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# Three Essays on the Economics of Spatial Price Discrimination: Strategic Delegation Under Spatial Price Discrimination ; Consistent Location Conjectures Under Spatial Price Discrimination ; How to License a Transport Innovation

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THREE ESSAYS ON THE ECONOMICS OF SPATIAL PRICE DISCRIMINATION:

I) STRATEGIC DELEGATION UNDER SPATIAL PRICE DISCRIMINATION

II) CONSISTENT LOCATION CONJECTURES UNDER SPATIAL PRICE DISCRIMINATION

III) HOW TO LICENSE A TRANSPORT INNOVATION

by

Zheng Wang

A Dissertation Submitted in

Partial Fulfillment of the

Requirements for the Degree of

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## ABSTRACT

### THREE ESSAYS ON THE ECONOMICS OF SPATIAL PRICE DISCRIMINATION:

I) STRATEGIC DELEGATION UNDER SPATIAL PRICE DISCRIMINATION

II) CONSISTENT LOCATION CONJECTURES UNDER SPATIAL PRICE DISCRIMINATION

III) HOW TO LICENSE A TRANSPORT INNOVATION

by

Zheng Wang

The University of Wisconsin – Milwaukee, 2014  
Under the Supervision of Professor John S. Heywood

This dissertation consists of three papers that use models of spatial price discrimination to explore issues of long-standing interest in microeconomics. The first essay introduces strategic delegation into the traditional model of spatial price discrimination. In both simultaneous and sequential location cases, delegating location choices to managers causes firms to move toward each other. This movement typically reduces social welfare. While exceptions exist for high cost convexity and for some cases of elastic demand, this reduction reverses the increase in welfare associated with delegation in common quantity games outside the spatial context. The second paper uniquely explores consistent location conjectures in a model of spatial price discrimination. With linear production cost, firms locate too close to the center resulting in reduced social welfare relative to Nash conjectures. Thus, the frequent association of spatial price discrimination with efficiency vanishes when firms anticipate their rival's response. With quadratic production cost, the

firms continue to locate closer to the center but the degree of convexity determines whether or not welfare increases or decreases relative to Nash. The third paper identifies the optimal method to license an innovation that reduces transport cost in a model of duopolists engaging in spatial price discrimination. An inside innovator finds licensing by either a typical fixed fee or an output royalty to be unprofitable. Instead, a fee based on distance is shown to be a profitable option. An outside innovator finds the fixed fee more profitable than either a royalty or distance fee. It will license to either one or both firms and when it does license to both firms, it exploits a prisoner's dilemma between the duopolists in order to license an innovation that reduces their profit.

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THREE ESSAYS ON THE ECONOMICS OF SPATIAL PRICE DISCRIMINATION:

I) STRATEGIC DELEGATION UNDER SPATIAL PRICE DISCRIMINATION

## 1. Introduction

The separation between ownership and control makes delegation, empowering someone to act on behalf of another, the hallmark of the modern corporation. Yet, economists view delegation in two very distinct fashions. First, it stands as an expression of agency problems in which monitoring, incentives, and control systems are needed to align the interests of the principal and agent (see Gibbons 2005 and Prendergast 1999). Second, it represents an opportunity to commit to competitive strategies that are in the interest of the principal, but which would not be credible to rivals without delegation. In this second view, delegation profitably *stops* the alignment of the interests of the principal and agent. In the classic case, the owner commits to an incentive contract that rewards a manager for output, sales, or market share, and thereby creates a more aggressive manager. As this manager responds to the incentive contract by producing more, it causes rivals to reduce their output, and can ultimately increase the owner's profit (Vickers 1985; Fershtman and Judd 1987; Sklivas 1987). While illustrations in the literature on the use of strategic delegation are enormous, its basic insights have not been applied to the standard model of spatial price discrimination.<sup>1</sup> As we make clear, such an application is fruitful, and provides predictions that would not typically be drawn from the literature outside the spatial context.

