarter Balanced item has coordinate axes that are not scaled to one unit; rather, the x-axis is scaled by 5 years and the y-axis is scaled by \$1,000. All sample graphs given to Math 095 students are scaled to one unit on each axis with convenient coordinates to use to compute the slope of the given line. Lastly, the Smarter Balanced graph represents information modeling (theoretically) real data as opposed to having students work off a given equation or coordinate points that do not have any association with an application. It is clear that the goals of Math 095 do not include evaluating rate of change on nonlinear (including exponential) functions, but having students look at real, approximately linear data could lead to a deeper understanding of slope as a representation of a rate of change.

3.4.3 Standard F-IF 7a

In typical algebra courses, one hopes that students not only know how to calculate and interpret the rate of change, but also how to identify other important features such as the intercepts and maxima and minima of functions. Standard F-IF 7a from the Common Core states students should "graph linear and quadratic functions and show intercepts, maxima, and minima" [14]. In Math 095, students attempt to master this standard for linear functions in Section 6.3 and Section 7.1 of the textbook [5]. In Chapter Six, students are asked to graph linear equations:

- Graph each set of equations on the same coordinate system. Do the lines intersect? What are the y-intercepts? $y = 3x$, $y = 3x + 4$, $y = 3x - 5$. [5] (Section 6.3)
- Graph $y = -1$. [3] (Exam 3)
- Graph $2x - 5y = 10$. [4] (Final)

In Chapter Seven, students construct the equation of a line using the slope and y-intercept:

- Write the equation of the line with given slope and y-intercept. Then graph the line using the slope and y-intercept. Slope: $-\frac{2}{3}$, y-intercept: $(0, 0)$. [5] (Section 7.1)

Neither Smarter Balanced nor Illustrative Mathematics has released assessment items for this specific sub-standard of Standard F-IF 7. Nevertheless, it is clear that students in Math 095 are not expected to graph quadratic equations, and, therefore, they do not find the maxima and minima of quadratic equations. In Math 095, students are limited to linear equations with integer coefficients, and since the functions are linear one of the only key features that students can extract from the given information is the y-intercept.

3.4.4 Standard F-IF 8

Section 6.3 from the textbook [5] focuses on graphing linear equations once the equation is in slope-intercept form. Some equations are given in slope-intercept form, while others are given in a different equivalent format and students are expected to rewrite them in slope-intercept form to ultimately graph the equation. This skill underlies Standard F-IF 8 from the Common Core, which states that students should be able to "write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function"[14]. Representative sample items from Chapter Six are as follows:

- Graph by first solving for y. $2x + 3y = 12$ [3] (Exam 3)
- Find the slope and y-intercept of the line represented by the following equation: $3x - 2y = -20$. [4] (Final)

Smarter Balanced released the following assessment item [13]:

> Write the function $y - 3 = \frac{2}{3}(x - 4)$ in the equivalent form most appropriate for identifying the slope and y-intercept of the function.

Math 095 gives students the opportunity to master writing linear equations in equivalent forms and extracting important properties of the equation (even though it is not presented as a "function"). At the same time, Standard F-IF 8 has sub-parts (a.) and (b.) which are not fully addressed with only linear functions: F-IF 8a entails quadratic functions and F-IF 8b involves exponential functions. The development of quadratic and exponential functions is addressed in Math 105.

3.5 Standards Not Addressed in Math 95

There are many standards that are not addressed in Math 095. This is partially due to the content that has been historically established in the course and is also due to the content in Math 105 and the time constraints of the course. Many standards which are not addressed in Math 095 are studied in Math 105. Please see Appendix A for a full table of all of the standards within the conceptual categories of Number and Quantity, Algebra, and Functions. Included in Appendix A is a designation of the standards that are addressed in Math 095 (as determined by the results in the previous analysis sections) and in Math 105 (as determined by analogous results in a thesis completed by Raymond Dempsey).

From these results we can begin to identify critical content that was missing from Math 095 but is present in Math 105. Generally, the main topics include the study of functions and function notation; graphing equations other than linear equations; utilizing other methods, namely, the quadratic formula and completing the square, to solve quadratic equations; the introduction of complex numbers; and functions and inequalities involving absolute value. Of these topics, perhaps the most important is the study of functions in general. Students in Math 105 rewrite, graph, and find important parameters of quadratic, exponential, and logarithmic functions; in comparison, Math 095 only delves into equivalent detail into linear equations. While functions are addressed heavily in Math 105, from reviewing the table it is evident that many of the function standards are yet absent from the course.

