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Article

# Survival Analysis of Bridge Superstructures in Wisconsin

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**Featured Application:** The methodology provided in this article can be used for probabilistic assessment of service life of bridge superstructures based on survival analysis of long-term field inspection data from various geographic locations. The effects of various parameters on the probability of survival at different ages can be explicitly assessed when those parameters are included in the analysed data.

**Abstract:** Although survival analyses have long been used in biomedical research, their application to engineering in general, and bridge engineering in particular, is a more recent phenomenon. In this research, survival (reliability) of bridge superstructures in Wisconsin was investigated using the Hypertabastic accelerated failure time model. The 2012 National Bridge Inventory (NBI) data for the State of Wisconsin were used for the analyses. A recorded NBI superstructure condition rating of 5 was chosen as the end of service life. The type of bridge superstructure, bridge age, maximum span length (MSL) and average daily traffic (ADT) were considered as possible risk factors in the survival of bridge superstructures. Results show that ADT and MSL can substantially affect the survival of bridge superstructures at various ages. The reliability of Wisconsin superstructures at the ages of 50 and 75 years is on the order of 63% and 18%, respectively, when the ADT and MSL values are at Wisconsin's mean values.

**Keywords:** survival analysis; reliability; bridge superstructures; service life; deterioration

## 1. Introduction

### 1.1. Background

Featured Application: The methodology provided in this article can be used for probabilistic assessment of service life of bridge superstructures based on survival analysis of long-term field inspection data from various geographic locations. The effects of various parameters on the probability of survival at different ages can be explicitly

Long-term reliability is one of the most important issues in bridge engineering and preservation. There is substantial interest in seeking better design, maintenance and rehabilitation methods to achieve longer service lives. Bridges deteriorate due to long-term effects of environmental exposure and loading. Durability-based design, preventive maintenance, and structural health monitoring can provide important long-term benefits in a variety of bridge types [1–4]. Optimal bridge management strategies require effective probabilistic assessment of future bridge conditions based on reliability analyses. Such reliability analyses benefit from understanding and quantifying the existing field performance data as influenced by various parameters.

The strength-based reliability approach (based on a probabilistic assessment of resistance versus load) is commonly utilized for prediction of maintenance requirements or remaining service life. The reliability index ( $\beta$ ) provides a measure of the likelihood of failure (loads exceeding resistance) for various structural elements. Kong and Frangopol [5] used time-dependent reliability index profiles to optimize life-cycle maintenance and evaluate the reliability of deteriorating structures. Lee [6] evaluated safety and approximate remaining service life of a reinforced concrete bridge deck strengthened with FRP using the reliability index in a time-dependent approach. Lu et al. [7] conducted time-dependent reliability assessments of deteriorated reinforced concrete bridges under service load. Sun and Hong [8] performed time-dependent reliability research on the effect of corrosion on strength-based reliability of bridge girders. Giorgio and Frangopol [9] investigated strength-based reliability approaches based on the annual reliability index to determine the optimized maintenance time for aging structures.

Stewart and Rosowsky [10] conducted a study on time-dependent reliability of deteriorating reinforced concrete bridge decks. They used a reliability model to assess the probability of flexural failure of a concrete slab bridge under corrosion. Monte Carlo simulation was used to simulate 75 years of service life.

It should be noted that the end of service life for almost all bridges comes about because of serviceability issues such as corrosion, and not because of a local or global structural failure (load exceeding resistance). Therefore, serviceability-based reliability models have also been developed [11–13]. For example, theoretical models to assess the service life of bridge decks due to corrosion consider factors such as penetration of chlorides, initiation and progression of corrosion, cracking, and progressive delamination. Effects of concrete properties, cover depth, diffusion coefficient, surface chloride concentration, aggregate type, bar layout, etc. can be considered in an analytical model. Assuming availability of such an effective serviceability-based theoretical model, Monte Carlo simulations or other approaches can then be performed to assess the service life in a probabilistic manner. The primary challenge, however, is effective verification and calibration of such theoretical models against field performance data in a meaningful probabilistic approach. To do so, large-scale observation-based (phenomenological) reliability models are needed based on field performance data.

Survival analysis techniques can provide such a global probabilistic model given availability of large-scale data. The National Bridge Inventory (NBI) records can be the source of such large-scale data. Unlike conventional bridge reliability analyses, survival analyses do not consider the relationship between load and resistance, or the theoretical basis behind delamination in bridge decks, in determining reliability. These comprehensive mathematical/statistical techniques are purely observation-based and data-driven. Although substantial research has been performed on the general topic of reliability of bridges [14–17], only limited studies exist regarding survival analyses of bridge structures [18–23]. This study is meant to provide such information for bridge superstructures in Wisconsin.

## 1.2. Survival Analysis

Survival analysis is a particular form of reliability analysis in which “time-to-event” information (such as the age of a bridge at the end of service life or survival time of a patient) is analysed along with their associated contributing parameters (covariates). This is a data-driven approach that can evaluate the effect of covariates on the outcome (time to reach the specified event).

Survival analysis techniques have long been used in biomedical research. The applications of this approach to bridge engineering, although limited, have expanded in recent years. Recent research efforts have been focused on developing survival models for bridges decks [18–20]

In survival analyses, the key variable is the “survival time” or “failure time”. In general, survival time is a non-negative random variable indicating the elapsed time from a reference time to the occurrence of a given event [24]. Examples of survival time include time to recurrence of cancer after administration of a drug, lifetime of an electronic component, etc. Bridge survival analysis could take

into account the time that takes it to reach an event such as initiation of corrosion, cracking, or the end of service life. The term “survival” is, by definition, equivalent to the term “reliability” commonly used in engineering. Similarly, the term “hazard” used in survival analyses is analogous to “failure rate” in engineering applications.

Non-parametric and semi-parametric survival models, such as Kaplan-Meier and Cox-regression models, have been widely used to estimate survival time in different fields. As the simplest form of survival analysis, the non-parametric Kaplan-Meier technique does not consider the simultaneous effect of multiple covariates on the outcome. Only the effect of a single covariate (time to reach an event) can be considered. Data on survival times (for all observations) are simply sorted and the survival and hazard rate are calculated based on the number of subjects that have survived at any given time. The semi-parametric Cox model considers the relative influence of covariates on the outcome and assumes proportionality of hazards. The parametric method is the most elaborate form of survival analysis, in which the survival function considers quantified effects of individual covariates. The parametric method can be a good substitute for the Cox model when the assumption of proportionality of hazards is not valid, and when a distribution function is available for the survival time and the baseline hazard function [24]. In such cases, parametric calculations are more descriptive and concise in comparison to non-parametric and semi-parametric methods.

