Bus Route Operational Efficiency Evaluation in Milwaukee County

Ebtesam Hazbavi
University of Wisconsin-Milwaukee

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BUS ROUTE OPERATIONAL EFFICIENCY EVALUATION IN MILWAUKEE COUNTY

by

Ebtesam Hazbavi

A Thesis Submitted in
Partial Fulfillment of the
Requirements for the Degree of

Master of Science
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ABSTRACT

BUS ROUTE OPERATIONAL EFFICIENCY EVALUATION IN MILWAUKEE COUNTY

by

Ebtesam Hazbavi

The University of Wisconsin-Milwaukee, 2015
Under the Supervision of Professor Yue Liu

Public transit systems are expanding in the majority of cities in the United States due to the increasing numbers of people who use public transportation as their main commute facility. Evaluating the efficiency of public transit systems results in providing better services for the residents on one hand and reduces the unnecessary costs on the other hand.

In this research, the efficiencies of bus routes options in County are evaluated. After reviewing different literature, Fuzzy AHP model is selected as the main decision making model. AHP is one of the preferred methods of decision making in solving complex problems, which provides the opportunity of involving all provided criteria in the final decisions. In this research, the efficiency was calculated for Fixed Routes and Free Flyer Routes, daily and during the peak hours.

The data of this research are obtained from Milwaukee county 2013 annual ridership report as routine reports published by Milwaukee county transit system (MCTS).

In conclusion, various solutions are suggested for improving the transit system in Milwaukee County.
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LIST OF DEFINITIONS

- Bus (used in tables): Refers to the number of busses
- Free Flyer Routes: System of Freeway Flyer routes providing express service between the suburbs and Downtown Milwaukee. Flyers run Monday through Friday during rush hours in the morning from 6:00 to 9:00 am and afternoon from 3:00 to 6:00 pm
- Headway: The time interval between buses on a route
- In Service Bus Hours: The hours spent carrying passengers on a route plus hours in layover. It does not include pull-in / pull-out time or deadhead time
- Passengers Per Bus Hour (PBH) or Productivity: The number of rides generated on a route divided by the number of in-service bus hours on that route
- Platform Bus Hours: The total bus hours spent carrying passengers on a route, layover, pull-in and pullout time, and deadhead (the time traveling from one route to another route)
- Productivity Frequency Index (PFI): PBH * Headway / 60
- Ridership: The average of the total number of boarding and alighting
### LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>AHP</td>
<td>Analytical Hierarchy Process</td>
</tr>
<tr>
<td>MADM</td>
<td>Multi Attribute Decision Making</td>
</tr>
<tr>
<td>MCDM</td>
<td>Multi Criteria Decision Making</td>
</tr>
<tr>
<td>MCTS</td>
<td>Milwaukee County Transit System</td>
</tr>
<tr>
<td>MODM</td>
<td>Multi Objective Decision Making</td>
</tr>
<tr>
<td>PBH</td>
<td>Passengers per Bus Hour</td>
</tr>
<tr>
<td>PFI</td>
<td>Productivity Frequently Index</td>
</tr>
<tr>
<td>PM-BH</td>
<td>Number of passengers per bus hour</td>
</tr>
<tr>
<td>PM-Bus</td>
<td>Number of buses during the peak of 3:00 pm to 6:00 pm</td>
</tr>
<tr>
<td>PM-RIDES</td>
<td>Number of rides during the peak of 3:00 pm to 6:00 pm</td>
</tr>
<tr>
<td>RNK</td>
<td>Rank</td>
</tr>
<tr>
<td>RT</td>
<td>Route</td>
</tr>
<tr>
<td>TL-RIDES</td>
<td>Total number of ridership</td>
</tr>
<tr>
<td>SAT</td>
<td>Saturday</td>
</tr>
<tr>
<td>SUN</td>
<td>Sunday</td>
</tr>
<tr>
<td>WK</td>
<td>Weekday</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENTS

I take this opportunity to express my deep sense of gratitude to my graduate advisor, Dr. Yue Liu, for his continuous encouragement, guidance and support during this research.

This work would not have been possible without the help of a few collaborators. I would like to thank Milwaukee County Transit System (MCTS) for providing me with the data required in this research.

I wish to thank my entire family for providing a loving environment for me. My parents, who bore me, raised me, supported me, taught me, and loved me.

Lastly, for my supportive husband Erfan, I find it difficult to express my appreciation because it is so boundless. He is my most enthusiastic cheerleader; he is my best friend; and he is an amazing husband. Without his love and support, I would be lost. He has shared this entire amazing journey with me, so it only seems right that I dedicate this dissertation to him.

I also want to thank my friend Xin Li at Transportation Lab, who helped me through this thesis.
1. Introduction

Public transportation is critical to the nation’s future. It is a crucial part of a stronger economy, as it encourages business and develops green jobs. Reducing the fuel consumption in the nations, providing mobility for all the people, improving the air quality by reducing carbon emissions and improving sustainability are other important aspects of public transportation.

Investment on public transit will result in creating new jobs, saving money, reducing dependence on foreign oil, and bringing greenhouse gas emissions down. In other words, investment in transportation is more than just a physical investment in infrastructure. It is a commitment to children’s and nation’s future (American Public Transportation Association 2013).

1.1. Transit System in US

New York City was the first city in the US that welcomed initial kind of mass transit. A regular steam ferry in the early 1810s and horse drawn buses in late 1820s are the two examples of this mass transit system. Gradually, by spread of horse railways networks in the mid-19th century, mass transit became part of urban development. One decade after Electric Street Car was the main kind of public transportation in all streets.

Significant impact of public transit on all aspects of urban development made it an inseparable part of urban planning. However, the growth of automobile ownership started to reduce the impact of public transportation in 1920s, these days' environmental sustainability concerns has renewed the interests in the benefits of mass transit (Young 2015).
US currently has population of around 321 million people, distributed over 50 states and the District of Columbia. There are slightly over 375 metropolitan areas with more than 50,000 people, and over 50 metropolitan areas with more than 1,000,000 people. Nowadays, this big population relies more and more on public transportation facilities. The Statistics below show the fact of this reliance (Thompson 2008) (Census Data n.d.):

- In 2013, Americans took 10.7 billion trips on public transportation, the highest annual ridership number in 57 years.
- 35 million times each weekday, people boarded public transportation.
- Public transportation is a $58 billion industry that employs nearly 400,000 people.
- More than 7,100 organizations provide public transportation in the United States.
- From 1995 through 2013, public transportation ridership increased by 37.2%, a growth rate higher than the 22.7% increase in US population and higher than the 20.3% growth in the use of the nation’s highways over the same period (American Public Transportation Association 2015).

**Largest US Public Transit Systems**

At this section, some statics about largest transit systems in US are provided for a better understanding of the impact of public transportation in US. New York, Chicago, Boston and Washington DC are considered as the largest public transit systems in US. Table 1, Table 2, Table 3 and Table 4 show the statics of their transit systems in 2013, respectively (UNC School of Governet 2014).
New York: The Metropolitan Transportation Authority (MTA) is North America’s largest transportation and includes subways, buses, and railroads.

<table>
<thead>
<tr>
<th>Table 1: Statics for New York Transit System in 2013</th>
</tr>
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<tbody>
<tr>
<td>Number of daily riders</td>
</tr>
<tr>
<td>Average weekly bus ridership</td>
</tr>
<tr>
<td>Average weekly subway ridership</td>
</tr>
<tr>
<td>Number of Bus Routes</td>
</tr>
</tbody>
</table>

Chicago: Chicago Transit Authority (CTA) is a regional transit system that serves 35 suburbs, six counties, and the city of Chicago operating urban bus and train system.

<table>
<thead>
<tr>
<th>Table 2: Statics for Chicago Transit System in 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average weekday bus ridership</td>
</tr>
</tbody>
</table>

Boston: The Massachusetts Bay Transit Authority (MBTA) operates most bus, subways, commuter rail, and ferry rail in the Boston area.

<table>
<thead>
<tr>
<th>Table 3: Statics for Boston Transit System in 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average weekday bus ridership</td>
</tr>
<tr>
<td>Number of Bus Routes</td>
</tr>
</tbody>
</table>

Washington DC: Washington Metropolitan area Transit Authority (WMATA) operates both Metrobus and Metrorail.
Table 4: Statics for Washington DC Transit System in 2013

<table>
<thead>
<tr>
<th>Average weekday bus ridership</th>
<th>750000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Bus Routes</td>
<td>325 routes + 169 lines</td>
</tr>
</tbody>
</table>

1.2. Milwaukee Transit System

N. Water Street from Erie Street to E. Juneau Avenue was host of the first street car in Milwaukee on May 30, 1860. Thirty years later, operation of the first electric car on April 3, 1890 on Wells Street revolutionized the mass transit system. The Milwaukee Electric Railway &Light Company (TMER&L) was founded on January 29, 1896. The company handled electrical utilities; however, it was created primarily as a transportation company. The TMER&L was broken down in 1938 to form the Wisconsin Electric Power Company (The Electric Company) and The Milwaukee Electric Railway & Transport. Figure 1 and Figure 2 show the first streetcar and the first electric streetcar used in Milwaukee in 1860 and 1890, respectively (Milwaukee County Transit System 2015).

Figure 1: First Streetcar in Milwaukee
Figure 2: First Electric Streetcar in Milwaukee

From 1975 Milwaukee County Transit System (MCTS) has been the main agency which has provided public transportation for 956,025 (2012) population of Milwaukee County. MCTS ranked among top 50 transit agencies in US and in 2013 provided 56 routes for the Milwaukee County, which included 34 regular routes, 3-limited stop MetroExpress routes, 7 school routes, 4 UBUS service for UWM and MATC, and 8 Free Flyers routes (Milwaukee County Transit System 2013). Table 5 shows some MCTS statics, which were presented at MCTS 2013 annual report. (Milwaukee County Transit System 2013)

Table 5: Statics from MCTS 2013 Annual Report

<table>
<thead>
<tr>
<th>Service Area/Milwaukee County</th>
<th>242 Square Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milwaukee County</td>
<td>Population 956,025</td>
</tr>
<tr>
<td>City of Milwaukee Population</td>
<td>599,164</td>
</tr>
<tr>
<td>Miles Served</td>
<td>17,244,868</td>
</tr>
<tr>
<td>Bus hours</td>
<td>1,328,034</td>
</tr>
<tr>
<td>Total Passenger Ridership</td>
<td>43,008,924</td>
</tr>
<tr>
<td>Passenger Trips on an Average Weekday</td>
<td>156,328</td>
</tr>
<tr>
<td>Average Passengers per Bus</td>
<td>Hour 32.4</td>
</tr>
<tr>
<td>Freeway Flyer Ridership</td>
<td>409,387</td>
</tr>
</tbody>
</table>
1.3. Purpose of This Study

In current days, accessibility to transit network is crucial for many of Milwaukee residents, as it is the main method for different generations to connect them to different places such as reaching the students to school, workers to job and the elderly residents to medical appointment and social events. Apart from environmental sustainability advantages, society gain economic benefit from effective and efficient transit system as well. "United Stated Census Bureau" demonstrated that individuals who ride public transportation could save on average $10,116 annually based on the April 13, 2011 national average gas price and the national unreserved monthly parking rate (U.S. Census Bureau 2011).

All the above reasons motivated this study to evaluate the public transit system in Milwaukee County and make comparisons between this study's results and Milwaukee County Transit System's results. In this study, data from MCTS 2013 annual ridership report are used and the relations between different criteria and the importance of setting the appropriate weight for each criterion are studied. The MCTS report evaluated the efficiency based on each criterion, separately and without considering the impacts of them on each other and on the result. Different methods of decision-making are explained and finally suggestions for the improvement of the transit system are made (U.S. Census Bureau 2011).
2. Literature

2.1. Methods of Transit Evaluation

A review of literature about evaluating the transit operator performance, ending with three main approaches, which are as below:

- Considering the user's satisfaction as the main option of efficiency evaluation
  In this approach, different surveys are taking to achieve the results
- Considering different criteria relevant to transit system demand and operation and making a comprehensive evaluation framework to calculate efficiency
  The mathematical equations are widely used in this approach for efficiency evaluation.
- Combination of both users opinions and efficiency indicators is another method of evaluation (Li, et al. 2014)

In terms of introducing the criteria for evaluation of an intermodal passenger transfer facility, Horowitz and Thompson introduced the list of 70 generic objectives after extensive literature review and interview with users (Horowitz and Thompson 1995). Some guidelines for evaluating transit operator performance are developed by public agencies. For example, the Federal Transit Administration developed the Bus Route Evaluation Standards to evaluate the bus route performance. The groups of indicators were developed by the International Association of Public Transport (UTIP) to compare the performance of public transport in terms of services among the different cities and areas (Li, et al. 2014). The “Transit Capability and Quality of service Manual” which was published by Transportation Research Board is a guidebook-included guideline to provide technical advice for evaluating the performance of public transport system and further compiling specific public transport planning (Li, et al. 2014).
The European Standard has specified the requirements to define, target and measure quality of service in public passenger transport (PPT). It also has provided the guidance for the selection of related measurement methods (Li, et al. 2014).

Based on availability of inputs and outputs and different types of criteria, each of the aforementioned approach could be used for evaluating the efficiency. Other effective point regarding efficiency evaluation of transportation system is selecting appropriate decision-making model. Several studies have been done about introducing different decision-making methods. Some literatures review about different approaches towards the decision-making subject is explained in Sections 2.2 and 2.3.

2.2. Brief Overview of Decision-Making

Decision-making is the study of identifying and choosing alternatives based on the values and preferences of the decision makers. Regarding achieving the project goals and value, there is a need of considering all the existing choices and identifying all the possible alternatives. The next important step is to choose the option that best fits with the goals and this is what decision-making implies (Fülöp 2012). Based on (Saaty 2008), decision-making consists of the following steps:

1. Identifying the problem and the purpose of the decision
2. Establishing the goal of the decision
3. Identifying the criteria and sub criteria
4. Identifying the stakeholders and groups affected and the alternative actions need to take
5. Determining the best alternatives
In other study, it was claim that, decision-making should start with the identification of the decision makers and stakeholders in the decision, and continue by reducing the possible disagreement about problem definition and requirements, goals and criteria (Fülöp 2012).

In today world, decision-making process that is the main reason of collecting our information became science that is more mathematical. It formalizes the thinking we use so what we have to do to make better decisions is transparent in all its aspects (Saaty 2008).