Examining delegation in spatial models is warranted because such models have proven especially useful in capturing aspects of competition in horizontally differentiated markets as emphasized by Tirole (1988). Specifically, they provide a general method for examining markets in which either actual location or an ordered product characteristic differentiates output. Thus, airline flights between city pairs differ by departure time from

early morning to late evening, the editorial policies of newspapers differ from liberal left to conservative right, and breakfast cereals differ in their sugar content. Among the more popular spatial models is that of spatial price discrimination, in which the price a firm is able to charge depends upon how "close" it is to its rivals. Thisse and Vives (1988) show that such pricing is the preferred alternative when firms are able to adopt it, and Greenhut (1981) identifies it as "nearly ubiquitous" among actual markets with substantial freight costs. Thus, our study of delegation in the context of spatial price discrimination has salience as well as being a sensible extension of the existing literature.

In the classic application outside the spatial context, there exists a prisoner's dilemma in which each owner has a competitive incentive to create more aggressive managers, but all owners suffer reduced profits (and enhanced social welfare) when all managers are aggressive. Thus, each owner rewards his manager for output, and as a consequence, total market output increases. In this way, the choice of strategic delegation becomes part of a subgame perfect equilibrium that improves social welfare relative to a simultaneous quantity game without delegation. We will show that this is typically not the result in spatial price discrimination. Although delegation and more aggressive managers result as part of the equilibrium, the consequence is often diminished social welfare.

There have been several examinations of delegation in the spatial context but none consider spatial price discrimination in which delivery is included in the price. Instead, they assume Hotelling type models with mill pricing. Matsumura and Matsushima (2012) imagine delegating both location and price choices in a setting analogous to zoning. They imagine Hotelling competition with a uniform market on a

line segment, and consider a policy that either prohibits or allows duopolists to locate outside the market. Under delegation, the prohibition diminishes consumer welfare but the locations themselves (both with and without the prohibition) are actually unchanged by delegation. As will be seen, locations change dramatically with delegation in our model of spatial price discrimination.

In addition to this examination of “complete delegation,” other researchers explore “partial delegation.” Liang et al. (2011) model delegation in an “uncovered” Hotelling model.<sup>2</sup> They assume owners adopt locations in a first stage, and then set a delegation contract for managers who compete in either price or quantity in a second stage. They show that the form of second stage competition does not influence location. Barcena-Ruiz and Casado-Izaga (2005) imagine owners delegate the price decision but can decide whether or not to delegate location decisions in the final stage. In equilibrium, neither firm delegates the location decision. As price choices are strategic complements to managers, the firms locate farther away from each other than without delegation. Zhao (2012) imagines three choice variables: location, R&D, and price. He assumes that owners retain the location choice but are able to delegate R&D investment and price. In equilibrium, the partial delegation involves delegating only the price decision to the managers. Each owner locates his firm farther away from his rival to increase the product variety, and the level of R&D investment and price are higher.

In the context of spatial price discrimination, we follow the early tradition of examining a delegation contract that rewards quantity. Yet, with the typical assumptions that the entire market is served and that demand is inelastic, a one-to-one correspondence exists between rewarding output and rewarding market share (Jansen et al. 2007; Ritz

2008). This may be reassuring because market share often stands as an explicit objective in the performance standards generated by compensation committees of boards of directors (Borkowski 1999).

In what follows, we start with two firms engaging in spatial price discrimination with linear production costs and simultaneous location choices on a linear market with inelastic demand. We show that delegation of location is adopted in equilibrium. This adoption causes the firms to locate closer together, and as a consequence, it increases social cost. We then modify this basic case along a series of dimensions. First, we allow for sequential location decisions. Following the literature outside the spatial context, we show that only the second mover has an incentive to delegate, but again delegation increases social cost. Second, we retain two firms with simultaneous location choices but assume convex production costs. This case is important, as Gupta (1994) shows that spatial price discrimination with convex costs generates inefficient locations even without delegation, as each firm tries to give away its high cost marginal customer to its rival. We show that allowing delegation in this case can either improve or hurt welfare depending upon the degree of cost convexity. Third, we return to the basic case, but generalize to  $n$  firms, showing that the basic insight remains with welfare being diminished. Finally, we consider elastic demand in a duopoly. Delegation continues to cause the firms to move toward each other and harms welfare as long as transport costs are not large. Thus, the consequences of delegation under spatial price discrimination depend on the particular assumptions, but we emphasize that they are clearly not those that would be anticipated from the literature on delegating quantity decisions in typical oligopolies.