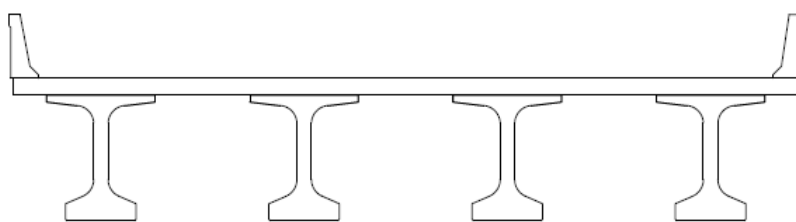
In this study, parametric survival analyses were conducted using the Hypertabastic distribution function first introduced by Tabatabai et al. [25]. This function has since been used in a variety of biomedical and engineering applications [25,26]. In a PhD dissertation, Tran [27] and Tahir et al. [28] studied the validity and effectiveness of both the accelerated failure time and proportional hazard forms of the Hypertabastic distribution and compared the Hypertabastic with classical models such as lognormal and loglogistic. They concluded that the Hypertabastic model was effective and accurate when compared with classical models.

### 1.3. Time-to-Event Data

Survival analyses typically require availability of sufficiently large datasets. In this research, the National Bridge Inventory (NBI) database was selected as the source of data for the analyses. NBI provides a comprehensive database of information on bridges from all across the U.S. The NBI data include geometric information, bridge type, conditions ratings of bridge components, traffic information, and inspection data. Bridge inspections include assignments of numerical ratings for the condition of major bridge components given by inspectors at two-year intervals. The process of extracting the NBI data used in this study is summarized in a following section.

The numerical ratings range from 0 to 9 and are given to individual components by bridge inspectors based on the Federal Highway Administration (FHWA) guidelines [29]. A rating of 0 for a bridge component indicates a failed condition, while other ratings are described as follows: imminent failure condition (1), critical condition (2), serious condition (3), poor condition (4), fair condition (5), satisfactory condition (6), good condition (7), very good condition (8), and excellent condition (9).

A typical slab-girder bridge cross section commonly used in Wisconsin is shown in Figure 1. The usual types of bridge girders are prestressed concrete I-shaped, prestressed concrete adjacent box-girders, steel I-shaped girders, and steel box girders.



**Figure 1.** Typical bridge cross section used in Wisconsin.

It is realized that many factors could influence the probability of survival of bridges. The NBI records provide substantial prior data that are needed for survival analyses. However, the number of independent parameters available in the NBI records that are relevant to superstructure deterioration are limited. For example, the NBI data do not include all parameters that can influence deterioration of concrete bridges such as compressive strength of concrete, cover depth, diffusion coefficient, deicing salt usage, reinforcing bar layout, etc. Although the number of independent and relevant parameters available from the NBI database is somewhat limited, the results can nonetheless serve as field performance benchmarks for probabilistic assessment and verification of analytical deterioration models and their statistical simulations.

The NBI data provide a valuable source of observation data, but only provide specific parametric data that are part of the dataset. This does not negate the fact that valuable survival information can be gathered using the available observation data, while the effects of various parameters on survival can be explicitly assessed when data on those parameters are available.

#### 1.4. End of Service Life

As indicated above, the end of service life is assumed to be a superstructure rating of 5 for non-reconstructed and non-rehabilitated bridges. A deck or superstructure rating of 4 would automatically classify a bridge as structurally deficient by FHWA guidelines [29]. Such a designation has significant policy and regulatory implications. Therefore, various states such as Wisconsin, Minnesota, and Virginia limit the number of bridges with ratings of 4 or below to less than a few percent of the total bridge population (e.g., 5% in Wisconsin and 8% in Minnesota and Virginia) [30–32]. A report on bridge management practices in Idaho, Michigan, and Virginia [33] states that, in general, structures with an NBI condition rating of 5 or lower are restored (defined as rehabilitated or replaced). According to Proctor et al. [34], Virginia Department of Transportation (VDOT) considers structures with a condition rating of 5 as candidates for restorative actions because of the risk of becoming structurally deficient. VDOT uses the percentage of structurally deficient bridges as a global performance measure for its bridges [31].

Hearn and Xi [35] used the time to reach rating of 5 as the “time to first rehabilitation of the bridge deck and, also the initial service life of the deck.” Tabatabai et al. [18–20] used the deck rating of 5 as indication of the end of service life for survival analyses of bridge decks. The Michigan Department of Transportation considers the NBI rating of 5 as the break point between fair and poor conditions [34].

The NBI ratings of 4 or 3 were not selected as the end of service life because a great majority of bridges are rehabilitated or replaced before reaching those ratings. For example, of the total of 12,011 Wisconsin bridges (from 2012 NBI data), the number of bridges with NBI superstructure ratings of 4 and 3 are 308 (2.5%) and 82 (0.7%), respectively. Abed-Al-Rahim and Johnston [36] state, “A bridge element only rarely receives a condition rating of 3 since . . . they are generally either rehabilitated or replaced before reaching this level.”

## 2. Prior Studies on Bridge Survival Analysis

Beng and Matsumoto [21] and Yang et al. [22] have used survival analysis to evaluate bridge infrastructure performance. Both used the Kaplan-Meier approach (non-parametric) and a parametric approach using the Weibull distribution. Yang et al. [22] used the Hong Kong Highway Department’s database to analyse survival of concrete bridge expansion joints. Beng and Matsumoto [21] analysed data on bridge replacement records from Japan and the United States to assess the time of replacement of bridges. Mauch and Madanat [23] used semiparametric survival models for deterioration of bridge decks. However, these studies did not address the survival analysis of bridge superstructure based on the end of service life without rehabilitation.

Sobanjo et al. [37] conducted research on reliability-based modelling of bridge deterioration in Florida. They used Florida’s bridge NBI data to predict the “natural deterioration” of bridges (without rehabilitation) during their lifetime. The authors conducted reliability assessments of bridge deck

and superstructure based on the type of roadway (interstate roadway or non-interstate roadway) and materials (steel or concrete). They determined that the Weibull distribution was the best fit for the data. The range of NBI rating data used was 7 to 9 (utilizing the 1992 to 2005 NBI data). The authors concluded that bridges would remain in their excellent condition (rating 9) a minimum of 1 or 2 years, 5 to 10 years in rating 8 (very good condition), and usually below 6 years in the rating 7 (good condition). It was also concluded that interstate highway bridges would deteriorate more rapidly than non-interstate bridges. This study did not address the time to NBI rating of 5 or lower.

In a preceding study, Tabatabai et al. [18] developed a Hypertabastic survival model for reliability assessments of bridge decks in the state of Wisconsin. Using the 2005 NBI data, the authors investigated the best fit model among Weibull, log-logistic, lognormal, and Hypertabastic survival functions [18]. Based on the Akaike Information Criteria (AIC) [38], the authors concluded that the Hypertabastic accelerated failure time model would best fit the data. Later, the same researchers extended the survival analysis of bridge decks to six states in northern United States [20] and to all fifty states and Puerto-Rico [19].