Chapter 4: Conclusions

We can use the findings from the above analysis chapter and Appendix A to gauge where the Math 095 and the CCSSM intersect and where they are disjoint. In general, it appears that given the quantity of practice and assessment items, students in Math 095 are likely to master standards that involve the verbs "solve," "rewrite," "graph (a linear equation or inequality)," "perform the operation," and "factor." Within the context of these verbs, students end up practicing these skills very frequently throughout the course. Since most equations and polynomials are presented with integer coefficients, students should be very comfortable working with integers. They spend a large portion of the semester working with linear equations, whether a singular linear equation or a system of linear equations, so we expect that students can solve and graph these fluently.

At the same time, the Math 095 homework and assessment items indicate that many standards containing verbs such as "understand," "explain," "interpret," "describe the context," and "model" are not fully mastered in the course when benchmarked according to the Smarter Balanced and Illustrative Mathematics items. If these standards (or portions of standards containing similar language) are not addressed in Math 095, students will miss general concepts or may have their reasoning reduced to a case of following procedures and solving algorithms. Since many practice and assessment items are very similar and are thus susceptible to rote memorization or a superficial understanding of the skeleton of a procedure, students have a hard time creating equations or modeling real-life situations if they fall outside of the frame of familiar problems. Many of these action verbs mirror the language of the Standards for Mathematical Practice. Without assessment items challenging students to higher order thinking, students do not engage with the practice standards.

In Appendix A, assessment items that are common to both Math 095 and Math 105 are indicated in bold. Below is a summary of items that they share:

- Factoring polynomial expressions, especially factoring quadratic polynomials; similarly, solving polynomial equations by factoring
- Applying the operations of addition, subtraction, multiplication, and division to polynomials
- Simplifying or rewriting rational expressions by dividing out common factors from the numerator and the denominator
- Rearranging linear and rational (square root) equations to highlight a quantity of interest
- Solving simple rational and radical (square root) equations in one variable; in both classes students check their solutions for extraneous solutions but it is unclear if students would be able to explain why the extraneous solutions arise
- Solving linear equations and inequalities in one variable
- Solving systems of linear equations in two variables graphically and exactly (by substitution or adding a multiple of one equation to the other)
- Calculating the slope of a linear equation
- Graphing linear equations and find intercepts

From this list, we can see that there is much overlap in content between the two courses. Furthermore, the overlap is very similar across courses in terms of the specific homework and assessments items of Math 095 and Math 105. For example, the assessment items for solving systems of linear equations in two variables are practically identical across the courses—it is not as though the assessment items are scaffolded or leveled in such a way that Math 095 presents simple cases and then in Math 105 students are exposed to much more complex problems.

Chapter 5: Recommendations

In order to more closely align the content of Math 095 to the expectations of the CCSSM, the Mathematics Department at UWM should start by first becoming aware of the Common Core and the language and formulation of the standards. The assessment items released by Smarter Balanced and Illustrative Mathematics give an objective starting point for members of the department to see the depth of the standards. Regardless of whether members agree or disagree with the standards, they are nonetheless adopted in Wisconsin and K-12 assessments will commence in just over a year. Since many future students will be a product of the curriculum designed to meet the Common Core standards, awareness should be raised through events such as professional development or visits to high-performing (or low-performing) high schools.

In the process of becoming more aware of the Common Core expectations, we should carefully examine the homework and assessment items asked of students in the course. These are the best indicators of what we actually expect students to be able to do. Assessment items should ideally support the goals of the course, so once the goals of the course are defined then it would be helpful to examine some of the most important standards addressed in Math 095 and compare to the sample Smarter Balanced and Illustrative Mathematics examples. We can compare assessment items side-by-side as we have done in the analysis chapter of this paper and determine what we expect our students to be able to do. Assessment items which require students to demonstrate understanding, model with mathematics, explain their reasoning, interpret results, and apply context to the underlying mathematics should be added to the course.

While a thorough analysis of the new textbook has not been completed, if we as instructors only rely on the textbook for homework and assessment items then we should ensure that the textbook contains comprehensive mathematical exercises for students. We should be supplement the course with materials from the outside the textbook should the textbook examples be insufficient. In most textbooks, there are often homework exercises that encourage students to answer "big idea" questions, but these questions are often not assigned. Adding a mix of exercises related to conceptual understanding and mastery of procedures would give students a well-rounded perspective. Once instructors have a solid idea of what we expect students to do, then instruction will be required to align with the assessment benchmarks.