Different methods of decision-making are categorized as below:

2.2.1. Group Decision-Making

Based on literature, group decision is defined as collecting different individual preferences on a given set of alternatives and selecting the alternative that is preferred with the majority of the members. Group decision method is based on the assumption that all the members who are involved in the project are facing the same problem and they have the same goal, which is realizing the best solution for the existing problem. In the group decision situation, multiple decision-makers are involved. These decision-makers benefit from various skills, experience and knowledge, which are related to different criteria of the problem. The competence of the different decision makers to the different professional fields should also be into consideration in a correct synthesized group decisions method. Each decision-maker should also consider the same sets of alternatives and criteria. A special decision-maker has the authority to establish consensus rules and to determine voting powers to the group members on the different criteria. Supra Decision-Maker (SDM) is a name given by Keeney and Raiffa in 1976 to this entity (Fülöp 2012). Finally, using SDM rules and priorities, synthesized opinions of the group members will derive and ruled as the final decision (Fülöp 2012).
2.2.2. Sensitivity Analysis

Sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or system (numerical or otherwise) can distribute to different sources of uncertainty in its inputs (Wikipedia n.d.). It is therefore an important question how the final ranking or the ranking values of the alternatives is sensitive to the changes of some input parameters of the decision model. The simplest case is when the value of the weight of a single criterion is allowed to vary. For additive multi attribute models, the ranking values of the alternatives are simple linear functions of this single variable and attractive graphical tools can be applied (Forman and Selly 2003). (Mareschal 1988) showed how to determine the stability intervals or regions for the weights of different criteria in Multi-Attribute Decision-Making Methods (MADM). In this approach, the value of the weight of one or more criteria can take out while the results, which were achieved with the initial set of weights, being kept constant. (Wolters and Mareschal 1995) proposed a linear programming model to find the minimum modification of the weights required to make a certain alternative ranked first. Meanwhile, (Triantaphyllou and Sanchez 1997) presented an approach of a more complex sensitivity analysis with the change of the scores of the alternatives against the criteria. (Mészáros and Rapcsák 1996) for a wide class of Multi-Attribute Utility Theory (MAUT) presented a general and comprehensive methodology where the aggregation is based on generalized means, including the additive and multiplicative models.

2.2.3. Multi-Criteria Decision-Making

Multi Criteria Decision-Making (MCDM) is both an approach and a set of techniques, which deals with the problems that involve multiple and conflicting criteria. The decision process is more complex when there is more than one objective involved in problem. MCDM
also deals with rating the option from the most preferred one to the least preferred one, because of that it provides a framework to evaluate different transport options on several criteria. Multi Criteria Decision-Making approaches also provide a systematic procedure to help decision makers choose the most desirable and satisfactory alternative under uncertain situation (OĞUZTİMUR n.d.).

Based on the number of alternatives involve in the project, MCDM approaches are classified into two groups: Multi Objective Decision-Making (MODM) and Multi Attribute Decision-Making (MADM). Generally, MADM problems contain relatively small number of alternatives and these alternatives are in terms of attributes.

**Multi-Objective Decision-Making (MODM)**

In MODM approach, there are a very large number of feasible alternatives, the objectives and the constraints. The number of each of these three categories depends on the decision variables that are involved in MODM problems. For dealing with multi-objective planning problems, MODM is adequate method. In MODM, unlimited numbers of continuous alternatives are defined by a set of constraints on a vector of decision variables, while MADM methods are used for the discrete alternatives (OĞUZTİMUR n.d.).

**Multi-Attribute Decision-Making (MADM)**

MADM method is useful in solving the problems that involve uncertain and subjective information by providing simple and intuitive tools (Cheng 2000). Eliminating the traditional restriction of converting all criteria’s unit to the same unit is a very important advantage of MADM method. The capacity to analyze both quantitative and qualitative criteria together is
another significant benefit of MADM methods. Each alternative are described by using multiple attributes and trying to choose the best alternative among a given set of alternatives. The next steps are ranking alternatives from the best to the worst or classifying them into classes. Generally, the MADM methods are used to solve discrete problems but some of them can also be used within the context of continuous decision problems (OĞUZTİMUR n.d.). Regarding resolving this difficulty Fuzzy set theory has been applied to the multi-criteria problems. Fuzzy set theory is based on selecting, prioritizing or ranking a limited number of courses of action by evaluating a group of predetermined criteria. For this reason, constructing an evaluation procedure is needed to rate and rank the set of alternatives. Among the MADM methods developed in the literature, AHP, multi-attribute utility theory and outranking methods are more frequently applied than all other methods to discrete decision problems (OĞUZTİMUR n.d.). This method is described in the following sections.

2.2.4. Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP) is one of the multi-criteria decision-making approaches, which was introduced by Saaty in 1970s. Based on the literature AHP is one of the preferred methods of decision-making in solving complex problem. AHP method breaks down the complex to its components by using multi-level hierarchical structure of objectives, criteria, sub criteria, and alternatives. To obtain the weights of different criteria a set of pairwise comparison matrices are also created (Stuart 1995). Based on Saaty theory, following steps should be taken to generate priorities involve in the final decision (Saaty 2008):

1. Specify the problem.
2. Organize the decision hierarchy, from top level including the goal to the lowest level, which contain the alternatives.

3. Construct the set of pairwise comparison matrices that provide the ability of comparison between different criteria. Each criterion in upper level will be compared with the criteria immediately lower with respect to it.

4. Weigh of the level below could assign by using the priorities obtain from comparison matrices. By adding the weighed values for each element in the level below, overall priority will be obtained. Doing the process of adding and weighing until the final priorities of the alternatives in the bottom most level are determined.

2.2.5. Problems of AHP Method

As mentioned earlier, AHP is the method of dividing the complicated decision problems and non-structural situations into hierarchical elements. In this model, the goal is located at the highest level and m alternatives, A₁, A₂... Aₘ, are locating at the lowest level. The criteria and sub criteria are applied at the middle levels, C₁, C₂...Cₙ (Figure 3). Based on the literature, the general method for weighting the criteria is applying the preferences of decision makers in the form of ratios by using pairwise comparison matrices. The judgment matrices of criteria or alternatives are defined by rating the relative importance of elements based on a standard scale where 1 = equally important; 3 = weak importance; 5 = strong importance; 7 = demonstrated importance; 9 = absolute importance. Numerical value aᵢⱼ, which is the ratio of the importance of the criteria at the final goal, is results of comparing any two elements at the same level of hierarchy. It is obvious that for a level with n element there are n (n – 1)/2 judgments required, which are further used to construct a positive reciprocal comparison matrix A = { aᵢⱼ} ∈ Rⁿ×ⁿ,
where $a_{ij} = 1/a_{ij}$, and $a_{ij} > 0$, for $j = 1, 2, \ldots, n$, $i = 1, 2, \ldots, n$ (MohammadJavanbarg, et al. 2012).

A priority vector $w = (w_1, w_2, \ldots, w_n)$ may be obtained from the comparison matrices by applying a prioritization method. Several methods for deriving the local weights of criteria and the local scores of alternatives from judgment matrices have been developed by (Mikhailov 2003). At this stage, the consistency of each pairwise comparison is checked and a final aggregation of local priorities is performed to rank the alternatives (MohammadJavanbarg, et al. 2012).

![Hierarchical Structure for AHP-Based MCDM](image)

Figure 3: Hierarchical Structure for AHP-Based MCDM

However AHP is consistent, structured and intuitive, different arguments have risen because of the reasons described below:

General approach to creating the pairwise matrices and estimating the weight of each criterion is relying on decision-makers knowledge, and the subject rises criticize because this reliance increases the probability of arbitrary and biased decision. Different set of unites for criteria especially in complex projects make it difficult to find the value of each criteria. Synthesizing weights is time-consuming process when the level of hierarchy increases. These
restrictions leads to various research to be done from 1980 till now for applying different methods to estimate the weights more realistic for all criteria involved in a project (MohammadJavanbarg, et al. 2012). Fuzzy set based approach has been suggested to overcome the inability of AHP to handle uncertainties. Fuzzy scale level makes it easier to assign nominal value to different indicators, because in this model numbers give a range of values for the concept and a membership quantity for each value. Other advantage is achieving total score by rolling up all the criteria by considering the value assigned to each of them (Li, et al. 2014).

Advantage of fuzzy AHP method compare with other decision making models, resulted in selecting this method as the main model of this research, which will be described more on chapter three and four.
3. Operational Efficiency Evaluation Framework

As it was mentioned in chapter one, purpose of this thesis is evaluating the efficiency of bus routes in Milwaukee County. The data was provided by MCTS, and the Methodology will be chose based on the available criteria and the goal. Before going further in terms of describing the methodology, this chapter (chapter three) will introduce the framework for efficiency evaluation of bus routes in Milwaukee County. However, due to the lack of some critical information this framework could not fully applied to the Milwaukee County Operational Efficiency Evaluating, which will be analyzed further in chapter four.

3.1. Different Viewpoints to the Transit Efficiency Evaluation

Efficiency is the relation between inputs and outputs and its aim is to produce a specific outcome effectively with a minimum amount or quantity of waste, expense, or unnecessary effort. In our today word, transport is the sector with the highest final energy consumption and, without any significant policy changes, is forecast to remain so (Kojima and Lisa 2010). In transit related area efficiency could be define from different views; the users' viewpoint and the transit operators' viewpoint. What users think about effectiveness and efficiency of the transit system is based on their satisfaction with the cost and availability of the buses to meet their travel demands. The efficiency from this viewpoint is measurable by considering the criteria like: ridership, service quality and accessibility to the services. Efficiency base on the transit operators' view is about minimizing the operational cost without preventing the daily travel demand of the people. Mainly in this method, the relation between input and output is critical.
The criteria for efficiency based on the viewpoint of transit operators could include overall cost, labor utilization, vehicle utilization, etc. Balancing between both viewpoints is crucial but not easy to achieve. From the transit system authorities viewpoint, it is important to consider the users satisfactions more than the operational cost (Hawas, Bayzid Khan and Nandita 2012). Evaluating the efficiency needs collecting related criteria and making the framework as the guideline for the study. In the following sections, different methods of collecting the criteria, which are included in the model, will be discussed more.

3.2. Different Sets of Criteria

To evaluate transit efficiency, different criteria have been set in different studies. Criteria for evaluating the transit system could include Daily Loads, Peak Hour Loads, Fleet Numbers, Capacity of each Bus, Number of Bus Stops and Operation Cost (Fuel Cost, Labor Cost) as inputs and Mileage, and Passenger satisfaction index as outputs. The problem in different studies is how to weight multiple criteria participated in the study, to see which one has greater impact on the final efficiency. To solve this problem this thesis chose the fuzzy AHP model for analyzing the efficiency of bus routes in Milwaukee County. Due to the choosing the fuzzy AHP model it needed to be mentioned only data that are completely unrelated to each other could be included in the process of fuzzy AHP model.
3.3. Data Collecting

Based on the each criterion different method of data collecting could be used. Ridership and the number of passenger per bus hour could be collected by using collector devices, usually maintained by the bus drivers. In terms of reducing the probability of faults in entering the data, it is better to not just focus on one specific day. Usually considering the average of a mount or a season could reduce the probability of the mistakes. For criteria like mileage, they could be directly derived from the buses. Using Google Map application and GIS software by focusing on the path would also provide these types of data. Fleet number and the cost are recorded by the agencies; however, there is always relation between fuel cost and the mileage, which should be considered as well. In terms of passenger satisfaction surveys are the best tools, and it also should be considered as an index in the analyzing the results.

3.4. Framework for Milwaukee Transit System

As it was mentioned, one of the purposes of this study is analyzing the efficiency and making comparison with the results of MCTS report. Unfortunately, no data in terms of cost or travel time was available. The other problem is the type of criteria was chose by the MCTS report. Ridership and Passenger per Bus Hour (productivity), which are considered in the report, are close in terms of definition and it is not what fuzzy AHP model seeks for.

However, this thesis looks at both ridership and productivity in the same time for efficiency evaluation; it is still recommended to the MCTS to collect different criteria (like what it was mentioned above) for evaluating the efficiency of bus routes in Milwaukee County. Considering different criteria would help in achieving more reasonable results.
4. Methodology – The Fuzzy AHP Model

The data which are analyzed in this research are from 2013 Milwaukee County Transit System (MCTS) Annual Ridership Statics Book provided by Milwaukee County Transit System. As it was mentioned in chapter one, the purpose of the study is evaluating the efficiency of bus routes in Milwaukee County based on different criteria. Routes are divided into two different categories based on the services they provide for the public; Fixed Routes and Free Flyer Routes. Efficiency of these categories will be evaluated both by weekday and during the rush hours. The results will be compared by the results of MCTS 2013 annual report. This report includes ranking of the bus routes based on ridership and productivity. A major problem with the MCTS report is that they have calculated the efficiency just based on each criterion separately without considering other critical criteria. For example, they have calculated the ridership and passenger per bus hour without considering the impact of both criteria together on the system, which directly affect the efficiency. In this thesis, efficiency is calculated for both of these criteria at the same time, which is a more precise way to obtain the efficiency. Rush hour, a very important criterion in efficiency is also studied in this thesis.

Briefly, this section is about the accurate efficiency calculation by analyzing different criteria, deciding about the most appropriate ones and using any preferred criteria simultaneously. Therefore, Fuzzy Analytic Hierarchy Process (AHP) model is used as one of the most popular Multi-criteria decision-making (MCDM) methods to measure the performance of different routes. Fuzzy AHP model provides the opportunity of involving all criteria in the final decision and analyzing them to detect the weight impact of each criterion on the efficiency as the final goal.
There are different criteria involving in this research, and participation of all criteria in the decision-making process is one of the main goals of this thesis. Therefore, in case of using the conventional AHP model it will be difficult to weight the criteria without biased judgment or facing uncertainty. Because of that, Fuzzy AHP Model is selected as a framework for this thesis. Unlike the conventional fuzzy AHP methods, the proposed method drives exact weights from consistent and inconsistent fuzzy comparison matrices. The fuzzy AHP method transforms a fuzzy prioritization problem into a constrained nonlinear optimization model. The model is extended in following sections (MohammadJavanbarg, et al. 2012).

4.1. Hierarchical Structure of AHP

This thesis constructs the hierarchy levels by resolving the complex system, based on the main goal, to the levels. The first level contains the goal, which is improving the efficiency and it will be remain constant during the study; however, the set of the criteria are possible to be changed. Criteria belong to the second level of AHP model. The last level contain the indicators which in this thesis are all 34-bus routes serving in Milwaukee County.

4.2. Selecting a Set of Criteria

Criteria that support the goal, which is transit evaluation and improving the efficiency of bus routes options, are selected for creating the AHP model. However, criteria in this study are based on the data came from MCTS, meanwhile this thesis in the process of selecting the preferred criteria will analyze each indicator and the probable consequences related with one set
of indicators over another. For example, ridership, passenger per bus hour (PBH) or productivity frequency index (PFI) is selectable choices for evaluating the efficiency in this research.

4.3. Fuzzy Scaling

Normalizing the weight of the criteria is achievable by introducing a set of fuzzy membership functions. Therefore, fuzzy scaling will resolve the problem of dealing with different indicators with different types of units. For the reason of measuring all criteria on similar numerical scales, all criteria even with different units, are divided in to two main groups; “the-lower-the-better” and “the-higher-the-better”. Base on the research's goal, each criterion will belong to one of the aforementioned group. The Two following equations will be applied in this research in terms of normalizing $X_{ik}$ (Liu and Yu 2012) (Li, et al. 2014);

For the-lower-the-better indicators: $\mu_{ij} = \frac{[x_i(max) + x_i(min) - x_{ij}]}{[x_i(max) + x_i(min)]}$ (1)

For the-higher-the-better indicators: $\mu_{ij} = \frac{x_{ij}}{[x_i(max) + x_i(min)]}$ (2)

4.4. Pairwise Fuzzy Comparison Matrices

In multicriteria decision-making, the pairwise comparisons are a useful point comes after normalizing the scale for determining a ranking on a set $X = \{x_1, x_2...x_n\}$ of alternatives or criteria. The pairwise comparison between $x_i$ and $x_j$ is quantified in a number $a_{ij}$ expressing how much $x_i$ is preferred to $x_j$ and the quantitative preference relation is represented by means of the
matrix $A = \left( a_{ij} \right)$ (Cavallo, D’Apuzzo and Gabriella 2010). Different methods are applied in literature in terms of calculating pairwise comparison matrices. This thesis focused on the following equations in terms of diagnosing the pairwise matrices:

Standard deviation $s_i$, given by the following equation:

$$s_i = \sqrt{\sum_{k=1}^{m} (\mu_{ik} - \bar{\mu}_i)^2 / (m - 1)}$$

(3)

Pairwise comparisons are quantified by using a scale, so matrix $A = (a_{ij})_{n \times n}$ is calculated by equations (4) and (5):

$$a_{ij} = \frac{s_i - s_j}{s_{\text{max}} - s_{\text{min}}} \times (a_m - 1) + 1, \quad s_i \geq s_j \quad (4)$$

$$a_{ji} = \frac{1}{\frac{s_j - s_i}{s_{\text{max}} - s_{\text{min}}} \times (a_m - 1) + 1} \quad s_i < s_j \quad (5)$$

In AHP model, one of the main topics is measuring all criteria on similar numerical scales. For the reason that, defining comparison scale for all criteria is important. In this study $a_m = \min \left\{ 9, \text{int} \left( \frac{s_{\text{max}}}{s_{\text{min}}} + 0.5 \right) \right\}$ is considered as a comparison scale for all criteria (Liu and Yu 2012).