### 3. Survival Model Development

#### 3.1. NBI Data Extraction

Numerical ratings given for the condition of major bridge components form some of the 117 data entries in the NBI records used in this study. The following procedures were used to extract data for this study. Further details on the data extraction and analysis are presented in a thesis by Nabizadeh [39]. A list of extracted data and the software code used to analyse the data are provided as supplementary materials accompanying this paper.

- Bridge records without a superstructure rating (NBI item 59) and/or construction date (NBI item 27) were removed.
- All reconstructed or rehabilitated superstructures (NBI item 106) were removed.
- Uncommon bridge types and superstructures were excluded. Steel superstructures (NBI item 43A, code 1, 2, 5, or 6) as well as concrete and pre-stressed concrete superstructures (NBI item 43A, code 3 or 4) were retained for analysis. All other structural materials and superstructures were excluded from the data.
- Bridges with more common structural systems such as slab, multi-beam, girder, tee beam, floor beam, and box beam (NBI item 43B, code 1 to 6) were retained for analysis. Data from bridges with less common structural systems such as truss, arch, and cable-stayed were removed.
- Bridges with superstructure ratings other than 5 were excluded from the dataset.
- The retained bridge records were then classified based on their superstructure material type (NBI, item 59) (steel or concrete).

There are several NBI parameters that may potentially affect the bridge superstructure reliability. These include bridge age, deck area, Average Daily Traffic (ADT), maximum span length (MSL), and superstructure type (steel or concrete).

Tabatabai et al. [18] showed that the deck area was an important factor in reliability of bridge decks. This parameter was initially considered as a covariate in this study. In a multi-beam bridge, a large deck area may indicate a large superstructure member length, a large deck width (more beams), or both. With a larger overall superstructure member length, the probability of appearance of defects would increase. The maximum span length could also be a relevant parameter as it has direct influence on bending moments. The ADT was included as a covariate because of its influence on load and “wear and tear” on the bridge. The Average Daily Truck Traffic (ADTT) could also be considered but was not selected because ADT and ADTT are not independent of each other (i.e., one must be selected). ADT may also be related to usage of corrosion-inducing de-icing salts. Finally, the type of superstructure (steel or concrete) may also be relevant to superstructure reliability. For the

superstructure type parameter, a discrete variable was assigned. The results of survival analyses can be used to assess whether a covariate is statistically significant.

A correlation analysis was first performed among considered covariates to determine if they were correlated. Results of this analysis are shown in Table 1.

**Table 1.** Correlation between covariates.

	ADT	MSL	Age	Deck Area
ADT	1.000			
MSL	0.240	1.000		
Age	−0.229	−0.207	1.000	
Deck area	0.385	0.665	−0.217	1.000

The correlation analysis indicated that there was moderate correlation between deck area and maximum span length parameters (Pearson correlation coefficient of 0.665), and therefore, the two parameters are not independent of each other. This indicates that both parameters cannot be used simultaneously and only one should be selected. The maximum span length can significantly affect the overall structural response of the bridge. Therefore, in this study, the maximum span length is included as a parameter (in lieu of deck area) for the survival analysis of bridge superstructures.

### 3.2. Choice of Distribution Function

A basic challenge in parametric survival analyses is selecting a baseline distribution model that can best represent the observations (collected data). Furthermore, a choice between Accelerated Failure Time (AFT) and Proportional Hazards (PH) forms of a model must be made when considering the effects of covariates. In this study, an effort was made to find the most suitable baseline distribution model for the Wisconsin superstructure data. The Weibull, log-logistic, lognormal, and Hypertabastic models were evaluated and compared using AIC. As shown in Table 2, the Hypertabastic model had the lowest AIC value indicating that it was the best fitting model. The AFT model was selected over the PH because the non-parametric hazard and survival functions were intersecting, thus indicating non-proportionality of hazards.

**Table 2.** Comparison of Akaike Information Criterion for various models.

Distribution Model	−2 Log-Likelihood	AIC
Hypertabastic	637.496	647.496
Log-logistic	649.628	659.628
Weibull	655.688	665.688
Lognormal	712.192	722.192

The main advantage of Hypertabastic survival model over the more common models such as Weibull and lognormal is in its flexibility to assume a wide variety of hazard shapes [25]. Tabatabai et al. [19] reported that the Hypertabastic hazard function can be “increasing followed by decreasing with time, increasing toward an asymptote, monotonically decreasing with time, increasing with upward concavity followed by increasing with downward concavity, increasing with upward concavity followed by a linear increase, or continuously increasing with upward concavity”.

### 3.3. Survival Function

Results of survival analyses can be used to determine the probability of survival (reliability) and instantaneous failure rates (hazard). Three distinct functions commonly used in survival analysis are: survival function  $S(t)$  (Equation (1)), probability density function  $f(t)$  (Equation (2)), and hazard or conditional failure rate  $h(t)$  (defined as failure rate at any given time assuming survival up to that time) (Equation (3)) [24].

$$S(t) = P(T > t) = 1 - F(t) \tag{1}$$

$$f(t) = \lim_{\Delta t \rightarrow 0} P(t < T < t + \Delta t) / \Delta t \tag{2}$$

$$h(t) = \lim_{\Delta t \rightarrow 0} p(t < T < t + \Delta t | T > t) / \Delta t \tag{3}$$

where  $T$  indicates the survival time as a random variable,  $t$  is the time, and  $F(t)$  denotes the cumulative probability of failure at various times.  $S(t) = 1$  at  $t = 0$  and  $S(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

The baseline Hypertabastic probability density function is defined as [18]:

$$f(t) = \begin{cases} Sech[W(t)] \left( \alpha t^{2\beta-1} Csch(t^\beta)^2 - \alpha t^{\beta-1} Coth(t^\beta) \right) Tanh[W(t)] & \text{for } t > 0 \\ 0 & \text{for } t > 0 \end{cases} \tag{4}$$

where parameters  $\alpha$  and  $\beta$  are both positive,  $Sech[\bullet]$ ,  $Csch[\bullet]$  and  $Coth[\bullet]$  are hyperbolic secant, hyperbolic cosecant, and hyperbolic cotangent, respectively; and  $W(t) = \alpha(1 - t^\beta Coth(t^\beta)) / \beta$ .