Not only are the assessments very important in Math 095, but so is the determination of the content taught in Math 095 (and subsequently in Math 105). One specific recommendation with regard to content would be to infuse more modeling across the course. Standards which go hand-in-hand with modeling include the following: A-SSE 3 and 3a; A-CED 1, 2, 3, and 4; and F-IF 6 and 7a. Infusing the content related to these standards would lead to a firmer conceptual understanding of topics, would give students a context behind the mathematics, would require students to reason and use critical thinking skills, and would involve the students more heavily in the *creation* of equations/functions/models as opposed to giving these to students. It is likely that instructors will feel overwhelmed by adding modeling content into the course, so a way to free up time and content would be to review the list of overlapping topics and separate these topics into one—as opposed to both—of the courses.

Finally, we recommend that an analysis be performed on other course formats. Courses utilizing ALEKS should be reviewed in depth standard-by-standard as we have done in this paper. The ALEKS software program has a simple tool where an instructor (and the students) can see a list of topics that are associated with each standard in the Common Core. This list is merely an association; the depth of the assessment items in ALEKS should be investigated to see if they have the same level of expectations as the CCSSM.

Post-secondary institutions across the country are engaging in the conversation about the impact of the Common Core. The ramifications of the discussion will be vast. Should we offer college credit for remedial mathematics courses offered at a given university? Since the Common Core standards are adopted in most of the country, will performance on the high school assessments factor into admissions requirements for various institutions? How will mathematics placement examinations change, if at all? These are questions that will likely be discussed in the higher education community in the coming decades as the Common Core States Standards are fully implemented.

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APPENDIX A: CCSSM STANDARDS IN COMPARISON WITH MATH 095 AND MATH 105

Please note that all of the standards are quoted directly from [CCSSM]. A standard with an asterisk (*) has been identified by the CCSSM authors as having particularly strong connections to the Modeling conceptual category.

Note that standards starting with "N" are from the Number and Quantity high school conceptual category; those starting with "A" refer to items from the Algebra high school conceptual category; those starting with "F" refer to the Function high school conceptual category

The brief summary comments in the Math 095 column were resulting from the Chapter 3 Analysis. The Math 105 column information was determined from Raymond Dempsey's thesis exploring the connections between Math 105 and the CCSSM standards [8].

A blank cell translates to content that is absent from a given course.

Bolded items represent some degree of overlap between content from the two courses.

Standard Number	Standard	Math 095	Math 105
N-RN 1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*		Students are given a formal definition of the nth root
N-RN 2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Rewrite radical expressions (square root only) and those involving rational exponents (integers only)	Rewrite radical expressions (nth roots) and those involving rational exponents
N-RN 3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.		The classification of rational and irrational numbers is addressed in the course; there are no assessment items for this standard and the said properties of sums and products are not addressed

Standard Number	Standard	Math 095	Math 105
N-Q 1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.		Addressed in the course but there are no specific assessment items
N-Q 2	Define appropriate quantities for the purpose of descriptive modeling.		Addressed in the course through application problems related to solving equations and systems of equations
N-Q 3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.		Addressed in the course but no specific assessment items
N-CN 1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.		Students know the existence of i and write complex numbers in the form $a + bi$
N-CN 2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.		Students apply the quadratic equation and find complex roots
N-CN 7	Solve quadratic equations with real coefficients that have complex solutions.		
A-SSE 1	Interpret expressions that represent a quantity in terms of its context. *		
A-SSE 1a	Interpret parts of an expression, such as terms, factors, and coefficients.		Students are not directly assessed on this but they identify terms, factors, and coefficients; it is unclear if they *interpret* these items; students also identify the degree of polynomials
A-SSE 1b	Interpret complicated expressions by viewing one or more of their parts		

Standard Number	Standard	Math 095	Math 105
	as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*		
A-SSE 2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*		Students see structure in linear equations of the form $y = mx + b$ and identify the real and imaginary components of a complex number $a + bi$; interpret the structure of quadratic equations in vertex form (identify vertex, maximum/minimum, opening up or down); rewrite polynomials in different forms in order to factor them
A-SSE 3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *	Limited to choosing and producing an equivalent form	See above
A-SSE 3a	Factor a quadratic expression to reveal the zeros of the function it defines.	**Limited to factoring quadratic expressions**	Students solve quadratic equations by **factoring** (and other methods).
A-SSE 3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.		Students complete the square to convert quadratic functions into vertex form so that they can identify the maximum or the minimum
A-SSE 3c	Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*		
A-SSE 4	Derive the formula for the		[Listed as being addressed