4.5. Consistency Check and Deriving Priorities

In $n \times n$ comparison matrix, like the matrix
\[
A = \begin{bmatrix}
1 & a_{12} & \cdots & a_{1n} \\
\frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1
\end{bmatrix}
\]

it could be recognized as a reciprocal matrix if \( a_{ij} = 1/ a_{ij} \) for each \( 1 \leq i, j \leq n \)

This matrix is consistent if \( a_{ij} \cdot a_{jk} = a_{ik} \), and \( 1 \leq i, j, k \leq n \) (Cavallo, D’Apuzzo and Gabriella 2010).

Checking for consistency is a significant advantage of fuzzy AHP model. However, there are not so much studies has been done in the past regarding the consistency check and extracting the priorities from the pairwise comparison matrices, this research applies the following nonlinear optimization equations to estimate the weight \( \{w_j ; j = 1, \ldots, m\} \) from the inconsistent \( a_{ij} : \)

\[
\min \text{CIC}(n) = \sum_{i=1}^{m} \sum_{i=1}^{m} \frac{|y_{ij} - b_{ij}|}{n^2} + \sum_{k=1}^{m} \sum_{i=1}^{n} \frac{|y_{ij} w_j - w_i|}{n^2}
\] (6)

The consistency of A is measured by calculating the Consistency Index. Eq.6 is a constrained non-linear optimization formulation applied for finding out the closest matrix compared with the original comparison matrix during the minimizing process of the consistency index coefficient.

\[
y_{ii} = 1 (i = 1 \ldots n)
\] (7)

\[
\frac{1}{y_{ij}} = y_{ij} \in [a_{ij} - da_{ij}, a_{ij} - da_{ij}] (i = 1, \ldots, n; j = i + 1, \ldots, n)
\] (8)

The consistency judgment matrix in forms of \( Y = (y_{ij})_{n \times n} \) is calculated by Eq.7 and Eq.8. In order to obey the rules of AHP theory the minimization of C.I.C. (n) is considering to ensure that \( Y = (y_{ij})_{n \times n} \) is as consistent as possible.
\[ w_i > 0 \ (j = 1, \ldots, m) \quad (9) \]
\[ \sum_{j=1}^{n} w_i = 1 \quad (10) \]
\[ S_k = \sum_{i=1}^{n} w_i \times \mu_{ik} \quad (11) \]

When the C.I.C. (n) is equal to one, the pairwise matrix is completely consistent and the minimum value of C.I.C (n) could be further guaranteed to be the unique based on the edge-restraint condition. (Liu and Yu 2012)
5. Analysis and Results

The purpose of this study is evaluating the efficiency of bus routes in city of Milwaukee and Milwaukee County. As it was mentioned in chapter three, the raw data was provided by 2013 MCTS annual ridership statics book and the results of this study will be compared with the results of MCTS, which is available on their report. Based on the methodology was described in chapter three, the Fuzzy AHP Model, is applied to this research. In the next steps, the results will be analyzed and recommendations will be offer to improve the efficiency. Fuzzy AHP model provides the participation opportunity for all criteria in the final decision. The problem of MCTS’s report was that the report focused on efficiency discontinuously. It means it does not show the efficiency when all criteria are involved, it shows efficiency based on each factor, like ridership and passenger per bus hour.

5.1. Data Collection and Data Analyzing

This thesis will find the efficiency for both Fixed Routes, which includes regular and MetroExpress Routes, and the Free Flyer Routes on weekdays and weekends separately. The reason of selecting these categories is these routes are public routes, which provide services for entire Milwaukee and Milwaukee County. Therefore, improving the efficiency will result in providing better public services for all the residents. Data analyzing started with data cleaning and contains following steps:
Categorizing the Bus Route Options

Buses are categorized in terms of type of services they provide for the county. Based on the MCTS report, there are two main classifications: Fixed Routes and Free Flyer Routes. Upcoming parts contain the routes and the name of all routes in each category.

**Fixed Routes:** Contains both Regular and MetroExpress Routes, which areas below:

- **Regular Routes:**

<table>
<thead>
<tr>
<th>Route</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt - Wisconsin</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia - Hampton</td>
</tr>
<tr>
<td>14</td>
<td>Holton – Forest Home</td>
</tr>
<tr>
<td>15</td>
<td>Oakland – Kinnickinnic</td>
</tr>
<tr>
<td>19</td>
<td>King Drive – S. 13th</td>
</tr>
<tr>
<td>21</td>
<td>North Avenue</td>
</tr>
<tr>
<td>22</td>
<td>Center Street</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac Avenue</td>
</tr>
<tr>
<td>27</td>
<td>27th Street</td>
</tr>
<tr>
<td>28</td>
<td>108th Street</td>
</tr>
<tr>
<td>30</td>
<td>Sherman – Wisconsin</td>
</tr>
<tr>
<td>31</td>
<td>Wisconsin – Mayfair</td>
</tr>
<tr>
<td>33</td>
<td>Vliet Street</td>
</tr>
<tr>
<td>35</td>
<td>35th Street</td>
</tr>
<tr>
<td>51</td>
<td>Oklahoma Avenue</td>
</tr>
<tr>
<td>52</td>
<td>Clement – 15th Avenue</td>
</tr>
<tr>
<td>53</td>
<td>Lincoln Avenue</td>
</tr>
<tr>
<td>54</td>
<td>Mitchell – Burnham</td>
</tr>
<tr>
<td>55</td>
<td>Layton Avenue</td>
</tr>
<tr>
<td>56</td>
<td>Greenfield Avenue</td>
</tr>
<tr>
<td>57</td>
<td>Walnut – Lisbon</td>
</tr>
<tr>
<td>60</td>
<td>Burleigh Street</td>
</tr>
<tr>
<td>62</td>
<td>Capitol Drive</td>
</tr>
<tr>
<td>63</td>
<td>Silver Spring Drive</td>
</tr>
<tr>
<td>64</td>
<td>S. 60th – Grange</td>
</tr>
</tbody>
</table>
5.2. Criteria Selection

Selecting the criteria is one of the most important steps in all approach of multicriteria decision making, and consequently in AHP model. This part contains two main categories, which are as below:

- Evaluating efficiency during 24 hours of an optional day:

  In this section, criteria are selected based on what MCTS considered as important factors for efficiency evaluating. The same selected criteria give the opportunity of making comparison and providing appropriate suggestions to improve the entire system. So in this section efficiency will be evaluated for daily ridership and passenger per bus hour.

- Evaluating efficiency during the peak hours:
This thesis will also focus on different relations between criteria to find out the efficiency of bus routes options during the rush hours. Rush hours always are the problem of wasting time and natural energy. Improving the efficiency during these times will provide more satisfaction to the residents. The importance of this subject leads to the research in evaluation of the efficiency during the rush hours. Next sections will provide better view for this topic.

5.3. Daily Efficiency Assessments

Daily efficiency assessments start with categorizing and dividing the statics of the routes to the weekdays, Saturdays and Sundays. Then will continue with applying the fuzzy AHP model to the data. The sections below cover all the necessary steps to achieve the final efficiency ranking.

Daily Bus Routes Statistics

Table 6 represents the row data of Fixed Routes serving on the weekdays. The criteria are Total Ride and Total Passenger per Bus Hour. As it is obvious, the rout 30 contains the most rides and the Routes 21 and 27 served the most number of passengers in an hour.

<table>
<thead>
<tr>
<th>RT</th>
<th>Name</th>
<th>SERVICE</th>
<th>TOTAL_RIDE</th>
<th>TOTAL-PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>WK</td>
<td>6156</td>
<td>34.9</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>WK</td>
<td>9699</td>
<td>51.8</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>WK</td>
<td>3359</td>
<td>32.9</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>WK</td>
<td>4871</td>
<td>31.2</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>WK</td>
<td>127</td>
<td>15.2</td>
</tr>
<tr>
<td></td>
<td>NAME</td>
<td>SERVICE</td>
<td>TOTAL_RIDE</td>
<td>TOTAL_PBH</td>
</tr>
<tr>
<td>---</td>
<td>-------------------------------------</td>
<td>-----------</td>
<td>------------</td>
<td>-----------</td>
</tr>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SAT</td>
<td>3489</td>
<td>34.7</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SAT</td>
<td>4165</td>
<td>40.2</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SAT</td>
<td>2157</td>
<td>33.4</td>
</tr>
</tbody>
</table>

Table 7 represents the row data of Fixed Routes, which provided services during the Saturday. The line 30 is in the best level in term of total ridership. Route 27 served the most number of passengers per hour in Saturday as well.
The significant feature of Sunday services as it is clear in Table 8 is the lowest number of total rides and total passengers per bus hour compare with weekdays and Saturdays. The best routes in terms of total ridership and total number of passenger are same with Table 7.

Table 8: SUN Fixed Routes Statistics

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TOTAL_RIDE</th>
<th>TOTAL_PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SUN</td>
<td>2443</td>
<td>25.9</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SUN</td>
<td>2774</td>
<td>27.0</td>
</tr>
</tbody>
</table>
The Free Flyers provides services only during the weekdays. In Free Flyer Routes based to the Table 9 greatest number of ridership belongs to the Route 143 but Route 48 commutes the majority of passengers.

Table 9: Free Flyer Statistics

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TOTAL_RIDE</th>
<th>TOTAL_PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>Holt-College Flyer</td>
<td>WK</td>
<td>286</td>
<td>15.5</td>
</tr>
<tr>
<td>43</td>
<td>Whitnall Flyer</td>
<td>WK</td>
<td>236</td>
<td>13.6</td>
</tr>
</tbody>
</table>
Scale Exhibition

This thesis applies Fuzzy AHP model to achieve the goal of evaluating and improving the efficiency. Therefore, the set of fuzzy scaling membership are applied to normalize the scale of different criteria. In this section, "Total Ridership" and "Total Passenger per Bus Hour" are considered as the indicators. It is obvious that the greater amount of these indicators is desirable for the operation system. Therefore equation one from chapter three will be use to normalize the scales. Standard deviation is also calculated from equation 3. All the fuzzy values and the standard deviations indicate as \{\mu_{ij} \mid i = 1, 2, j = 1, 2\} and \{\sigma_j \mid j = 1, 2\} respectively, are listed in tables below:

Table 10 represents the normalized scale data for Fixed Routes during the weekdays. Route 30 and both Routes 21 and 27 are still in the top level of providing the biggest ridership and serving the largest number of commuters respectively.

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TOTAL_RIDE</th>
<th>TOTAL_PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>WK</td>
<td>0.4178</td>
<td>0.5336</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>WK</td>
<td>0.6582</td>
<td>0.7924</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>WK</td>
<td>0.2280</td>
<td>0.5033</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>WK</td>
<td>0.3306</td>
<td>0.4778</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>WK</td>
<td>0.0086</td>
<td>0.2328</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S, 13th &amp; S. 20th</td>
<td>WK</td>
<td>0.5410</td>
<td>0.5827</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>WK</td>
<td>0.5378</td>
<td>0.8882</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>WK</td>
<td>0.2755</td>
<td>0.8421</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>WK</td>
<td>0.4540</td>
<td>0.5930</td>
</tr>
</tbody>
</table>
Normalizing the scale of the criteria should be applied for Saturdays and Sundays as well.

Table 11 shows the related data for Saturdays.

Table 11: Fuzzy Scaling Fixed Routes SAT

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TOTAL_RIDE</th>
<th>TOTAL_PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SAT</td>
<td>0.4056</td>
<td>0.6434</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SAT</td>
<td>0.4842</td>
<td>0.7454</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SAT</td>
<td>0.2508</td>
<td>0.6197</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>SAT</td>
<td>0.3246</td>
<td>0.5529</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>SAT</td>
<td>0.0047</td>
<td>0.1520</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>SAT</td>
<td>0.3081</td>
<td>0.5122</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>SAT</td>
<td>0.4309</td>
<td>0.6903</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>SAT</td>
<td>0.2362</td>
<td>0.7069</td>
</tr>
</tbody>
</table>
Sundays are the least active day of a week. Table 12 shows what are the normalizing scales of the criteria appear on Sundays. Routes 30 and 27 by significant gap with the other routes are in the best place.