The baseline hypertabastic survival function is defined as

$$S(t) = Sech \left[ \alpha(1 - t^\beta Coth(t^\beta)) / \beta \right] \tag{5}$$

The baseline hypertabastic failure rate function  $h(t)$  is defined as

$$h(t) = \alpha \left( t^{2\beta-1} Csch(t^\beta)^2 - t^{\beta-1} Coth(t^\beta) \right) Tanh[W(t)] \tag{6}$$

Using the proposed Hypertabastic accelerated failure time model and after removal of small terms, survival and failure rates can be determined using Equations (7) through (9) [18]:

$$S(t_g) = Sech \left[ \alpha \left( 1 - t_g^\beta Coth(t_g^\beta) \right) / \beta \right] \tag{7}$$

$$h(t_g) = \alpha \left( -t_g^{\beta-1} Coth(t_g^\beta) \right) Tanh[W(t_g)] e^{[c(MSL)+d(ADT)+h(\text{Type})]} \tag{8}$$

$$t_g = (\text{AGE}) e^{[c(MSL)+d(ADT)+h(\text{Type})]} \tag{9}$$

Parameters “AGE”, “MSL”, and “ADT” indicate the age of the bridge superstructure, maximum span length, and average daily traffic, respectively. “Type” is a discrete variable representing the type of bridge superstructure. This variable is equal to 1 for steel, 0 for concrete, and 2 when both types of superstructures are combined (i.e., when superstructure types are not distinguished).

The parameters  $\alpha$ ,  $\beta$ ,  $c$ ,  $d$ , and  $h$  are determined using the method of maximum likelihood. The log-likelihood function was maximized within the Mathematica software. Detailed equations and procedures used in finding these parameters are described by Tabatabai et al. [18]. These parameters and the results of analyses are presented in the following section.

### 4. Results and Discussion

Statistical information on various parameters and superstructure types (steel, concrete or combined) for the data used in the analyses is shown in Table 3. These parameters were calculated for bridges in the State of Wisconsin.

**Table 3.** Statistical information on the Wisconsin NBI data used in the analyses.

Statistical Parameter	Wisconsin Bridge with Superstructure Rating of 5									All Wisconsin Bridges Combined *	
	Steel Superstructure			Concrete Superstructure			Both Superstructure Types			ADT	MSL (m)
	ADT	MSL (m)	Age (yrs)	ADT	MSL (m)	Age (yrs)	ADT	MSL (m)	Age (yrs)		
Mean	1911	14.0	61.8	6014	14.5	50.9	3879	14.2	56.6	4872	18.5
Median	210	9.8	60.0	1200	12.8	47.0	450	11.9	52.0	690	14.9
Standard deviation	4971	13.0	17.0	11,035	8.4	21	8682	11.0	19.6	9699	11.5
No. of bridges	472			435			907			9835	

\* Some data in this column are taken from reference [18].



The calculated parameters for Equations (4) through (6) are shown in Table 4. In Table 4, the parameters  $\alpha$ ,  $\beta$ ,  $c$ ,  $d$ , and  $h$  are identical for steel and concrete superstructures. The value of the parameter “Type” (0 or 1) describes the difference between the two superstructure types.

**Table 4.** Reliability and failure rate parameters.

Parameters Used in Survival Analysis, Equations (4) through (9)			
Type of Superstructure	Steel	Concrete	Combined
Parameter “Type”	1	0	2
$\alpha$	$5.80 \times 10^{-04}$	$5.80 \times 10^{-04}$	$5.40 \times 10^{-04}$
$\beta$	$2.12 \times 10^{+00}$	$2.12 \times 10^{+00}$	$2.09 \times 10^{+00}$
$c$	$3.74 \times 10^{-03}$	$3.74 \times 10^{-03}$	$3.37 \times 10^{-03}$
$d$	$6.28 \times 10^{-06}$	$6.28 \times 10^{-06}$	$8.31 \times 10^{-06}$
$h$	$-1.50 \times 10^{-01}$	$-1.50 \times 10^{-01}$	$0.00 \times 10^{+00}$

Calculated  $p$ -values are shown in Table 5. Smaller  $p$ -values indicate that the effect of that parameter is statistically significant. The  $p$ -values in Table 5 indicate that covariates Type, MSL and ADT are all statistically significant parameters affecting superstructure reliability.

**Table 5.** Parameter and standard error estimates for the Hypertabastic accelerated failure time model.

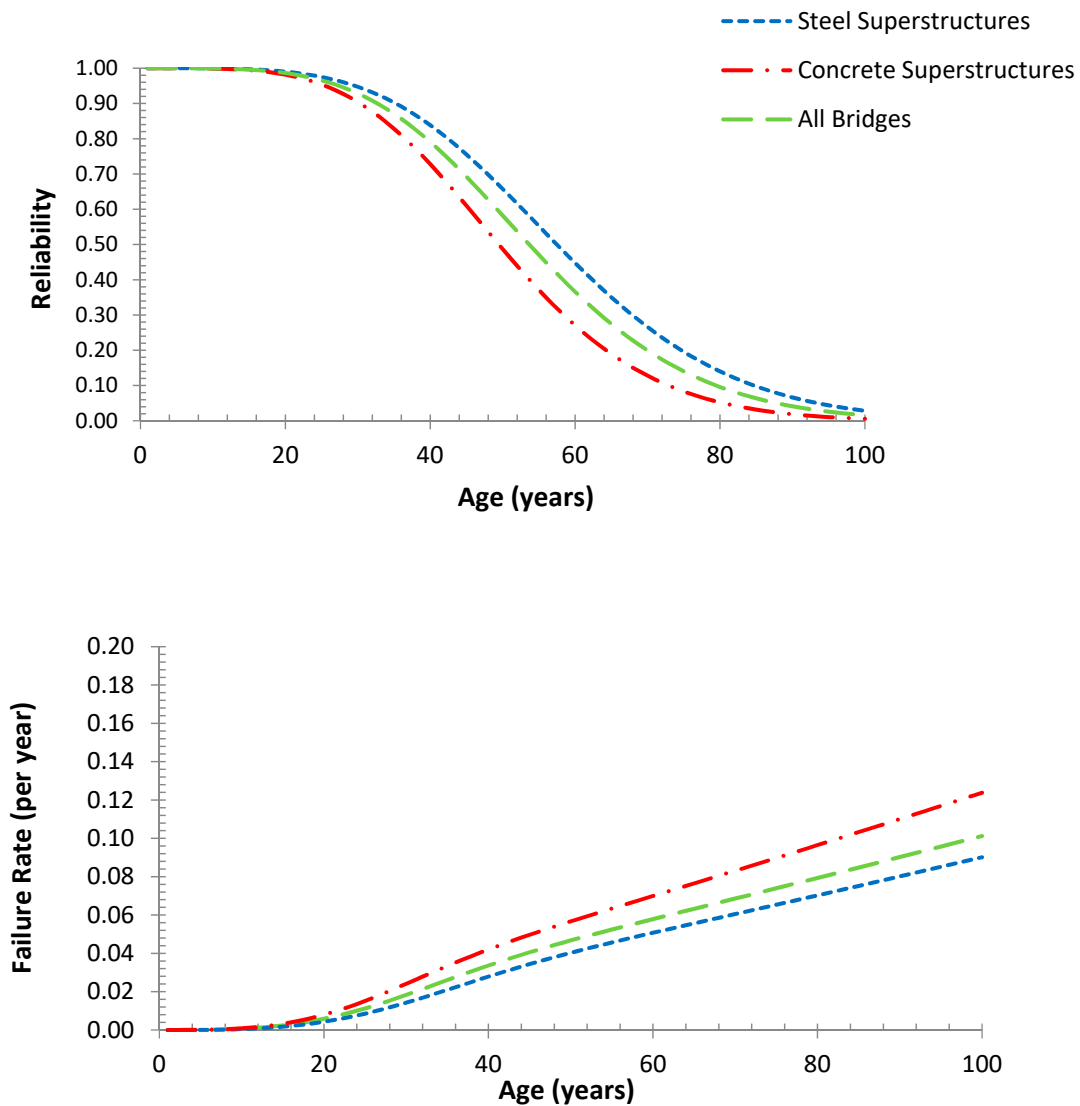
Parameter	Estimate	Standard Error	Wald	$p$ -Value
$\alpha$	$5.80 \times 10^{-04}$	$1.17 \times 10^{-04}$	24.68	$6.76 \times 10^{-07}$
$\beta$	$2.12 \times 10^{+00}$	$5.35 \times 10^{-02}$	1566.19	$1.63 \times 10^{-342}$
Type	$-1.50 \times 10^{-01}$	$2.25 \times 10^{-02}$	44.26	$2.88 \times 10^{-11}$
MSL	$3.75 \times 10^{-03}$	$6.61 \times 10^{-04}$	32.09	$1.47 \times 10^{-08}$
ADT	$6.28 \times 10^{-06}$	$1.16 \times 10^{-06}$	29.17	$6.62 \times 10^{-08}$