Standard Number	Standard	Math 095	Math 105
	sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* *		in the course but no assessment items or discussion presented]
A-APR 1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	**Students add, subtract, and multiply polynomials**	**Students add, subtract, and multiply polynomials**; they also multiply *functions* represented by polynomials
A-APR 2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.		Addressed in one homework problem, but students can solve the homework problem without applying the Remainder Theorem; the Remainder Theorem is used more as a practice tool for synthetic division
A-APR 3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	**Students solve polynomial equations by factoring**	**Students solve polynomials equations by factoring**
A-APR 4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*		These ideas are developed in the lessons in class, but no assessment items are given
A-APR 6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and	**Students write rational expressions in the form** $\frac{\boldsymbol{a}(\square)}{\boldsymbol{b}(\boldsymbol{x})}$ **in simplified form by factoring**; the degree of the polynomial is not	**Students simplify rational expressions by dividing out common factors**; students perform long division and

Standard Number	Standard	Math 095	Math 105
	$r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	addressed and, as such, the division algorithm for polynomials is not utilized; long division is not performed	synthetic division; students are asked to state the quotient and remainder
A-CED 1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.*	Create linear equations and linear inequalities in one variable and use them to solve problems; also create rational equations	
A-CED 2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*	Create linear equations in two variables; graph linear equations on coordinate axes with labels and scales; more emphasis on the solving than the creating	Students create simple linear equations and systems of linear equations; uses distance/rate/time problems, constraint problems relating area and perimeter, and rational equations involving rates; more emphasis on the solving rather than creating
A-CED 3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.*	Represent constraints by linear inequalities in one variable	See above
A-CED 4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V =*	**Solve linear and rational equations for a quantity of interest**	**Solve linear and rational equations for a quantity of interest**

Standard Number	Standard	Math 095	Math 105
	*IR to highlight resistance R.**		
A-REI 1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.		Students use steps but there is no assessment of understanding each step in solving an equation; students solve many different types of equations
A-REI 2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	**Solve simple rational and radical (square root) equations in one variable; students test their answers but are not asked to explain or describe why the extraneous solutions arise in the first place**	**Students solve rational and radical (square root) equations in one variable; students understand they should test solutions but assessment items to do not individually assess the extraneous solution component**
A-REI 3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	**Solve linear equations and inequalities in one variable**	**Solve linear inequalities in one variable**; solve inequalities involving absolute value in one variable
A-REI 4	Solve quadratic equations in one variable.		Solve quadratic equations and find inputs resulting in a given output of a quadratic function
A-REI 4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.		Solve by completing the square; the quadratic formula is used in the course but not derived
A-REI 4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate	**Quadratic equations are solved only by factoring**.	**Quadratic equations are solved** using all methods noted here; students write complex solutions in the prescribed form

Standard Number	Standard	Math 095	Math 105
	to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.		
A-REI 5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.		Addressed but students are not assessed
A-REI 6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	**Solve systems of linear equations in two variables exactly using substitution and the method of adding a multiple of one equation to another; graphically show solutions**	**Solve systems graphically or algebraically (elimination and substitution methods)**
A-REI 7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.		Addressed in the chapter but the assessment items deemphasize the nonlinear case; graphing is not completed
A-REI 10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).		Students graph linear, quadratic, exponential, and logarithmic functions; the understanding is not officially assessed
A-REI 11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to		Systems of linear (only) equations are solved graphically. Function notation is not used in the assessment items.

Standard Number	Standard	Math 095	Math 105
	graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. *		
A-REI 12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.		
F-IF 1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.		Students identify the domain and range (rather than the domain being given in advance)
F-IF 2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.		Students use function notation for a variety of types of common families of functions; students evaluate functions at given inputs
F-IF 3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci*		

Standard Number	Standard	Math 095	Math 105
	sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.		
F-IF 4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. *		Students distinguish between positive and negative slope of a linear equation; students describe the symmetry and maximum/minimum of a parabola; symmetries of inverse functions are addressed [Note there are no assessment items for any of these topics]
F-IF 5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *		
F-IF 6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *	**Calculate** and (on occasion) interpret **the slope of a linear equation**, including estimating the rate of change from a graph	**Students determine the slope of a linear equation**
F-IF 7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more		