Table 12: Fuzzy Scaling Fixed Routes SUN

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TOTAL_RIDE</th>
<th>TOTAL_PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SUN</td>
<td>0.4436</td>
<td>0.6781</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SUN</td>
<td>0.5037</td>
<td>0.7062</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SUN</td>
<td>0.2856</td>
<td>0.6474</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>SUN</td>
<td>0.3312</td>
<td>0.6291</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>SUN</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>Route Description</td>
<td>Day</td>
<td>Flux 1</td>
<td>Flux 2</td>
</tr>
<tr>
<td>---</td>
<td>-----------------------------------------</td>
<td>-----</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>SUN</td>
<td>0.5139</td>
<td>0.5641</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>SUN</td>
<td>0.4019</td>
<td>0.7746</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>SUN</td>
<td>0.2686</td>
<td>0.8154</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>SUN</td>
<td>0.4378</td>
<td>0.7458</td>
</tr>
<tr>
<td>27</td>
<td>27th St.</td>
<td>SUN</td>
<td>0.9424</td>
<td>1.0000</td>
</tr>
<tr>
<td>28</td>
<td>108th St.</td>
<td>SUN</td>
<td>0.0556</td>
<td>0.4346</td>
</tr>
<tr>
<td>30</td>
<td>Sherman-Wisconsin</td>
<td>SUN</td>
<td>1.0000</td>
<td>0.9572</td>
</tr>
<tr>
<td>31</td>
<td>State-Highland</td>
<td>SUN</td>
<td>0.1260</td>
<td>0.3272</td>
</tr>
<tr>
<td>33</td>
<td>Vliet Street</td>
<td>SUN</td>
<td>0.0532</td>
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</tr>
<tr>
<td>35</td>
<td>35th St.</td>
<td>SUN</td>
<td>0.3519</td>
<td>0.6663</td>
</tr>
<tr>
<td>51</td>
<td>Oklahoma Ave.</td>
<td>SUN</td>
<td>0.2068</td>
<td>0.6450</td>
</tr>
<tr>
<td>52</td>
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<td>SUN</td>
<td>0.0439</td>
<td>0.1997</td>
</tr>
<tr>
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<td>SUN</td>
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<td>0.5688</td>
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<tr>
<td>54</td>
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<td>SUN</td>
<td>0.1371</td>
<td>0.5033</td>
</tr>
<tr>
<td>55</td>
<td>Layton Ave.</td>
<td>SUN</td>
<td>0.0819</td>
<td>0.5533</td>
</tr>
<tr>
<td>56</td>
<td>Greenfield Ave.</td>
<td>SUN</td>
<td>0.1790</td>
<td>0.5289</td>
</tr>
<tr>
<td>57</td>
<td>Walnut-N. 92nd</td>
<td>SUN</td>
<td>0.1129</td>
<td>0.3534</td>
</tr>
<tr>
<td>60</td>
<td>Burleigh St.</td>
<td>SUN</td>
<td>0.3165</td>
<td>0.7196</td>
</tr>
<tr>
<td>62</td>
<td>Capitol Dr.</td>
<td>SUN</td>
<td>0.2017</td>
<td>0.8217</td>
</tr>
<tr>
<td>63</td>
<td>Silver Spring-Pt. Washington</td>
<td>SUN</td>
<td>0.2466</td>
<td>0.6524</td>
</tr>
<tr>
<td>64</td>
<td>S. 60th St.</td>
<td>SUN</td>
<td>0.0171</td>
<td>0.2389</td>
</tr>
<tr>
<td>67</td>
<td>N. 76th-S. 84th St.</td>
<td>SUN</td>
<td>0.2379</td>
<td>0.5871</td>
</tr>
<tr>
<td>76</td>
<td>N. 60th-S. 70th</td>
<td>SUN</td>
<td>0.4402</td>
<td>0.6074</td>
</tr>
<tr>
<td>80</td>
<td>6th St.</td>
<td>SUN</td>
<td>0.3782</td>
<td>0.5414</td>
</tr>
<tr>
<td>BLU</td>
<td>Fond du Lac-National MetroExpress</td>
<td>SUN</td>
<td>0.4672</td>
<td>0.8155</td>
</tr>
<tr>
<td>GRN</td>
<td>Oakland-Howell MeterExpress</td>
<td>SUN</td>
<td>0.5804</td>
<td>0.7631</td>
</tr>
<tr>
<td>RED</td>
<td>Capitol Dr. MetroExpress</td>
<td>SUN</td>
<td>0.3497</td>
<td>0.8090</td>
</tr>
<tr>
<td>SP</td>
<td></td>
<td>SUN</td>
<td>0.2368</td>
<td>0.2258</td>
</tr>
</tbody>
</table>

The fuzzy scaled criteria for Free Flyers are contained in Table 13. As it is clear in the table, for all these eight routes, the rate of changes in number of total rides that made by Free Flyer routes, as well as the rate of changes in the quantity of passengers that were carried by the buses in each hour of a day, are small. It might be because these routes are only commute during rush hours daily in the morning and afternoon and provided accessibility to almost popular destination in that specific time of a day.
Table 13: Fuzzy Scaling Free Flyer

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TOTAL_RIDE</th>
<th>TOTAL_PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>Holt-College Flyer</td>
<td>WK</td>
<td>0.5663</td>
<td>0.4985</td>
</tr>
<tr>
<td>43</td>
<td>Whitnall Flyer</td>
<td>WK</td>
<td>0.4673</td>
<td>0.4375</td>
</tr>
<tr>
<td>44</td>
<td>Fair Park-National Flyer</td>
<td>WK</td>
<td>0.4178</td>
<td>0.4741</td>
</tr>
<tr>
<td>46</td>
<td>Loomis-Southridge Flyer</td>
<td>WK</td>
<td>0.4713</td>
<td>0.3793</td>
</tr>
<tr>
<td>48</td>
<td>South Shore Flyer</td>
<td>WK</td>
<td>0.4851</td>
<td>0.6207</td>
</tr>
<tr>
<td>49</td>
<td>Brown Deer-Northshore Flyer</td>
<td>WK</td>
<td>0.6317</td>
<td>0.5166</td>
</tr>
<tr>
<td>79</td>
<td>Menomonee Falls Flyer</td>
<td>WK</td>
<td>0.2950</td>
<td>0.3954</td>
</tr>
<tr>
<td>143</td>
<td>Ozaukee County Express</td>
<td>WK</td>
<td>0.7050</td>
<td>0.4701</td>
</tr>
<tr>
<td>SP</td>
<td></td>
<td></td>
<td>0.1278</td>
<td>0.0759</td>
</tr>
</tbody>
</table>

Construct a Set of Pairwise Comparison Matrixes

After normalizing the scale of criteria, the comparison matrixes in the form of $A = (a_{ij})_{n \times n}$ are achievable by using equation 4 and 5. Comparison matrixes for criteria "Total ridership" and "Total passenger per bus hour" for fixed and free flyer routes are as below respectively:

Fixed Routes; Weekday, Saturday, Sunday $= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, Free Flyer $= \begin{bmatrix} 1 & 2 \\ 0.5 & 1 \end{bmatrix}$

Pairwise Comparison Optimization

Based on Fuzzy AHP model, after a set of pairwise comparison matrixes was obtained, the optimized pairwise matrixes should be calculated by Eq. 6 to 10 from chapter three. For this, section Lingo: Optimization Modeling Software for Linear, Nonlinear and Integer Programming is used to create comparison and optimization matrixes. Comparison matrixes for indicators; Total ridership and Total passenger per bus hour are found as $(y)_{2 \times 2}$ and are described below for both Fixed and Free Flyers routes;

Fixed Routes; Weekday, Saturday, Sunday $= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, Free Flyer $= \begin{bmatrix} 1 & 2 \\ 0.5 & 1 \end{bmatrix}$
**Code Description in Lingo Software**

1. Define variable: description of variable used in the code.

2. Input and output: SP, which is the standard deviation and based on knowledge of AHP, is defined as an input for the code and as a result outputs are:
   - Pairwise comparison matrix which named as a solution
   - Pairwise comparison optimization matrix, named Y
   - Weight, is the standard value of each criterion and it is what AHP model tries to find.

3. $a_m$: The equation was described in chapter 3 about using the pairwise comparison

4. Fuzzy Scaling: Importing the function in chapter 3, finding the pairwise comparison matrix A

5. Min CIC (n): Equation 6 described in chapter 3 about how to weight the priorities.

Figure 4 and Figure 5 are pictures from Lingo software.
Figure 4: Code in Lingo
Figure 5: Result of Applying Lingo

Synthesis

By performing the model with Lingo, weight will be obtained and it is the important point for synthesis. Weight shows which criterion has the most influence on efficiency of bus routes options.

Table 14 represents the weight achieved by applying Lingo 11.0 for both ridership and quantity of passengers in each bus hour. The results show that both these criteria have the same impact on the efficiency of the Fixed Routes in a week.
Table 14: Weight Fixed Routes

<table>
<thead>
<tr>
<th>Fixed Routes WK, SAT, SUN</th>
<th>Total-Ride</th>
<th>Total-PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 15 shows the value of the aforementioned criteria on the efficiency of the Free Flyer routes. As it is understandable from Table 15, Total Ride has almost two times more impact on the efficiency of Free Flyer routes. It might be because the Free Flyers are active for only six hours a week, during the time with the highest public demand for transportation. It means there are always passengers waiting to take the bus for commuting with Free Flyers. Therefore, the more ridership results in the more level of efficiency.

Table 15: Weight Free Flyer Routes

<table>
<thead>
<tr>
<th>Free Flyer-WK</th>
<th>Total-Ride</th>
<th>Total-PBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.66</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Efficiency Ranking for Fixed Routes on Weekday

After obtaining the weight of each criterion, efficiency is calculated by applying Eq. 11. So as a result, efficiency for both types of routes is calculated and represented in tables below:

Table 16 shows the ranking of the efficiency for Fixed Routes on weekdays. The results reviled that Routes 30 and 27 are the most efficient routes based on the selected criteria. Previous sections of this study determined that the Route 30 provided the highest amount of rides in a day and Routes 27, after Route 21 carried the maximum number of passengers in an hour. Therefore, these results make completely sense, as the value of the two criteria is obtained, equal as well.
### Table 16: Efficiency WK Fixed Routes

<table>
<thead>
<tr>
<th>RT</th>
<th>Name</th>
<th>SERVICE</th>
<th>TL_RIDE</th>
<th>TL_PBH</th>
<th>Efficiency</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Sherman-Wisconsin</td>
<td>WK</td>
<td>0.4987</td>
<td>0.3963</td>
<td>0.8951</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>27th St.</td>
<td>WK</td>
<td>0.4449</td>
<td>0.4415</td>
<td>0.8864</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>WK</td>
<td>0.3291</td>
<td>0.3962</td>
<td>0.7253</td>
<td>3</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>WK</td>
<td>0.2689</td>
<td>0.4441</td>
<td>0.7130</td>
<td>4</td>
</tr>
<tr>
<td>BLU</td>
<td>Fond du Lac-National MetroExpress</td>
<td>WK</td>
<td>0.2654</td>
<td>0.3744</td>
<td>0.6399</td>
<td>5</td>
</tr>
<tr>
<td>35</td>
<td>35th St.</td>
<td>WK</td>
<td>0.1795</td>
<td>0.4035</td>
<td>0.5830</td>
<td>6</td>
</tr>
<tr>
<td>19</td>
<td>ML. King-S. 13th &amp; S. 20th</td>
<td>WK</td>
<td>0.2705</td>
<td>0.2913</td>
<td>0.5618</td>
<td>7</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>WK</td>
<td>0.1377</td>
<td>0.4211</td>
<td>0.5588</td>
<td>8</td>
</tr>
<tr>
<td>80</td>
<td>6th St.</td>
<td>WK</td>
<td>0.2537</td>
<td>0.2982</td>
<td>0.5520</td>
<td>9</td>
</tr>
<tr>
<td>63</td>
<td>Silver Spring-Pt. Washington</td>
<td>WK</td>
<td>0.1285</td>
<td>0.4130</td>
<td>0.5415</td>
<td>10</td>
</tr>
<tr>
<td>GRN</td>
<td>Oakland-Howard MeterExpress</td>
<td>WK</td>
<td>0.2516</td>
<td>0.2732</td>
<td>0.5249</td>
<td>11</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>WK</td>
<td>0.2270</td>
<td>0.2965</td>
<td>0.5235</td>
<td>12</td>
</tr>
<tr>
<td>60</td>
<td>Burleigh St.</td>
<td>WK</td>
<td>0.1412</td>
<td>0.3455</td>
<td>0.4867</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
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<td>0.4757</td>
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</tr>
<tr>
<td>RED</td>
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<td>WK</td>
<td>0.1538</td>
<td>0.3062</td>
<td>0.4600</td>
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</tr>
<tr>
<td>67</td>
<td>N. 76th-S. 84th St.</td>
<td>WK</td>
<td>0.1608</td>
<td>0.2960</td>
<td>0.4569</td>
<td>16</td>
</tr>
<tr>
<td>76</td>
<td>N. 60th-S. 70th</td>
<td>WK</td>
<td>0.1915</td>
<td>0.2507</td>
<td>0.4422</td>
<td>17</td>
</tr>
<tr>
<td>62</td>
<td>Capitol Dr.</td>
<td>WK</td>
<td>0.0969</td>
<td>0.3165</td>
<td>0.4134</td>
<td>18</td>
</tr>
<tr>
<td>53</td>
<td>Lincoln Ave.</td>
<td>WK</td>
<td>0.0906</td>
<td>0.3168</td>
<td>0.4074</td>
<td>19</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>WK</td>
<td>0.1653</td>
<td>0.2389</td>
<td>0.4042</td>
<td>20</td>
</tr>
<tr>
<td>54</td>
<td>Mitchell-Burnham</td>
<td>WK</td>
<td>0.0974</td>
<td>0.2994</td>
<td>0.3968</td>
<td>21</td>
</tr>
<tr>
<td>51</td>
<td>Oklahoma Ave.</td>
<td>WK</td>
<td>0.0915</td>
<td>0.2749</td>
<td>0.3664</td>
<td>22</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>WK</td>
<td>0.1140</td>
<td>0.2516</td>
<td>0.3656</td>
<td>23</td>
</tr>
<tr>
<td>56</td>
<td>Greenfield Ave.</td>
<td>WK</td>
<td>0.0680</td>
<td>0.2034</td>
<td>0.2714</td>
<td>24</td>
</tr>
<tr>
<td>55</td>
<td>Layton Ave.</td>
<td>WK</td>
<td>0.0445</td>
<td>0.2246</td>
<td>0.2691</td>
<td>25</td>
</tr>
<tr>
<td>31</td>
<td>State-Highland</td>
<td>WK</td>
<td>0.0714</td>
<td>0.1794</td>
<td>0.2508</td>
<td>26</td>
</tr>
<tr>
<td>57</td>
<td>Walnut-N. 92nd</td>
<td>WK</td>
<td>0.0608</td>
<td>0.1881</td>
<td>0.2489</td>
<td>27</td>
</tr>
<tr>
<td>33</td>
<td>Vliet Street</td>
<td>WK</td>
<td>0.0271</td>
<td>0.1565</td>
<td>0.1837</td>
<td>28</td>
</tr>
<tr>
<td>28</td>
<td>108th St.</td>
<td>WK</td>
<td>0.0281</td>
<td>0.1477</td>
<td>0.1759</td>
<td>29</td>
</tr>
<tr>
<td>64</td>
<td>S. 60th St.</td>
<td>WK</td>
<td>0.0136</td>
<td>0.1131</td>
<td>0.1267</td>
<td>30</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>WK</td>
<td>0.0043</td>
<td>0.1164</td>
<td>0.1207</td>
<td>31</td>
</tr>
<tr>
<td>52</td>
<td>Clement-15th Ave.</td>
<td>WK</td>
<td>0.0151</td>
<td>0.1039</td>
<td>0.1189</td>
<td>32</td>
</tr>
<tr>
<td>223</td>
<td>Park Place-Bradley Woods Shuttle</td>
<td>WK</td>
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<td>0.0560</td>
<td>0.0593</td>
<td>33</td>
</tr>
<tr>
<td>219</td>
<td>Oak Creek Shuttle</td>
<td>WK</td>
<td>0.0013</td>
<td>0.0559</td>
<td>0.0571</td>
<td>34</td>
</tr>
</tbody>
</table>
Figure 6 shows the graph obtained from assessing the daily efficiency for the Fixed Routes, in this research. The significant feature of this graph is its ability to rank the routes based on both ridership and number of passenger per hour.

Figure 6: Efficiency based on Total Ridership & Total PBH - Fixed Routes WK

Figure 7 represents the ranking for the Fixed Routes based on the results of the MCTS report. This graph considered the ridership as the only main criteria for decision-making process. Comparing the graph on Figure 7 and Figure 6 unveiled that the first three efficient routes are common. The Routes 30 and 27 are in a close competition for best level of efficiency.
Figure 8 shows the ranking based on the productivity (passenger per bus in an hour) which is results of the MCTS report. The close competition between productivity of the routes is obvious from the graph. The first five most efficient routes in terms of productivity, is comparable with the results of this study. Routes 21 and 27, which are ranked as the most efficient routes in Figure 8, respectively belong to the rank of four and two of the Table 16.
Efficiency Ranking for Fixed Routes on Saturday

Table 17 represents the ranking for efficiency of Fixed Routes on Saturdays. Routes 30 and 27 are the most efficient routes. Route 30 provided the highest amount of rides in a day and Route 27 carries the maximum numbers of passengers in each hour. The first three top ranking routes on Saturdays are the same with weekdays.