Using Equations (4) and (5), the developed Hypertabastic survival model can estimate the reliability and failure rates at any age as a function of Type, ADT and MSL. The estimated bridge age corresponding to a reliability of 0.5 (50%) at the end of service life for steel and concrete superstructures was 57 and 49 years, respectively, when mean values of covariates ADT and MSL were used in the equations. Table 6 shows percentiles for age at the end of service life when either mean or median values of covariates (ADT and MSL) were used for concrete and steel bridges.

**Table 6.** Deciles for estimated age at the end of service Life using Hypertabastic accelerated failure time model.

Percentile	Mean (ADT and MSL)		Median (ADT and MSL)	
	Concrete (years)	Steel (years)	Concrete (years)	Steel (years)
0.1	73	85	76	88
0.2	64	74	67	78
0.3	58	68	61	70
0.4	53	62	56	65
0.5	49	57	51	60
0.6	46	53	47	55
0.7	41	48	43	50
0.8	36	42	38	44
0.9	30	35	31	37

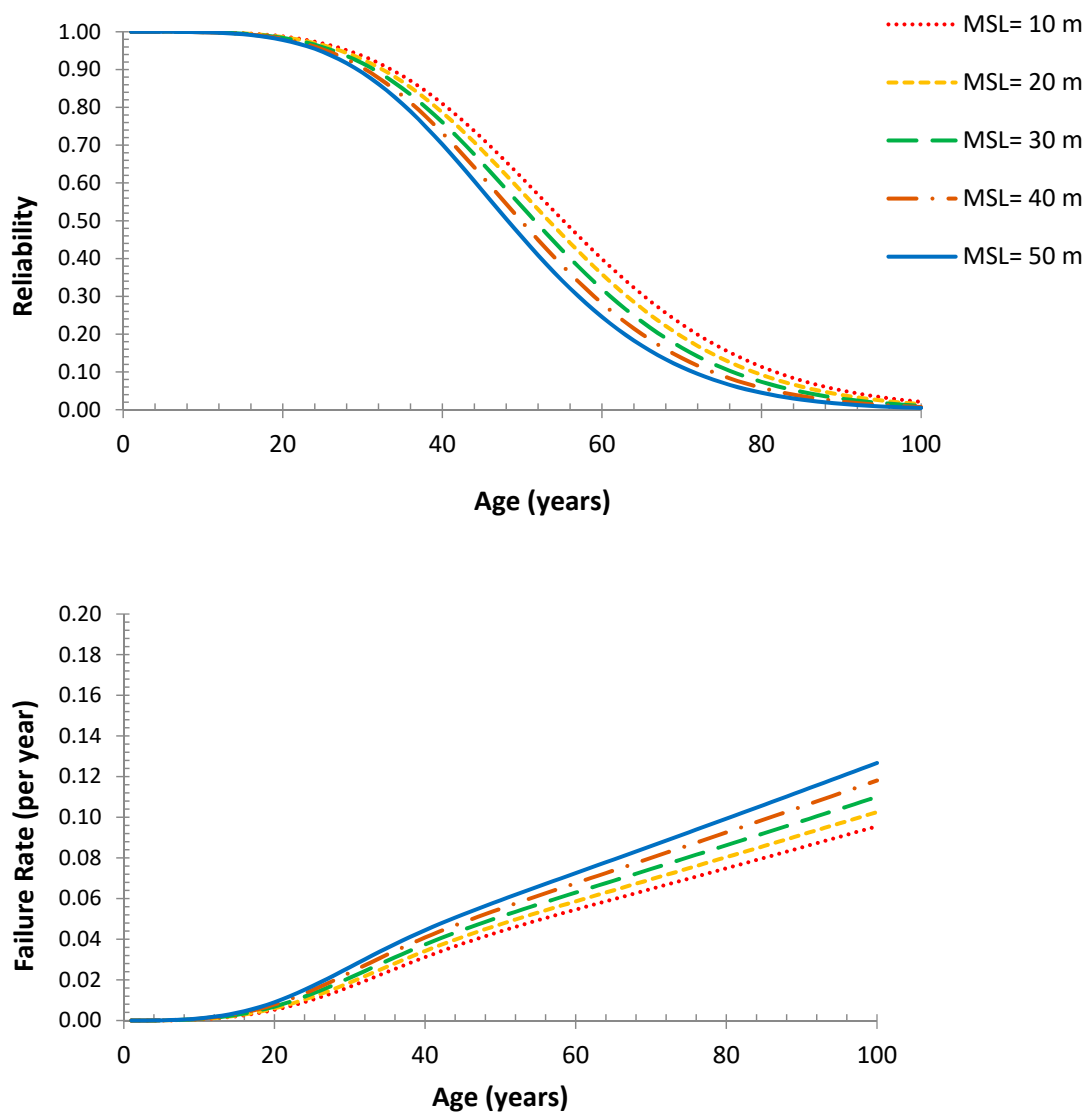
Plots of reliability and failure rate associated with steel, concrete, and combined superstructures are shown as a function of age in Figure 2. In these curves, the ADT and MSL are assumed to be at their mean values for Wisconsin (ADT = 4872 and MSL = 18.5 m).