In the next section, we set up the basic duopoly model of spatial price discrimination with linear production costs and delegation. We next derive the subgame perfect equilibrium, and compare to the case without the possibility of delegation. Subsequent sections consider each of the extensions isolated above, and a final section concludes the paper by suggesting avenues for further research.

## 2. Solving for Equilibrium in the Basic Model

In the basic model, two firms engage in spatial price discrimination, and the game is structured with four potential stages. In the first stage, the owner decides whether or not to delegate the location choice to a manager by providing an incentive contract. If they decide to do so, the terms of the incentive contract are simultaneously chosen in the second stage. We imagine owners provide an incentive contract that has convex weights on profit and output. Given the chosen incentive weights, managers adopt locations in the third stage and the price schedule is determined in the final stage given the locations. The subgame perfect equilibrium is determined by backward induction. Thus, our model represents a case of partial delegation because we retain spatial price discrimination rather than delegating price schedules as a strategic variable.<sup>3</sup>

To show the backward induction, we first describe the nature of the pricing equilibrium given locations. We imagine consumers are uniformly distributed along a unit line segment. Each consumer has inelastic demand for one unit of the good, with reservation price  $r$ . Two firms compete with a common constant marginal cost of production which we normalize to zero without loss of generality. Transport cost is  $t$  per unit of distance.  $L_1$  and  $L_2$  are firm locations with  $L_1 \leq L_2$ . The equilibrium delivered



price schedule for any consumer located at  $x$  is  $p^*(x, L_1, L_2) = \max \{t(L_2 - x), t(x - L_1)\}$ . As shown in Figure 1, the bold envelope represents the price schedule. It is the upper envelope of the rival's transport cost. By convention, the indifferent customer (faces the same delivered price from either firm) can purchase from either firm. That customer has location  $x^* = \frac{(L_1 + L_2)}{2}$  and defines the output of firm 1 as  $q_1 = x^*$  and of firm 2 as  $q_2 = 1 - x^*$ .

<Insert Figure 1 about here>

Given the pricing equilibrium, the profits of the two firms follow as:

$$\pi_1 = \int_0^{L_1} t(L_2 - x) - t(L_1 - x) dx + \int_{L_1}^{x^*} t(L_2 - x) - t(x - L_1) dx \quad (1)$$

$$\pi_2 = \int_{x^*}^{L_2} t(x - L_1) - t(L_2 - x) dx + \int_{L_2}^1 t(x - L_1) - t(x - L_2) dx \quad (2)$$

and these are identified in Figure 1. Note that for any given locations, owners would not delegate the pricing decision to managers.<sup>4</sup>

Managers determine locations under delegation and they do so to maximize their respective incentive contracts. Managers are paid based on incentive contracts that retain weight  $\alpha$  on firm profit and weight  $(1 - \alpha)$  on firm's output, with  $0 < \alpha \leq 1$ :

$$I_1 = \alpha_1 \pi_1 + (1 - \alpha_1) q_1 \quad (3)$$

$$I_2 = \alpha_2 \pi_2 + (1 - \alpha_2) q_2 \quad (4)$$

Thus, (1) and (2) are substituted into (3) and (4) and each manager maximizes their earnings (incentive contract) with respect to their location. This yields two best response functions in which the location of one firm depends upon the location of the other firm:

$$L_1 = (1 + \alpha_1 t L_2 - \alpha_1) / 3\alpha_1 t \quad (5)$$

$$L_2 = (-1 + \alpha_2 t L_1 + 2\alpha_2 t + \alpha_2) / 3\alpha_2 t \quad (6)$$

Solving simultaneously yields the optimal locations:

$$L_1 = (3\alpha_2 - \alpha_1 + 2\alpha_1\alpha_2 t - 2\alpha_1\alpha_2) / 8\alpha_1\alpha_2 t \quad (7)$$

$$L_2 = (\alpha_2 - 3\alpha_1 + 6\alpha_1\alpha_2 t + 2\alpha_1\alpha_2) / 8\alpha_1\alpha_2 t \quad (8)$$

We note that  $\frac{\partial L_1}{\partial t} < 0$  and  $\frac{\partial L_2}{\partial t} > 0$  for any  $\alpha_1 = \alpha_2 < 1$ . These movements toward the corners reflect the change that  $t$  makes in managers balancing the loss in profit and the gain in output caused by the incentive parameter. For given locations, as  $t$  increases, profit increases meaning that for a given incentive parameter movement toward the middle now entails a greater reduction in profit for a given increase in output.<sup>5</sup> Thus, all else equal, managers do not move as far toward the middle when  $t$  is large.