Table 17: Efficiency SAT Fixed Routes

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TL_RIDE</th>
<th>TL_PBH</th>
<th>Efficiency</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Sherman-Wisconsin</td>
<td>SAT</td>
<td>0.4977</td>
<td>0.3891</td>
<td>0.8867</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>27th St.</td>
<td>SAT</td>
<td>0.4491</td>
<td>0.4240</td>
<td>0.8731</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SAT</td>
<td>0.2421</td>
<td>0.3727</td>
<td>0.6148</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Fond du Lac-National MetroExpress</td>
<td>SAT</td>
<td>0.2444</td>
<td>0.3693</td>
<td>0.6137</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Oakland-Howell MeterExpress</td>
<td>SAT</td>
<td>0.2719</td>
<td>0.3156</td>
<td>0.5875</td>
<td>5</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>SAT</td>
<td>0.2155</td>
<td>0.3452</td>
<td>0.5606</td>
<td>6</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>SAT</td>
<td>0.2177</td>
<td>0.3109</td>
<td>0.5287</td>
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</tr>
<tr>
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<td>0.1193</td>
<td>0.4063</td>
<td>0.5256</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
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<td>0.2028</td>
<td>0.3217</td>
<td>0.5245</td>
<td>9</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
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<td>0.5102</td>
<td>10</td>
</tr>
<tr>
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<td>0.3508</td>
<td>0.4948</td>
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</tr>
<tr>
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<td>0.4716</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.4465</td>
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</tr>
<tr>
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<td>0.2765</td>
<td>0.4388</td>
<td>17</td>
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<tr>
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<td>18</td>
</tr>
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<td>0.3096</td>
<td>0.4316</td>
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<tr>
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<td>Oklahoma Ave.</td>
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<td>0.4063</td>
<td>20</td>
</tr>
<tr>
<td>67</td>
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<td>0.2913</td>
<td>0.3946</td>
<td>21</td>
</tr>
<tr>
<td>54</td>
<td>Mitchell-Burnham</td>
<td>SAT</td>
<td>0.0624</td>
<td>0.2644</td>
<td>0.3268</td>
<td>22</td>
</tr>
<tr>
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<td>Greenfield Ave.</td>
<td>SAT</td>
<td>0.0872</td>
<td>0.2231</td>
<td>0.3103</td>
<td>23</td>
</tr>
<tr>
<td>53</td>
<td>Lincoln Ave.</td>
<td>SAT</td>
<td>0.0553</td>
<td>0.2497</td>
<td>0.3050</td>
<td>24</td>
</tr>
<tr>
<td>55</td>
<td>Layton Ave.</td>
<td>SAT</td>
<td>0.0458</td>
<td>0.1852</td>
<td>0.2310</td>
<td>25</td>
</tr>
<tr>
<td>28</td>
<td>108th St.</td>
<td>SAT</td>
<td>0.0250</td>
<td>0.1891</td>
<td>0.2140</td>
<td>26</td>
</tr>
<tr>
<td>31</td>
<td>State-Highland</td>
<td>SAT</td>
<td>0.0557</td>
<td>0.1575</td>
<td>0.2132</td>
<td>27</td>
</tr>
<tr>
<td>57</td>
<td>Walnut-N. 92nd</td>
<td>SAT</td>
<td>0.0443</td>
<td>0.1537</td>
<td>0.1980</td>
<td>28</td>
</tr>
<tr>
<td>33</td>
<td>Vliet Street</td>
<td>SAT</td>
<td>0.0249</td>
<td>0.1097</td>
<td>0.1346</td>
<td>29</td>
</tr>
<tr>
<td>137</td>
<td>House of Correction</td>
<td>SAT</td>
<td>0.0028</td>
<td>0.1012</td>
<td>0.1040</td>
<td>30</td>
</tr>
</tbody>
</table>
Figure 9 represents a graph of ranking for Saturday efficiency. From the graph, it is clear that efficiency of the Routes 30 and 27 are on the top and very close to each other. There is a significant difference in the level of efficiency from Route 12 to the rest of the graph, with the routes in the top.

Figure 9: Efficiency based on Total Ridership & Total PBH - Fixed Routes SAT

Figure 10 shows efficiency of the routes based on the ridership, derived from MCTS annual report. It is obvious the two top ranking routes are the same with the results of Figure 9 and the gap between efficiency of the first two routes and other routes is great in this graph too. For the top five routes, the results do not have great diversity with the results of this thesis. The point is however, the sequences of the routes are not completely similar in Table of Ranking they are still show very similar trend.
Figure 10: Efficiency based on Ridership. Fixed Routes SAT - Source MCTS 2013 Report

Figure 11 is a graph that ranked the routes based on the number of passengers commute with buses in every bus hour. This graph is come from the MCTS report and shows very close competition between routes. The Route 27 and 30 are in the first three top levels of the most efficient routes in graphs of Figure 9, Figure 10 and Figure 11. The noticeable point is the Route 62, which in the graph below is in the second place of the highest efficient routes, is ranked under the place of eight and eighteen on the graphs of Figure 9 and Figure 10 respectively.

Figure 11: Efficiency Based on PBH. Fixed Routes SAT - Source MCTS 2013 Report
Efficiency Ranking for Fixed Routes on Sunday

Table 18 represents the ranking for efficiency of Fixed Routes on Sunday, based on the aforementioned criteria. It was expectable that Routes 30 and 27 to be the routes with highest level of efficiency. These two routes are in very close competition in terms of efficiency and it is obvious there is a significant gap between route in second and the route in third level of efficiency. Other noticeable point is the Routes 30 and 27 are the most efficient routes for daily services for both weekdays and weekends. From the level third to the fifth, which belongs to the Routes GRN, BLU and 12, there is not considerable difference in amount of efficiency for Table 16, and Table 17 and Table 18.

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TL_RIDE</th>
<th>TL_PBH</th>
<th>Efficiency</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>Sherman-Wisconsin</td>
<td>SUN</td>
<td>0.5000</td>
<td>0.4786</td>
<td>0.9786</td>
<td>1</td>
</tr>
<tr>
<td>27</td>
<td>27th St.</td>
<td>SUN</td>
<td>0.4712</td>
<td>0.5000</td>
<td>0.9712</td>
<td>2</td>
</tr>
<tr>
<td>GRN</td>
<td>Oakland-Howell MeterEXPress</td>
<td>SUN</td>
<td>0.2902</td>
<td>0.3816</td>
<td>0.6717</td>
<td>3</td>
</tr>
<tr>
<td>BLU</td>
<td>Fond du Lac-National MetroEXpress</td>
<td>SUN</td>
<td>0.2336</td>
<td>0.4078</td>
<td>0.6414</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SUN</td>
<td>0.2519</td>
<td>0.3531</td>
<td>0.6050</td>
<td>5</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>SUN</td>
<td>0.2189</td>
<td>0.3729</td>
<td>0.5918</td>
<td>6</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>SUN</td>
<td>0.2099</td>
<td>0.3873</td>
<td>0.5882</td>
<td>7</td>
</tr>
<tr>
<td>RED</td>
<td>Capitol Dr. MetroEXpress</td>
<td>SUN</td>
<td>0.1749</td>
<td>0.4045</td>
<td>0.5794</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SUN</td>
<td>0.2218</td>
<td>0.3391</td>
<td>0.5609</td>
<td>9</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>SUN</td>
<td>0.1343</td>
<td>0.4077</td>
<td>0.5420</td>
<td>10</td>
</tr>
<tr>
<td>19</td>
<td>ML. King-S. 13th &amp; S. 20th</td>
<td>SUN</td>
<td>0.2569</td>
<td>0.2820</td>
<td>0.5390</td>
<td>11</td>
</tr>
<tr>
<td>76</td>
<td>N. 60th-S. 70th</td>
<td>SUN</td>
<td>0.2201</td>
<td>0.3037</td>
<td>0.5238</td>
<td>12</td>
</tr>
<tr>
<td>60</td>
<td>Burleigh St.</td>
<td>SUN</td>
<td>0.1583</td>
<td>0.3598</td>
<td>0.5181</td>
<td>13</td>
</tr>
<tr>
<td>62</td>
<td>Capitol Dr.</td>
<td>SUN</td>
<td>0.1009</td>
<td>0.4109</td>
<td>0.5117</td>
<td>14</td>
</tr>
<tr>
<td>35</td>
<td>35th St.</td>
<td>SUN</td>
<td>0.1760</td>
<td>0.3331</td>
<td>0.5091</td>
<td>15</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>SUN</td>
<td>0.1656</td>
<td>0.3146</td>
<td>0.4802</td>
<td>16</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SUN</td>
<td>0.1428</td>
<td>0.3237</td>
<td>0.4665</td>
<td>17</td>
</tr>
<tr>
<td>80</td>
<td>6th St.</td>
<td>SUN</td>
<td>0.1891</td>
<td>0.2707</td>
<td>0.4598</td>
<td>18</td>
</tr>
<tr>
<td>63</td>
<td>Silver Spring-Pt. Washington</td>
<td>SUN</td>
<td>0.1233</td>
<td>0.3262</td>
<td>0.4495</td>
<td>19</td>
</tr>
<tr>
<td>51</td>
<td>Oklahoma Ave.</td>
<td>SUN</td>
<td>0.1034</td>
<td>0.3225</td>
<td>0.4259</td>
<td>20</td>
</tr>
<tr>
<td>67</td>
<td>N. 76th-S. 84th St.</td>
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<td>0.2935</td>
<td>0.4125</td>
<td>21</td>
</tr>
<tr>
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<td>0.0702</td>
<td>0.2844</td>
<td>0.3546</td>
<td>22</td>
</tr>
<tr>
<td>56</td>
<td>Greenfield Ave.</td>
<td>SUN</td>
<td>0.0895</td>
<td>0.2645</td>
<td>0.3540</td>
<td>23</td>
</tr>
</tbody>
</table>
Figure 12 represents the graph of efficiency for routes served on Sundays. This graph reveal that the first three most efficient routes, which are achieved in this study is the same with the most efficient routes in terms of ridership, which is showed in Figure 13 derived from MCTS annual report.

Figure 12: Efficiency based on Total Ridership & Total PBH - Fixed Routes SUN

Figure 13 shows efficiency in terms of ridership for fixed buses on Sunday. This graph derived from MCTS report. Comparing the results of this graph with the results was achieved by the graph of Figure 12, revealed close similarity in efficiency ranking of the routes. In addition, the rate of changes in both graphs is in the close equality.
Figure 13: Efficiency based on Ridership. Fixed Routes SUN - Source MCTS 2013 Report

Figure 14 represents the efficiency based on the productivity and comes from MCTS report. The Routes 27, 30, 12 and BLU are in the top five of the best efficient routes, both in this graph and in the graph in Figure 12. Route 62, which based on the productivity is claimed to be the second route with the highest level of efficiency, belongs to the rank of fourteenth and twenty-first of the graphs in Figure 12 and Figure 13.

Figure 14: Efficiency Based on PBH. Fixed Routes SUN - Source MCTS 2013 Report
Efficiency Ranking of Free Flyer Routes

Table 19 shows the rank of the efficiency for Free Flyer Routes. It is clear that, in Free Flyer Routes unlike the Fixed Routes, there is small rate of efficiency changes. The amplitude of changes is between 0.62 and 0.44, which shows the efficiency of the Free Flyers Routes is almost close to each other.

Table 19: Efficiency Free Flyer Routes

<table>
<thead>
<tr>
<th>RT</th>
<th>Name</th>
<th>Service</th>
<th>TL- Ride</th>
<th>TL_PBH</th>
<th>Efficiency</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>143</td>
<td>Ozaukee County Express</td>
<td>WK</td>
<td>0.4699</td>
<td>0.1566</td>
<td>0.6266</td>
<td>1</td>
</tr>
<tr>
<td>49</td>
<td>Brown Deer-Northshore Flyer</td>
<td>WK</td>
<td>0.4211</td>
<td>0.1721</td>
<td>0.5933</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
<td>Holt-College Flyer</td>
<td>WK</td>
<td>0.3775</td>
<td>0.1661</td>
<td>0.5437</td>
<td>3</td>
</tr>
<tr>
<td>48</td>
<td>South Shore Flyer</td>
<td>WK</td>
<td>0.3234</td>
<td>0.2068</td>
<td>0.5303</td>
<td>4</td>
</tr>
<tr>
<td>43</td>
<td>Whitnall Flyer</td>
<td>WK</td>
<td>0.3115</td>
<td>0.1458</td>
<td>0.4573</td>
<td>5</td>
</tr>
<tr>
<td>46</td>
<td>Loomis-Southridge Flyer</td>
<td>WK</td>
<td>0.3141</td>
<td>0.1264</td>
<td>0.4406</td>
<td>6</td>
</tr>
<tr>
<td>44</td>
<td>Fair Park-National Flyer</td>
<td>WK</td>
<td>0.2785</td>
<td>0.1580</td>
<td>0.4365</td>
<td>7</td>
</tr>
<tr>
<td>79</td>
<td>Menomonee Falls Flyer</td>
<td>WK</td>
<td>0.1967</td>
<td>0.1317</td>
<td>0.3284</td>
<td>8</td>
</tr>
</tbody>
</table>

Figure 15 represented the efficiency of Free Flyers based on the ridership and the number of passengers per bus hour. It is obvious that the results of this graph are very close to the results of Figure 16, which represents the efficiency in terms of ridership. The Route 143 is the most efficient routes among the Free Flyers Routes.
Figure 15: Efficiency based on Total Ridership & Total PBH - Free Flyer Routes

Figure 16 shows the efficiency of Free Flyer Routes in terms of ridership. This graph is from the MCTS report. Number of rides varied from 150 to 350 on the weekdays. The results are similar with the ranking was represented in Figure 15. The reason of this similarity might be due to the greater impact of ridership in the final results of graph in Figure 15, which represented efficiency by considering both ridership and number of passengers per bus hour.

Figure 16: Efficiency based on Ridership. Free Flyer Routes - Source MCTS 2013 Report
Figure 17 shows the efficiency based on the productivity of Free Flyer Routes. The weight of productivity, which was achieved in previous sections, was almost half of the weight of ridership and it is the reason of incompatibility between the results of Figure 17, which only focused on productivity, and the results of Figure 16, which includes both ridership and productivity together.

![Freeway Flyer Service](image)

Figure 17: Efficiency based on PBH. Free Flyer Routes - Source MCTS 2013 Report

5.4. Evaluating the Efficiency during the Peak Hours

Another part of this thesis is evaluating the efficiency for Fixed Routes and Free Flyer Routes during the rush hours. This thesis chooses the peak hours because the highest demand for travelling belongs to these times, which results in traffic jams, increasing the travel times and delays. Providing reasonable suggestions for improving the efficiency during the peak hours will result in more satisfaction with the society.

Selecting Different Criteria
In this section, criteria have been changed to make the results more accurate. The result of section 4.1.3 showed the important impact of ridership on the efficiency. Furthermore, in previous sections it was clear the rate of changes for total ridership is big. Therefore, even small changes in ridership would result in big changes in efficiency. Because of that, this section seeks to find number of rides made with each bus and average number of rides in an hour, to achieve this results criteria are selected as (Number of Rides/Number of Bus), (PBH on peak hours = Number of Rides/Bus hours). Productivity Frequently Index (PFI) = PBH * Headway / 60 is another selected criteria for this part, which shows the importance of headway between buses as well as productivity.

Statistics of Bus Routes during the Rush Hours

It should be noticed the categories of the buses are the same with section 4.2.1. Tables below contain the row data of the selected criteria.