**Figure 2.** Hypertabastic reliability and hazard rate curves for bridge superstructures with covariates at their mean values (ADT = 4872 and MSL = 18.5 m).

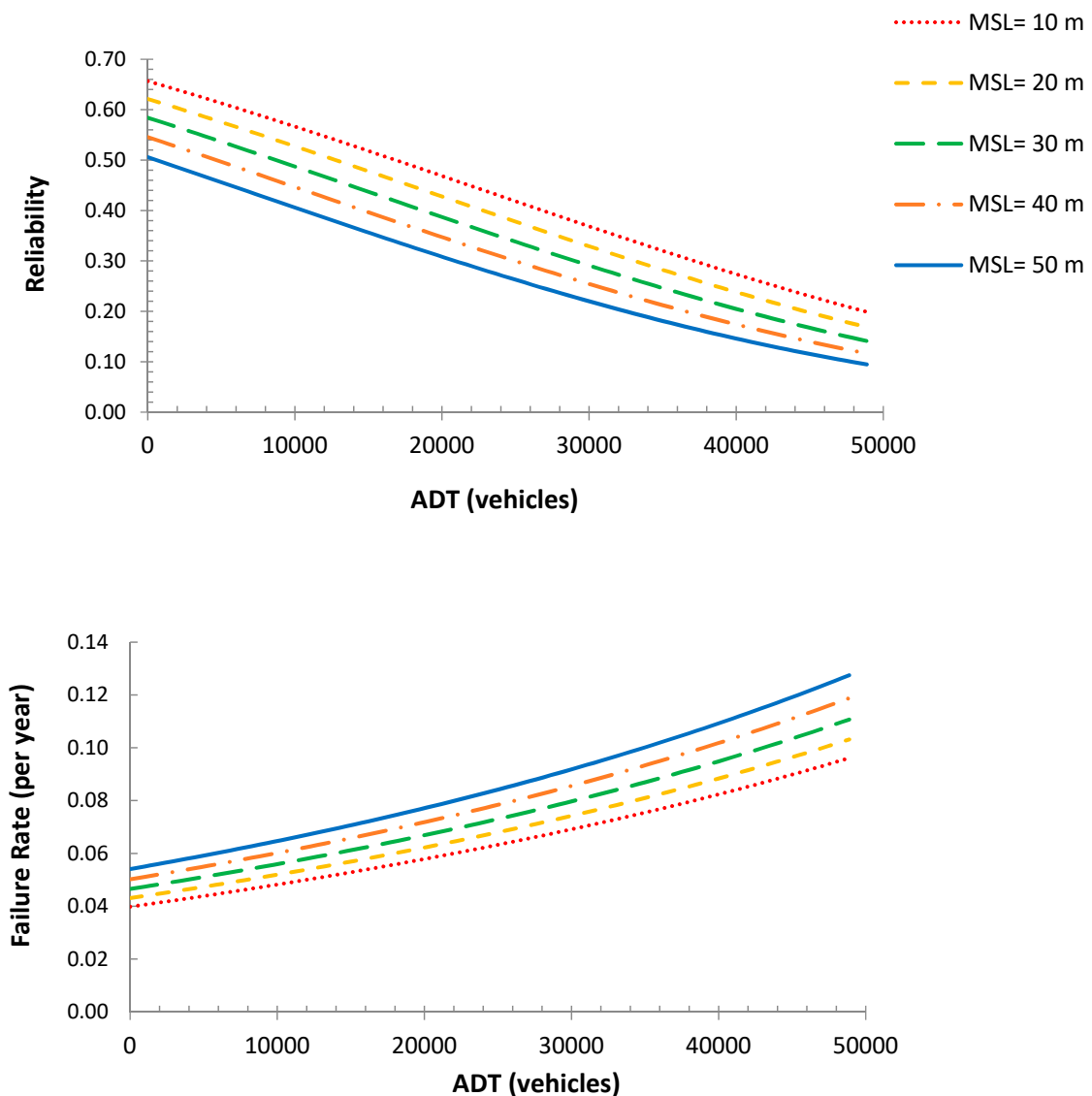
Based on the results shown in Figure 2, the reliability (survival) of steel superstructures is slightly higher than concrete superstructures in Wisconsin. Reliability starts from a value of 1 or 100% at the beginning of the service life and decreases as the age increases. At 75 years of age, the reliability of Wisconsin bridges with steel, concrete, and combined superstructures is 0.23, 0.11, and 0.18, respectively (when covariates are held at Wisconsin’s mean values). It should be noted that this observation does not necessarily indicate superiority of one system over the other. This comparison does not address initial and long-term costs, and has not been consistently preferential to one superstructure system in different states in a similar study on bridge deck reliability [20].

To investigate the effect of MSL on the reliability and failure rate of superstructures, various maximum span lengths ranging from 10 m to 50 m were analysed for all superstructure types while holding ADT at a constant level of 5000. Results are shown in Figure 3. As the maximum span length increases, the reliability of superstructure decreases, and the failure rate increases at a given age. As shown in Figure 3, the slope of failure rate remains approximately constant after the age of 40. This slope is estimated here as 0.0014 per year for MSL of 50 m and 0.0011 per year for MSL of 10 m. Figure 4 shows plots of reliability and failure rate as a function of ADT for steel and concrete superstructures at a bridge age of 50 years.



**Figure 3.** Superstructure reliability and hazard rate versus age at different maximum span lengths for ADT of 5000 (All superstructure types).

As expected, at the age of 50 years, superstructure reliability decreases rapidly as ADT increases. Similarly, increasing ADT increases the failure rate. Increasing ADT indicates increased frequency of load application. Higher ADT may also lead to an increase in deicing salt usage during the winter (chloride exposure). The reinforcing steel bars embedded in concrete may thus be at increased risk of chloride-induced corrosion. Figure 4 shows superstructure reliability curves that cover a wide range of values from 0.66 to 0.09 (at age 50), depending on the ADT and MSL.



**Figure 4.** Superstructure reliability and hazard rate versus ADT at different maximum span length at the age of 50 years (All superstructure types).

Figure 4 shows an approximately linear relationship for both reliability and failure rate as a function of ADT (with different MSL values) at the age of 50 years. The rate of decrease in superstructure reliability is approximately 0.8% to 0.9% per 1000 vehicles, and the rate of increase in the failure rate is roughly 0.12% to 0.16% per year per 1000 vehicles.

Figure 5 shows superstructure reliability and failure rates as a function of MSL at a bridge age of 50 years. The analysis was run at different ranges of ADT, and the superstructures types were not separated into steel and concrete. The rate of decrease in superstructure reliability is approximately 0.38% to 0.34% per meter of MSL, and the rate of increase in the failure rate is roughly 0.04% to 0.06% per year per meter of MSL.