Standard Number	Standard	Math 095	Math 105
	complicated cases. *		
F-IF 7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.	**Graph linear equations and find intercepts.**	**Students find intercepts of linear equations**; graph quadratic functions and show intercepts, maxima, and minima
F-IF 7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.		
F-IF 7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.		Students identify zeros for suitable factorizations
F-IF 7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.		Graph exponential and logarithmic functions (only one assessment item for graphing an exponential function)
F-IF 8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	Write linear equations in multiple forms (for example, slope-intercept and standard form)	
F-IF 8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.		Students convert quadratic functions to vertex form
F-IF 8b	Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10,		

Standard Number	Standard	Math 095	Math 105
	and classify them as representing exponential growth or decay.		
F-IF 9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.		
F-BF 1	Write a function that describes a relationship between two quantities. *		
F-BF 1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.		
F-BF 1b	Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*.		
F-BF 2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. *		
F-BF 3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value		Intrinsically studied when working with quadratic functions in vertex form

Standard Number	Standard	Math 095	Math 105
	of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.		
F-BF 4	Find inverse functions.		Inverse functions are found
F-BF 4a	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.		
F-LE 1	Distinguish between situations that can be modeled with linear functions and with exponential functions.		
F-LE 1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.		
F-LE 1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.		
F-LE 1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.		
F-LE 2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a		Students construct linear functions given a point and the slope or given two points

Standard Number	Standard	Math 095	Math 105
	relationship, or two input-output pairs (include reading these from a table).		
F-LE 3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.		
F-LE 4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.		Students solve exponential equations using logarithms
F-LE 5	Interpret the parameters in a linear or exponential function in terms of a context.		Students interpret parameters in linear equations.

APPENDIX B: EXAM ONE

Below are the exam items from Exam One from the Fall 2012 semester for Math 095.

1. Is -9 a solution to the equation $4 - 8x = 76$?
2. If the perimeter of a rectangle is 22 ft and the width is 5 ft, find its length.
3. Solve for x and show check. $-4(x + 4) + 7x = 8x + 24$
4. Solve for x. $-10x - 10 - 3x = -13x - 6$
5. An arithmetic student needs an average of 70 or more to receive credit for the course. She scored 79, 75, and 81 on the first three exams. Write a simplified inequality representing the score she must get on the last test to receive credit for the course.
6. Solve and graph the solution set. $5x + 7 \geq 4x + 11$
7. Simplify. $(a^3b^6)^5$
8. Solve for x and show check. $\frac{7}{4}x - 4 = \frac{9}{4}x - 7$
9. Simplify. Write your answer with only positive exponents. $\frac{(a^5)^{-5}}{a(a^5)^{-4}}$
10. Evaluate (assume $x \neq 0$):

    a. $-10x^0$

    b. $\frac{-10}{0}$

11. Simplify. $7a - (6a - 5b)$
12. Evaluate $-x^2 + 10x - 2$ for $x = -1$.
13. Divide. $\frac{20x^2+63x+55}{4x+7}$
14. Solve for y. $x + 5y = 25$
15. Multiply. $(-4x - 2)^2$

16. One number is 10 less than another. If 5 times the smaller number minus 2 times the larger number is 7, find the two numbers.

17. Multiply. $3xy^2(5xy - 4x + 6y)$

18. Divide. $\frac{27x^3 - 3x^2}{3x}$

19. Find two consecutive integers such that the sum of 3 times the first integer and 2 times the second integer is 22.

20. Subtract $6y^2 + 3y - 4$ from $9y^2 + 5y + 1$.

APPENDIX C: EXAM TWO

Below are the exam items from Exam Two from the Fall 2012 semester for Math 095.