Table 20 shows the statics for Fixed Routes on weekdays during rush hours. Table reveals that the Route 63 is in the first place of the largest number of rides for each bus, number of rides in each bus hour (PBH) and for the productivity frequently index. Compare with the results of Table 16 the Route 63 was in the tenth place of daily efficiency ranking.

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TL-Rides/PM-Bus</th>
<th>TL-Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>WK</td>
<td>117.5287</td>
<td>39.1762</td>
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<td>12</td>
<td>Teutonia-Hampton</td>
<td>WK</td>
<td>180.1625</td>
<td>60.0542</td>
<td>11.0099</td>
</tr>
<tr>
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<td>Forest Home</td>
<td>WK</td>
<td>126.8065</td>
<td>42.2688</td>
<td>14.7941</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>WK</td>
<td>112.2500</td>
<td>37.4167</td>
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</tr>
<tr>
<td>17</td>
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<td>WK</td>
<td>17.2500</td>
<td>5.7500</td>
<td>0.9583</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>WK</td>
<td>132.9769</td>
<td>44.3256</td>
<td>11.0814</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>WK</td>
<td>174.4105</td>
<td>58.1368</td>
<td>11.6274</td>
</tr>
</tbody>
</table>
Table 21 represents the data for Fixed Routes on Saturday during the rush hours. From the data, it is obvious that the Route BLU has the greatest number of riders per bus and the greatest quantity of PFI. Route 62 has the most number of rides per bus hour. Comparing this table with Table 17 showed the Routes BLU and 62 were respectively in the levels of fourth and eighth of Saturday daily efficiency ranking.

Table 21: Statics for Fixed Routes - SAT - Rush Hours

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TL-Rides/PM-Bus</th>
<th>TL-Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SAT</td>
<td>125.1667</td>
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</tbody>
</table>

54
<table>
<thead>
<tr>
<th></th>
<th>Route Description</th>
<th>SAT</th>
<th>BLU</th>
<th>GRN</th>
<th>RED</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SAT</td>
<td>112.0000</td>
<td>37.3333</td>
<td>11.2000</td>
</tr>
<tr>
<td>14</td>
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<td>SAT</td>
<td>117.6536</td>
<td>39.2179</td>
<td>19.6089</td>
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<td>15</td>
<td>Holton-Kinnickinnic</td>
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<td>18.6633</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
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<td>ML King-S. 13th &amp; S. 20th</td>
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<td>93.5556</td>
<td>31.1852</td>
<td>9.8753</td>
</tr>
<tr>
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<td>43.2222</td>
<td>13.6870</td>
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<tr>
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<td>SAT</td>
<td>136.0000</td>
<td>45.3333</td>
<td>17.3778</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>SAT</td>
<td>116.0368</td>
<td>38.6789</td>
<td>18.6948</td>
</tr>
<tr>
<td>27</td>
<td>27th St.</td>
<td>SAT</td>
<td>148.6047</td>
<td>49.5349</td>
<td>11.5581</td>
</tr>
<tr>
<td>28</td>
<td>108th St.</td>
<td>SAT</td>
<td>58.5000</td>
<td>19.5000</td>
<td>17.2250</td>
</tr>
<tr>
<td>30</td>
<td>Sherman-Wisconsin</td>
<td>SAT</td>
<td>137.0833</td>
<td>45.6944</td>
<td>9.1389</td>
</tr>
<tr>
<td>31</td>
<td>State-Highland</td>
<td>SAT</td>
<td>58.2500</td>
<td>19.4167</td>
<td>8.4139</td>
</tr>
<tr>
<td>33</td>
<td>Vliet Street</td>
<td>SAT</td>
<td>40.0000</td>
<td>13.3333</td>
<td>6.6667</td>
</tr>
<tr>
<td>35</td>
<td>35th St.</td>
<td>SAT</td>
<td>109.2000</td>
<td>36.4000</td>
<td>13.3467</td>
</tr>
<tr>
<td>51</td>
<td>Oklahoma Ave.</td>
<td>SAT</td>
<td>131.7791</td>
<td>43.9264</td>
<td>21.2311</td>
</tr>
<tr>
<td>52</td>
<td>Clement-15th Ave.</td>
<td>SAT</td>
<td>34.0000</td>
<td>11.3333</td>
<td>7.9333</td>
</tr>
<tr>
<td>53</td>
<td>Lincoln Ave.</td>
<td>SAT</td>
<td>82.0000</td>
<td>27.3333</td>
<td>18.6778</td>
</tr>
<tr>
<td>54</td>
<td>Mitchell-Burnham</td>
<td>SAT</td>
<td>117.5000</td>
<td>39.1667</td>
<td>29.3750</td>
</tr>
<tr>
<td>55</td>
<td>Layton Ave.</td>
<td>SAT</td>
<td>62.5191</td>
<td>20.8397</td>
<td>10.7672</td>
</tr>
<tr>
<td>56</td>
<td>Greenfield Ave.</td>
<td>SAT</td>
<td>90.5660</td>
<td>30.1887</td>
<td>12.5786</td>
</tr>
<tr>
<td>57</td>
<td>Walnut-N. 92nd</td>
<td>SAT</td>
<td>60.0000</td>
<td>20.0000</td>
<td>11.0000</td>
</tr>
<tr>
<td>60</td>
<td>Burleigh St.</td>
<td>SAT</td>
<td>125.7500</td>
<td>41.9167</td>
<td>16.0681</td>
</tr>
<tr>
<td>62</td>
<td>Capitol Dr.</td>
<td>SAT</td>
<td>169.0000</td>
<td>56.3333</td>
<td>20.6556</td>
</tr>
<tr>
<td>63</td>
<td>Silver Spring-Pt. Washington</td>
<td>SAT</td>
<td>120.2500</td>
<td>40.0833</td>
<td>16.0333</td>
</tr>
<tr>
<td>64</td>
<td>S. 60th St.</td>
<td>SAT</td>
<td>26.3415</td>
<td>8.7805</td>
<td>8.6341</td>
</tr>
<tr>
<td>67</td>
<td>N. 76th-S. 84th St.</td>
<td>SAT</td>
<td>112.0000</td>
<td>37.3333</td>
<td>20.5333</td>
</tr>
<tr>
<td>76</td>
<td>N. 60th-S. 70th</td>
<td>SAT</td>
<td>79.4118</td>
<td>26.4706</td>
<td>9.2647</td>
</tr>
<tr>
<td>80</td>
<td>6th St.</td>
<td>SAT</td>
<td>91.9364</td>
<td>30.6455</td>
<td>11.7474</td>
</tr>
<tr>
<td>137</td>
<td>House of Correction</td>
<td>SAT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BLU</td>
<td>Fond du Lac-National MetroExpress</td>
<td>SAT</td>
<td>144.4464</td>
<td>48.1488</td>
<td>39.3215</td>
</tr>
<tr>
<td>GRN</td>
<td>Oakland-Howell MeterExpress</td>
<td>SAT</td>
<td>130.7500</td>
<td>43.5833</td>
<td>13.8014</td>
</tr>
<tr>
<td>RED</td>
<td>Capitol Dr. MetroExpress</td>
<td>SAT</td>
<td>129.7500</td>
<td>43.2500</td>
<td>19.4625</td>
</tr>
</tbody>
</table>

Table 22 represents the statics for Fixed Routes on Sunday during the rush hours. The amount of these three criteria on Sundays, compare with Table 20 and Table 21, shows the significant decrease in ridership and the number of passengers per bus. Furthermore, Table 22 revealed that the maximum number of rides per bus and the greatest number of rides per bus hour belongs to Routes 62 and 30. The quantity of PFI is the maximum for Route BLU. Comparing the results with Table 18, about the daily efficiency ranking on Sundays, it was
determined that Routes 30, 62 and BLU are under levels of first, fourteenth and fifth respectively.

Table 22: Statics for Fixed Routes - SUN - Rush Hours

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>Rides/PM-Bus</th>
<th>Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SUN</td>
<td>85.1667</td>
<td>28.3889</td>
<td>13.7213</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SUN</td>
<td>89.7143</td>
<td>29.9048</td>
<td>8.4730</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SUN</td>
<td>96.9635</td>
<td>32.3212</td>
<td>15.6219</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kimickinnic</td>
<td>SUN</td>
<td>85.0000</td>
<td>28.3333</td>
<td>20.7778</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>SUN</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>SUN</td>
<td>65.0000</td>
<td>21.6667</td>
<td>6.8611</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>SUN</td>
<td>105.6000</td>
<td>35.2000</td>
<td>11.7333</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>SUN</td>
<td>92.6667</td>
<td>30.8889</td>
<td>11.3259</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>SUN</td>
<td>101.0000</td>
<td>33.6667</td>
<td>20.7611</td>
</tr>
<tr>
<td>27</td>
<td>27th St.</td>
<td>SUN</td>
<td>109.4026</td>
<td>36.4675</td>
<td>10.3325</td>
</tr>
<tr>
<td>28</td>
<td>108th St.</td>
<td>SUN</td>
<td>48.0000</td>
<td>16.0000</td>
<td>14.4000</td>
</tr>
<tr>
<td>30</td>
<td>Sherman-Wisconsin</td>
<td>SUN</td>
<td>121.0000</td>
<td>40.3333</td>
<td>9.4111</td>
</tr>
<tr>
<td>31</td>
<td>State-Highland</td>
<td>SUN</td>
<td>42.7500</td>
<td>14.2500</td>
<td>5.9375</td>
</tr>
<tr>
<td>33</td>
<td>Vliet Street</td>
<td>SUN</td>
<td>30.5000</td>
<td>10.1667</td>
<td>4.9139</td>
</tr>
<tr>
<td>35</td>
<td>35th St.</td>
<td>SUN</td>
<td>85.0000</td>
<td>28.3333</td>
<td>9.9167</td>
</tr>
<tr>
<td>51</td>
<td>Oklahoma Ave.</td>
<td>SUN</td>
<td>84.4693</td>
<td>28.1564</td>
<td>13.1397</td>
</tr>
<tr>
<td>52</td>
<td>Clement-15th Ave.</td>
<td>SUN</td>
<td>29.0000</td>
<td>9.6667</td>
<td>6.7667</td>
</tr>
<tr>
<td>53</td>
<td>Lincoln Ave.</td>
<td>SUN</td>
<td>65.5000</td>
<td>21.8333</td>
<td>14.5556</td>
</tr>
<tr>
<td>54</td>
<td>Mitchell-Burnham</td>
<td>SUN</td>
<td>77.0000</td>
<td>25.6667</td>
<td>19.6778</td>
</tr>
<tr>
<td>55</td>
<td>Layton Ave.</td>
<td>SUN</td>
<td>70.5000</td>
<td>23.5000</td>
<td>18.0167</td>
</tr>
<tr>
<td>56</td>
<td>Greenfield Ave.</td>
<td>SUN</td>
<td>62.6667</td>
<td>20.8889</td>
<td>11.4889</td>
</tr>
<tr>
<td>57</td>
<td>Walnut-N. 92nd</td>
<td>SUN</td>
<td>46.0000</td>
<td>15.3333</td>
<td>8.1778</td>
</tr>
<tr>
<td>60</td>
<td>Burleigh St.</td>
<td>SUN</td>
<td>103.2500</td>
<td>34.4167</td>
<td>13.1931</td>
</tr>
<tr>
<td>62</td>
<td>Capitol Dr.</td>
<td>SUN</td>
<td>121.5000</td>
<td>40.5000</td>
<td>20.9250</td>
</tr>
<tr>
<td>63</td>
<td>Silver Spring-Pt. Washington</td>
<td>SUN</td>
<td>72.1541</td>
<td>24.0514</td>
<td>10.4223</td>
</tr>
<tr>
<td>64</td>
<td>S. 60th St.</td>
<td>SUN</td>
<td>24.6701</td>
<td>8.2234</td>
<td>8.0863</td>
</tr>
<tr>
<td>67</td>
<td>N. 76th St.- 84th St.</td>
<td>SUN</td>
<td>75.7500</td>
<td>25.2500</td>
<td>13.8875</td>
</tr>
<tr>
<td>76</td>
<td>N. 60th-S. 70th</td>
<td>SUN</td>
<td>73.4351</td>
<td>24.4784</td>
<td>10.1993</td>
</tr>
<tr>
<td>80</td>
<td>6th St.</td>
<td>SUN</td>
<td>70.8333</td>
<td>23.6111</td>
<td>8.6574</td>
</tr>
<tr>
<td>BLU</td>
<td>Fond du Lac-National MetroEXP</td>
<td>SUN</td>
<td>111.4000</td>
<td>37.1333</td>
<td>22.8989</td>
</tr>
<tr>
<td>GRN</td>
<td>Oakland-Howell MeterEXP</td>
<td>SUN</td>
<td>114.1667</td>
<td>38.0556</td>
<td>15.8565</td>
</tr>
<tr>
<td>RED</td>
<td>Capitol Dr. MetroEXP</td>
<td>SUN</td>
<td>103.0000</td>
<td>34.3333</td>
<td>14.3056</td>
</tr>
</tbody>
</table>
Table 23 represents the statics for Free Flyer Routes during rush hours. Rate of changes in quantity of selected criteria continue to be very small, like what it was in Table 9 for daily efficiency. The Route 48 has the maximum quantity for rides per bus and rides per bus hour; the greatest amount of PFI is for Route 49. Making a comparison by Table 19 revealed that Routes 48 and 49 respectively are in the level of fourth and second for daily evaluation of efficiency.

Table 23: Statics for Free Flyer Routes - WK - Rush Hours

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TL-Rides/PM-Bus</th>
<th>TL-Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>Holt-College Flyer</td>
<td>WK</td>
<td>46.4516</td>
<td>15.4839</td>
<td>2.8387</td>
</tr>
<tr>
<td>43</td>
<td>Whitnall Flyer</td>
<td>WK</td>
<td>33.4954</td>
<td>11.1651</td>
<td>1.6748</td>
</tr>
<tr>
<td>44</td>
<td>Fair Park-National Flyer</td>
<td>WK</td>
<td>39.9523</td>
<td>13.3174</td>
<td>1.7757</td>
</tr>
<tr>
<td>46</td>
<td>Loomis-Southridge Flyer</td>
<td>WK</td>
<td>32.8507</td>
<td>10.9502</td>
<td>2.3725</td>
</tr>
<tr>
<td>48</td>
<td>South Shore Flyer</td>
<td>WK</td>
<td>56.9959</td>
<td>18.9986</td>
<td>2.2165</td>
</tr>
<tr>
<td>49</td>
<td>Brown Deer-Northshore Flyer</td>
<td>WK</td>
<td>44.5447</td>
<td>14.8482</td>
<td>3.2171</td>
</tr>
<tr>
<td>79</td>
<td>Menomonee Falls Flyer</td>
<td>WK</td>
<td>35.7410</td>
<td>11.9137</td>
<td>1.1914</td>
</tr>
<tr>
<td>143</td>
<td>Ozaukee County Express</td>
<td>WK</td>
<td>40.1717</td>
<td>13.3906</td>
<td>2.0086</td>
</tr>
</tbody>
</table>

Scale Exhibition

Tables below represent the data after normalizing the scales of bus routes involved in this study. Table 24 shows the fuzzy scaling for the routes on weekdays. As it was expected, Route 63 has the largest amount of scaling between other routes.