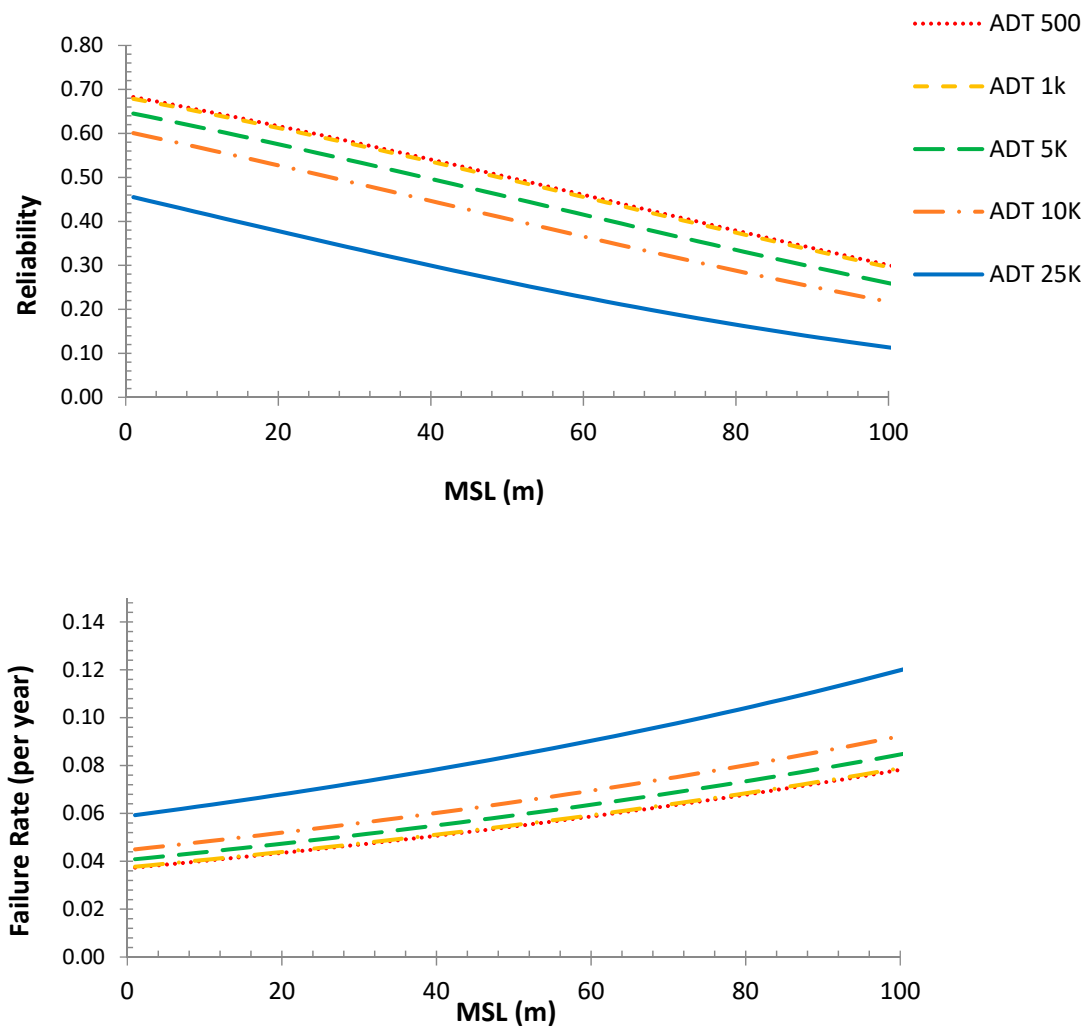


Figure 5. Superstructure reliability and hazard rate versus MSL with different ADT values at the age of 50 years (All superstructure types).

The superstructure reliability and failure rate are similarly examined at an age of 75 years. The analyses were done for different MSL and ADT values. The results are shown in Figures 6 and 7.

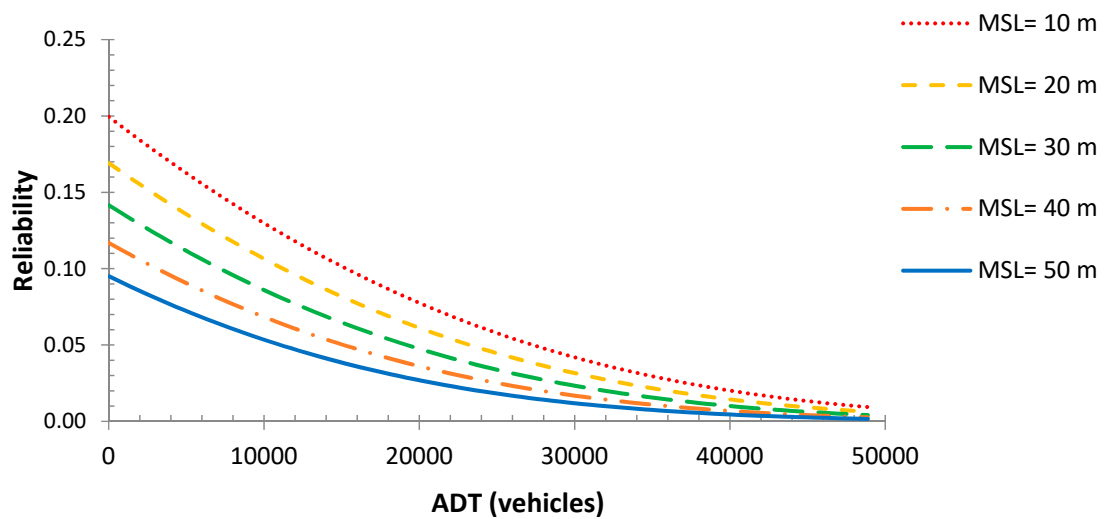


Figure 6. Cont.

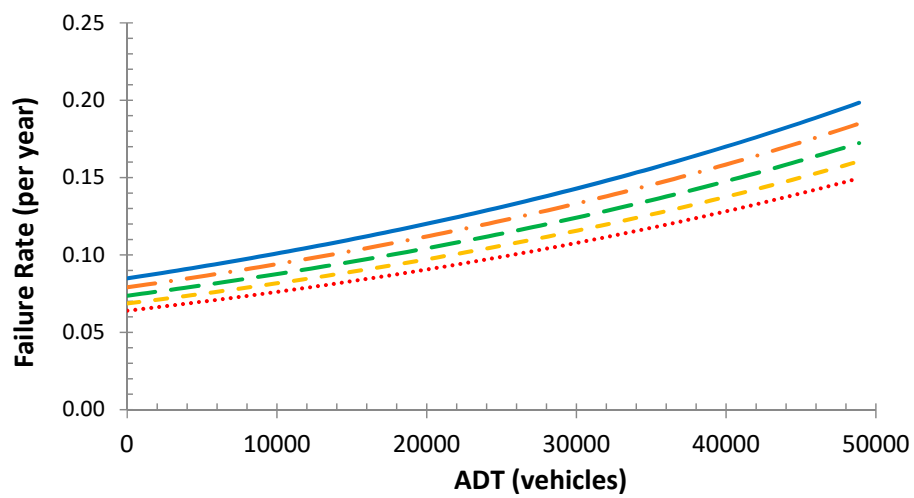


Figure 6. Superstructure reliability and hazard rate versus ADT at different MSL values at the age of 75 years (All superstructure types).