Given these locations, owners determine the optimal incentive parameters  $\alpha_1$  and  $\alpha_2$ . Plugging equations (7) and (8) into equations (1) and (2) each owner maximizes his own profit with respect to his manager's incentive parameter. This yields two best response functions of one owner's incentive parameter as a function of the other owner's parameter. Solving the response functions simultaneously yields the equilibrium incentive parameters:

$$\alpha_1 = \alpha_2 = 3/(3 + t) \quad (9)$$

The incentive parameters on profit decrease as  $t$  increases. Owners who want managers to be more aggressive and locate toward the middle, know that smaller weights on profit are required when  $t$  is large. The larger  $t$  increases the size of profit and otherwise makes managers less willing to move toward the center as described above. The owner responds by reducing the weight on profit.

Returning (9) to (7) and (8) yields the equilibrium locations at the thirds:

$$L_1 = .333 \text{ and } L_2 = .667 \quad (10)$$

Although the incentive parameters depend on  $t$ , the firms' locations do not depend on  $t$ . This happens because  $t$  cancels out as a result of the linearity of transport and production costs and the symmetry of the problem. In essence, the owner wishes his manager to seek an optimal location, and adjusts the incentive parameter to achieve this location.

Returning (10) to (1) and (2) yields:

$$\pi_1 = \pi_2 = .139t \quad (11)$$

Thus, (9), (10), and (11) characterize the equilibrium on the assumption that each owner adopts delegation if the choice is available. We now show that such adoption will occur.

**Proposition 1:** When delegation is available, it will be chosen and the equilibrium will be as identified in (9), (10), and (11).

**Proof:** The payoffs for the choices of each firm are shown in Table 1.

<Table 1 about here>

The profit for both owners delegating is in (11), and that for both choosing no delegation comes from Hurter and Lederer (1985) and equals  $\pi_i = .188t$ . The off diagonal terms have only one firm adopting delegation and are symmetric. Their derivation is presented in Appendix 1. Delegating is the dominant strategy.

Given that the delegation game will emerge in equilibrium, we compare it to the traditional equilibrium without delegation. While profits and locations can be immediately compared, social welfare ( $SW$ ) follows as the difference between total willingness to pay and the transport cost ( $TC$ ):

$$SW = r - TC \quad \text{where} \quad TC = \frac{1}{2}t(L_1)^2 + \frac{1}{4}t(L_2 - L_1)^2 + \frac{1}{2}t(1 - L_2)^2 \quad (12)$$

With this, we summarize the comparison.

**Proposition 2:** With delegation, i) locations are closer to the center, ii) profits are lower, and iii) social welfare is lower than without delegation.

**Proof:** The equilibrium in (9), (10), and (11) is compared to that without delegation.

i) Without delegation,  $L'_1 = .25$  and  $L'_2 = .75$  (Hurter and Lederer 1985). Thus,

$$(L_2 - L_1) - (L'_2 - L'_1) = -.167 < 0.$$

ii)  $\pi_i - \pi'_i = .139t - .188t = -.049t < 0$ .

iii) From (12),  $SW(L_1 = .333, L_2 = .667) - SW(L'_1 = .25, L'_2 = .75) = (r - .139t) - (r - .125t) < 0$ .