Table 24: Fuzzy Scaling Fixed Routes - WK - Rush Hours

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TL-Rides/PM-Bus</th>
<th>TL-Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>WK</td>
<td>0.5057</td>
<td>0.5057</td>
<td>0.3056</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>WK</td>
<td>0.7752</td>
<td>0.7752</td>
<td>0.3435</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>WK</td>
<td>0.5456</td>
<td>0.5456</td>
<td>0.4616</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kimmickinnic</td>
<td>WK</td>
<td>0.4830</td>
<td>0.4830</td>
<td>0.4475</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>WK</td>
<td>0.0742</td>
<td>0.0742</td>
<td>0.0299</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>WK</td>
<td>0.5721</td>
<td>0.5721</td>
<td>0.3458</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>WK</td>
<td>0.7504</td>
<td>0.7504</td>
<td>0.3628</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>WK</td>
<td>0.8501</td>
<td>0.8501</td>
<td>0.5138</td>
</tr>
</tbody>
</table>
Table 25 represents fuzzy scaling for Fixed Routes on Saturdays. The Routes BLU and 62 are in the same rank with what it was explained in the Table 21.

Table 25: Fuzzy Scaling Fixed Routes - SAT - Rush Hours

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>TL-Rides/PM-Bus</th>
<th>TL-Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SAT</td>
<td>0.7406</td>
<td>0.7406</td>
<td>0.5482</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SAT</td>
<td>0.6627</td>
<td>0.6627</td>
<td>0.2848</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SAT</td>
<td>0.6962</td>
<td>0.6962</td>
<td>0.4987</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>SAT</td>
<td>0.6024</td>
<td>0.6024</td>
<td>0.4746</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>SAT</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>SAT</td>
<td>0.5536</td>
<td>0.5536</td>
<td>0.2511</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>SAT</td>
<td>0.7673</td>
<td>0.7673</td>
<td>0.3481</td>
</tr>
<tr>
<td>22</td>
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<td>SAT</td>
<td>0.8047</td>
<td>0.8047</td>
<td>0.4419</td>
</tr>
<tr>
<td>RT</td>
<td>NAME</td>
<td>SERVICE</td>
<td>TL-Rides/PM-Bus</td>
<td>TL-Rides/PM-BH</td>
<td>PFI</td>
</tr>
<tr>
<td>----</td>
<td>-------------------------------</td>
<td>---------</td>
<td>------------------</td>
<td>----------------</td>
<td>-------</td>
</tr>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SUN</td>
<td>0.7010</td>
<td>0.7010</td>
<td>0.5992</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>SUN</td>
<td>0.7384</td>
<td>0.7384</td>
<td>0.3700</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SUN</td>
<td>0.7981</td>
<td>0.7981</td>
<td>0.6822</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>SUN</td>
<td>0.6996</td>
<td>0.6996</td>
<td>0.9074</td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>SUN</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>SUN</td>
<td>0.5350</td>
<td>0.5350</td>
<td>0.2996</td>
</tr>
</tbody>
</table>

Table 26 includes the data of fuzzy scaling for Fixed Routes during rush hours on Sundays. Like Table 22 the greatest amount of the criteria in fuzzy scaling belongs to the Routes 30 and BLU.

Table 26: Fuzzy Scaling Fixed Routes - SUN - Rush Hours
Table 27 is for fuzzy scaling of Free Flyer services during the rush hours. Routes 48 and 49 have the maximum amount of the criteria.

Table 27: Fuzzy Scaling Free Flyer Routes - WK - Rush Hours

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>Rides/PM-Bus</th>
<th>Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>Holt-College Flyer</td>
<td>WK</td>
<td>0.5170</td>
<td>0.5170</td>
<td>0.6439</td>
</tr>
<tr>
<td>43</td>
<td>Whitnall Flyer</td>
<td>WK</td>
<td>0.3728</td>
<td>0.3728</td>
<td>0.3799</td>
</tr>
<tr>
<td>44</td>
<td>Fair Park-National Flyer</td>
<td>WK</td>
<td>0.4447</td>
<td>0.4447</td>
<td>0.4028</td>
</tr>
<tr>
<td>46</td>
<td>Loomis-Southridge Flyer</td>
<td>WK</td>
<td>0.3656</td>
<td>0.3656</td>
<td>0.5382</td>
</tr>
<tr>
<td>48</td>
<td>South Shore Flyer</td>
<td>WK</td>
<td>0.6344</td>
<td>0.6344</td>
<td>0.5028</td>
</tr>
<tr>
<td>49</td>
<td>Brown Deer-Northshore Flyer</td>
<td>WK</td>
<td>0.4958</td>
<td>0.4958</td>
<td>0.7298</td>
</tr>
<tr>
<td>79</td>
<td>Menomonee Falls Flyer</td>
<td>WK</td>
<td>0.3978</td>
<td>0.3978</td>
<td>0.2702</td>
</tr>
<tr>
<td></td>
<td>Ozaukee County Express</td>
<td>WK</td>
<td>0.4471</td>
<td>0.4471</td>
<td>0.4556</td>
</tr>
<tr>
<td>----</td>
<td>----------------------</td>
<td>----</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>SP</td>
<td></td>
<td></td>
<td>0.0892</td>
<td>0.0892</td>
<td>0.1478</td>
</tr>
</tbody>
</table>

**Construct a Set of Pairwise Comparison Matrixes**

Applying equations 4 and 5 ends in a set of comparison matrixes based on the three selected criteria; (Number of Rides/Number of Bus), (Number of Rides/Bus hour) and Productivity Frequently Index. Matrixes for Fixed Routes and Free Flyer Routes are respectively as below.

Fixed Routes, WK, SUN, SAT = \[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\] and Free Flyer = \[
\begin{bmatrix}
1 & 1 & 0.5 \\
1 & 1 & 0.5 \\
2 & 2 & 1 \\
\end{bmatrix}
\]

**Pairwise Comparison Optimization**

Comparison matrixes for the three indicators is in the form of \((y)_{3\times3}\) and are described below for both fixed and free flyers routes;

Fixed Routes, WK, SUN, SAT = \[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\] and Free Flyer = \[
\begin{bmatrix}
1 & 1 & 0.5 \\
1 & 1 & 0.5 \\
2 & 2 & 1 \\
\end{bmatrix}
\]

**Synthesis**

Weight of each criterion is the key solution of evaluating the efficiency. The value of each criterion based on the type of routes; Fixed Routes and Free Flyer Routes are represented in Table 28 and Table 29.

From Table 28 it is obvious that the weight of all three certain is same, which means they supposed to have same influence on the results.
Table 28: Weight Fixed Routes- Rush Hours

<table>
<thead>
<tr>
<th>Fixed Routes (WK, SAT, SUN)</th>
<th>PM-Rides/PM-Bus</th>
<th>PM-Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 29 determined the influence of "Productivity Frequently Index" on the results is two times more than impact of each criteria "number of rides per bus" and "number of rides per hour".

Table 29: Weight Free Flyer Routes- Rush Hours

<table>
<thead>
<tr>
<th>Free Flyer (WK)</th>
<th>PM-Rides/PM-Bus</th>
<th>PM-Rides/PM-BH</th>
<th>PFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Efficiency Ranking for Fixed Routes during Rush Hours

In this section, final ranking will be obtained by applying the Eq.11 to the previous results. Routes are ranked as Fixed Routes on weekdays, Saturdays and Sundays, and as the Free Flyer routes on weekdays. Table 30 represents the efficiency for Fixed Routes on the weekdays and during the rush hours. Route 63, which was mentioned during the analysis of Table 20 is in the first place of most efficient routes and has the largest amount for the three indicators in both row data and after normalizing the scale in Table 24.

Table 30: Fixed Routes Efficiency Ranking - WK - Rush Hours

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>PM- Rides / PM-Bus</th>
<th>PM- Rides / PM-BH</th>
<th>PFI</th>
<th>Efficiency</th>
<th>RNK</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>Silver Spring-Pt. Washington</td>
<td>WK</td>
<td>0.3162</td>
<td>0.3162</td>
<td>0.3313</td>
<td>0.9637</td>
<td>1</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>WK</td>
<td>0.2834</td>
<td>0.2834</td>
<td>0.1713</td>
<td>0.7380</td>
<td>2</td>
</tr>
<tr>
<td>35</td>
<td>35th St.</td>
<td>WK</td>
<td>0.2538</td>
<td>0.2538</td>
<td>0.1738</td>
<td>0.6814</td>
<td>3</td>
</tr>
<tr>
<td>BLU</td>
<td>Fond du Lac-National MetroExpress</td>
<td>WK</td>
<td>0.2338</td>
<td>0.2338</td>
<td>0.1696</td>
<td>0.6372</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>WK</td>
<td>0.2584</td>
<td>0.2584</td>
<td>0.1145</td>
<td>0.6313</td>
<td>5</td>
</tr>
<tr>
<td>54</td>
<td>Mitchell-Burnham</td>
<td>WK</td>
<td>0.1994</td>
<td>0.1994</td>
<td>0.2249</td>
<td>0.6236</td>
<td>6</td>
</tr>
</tbody>
</table>
Figure 18 shows the graph of efficiency for Fixed Routes during the rush hour. It is easy to identify the considerable gap between efficiency of Route 63 and the other routes. The Route 63 belongs to the level of tenth in daily assessment of efficiency, based on Table 16. Furthermore, the quantity of efficiency has small amplitude of changes from the second level to eighth.
Table 31 shows the efficiency ranking for Fixed Routes on Saturday and during Rush Hours. The Route BLU and 62, as it was expectable from Table 21, are in the best efficient level. The significant point is that the Route 63, which was the most efficient route during rush hour on weekdays, fell to the rank of sixteenth in Table 31. The common reason lies in the feature of the path. The route goes from Silver Spring to the Port Washington and reverse. In the path, there are stops near US bank, Bayshore Town Centre and Cardinal Stritch College, which are supposed to have more travelling demand on rush hours of the weekdays than weekends.

Table 31: Efficiency Ranking Fixed Routes - SAT - Rush Hours

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>PM- Rides / PM-Bus</th>
<th>PM- Rides / PM-BH</th>
<th>PFI</th>
<th>Efficiency</th>
<th>RNK</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLU</td>
<td>Fond du Lac-National MetroExpress</td>
<td>SAT</td>
<td>0.2849</td>
<td>0.2849</td>
<td>0.3333</td>
<td>0.9031</td>
<td>1</td>
</tr>
<tr>
<td>62</td>
<td>Capitol Dr.</td>
<td>SAT</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.1751</td>
<td>0.8418</td>
<td>2</td>
</tr>
<tr>
<td>54</td>
<td>Mitchell-Burnham</td>
<td>SAT</td>
<td>0.2318</td>
<td>0.2318</td>
<td>0.2490</td>
<td>0.7125</td>
<td>3</td>
</tr>
<tr>
<td>51</td>
<td>Oklahoma Ave.</td>
<td>SAT</td>
<td>0.2599</td>
<td>0.2599</td>
<td>0.1800</td>
<td>0.6998</td>
<td>4</td>
</tr>
<tr>
<td>27</td>
<td>27th St.</td>
<td>SAT</td>
<td>0.2931</td>
<td>0.2931</td>
<td>0.0980</td>
<td>0.6842</td>
<td>5</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>SAT</td>
<td>0.2682</td>
<td>0.2682</td>
<td>0.1473</td>
<td>0.6838</td>
<td>6</td>
</tr>
<tr>
<td>RED</td>
<td>Capitol Dr. MetroExpress</td>
<td>SAT</td>
<td>0.2559</td>
<td>0.2559</td>
<td>0.1650</td>
<td>0.6768</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SAT</td>
<td>0.2469</td>
<td>0.2469</td>
<td>0.1827</td>
<td>0.6765</td>
<td>8</td>
</tr>
<tr>
<td>GRN</td>
<td>Oakland-Howell MeterExpress</td>
<td>SAT</td>
<td>0.2579</td>
<td>0.2579</td>
<td>0.1170</td>
<td>0.6328</td>
<td>9</td>
</tr>
<tr>
<td>60</td>
<td>Burleigh St.</td>
<td>SAT</td>
<td>0.2480</td>
<td>0.2480</td>
<td>0.1362</td>
<td>0.6323</td>
<td>10</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SAT</td>
<td>0.2321</td>
<td>0.2321</td>
<td>0.1662</td>
<td>0.6303</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Route Description</td>
<td>SAT</td>
<td>SAT</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>------------------</td>
<td>------</td>
<td>------</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>0.2558</td>
<td>0.2558</td>
<td>0.1160</td>
<td>0.6275</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Sherman-Wisconsin</td>
<td>0.2704</td>
<td>0.2704</td>
<td>0.0775</td>
<td>0.6182</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>0.2289</td>
<td>0.2289</td>
<td>0.1585</td>
<td>0.6162</td>
<td></td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>N. 76th-S. 84th St.</td>
<td>0.2209</td>
<td>0.2209</td>
<td>0.1741</td>
<td>0.6159</td>
<td></td>
<td></td>
</tr>
<tr>
<td>63</td>
<td>Silver Spring-Pt. Washington</td>
<td>0.2372</td>
<td>0.2372</td>
<td>0.1359</td>
<td>0.6103</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>0.2008</td>
<td>0.2008</td>
<td>0.1582</td>
<td>0.5598</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>35th St.</td>
<td>0.2154</td>
<td>0.2154</td>
<td>0.1131</td>
<td>0.5439</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Teutonia-Hampton</td>
<td>0.2209</td>
<td>0.2209</td>
<td>0.0949</td>
<td>0.5368</td>
<td></td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>Lincoln Ave.</td>
<td>0.1617</td>
<td>0.1617</td>
<td>0.1583</td>
<td>0.4818</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>Greenfield Ave.</td>
<td>0.1786</td>
<td>0.1786</td>
<td>0.1066</td>
<td>0.4639</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>6th St.</td>
<td>0.1813</td>
<td>0.1813</td>
<td>0.0996</td>
<td>0.4623</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>0.1845</td>
<td>0.1845</td>
<td>0.0837</td>
<td>0.4528</td>
<td></td>
<td></td>
</tr>
<tr>
<td>76</td>
<td>N. 60th-S. 70th</td>
<td>0.1566</td>
<td>0.1566</td>
<td>0.0785</td>
<td>0.3918</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>108th St.</td>
<td>0.1154</td>
<td>0.1154</td>
<td>0.1460</td>
<td>0.3768</td>
<td></td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>Layton Ave.</td>
<td>0.1233</td>
<td>0.1233</td>
<td>0.0913</td>
<td>0.3379</td>
<td></td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>Walnut-N. 92nd</td>
<td>0.1183</td>
<td>0.1183</td>
<td>0.0932</td>
<td>0.3299</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>State-Highland</td>
<td>0.1149</td>
<td>0.1149</td>
<td>0.0713</td>
<td>0.3011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Vliet Street</td>
<td>0.0789</td>
<td>0.0789</td>
<td>0.0565</td>
<td>0.2143</td>
<td></td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>Clement-15th Ave.</td>
<td>0.0671</td>
<td>0.0671</td>
<td>0.0673</td>
<td>0.2014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>64</td>
<td>S. 60th St.</td>
<td>0.0520</td>
<td>0.0520</td>
<td>0.0732</td>
<td>0.1771</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Canal St.</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>137</td>
<td>House of Correction</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 16 shows the graph of efficiency ranking for Fixed Routes on Saturdays. From the graph, it is clear that the rate of changes in efficiency for the routes at the middle level is very slow and they are very close in the level of efficiency.