1. Simplify. $\frac{1+\frac{4}{5}}{1-\frac{1}{4}}$

2. If one-half a number is subtracted from five-sixths of the number, the difference is 6. Find the number.

3. The product of two consecutive positive even integers is 224. Find the integers.

4. Divide. Write your answer in simplest form. $\frac{4}{x^5} \div \frac{8}{x^2}$

5. Factor completely. $35x^2 + 20x - 15$

6. Solve the quadratic equation. $3x^2 + 28x = -49$

7. Factor completely. $7x^2 - 58x + 16$

8. The sum of an integer and its square is 12. Find the integer.

9. Subtract. Express your answer in simplest form. $\frac{3x}{x-6} - \frac{18}{x-6}$

10. Factor completely. $9x^2 - 64$

11. Add. Express your answer in simplest form. $\frac{2x}{x^2+15x+54} + \frac{4}{x+6}$

12. Solve for x. $\frac{x-5}{4} = \frac{5}{2}$

13. Factor completely. $7x^2 + 70x + 175$

14. One number is 3 times another. If the sum of their reciprocals is $\frac{1}{6}$, find the two numbers.

15. Solve for x. $\frac{8}{x+4} + 3 = \frac{5}{x+4}$

16. Write in simplest form. $\frac{x^2-2x-24}{x^2-36}$

17. Factor completely. $x(x+4) - 5(x+4)$

18. Factor completely. $40x^3 + 15x^2 - 8x - 3$

19. What values for x, if any, must be excluded in the following algebraic function?
$\frac{x+2}{x^2-9x+14}$

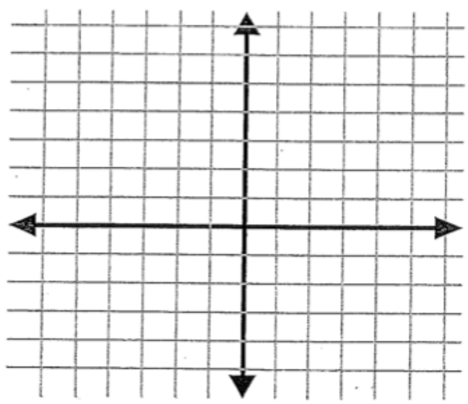
20. Multiply. Write your answer in simplest form. $\frac{5x-45}{x^2+9x} \cdot \frac{4x}{9-x}$

APPENDIX D: EXAM THREE

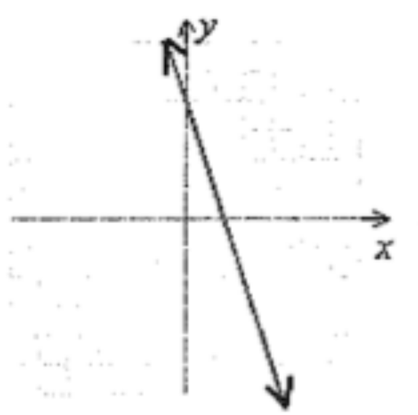
Below are the exam items from Exam Three from the Fall 2012 semester for Math 095.

1. The equation $y = 18x - 50$ describes the amount of money a class of students might earn from candy bar sales. What are the slope and y-intercept of this line?

2. Graph $y = -1$



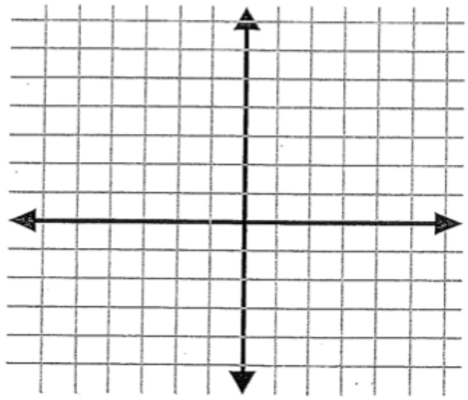
3. Find the slope of the graphed line.



(Gridlines are spaced one unit apart.)

4. Write the equation of the line passing through $(1, -6)$ and $(3, 0)$. Write your results in slope-intercept form, if possible.

5. Graph by first solving for y. $2x + 3y = 12$



6. Solve the system by substitution.

$-3x - y = -2$
$y = -3x + 4$

7. Find four solutions for the equation $3x + 5y = 15$.

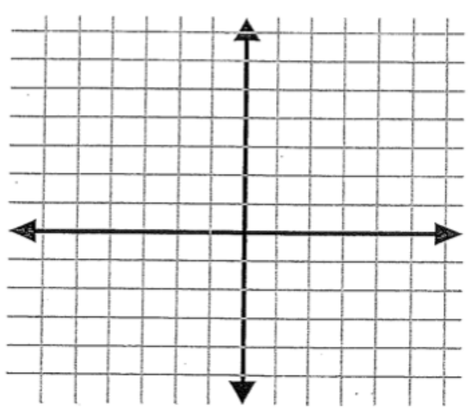
8. The length of a rectangle is 8 in. more than twice its width. If the perimeter of the rectangle is 28 in., find the width of the rectangle.

9. Find the slope of the line through the points $(0, -4)$ and $(0, -2)$.

10. Determine which of the ordered pairs $(-3, 1), (2, 0), (-13, -3), (-18, 4)$ are solutions for the equation $x + 5y = 2$.