Our presentation reproduces much of the logic from outside the spatial context but with rather different outcomes. There remains an inherent prisoner's dilemma at the heart of our delegation game. Each owner benefits from unilaterally providing an incentive for his manager to locate aggressively but each owner suffers losses when both managers receive such incentives. Under delegation, each manager moves toward the middle under delegation in an effort to capture output. Moreover, just as incentive contracts need not alter the market shares outside the spatial context, our two firms continue to split the market evenly even with incentive contracts that serve to move the two firms toward each other. Yet, the typical assumption of inelastic demand in our presentation means that the ability to delegate actually wastes resources. Delegation results in an 11% increase in transport costs. Thus, while a social planner prefers delegation in a typical quantity game outside the spatial context, the planner would prefer its absence in a market characterized by spatial price discrimination.<sup>6</sup>

### **3. First Extension: Sequential Location**

In this extension, we retain the set-up of the basic model but imagine that locations choices are sequential rather than simultaneous. Sequential location can be seen as the analog to Stackelberg leadership in which one firm commits earlier to an output (Neven 1987). This early commitment typically results in the leader producing more than the follower and in total output and so social welfare being higher than in the simultaneous Cournot model.<sup>7</sup> Yet, sequential locations in spatial price discrimination deviate substantially from the first best locations associated with simultaneous choice and so are associated with lower social welfare. Gupta (1992) shows that in duopoly, the

location leader locates at .4 and the follower at .8. This opens the possibility that delegation may either improve or harm welfare in the context of spatial price discrimination.

The derivation remains the same except that within the location subgame firm 1 now locates first. Thus, the manager of firm 2 maximizes (4) with respect to  $L_2$  knowing  $L_1$  the location of firm 1. The best response function is:

$$L_2 = \frac{-1 + \alpha_2 t L_1 + 2\alpha_2 t + \alpha_2}{3\alpha_2 t} \quad (13)$$

firm 1's manager returns this best response function to (3) and maximizes with respect to  $L_1$ . Firm 1's location is:

$$L_1 = (3\alpha_2 - \alpha_1 + 2\alpha_1\alpha_2 t - 2\alpha_1\alpha_2)/5\alpha_2\alpha_1 t \quad (14)$$

Returning equation (14) to equation (13), firm 2's location in terms of the incentive parameters is:

$$L_2 = (-2\alpha_1 + \alpha_2 + 4\alpha_1\alpha_2 t + \alpha_1\alpha_2)/5\alpha_1\alpha_2 t \quad (15)$$

Given equations (14) and (15), each owner maximizes profit with respect to their own incentive parameters. This yields two best response functions of one firm's incentive parameter as function of the other. Solving simultaneously yields the equilibrium incentive parameters:

$$\alpha_1 = 1 \text{ and } \alpha_2 = 7/(7 + 4t) \quad (16)$$

Note that the location leader ultimately does not delegate and simply maximizes profit. This result carries over from the Stackelberg game in which the leader also never has an incentive to delegate (see Kopel and Löffler 2008 and Lekeas and Stamatopoulos 2011). In our case, the leader knows the complete best response function of the follower including exactly how far the follower will retreat in the face of its advances. As a consequence, it has no incentive to tell its manager to locate any closer to the middle than implied by profit maximization.<sup>8</sup>

Note that the follower's incentive parameter gets smaller (more emphasis is placed on output) as  $t$  increases. As in the simultaneous case, as  $t$  increases the follower earns more profit and so its manager is more reluctant to move toward the middle. This can be overcome only by the owner placing more emphasis on output.<sup>9</sup>

Returning equation (16) to equations (14) and (15) yields:

$$L_1 = .286 \text{ and } L_2 = .571 \quad (17)$$

Again the linearity and symmetry of the problem causes  $t$  to cancel yielding unique locations. Returning these to equations (1) and (2) yields:

$$\pi_1 = .102t \text{ and } \pi_2 = .143t \quad (18)$$

As in the basic model, the equilibrium characterized by (16), (17), and (18) will be that adopted by the two firms if given a choice over the ability to delegate.<sup>10</sup>

This allows us to identify the consequences of delegation with sequential location.

**Proposition 3:** When compared to the simultaneous location model, sequential location choice results in locations that i) provide greater output to the follower, ii) have lower total profit, and iii) have lower social welfare.