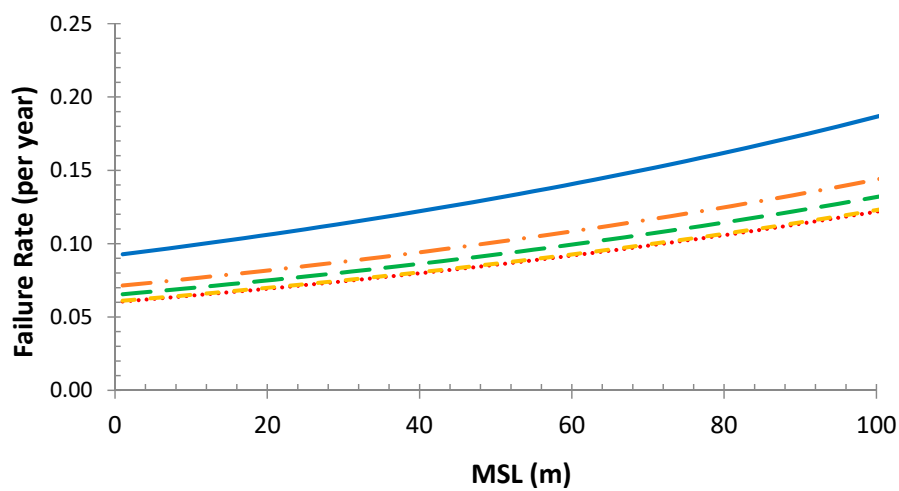
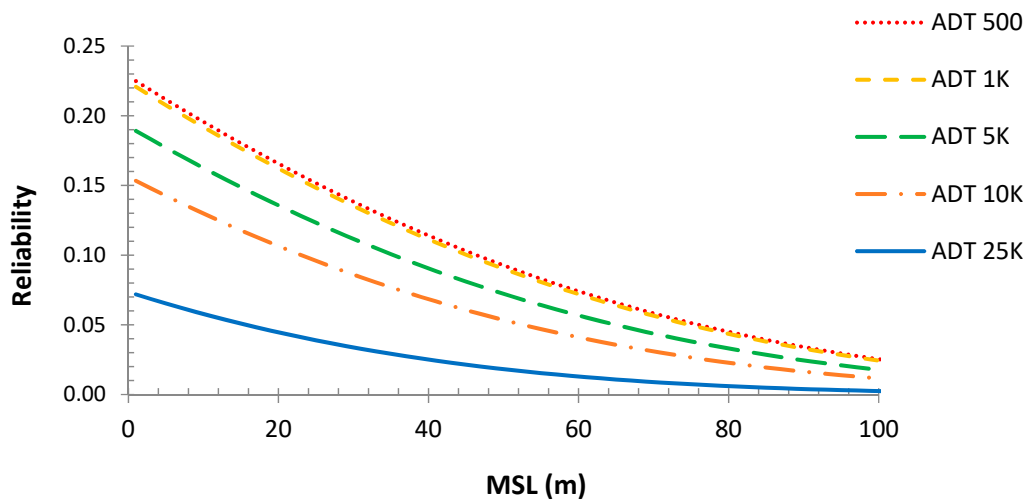


Figure 7. Superstructure reliability and hazard rate versus MSL with different ADT values at the age of 75 years (All superstructure types).

Comparison of results for 50- and 75-year-old bridge structures shows a substantially higher reliability for bridge superstructures at the age of 50 years. For example, the superstructure reliability at the age of 75 years for ADT of 1000 and MSL of 30 m is 14%, whereas the corresponding reliability value at the age of 50 years is 57%.

## 5. Summary and Conclusions

In this research, the Hypertabastic survival model was used to analyse the reliability and failure rates of bridge superstructures in Wisconsin. The 2012 NBI records were used for statistical analysis and parameter estimations for the survival model, considering a recorded superstructure rating of 5 as the end of service life.

As bridges age, the superstructure reliability decreases and failure rate increases. The failure rate is relatively small prior to age 20, after which the rate rises more rapidly. At ages older than 40, the superstructure failure rate increases are nearly linear with respect to age.

Results indicate that ADT and MSL both influence the bridge survival time and failure rate significantly. Increasing MSL for a given ADT has a significant negative effect on the superstructure reliability and failure rate. Similarly, increasing ADT at a fixed value of MSL results in a decreased superstructure reliability and an increased failure rate.

At age 50 years, the rate of decrease in superstructure reliability is approximately 0.8% to 0.9% per 1000 vehicles, and the rate of increase in the failure rate is roughly 0.12% per year per 1000 vehicles. Similar trends exist for the effect of increase in MSL on reliability and failure rate. However, variations at that age deviate from a linear response for higher ADT levels. The bridge reliability at the age of 75 was 18% (for mean values of covariates), showing relatively low reliability of bridges (from a serviceability standpoint) at the design service life. On the other hand, at an age of 50 years, the reliability of a Wisconsin bridge under average covariate conditions is on the order of 63%. This indicates that the American Association of State Highway and Transportation Officials (AASHTO) design service life of 75 years is not currently achieved by most non-rehabilitated Wisconsin bridges that are reaching the end of their service life (defined as a recorded NBI superstructure rating of 5).

Survival analyses of recorded data based on bridge element ratings can provide useful insight into probabilistic assessment of bridge service life. The results of this study can be further used as benchmarks to calibrate and verify current and future analytical deterioration models and their statistical simulation results.

## Notation

The following symbols are used in this paper:

ADT	Average Daily Traffic (vehicles per day)
$c$	Numerical coefficient for variable MSL
$d$	Numerical coefficient for variable ADT
$F(t)$	Cumulative distribution function
$f(t)$	Probability density function
$h(t)$	Hazard (failure rate) function
$h(t_g)$	Hypertabastic hazard (failure rate) function
MSL	Maximum Span Length (m)
$S(t)$	Survival (reliability) function
$S(t_g)$	Hypertabastic survival (reliability) function
Type	Type of bridge superstructure [steel (1), concrete (0)]
$t$	Time (years)
AGE	Age of bridge (years)
$t_g$	A function of AGE, MSL, ADT, Type, $c$ , $d$ , and $h$
$W(t)$	A function of $\alpha$ , $\beta$ , and $t$
$\alpha$	A positive constant
$\beta$	A positive constant

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