11. Solve the system by graphing.

$3x + y = 6$
$3x - y = 0$

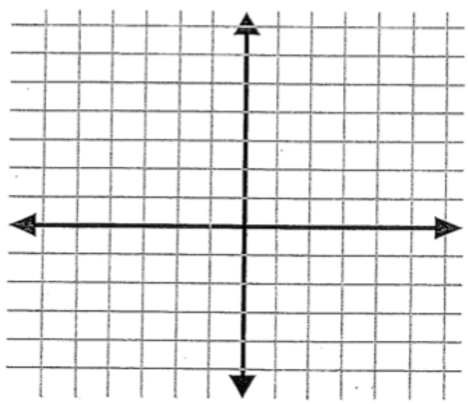


12. Sally bought three chocolate bars and a pack of gum and paid \$1.75. Jake bought two chocolate bars and four packs of gum and paid \$2.00. Find the cost of a chocolate bar and the cost of a pack of gum.

13. The difference of two numbers is 80. The larger is 8 less than 5 times the smaller. What are the two numbers?

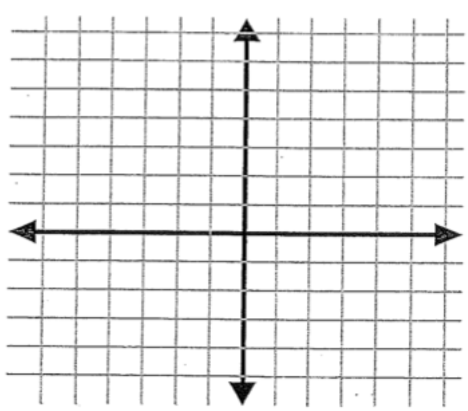
14. Graph the inequality.

$x - y \geq 2$



15. Find the slope and y-intercept of the line represented by the following equation.
$-x - 4y = -32$

16. Graph $x = 3$.



17. Solve the system by addition.

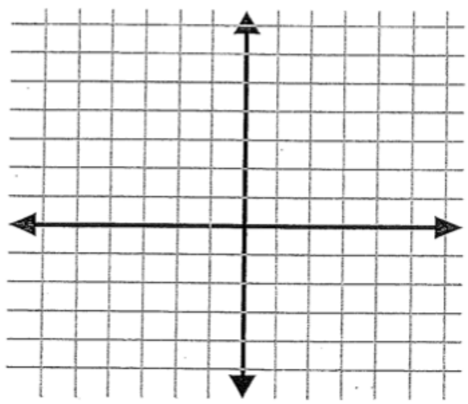
$3x + 2y = -7$
$6x - y = -1$

18. Without graphing, determine whether the following lines are parallel, perpendicular, or neither. Show work for all calculations for full credit.

    L_1 with equation $x - 3y = 12$
    L_2 with equation $3x + y = 3$

19. Complete the ordered pairs so that each is a solution for the equation $y = 4x + 12$: (,0), $(\frac{1}{4},$), (0,), $(-\frac{1}{4},$)

20. Graph $2x - y = 4$.

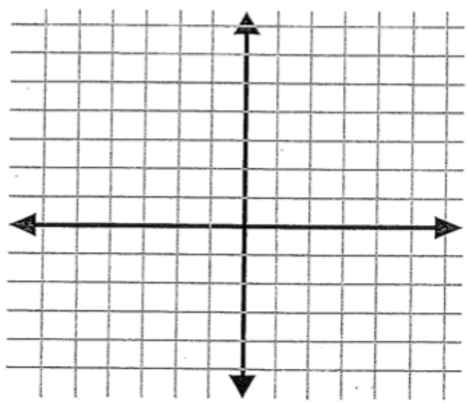


APPENDIX E: FINAL EXAMINATION

Below are the exam items from the Final Examination from the Fall 2012 semester for Math 095.

1. Solve the system by graphing.

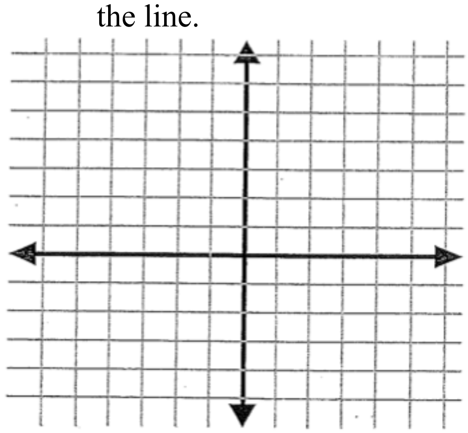
   $3x + y = 6$
   $3x - y = 0$



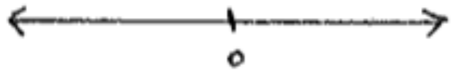
2. Find two consecutive integers such that the sum of 6 times the first integer and 2 times the second integer is 58.