![Efficiency Fixed Routes SAT. PM](image)

Figure 19: Efficiency Ranking of Fixed Routes - SAT - Rush Hours

65
Table 32 represents data of efficiency level for Fixed Routes on Sundays and during rush hours. Routes 62, BLU, GRN, 23 and 30 are in the first fifth of best ranking place for Sundays. Compare with Table these routes are respectively in the level of fourteenth, fourth, third, sixth and first of daily efficiency for Sundays. Generally compare with other days a week, the lowest amount of rides and passengers in each bus even in rush hours belongs to Sundays.

**Table 32: Efficiency Ranking of Fixed Routes - SUN - Rush Hours**

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>PM- Rides / PM-Bus</th>
<th>PM- Rides / PM-BH</th>
<th>PFI</th>
<th>Efficiency</th>
<th>RNK</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>Capitol Dr.</td>
<td>SUN</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3046</td>
<td>0.9713</td>
<td>1</td>
</tr>
<tr>
<td>23</td>
<td>Fond du Lac-National</td>
<td>SUN</td>
<td>0.2771</td>
<td>0.2771</td>
<td>0.3022</td>
<td>0.8564</td>
<td>4</td>
</tr>
<tr>
<td>30</td>
<td>Sherman-Wisconsin</td>
<td>SUN</td>
<td>0.3320</td>
<td>0.3320</td>
<td>0.1370</td>
<td>0.8009</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>Holton-Kinnickinnic</td>
<td>SUN</td>
<td>0.2332</td>
<td>0.2332</td>
<td>0.2082</td>
<td>0.7734</td>
<td>6</td>
</tr>
<tr>
<td>14</td>
<td>Forest Home</td>
<td>SUN</td>
<td>0.2660</td>
<td>0.2660</td>
<td>0.2274</td>
<td>0.7594</td>
<td>8</td>
</tr>
<tr>
<td>60</td>
<td>Burleigh St.</td>
<td>SUN</td>
<td>0.2833</td>
<td>0.2833</td>
<td>0.1920</td>
<td>0.7586</td>
<td>9</td>
</tr>
<tr>
<td>27</td>
<td>27th St.</td>
<td>SUN</td>
<td>0.3001</td>
<td>0.3001</td>
<td>0.1504</td>
<td>0.7507</td>
<td>10</td>
</tr>
<tr>
<td>21</td>
<td>North Ave.</td>
<td>SUN</td>
<td>0.2897</td>
<td>0.2897</td>
<td>0.1708</td>
<td>0.7502</td>
<td>11</td>
</tr>
<tr>
<td>54</td>
<td>Mitchell-Burnham</td>
<td>SUN</td>
<td>0.2112</td>
<td>0.2112</td>
<td>0.2864</td>
<td>0.7089</td>
<td>12</td>
</tr>
<tr>
<td>22</td>
<td>Center St.</td>
<td>SUN</td>
<td>0.2542</td>
<td>0.2542</td>
<td>0.1649</td>
<td>0.6733</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>Humboldt-Wisconsin</td>
<td>SUN</td>
<td>0.2337</td>
<td>0.2337</td>
<td>0.1997</td>
<td>0.6670</td>
<td>14</td>
</tr>
<tr>
<td>51</td>
<td>Oklahoma Ave.</td>
<td>SUN</td>
<td>0.2317</td>
<td>0.2317</td>
<td>0.1913</td>
<td>0.6548</td>
<td>15</td>
</tr>
<tr>
<td>55</td>
<td>Layton Ave.</td>
<td>SUN</td>
<td>0.1934</td>
<td>0.1934</td>
<td>0.2623</td>
<td>0.6491</td>
<td>16</td>
</tr>
<tr>
<td>67</td>
<td>N. 76th-S. 84th St.</td>
<td>SUN</td>
<td>0.2078</td>
<td>0.2078</td>
<td>0.2022</td>
<td>0.6178</td>
<td>17</td>
</tr>
<tr>
<td>12</td>
<td>Teutonia- Hampton</td>
<td>SUN</td>
<td>0.2461</td>
<td>0.2461</td>
<td>0.1233</td>
<td>0.6156</td>
<td>18</td>
</tr>
<tr>
<td>35</td>
<td>35th St.</td>
<td>SUN</td>
<td>0.2332</td>
<td>0.2332</td>
<td>0.1444</td>
<td>0.6107</td>
<td>19</td>
</tr>
<tr>
<td>53</td>
<td>Lincoln Ave.</td>
<td>SUN</td>
<td>0.1797</td>
<td>0.1797</td>
<td>0.2119</td>
<td>0.5713</td>
<td>20</td>
</tr>
<tr>
<td>76</td>
<td>N. 60th-S. 70th</td>
<td>SUN</td>
<td>0.2015</td>
<td>0.2015</td>
<td>0.1485</td>
<td>0.5514</td>
<td>21</td>
</tr>
<tr>
<td>63</td>
<td>Silver Spring-Pt. Washington</td>
<td>SUN</td>
<td>0.1980</td>
<td>0.1980</td>
<td>0.1517</td>
<td>0.5476</td>
<td>22</td>
</tr>
<tr>
<td>80</td>
<td>6th St.</td>
<td>SUN</td>
<td>0.1943</td>
<td>0.1943</td>
<td>0.1260</td>
<td>0.5147</td>
<td>23</td>
</tr>
<tr>
<td>56</td>
<td>Greenfield Ave.</td>
<td>SUN</td>
<td>0.1719</td>
<td>0.1719</td>
<td>0.1672</td>
<td>0.5111</td>
<td>24</td>
</tr>
<tr>
<td>28</td>
<td>108th St.</td>
<td>SUN</td>
<td>0.1317</td>
<td>0.1317</td>
<td>0.2096</td>
<td>0.4730</td>
<td>25</td>
</tr>
<tr>
<td>19</td>
<td>ML King-S. 13th &amp; S. 20th</td>
<td>SUN</td>
<td>0.1783</td>
<td>0.1783</td>
<td>0.0999</td>
<td>0.4565</td>
<td>26</td>
</tr>
<tr>
<td>57</td>
<td>Walnut-N. 92nd</td>
<td>SUN</td>
<td>0.1262</td>
<td>0.1262</td>
<td>0.1190</td>
<td>0.3714</td>
<td>27</td>
</tr>
<tr>
<td>31</td>
<td>State-Highland</td>
<td>SUN</td>
<td>0.1173</td>
<td>0.1173</td>
<td>0.0864</td>
<td>0.3210</td>
<td>28</td>
</tr>
</tbody>
</table>
Figure 20 shows a graph of efficiency for Sundays. From the graph, it is clear Routes 62 and BLU are significantly in higher level of efficiency compare with other routes.

![Efficiency Fixed Routes PM-Sunday](image)

Figure 20: Efficiency Ranking for Fixed Routes - SUN - Rush Hour

Table 33 represents data regarding the final efficiency ranking for eight Free Flyer Routes during rush hours. Comparisons by Table 19 showed the results have reasonable similarity in ranking. The other noticeable point is, because the weight for PFI was achieved to be almost two times more important than the value of the other two criteria; the routes by largest amount of productivity index are ranked in the higher level of efficiency.

<table>
<thead>
<tr>
<th>RT</th>
<th>NAME</th>
<th>SERVICE</th>
<th>PM- Rides / PM-Bus</th>
<th>PM- Rides / PM-BH</th>
<th>PFI</th>
<th>Efficiency</th>
<th>RNK</th>
</tr>
</thead>
<tbody>
<tr>
<td>49</td>
<td>Brown Deer-Northshore Flyer</td>
<td>WK</td>
<td>0.1239</td>
<td>0.1239</td>
<td>0.3649</td>
<td>0.6128</td>
<td>1</td>
</tr>
<tr>
<td>40</td>
<td>Holt-College Flyer</td>
<td>WK</td>
<td>0.1293</td>
<td>0.1293</td>
<td>0.3220</td>
<td>0.5805</td>
<td>2</td>
</tr>
<tr>
<td>48</td>
<td>South Shore Flyer</td>
<td>WK</td>
<td>0.1586</td>
<td>0.1586</td>
<td>0.2514</td>
<td>0.5686</td>
<td>3</td>
</tr>
<tr>
<td>#</td>
<td>Route</td>
<td>WK</td>
<td>Efficiency 1</td>
<td>Efficiency 2</td>
<td>Efficiency 3</td>
<td>Efficiency 4</td>
<td></td>
</tr>
<tr>
<td>----</td>
<td>------------------------------</td>
<td>-----</td>
<td>--------------</td>
<td>--------------</td>
<td>--------------</td>
<td>--------------</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>Loomis-Southridge Flyer</td>
<td>WK</td>
<td>0.0914</td>
<td>0.0914</td>
<td>0.2691</td>
<td>0.4519</td>
<td></td>
</tr>
<tr>
<td>143</td>
<td>Ozaukee County Express</td>
<td>WK</td>
<td>0.1118</td>
<td>0.1118</td>
<td>0.2278</td>
<td>0.4514</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>Fair Park-National Flyer</td>
<td>WK</td>
<td>0.1112</td>
<td>0.1112</td>
<td>0.2014</td>
<td>0.4237</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>Whitnall Flyer</td>
<td>WK</td>
<td>0.0932</td>
<td>0.0932</td>
<td>0.1899</td>
<td>0.3764</td>
<td></td>
</tr>
<tr>
<td>79</td>
<td>Menomonee Falls Flyer</td>
<td>WK</td>
<td>0.0995</td>
<td>0.0995</td>
<td>0.1351</td>
<td>0.3340</td>
<td></td>
</tr>
</tbody>
</table>

Figure 21 shows the graph of efficiency for Free Flyer Routes. From the graph, it is clear the rate of changes for the routes is almost from 0.3 to 0.6. The efficiency of the route in the first level is two times more than the efficiency of the route in the last level.

![Efficiency Ranking of Free Flyer Routes - Rush Hours](image)
6. Conclusion

The objectives of this thesis are evaluating the efficiency of bus routes in Milwaukee County and making comparison between this thesis' results and the results produced by Milwaukee County Transit System (MCTS). The study started by reviewing various transit systems in US. Then, different models used in transit systems analysis, were discussed. In chapter three, operational efficiency framework was introduced. However the framework, make some suggestions about different criteria could be used in efficiency assessments as well as describing different viewpoints to this subject, finally AHP fuzzy model was selected for the methodology of this study. Fuzzy AHP analyze multiple criteria at a time by considering their impact on the results. Using this model, efficiency for Fixed Routes and Free Flyer Routes are evaluated. 24 hours a day with the emphasis on the rush hours are considered in efficiency calculations. The criteria picked in calculations are "passenger per bus hour (PBH)" and "Ridership" which are the same criteria used at MCTS annual report (as mentioned earlier, the term "Productivity" is used instead of "PBH" in MCTS reports). For aforementioned criteria, the weight achieved by the AHP model for Fixed Routes on both weekdays and weekends is calculated as 0.5, which means PBH and Ridership have the same impact on the total efficiency of Fixed Routes in a day. For Free Flyer Route, the weight of these criteria changed to 0.33 and 0.66 for PBH and Ridership respectively, which means Ridership has a greater impact on the efficiency at Free Flyer Routes.

In terms of efficiency during the rush hours, the criteria were changed to "Number of Rides/Number of Bus", "Number of Rides/Bus hour" and "Productivity Frequently Index (PFI)". The results showed that these criteria have the same impact on the efficiency of Fixed Routes.
However, Free Flyers Routes showed different results. For Free Flyer Routes, PFI is the criterion that has the most important role in the final efficiency while the other two criteria have the same impact on the efficiency.

**Daily Efficiency:** In daily analyzing of efficiency for Fixed Routes, although the weight for both Ridership and Productivity was calculated as 0.5, the results showed routes with more number of Ridership obtained better position in final ranking. The reason might be due to the higher rates of change for Ridership than the Productivity. In other words, the very close number of Productivity between buses, made this factor to have less impact than the Number of Rides on the final decision. For Fixed Routes, the Route 30 and 27 appeared as the most efficient routes for weekdays and weekends. Both have high values for Ridership and Number of Passenger carried in each bus hour. The Route 27 goes from north to the south and Route 30 cover the downtown all the way to the UWM. The other routes with high efficiency in Fixed Routes are Routes 12, 21, BLU and GRN. The noticeable point is that the Route GRN is ranked the eleventh on efficiency table on weekdays. However, it achieved significantly higher position in the table of efficiency ranking for weekends, which are fifth on Saturdays and third on Sundays, respectively. This might be because it has stops on both Bayshore Mall and Milwaukee Airport as well as many stops at Waters and Brady streets where many bar-restaurants are located and distinguish it as a more desirable route for the weekends. Route 23 is somehow like Route GRN. This route also has a higher ranking for weekends than the weekdays despite Sundays are the least crowded day of the week.

In terms of efficiency for Free Flyer Routes, Ridership has double impact on the final efficiency. The high demand for the Free Flyers is because these routes only provide services
during the peak hours in the morning and afternoon. The results also show that the rates of changes in efficiency of the Free Flyer Routes are small.

**Efficiency during Peak Hours:** At this study, the efficiency is also analyzed for the peak hours. The reason is the high impact that travel demand has on the transportation system during these times. The study shows different results in efficiency analysis than the routines daily analysis. For instance, Route 30, which ranked the first at the most efficient route on the weekdays, dropped to tenth for efficiency ranking at rush hour. It might be because of the higher demands than the provided services. Route 63 has significantly lower level of efficiency on weekend's rush hours than during daily rush hours. It could be due to the significant decrease in Number of Rides per Bus Hour (PBH) and amount of PFI during the weekends. Generally, routes with higher number of PBH have greater level of efficiency, like Route 62 on the weekends.

Efficiency results of Free Flyers during the peak hours were pretty much the same as the daily efficiency. The only considerable example was Route 143, which is Ozaukee County Express Route. It is in the first rank of daily efficiency, but ranked fifth during the peak hours. This decrease could be a result of lower demand in travelling in the evenings than the mornings.

### 6.1. Recommendations and Suggestions

Analyzing the data and providing different comparisons between the results of this study and the results presented on MCTS report, led to offer the suggestions below to improve the efficiency of the Milwaukee County Transit System:
• Adding fleet to the paths that have higher travel demand on the weekdays; one example is Route 30, which travels between UWM and downtown Milwaukee.

• Sundays and Saturdays show different statics. Therefore, it is needed to schedule different number of fleet for some routes on the weekends. Route 23 and GRN are two examples that by increasing the number of buses on weekends the efficiency will improve.

• Necessity of setting different schedules for some routes during the rush hours; some routes, like Route 63, show great level of efficiency during the rush hours, but doesn't appear to be in the best level of daily efficiency ranking. Hence, it is required to add a route only for the rush hours or include this path in Free Flyer Routes.

• Eliminating unnecessary bus stops for the routes with higher Number of Passenger per Bus Hour than the Number of Ridership; these routes probably have some desirable common stops, which are the main destinations of the passengers, like Route 62.

• In Free Flyer Routes, during peak hours both Productivity Frequency Index (PFI) and Ridership have great impacts on the efficiency. Improving these two factors could result in improving the efficiency.

6.2. Future Research

• This study shows two sets of different relations between the available criteria. Applying other relations would provide other visions to the efficiency of the system.

• Involving the residents in the process of evaluating the efficiency by different methods of surveys
• Adding other data like cost and mileage of each bus to the study; Data from MCTS does not include any information about these important factors and it seems there are no data recorded for cost of each bus routes. Adding expenses or at least fuel consumptions could help in achieving exact results in efficiency.

• Examine different methods of decision-making, like General AHP Model, which relies on decision-makers' opinions. However, it might not lead to the more accurate results; it would give another view to the problem and possible solutions.
7. Bibliography