**Proof:**

- i) Given (17),  $q_1 = .429$  and  $q_2 = .571$  rather than  $q_1 = q_2 = 0.5$  as in (10).
- ii) Subtracting the sum of profits in (11) from that in (18) yields  $-.033t < 0$ .
- iii) From (12),  $SW(L_1 = .286, L_2 = .571) - SW(L_1 = .333, L_2 = .667) = (r - .153t) - (r - .139t) < 0$ .

Allowing sequential location lowers profit and welfare while causing the follower to earn more than the leader. While the disadvantage for the leader also happens under delegation in a Stackelberg game, the other consequences differ. In the Stackelberg game, total output and social welfare are larger than in the Cournot game with delegation (Lekeas and Stamatopoulos 2011). In spatial price discrimination, location leadership lowers welfare. Thus, the social planner prefers leadership in a quantity game with delegation but prefers simultaneous choice in a location game with delegation.

We now examine the implication of delegation given sequential location choices.

**Proposition 4:** Allowing delegation in a model of sequential location, i) causes the follower to produce more than the leader, ii) reduces total profit, and iii) reduces social welfare.

**Proof:**

- i) Without delegation  $L_1 = .4$  and  $L_2 = .8$  (Gupta 1992) and  $q_1 - q_2 = .2 > 0$ . With delegation (17) implies that  $q_1 - q_2 = -.143 < 0$ .



- ii) Subtracting the sum of profits given  $L_1 = .4$  and  $L_2 = .8$  from (18) yields  $-.075t < 0$ .
- iii) From (12),  $SW(L_1 = .286, L_2 = .571) - SW(L_1 = .4, L_2 = .8) = (r - .153t) - (r - .14t) < 0$ .

The critical insight is that delegation allows the follower to commit to behavior that would not make sense in a simultaneous location game. Thus, as the follower puts more and more emphasis on output, it is telling its manager to locate increasingly close to the market center regardless of the location of the leader. This commitment gets built into the profit maximizing behavior of the leader who backs further away from the middle knowing the follower will be close to it. This yields the asymmetric locations and large advantage to the follower.

Comparisons to the typical Stackelberg model are again valuable. In such a model allowing delegation also causes the outputs of the leader and follower to reverse such that the follower produces more than the leader. Delegation also causes total profits to be reduced. Nonetheless, the critical social cost comparison differs. In the Stackelberg model, social welfare is higher when delegation is available. Allowing leadership in our location game generates an aggressive strategy by the follower's owner that causes greater asymmetry and lower social welfare. This is important as even absent delegation, sequential entry generated highly asymmetric and wasteful locations. Adding leadership lowers social welfare even further.

It is worth noting that while we have followed the literature modeling delegation in a Stackelberg quantity game (Kopel and Löffler 2008; Lekeas and Stamatopoulos 2011), other timings may be reasonable. The critical point is that time passes between any firm's delegation and location decision. Thus, one alternative timing could have firm 1 set

its incentive parameter, firm 2 then set its incentive parameter and then the sequential locations are chosen. We briefly consider this timing.

The profit functions and the incentive contracts are as in (1), (2), (3), and (4).

With backward induction, we first solve the sequential location game and so (14) and (15) remain the same. The owner of firm 2 maximizes profit with respect to  $\alpha_2$  yielding

$$\alpha_2 = \frac{7\alpha_1}{-4+4t\alpha_1+11\alpha_1}. \text{ This is returned to (14) and (15) and placed in the profit function of}$$

firm 1. Maximizing, the owner of firm 1 sets  $\alpha_1 = \frac{9}{9+2t}$  yielding  $\alpha_2 = \frac{9}{9+4t}$ ,  $L_1 = 0.44$ ,

$L_2 = 0.67$ ,  $\pi_1 = 0.11t$ , and  $\pi_2 = 0.086t$ . In this case, firm 1 retains a leadership

advantage as it anticipates firm 2's best response in both the delegation stage and the

location game. Yet, in this timing firm 2 retains the ability to commit through delegation

and the equilibrium in some ways mirrors the simultaneous location game with the firms

ultimately moving toward each other.

A second alternative timing could have firm 1 adopting both its incentive

parameter and location before firm 2 adopts an incentive parameter. Clearly, there is no

ability for firm to commit through delegating. Returning (13) to (2) yields firm 2's profit

as a function of  $\alpha_2$ ,  $L_1$ , and  $t$  and the owner of firm 2 maximizes profit by setting  $\alpha_2 = 1$ .