3. Solve. $\sqrt{y+4} = 5$

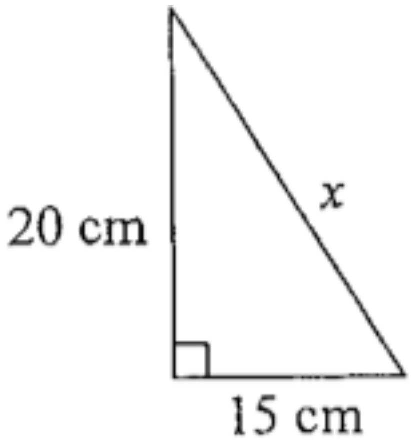
4. Write the equation of the line with slope $-\frac{1}{2}$ and y-intercept $(0, 3)$. Then graph the line.



5. Solve and graph the solution set. $3(x-4) > 7x - 9$



6. Find the length x. Express your answer in simplified radical form.



7. Divide. $\frac{16x^2-28x-38}{4x+3}$

8. Factor completely. $48x^3 + 6x^2 - 8x - 1$

9. The sum of two numbers is 110. The second is 2 more than 5 times the first. What are the two numbers?

10. The sum of an integer and its square is 56. Find the integer.

11. Tickets for a play at the community theater cost $12 for an adult and $10 for a child. If 150 tickets were sold and the total receipts were $1720, how many of each type of ticket were sold?

12. Write the equation of the line passing through $(-4, 27)$ and $(1, 2)$. Write your results in slope-intercept form, if possible.

13. Multiply. $(2x-4)^2$

14. Divide. $\frac{x^2+2x-15}{4x^2} \div \frac{x^2-25}{2x-10}$

15. Solve for x and show check. $2(x+10) + 3x = 4x + 11$

16. Factor completely. $15x^2 - x - 2$

17. Solve for x. $\frac{x}{x-6} + 3 = \frac{6}{x-6}$

18. Solve the quadratic equation. $x^2 + 2x - 80 = 0$

19. A car uses 7 gallons of gasoline on a trip of 168 miles. At that rate, how much gasoline will a trip or 312 miles require?

20. Simplify. $\sqrt{48} - \sqrt{3}$

21. Find the slope and y-intercept of the line represented by the following equation. $3x - 2y = -20$

22. Simplify. Write your answer with only positive exponents. $\frac{(b^5)^{-6}}{b(b^5)^{-5}}$

23. Subtract. Write your answer in simplest form. $\frac{5}{y-3} - \frac{1}{y+1}$

24. Subtract $6w^2 + w - 5$ from $9w^2 + 6w + 6$.

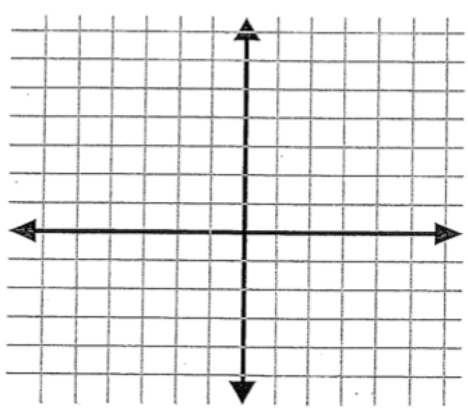
25. Solve the system by addition.

$$\begin{aligned} -5x - 4y &= -15 \\ x + 2y &= -3 \end{aligned}$$

26. A triangle has sides $2x - 5$, $3x + 1$, and $4x + 2$. Find the polynomial that represents its perimeter. Simplify your final answer.

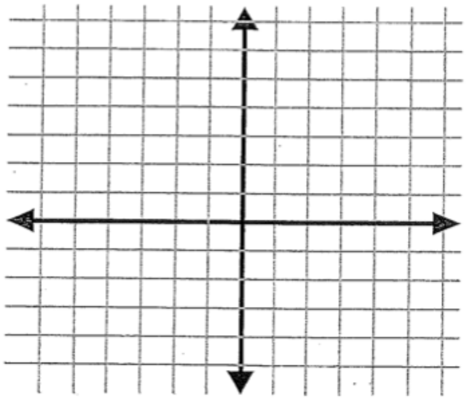
27. Graph the inequality.

$2x + 3y > 6$



28. Factor completely. $6b^4 - 18b^3 - 60b^2$

29. Graph $2x - 5y = 10$.



30. Evaluate $4p^2 - 2q$ if $p = -4$ and $q = 3$.