As a consequence, the owner of firm 1 has no incentive to have his manager locate more

aggressively than implied by profit maximization. The equilibrium results are thus

identical to those without delegation (Gupta 1992):  $L_1 = 0.4$ ,  $L_2 = 0.8$ ,  $\pi_1 = 0.2t$ , and

$\pi_2 = 0.12t$ .

As this discussion shows, the extent to which firm 2 can use delegation to

credibly commit to aggressive behavior depends on the exact timing of the game. Our

propositions focus on the case analogous to that examined in the Stackelberg game and

that which allows the greatest ability to commit. While we think it is one possible timing, we recognize that others could also be reasonable.

#### 4. Second Extension: Convex Production Cost

We now return to the assumption of simultaneous location from the basic model but substitute linear production costs with convex production costs. Such a substitution is potentially important as Gupta (1994) shows that under spatial price discrimination, firms with convex production costs locate inefficiently. Convex costs generate an incentive for the firms to move toward their respective corners in order to yield customers to their rival. This movement increases the rival's marginal cost and so their delivered price. In turn, the increased rival's price increases the profit earned on the infra-marginal customers retained by the original firm. Yet, in equilibrium each firm behaves this way causing the locations to remain symmetric but outside the efficient quartiles. As we have shown that delegation generates more aggressive managers that move toward each other, assuming convex production costs might provide a case of delegation improving welfare, reversing the demonstrations of the earlier sections.

We follow Gupta (1994) assuming each firm's delivered price is the sum of marginal production cost plus transport cost and we adopt quadratic production costs  $C_i = \frac{1}{2}kq_i^2$  for  $i=1,2$ . As a consequence, the equilibrium delivered price schedule for any consumer located at  $x$  is  $p^*(x, L_1, L_2) = \max(\{t|x - L_1| + kx^*\}, \{t|L_2 - x| + k(1 - x^*)\})$ , where  $x^*$  still represents firm 1's output and  $1 - x^*$  is firm 2's output. The price schedule is the sum of rival's transport cost and marginal production cost. Utilizing the price charged to the indifferent consumer, it follows that firm 1's output is  $q_1 = x^* =$

$\frac{k+t(L_1+L_2)}{2(k+t)}$  and so potentially depends on the degree of convexity,  $k$ .<sup>11</sup> The resulting

profits functions are:

$$\pi_1 = \int_0^{L_1} t(L_2 - x) + k(1 - x^*) - t(L_1 - x) dx + \int_{L_1}^{x^*} t(L_2 - x) + k(1 - x^*) - t(x - L_1) dx - \frac{1}{2}k(x^*)^2 \quad (19)$$

$$\pi_2 = \int_{x^*}^{L_2} t(x - L_1) + kx^* - t(L_2 - x) dx + \int_{L_2}^1 t(x - L_1) + kx^* - t(x - L_2) dx - \frac{1}{2}k(1 - x^*)^2 \quad (20)$$

Placing (19) and (20) into (3) and (4), each manager maximizes his incentive contract with respect to his location. Simultaneously solving the resulting best response functions yields:

$$L_1 = (4\alpha_2 k^3 \alpha_1 + 8\alpha_2 k^2 - 8k^2 \alpha_2 \alpha_1 + 15\alpha_1 k^2 t \alpha_2 - 14k\alpha_1 t \alpha_2 + 16\alpha_1 k t^2 \alpha_2 - \alpha_1 k t + 15\alpha_2 k t + 6\alpha_2 t^2 - 4\alpha_1 t^2 \alpha_2 + 4\alpha_1 t^3 \alpha_2 - 2\alpha_1 t^2) / 8\alpha_2 \alpha_1 (2t^3 + 9kt^2 + 4k^3 + 11k^2 t) \quad (21)$$

$$L_2 = (28\alpha_2 k^3 \alpha_1 + 73\alpha_1 k^2 t \alpha_2 + 8k^2 \alpha_2 \alpha_1 - 8\alpha_1 k^2 - 15k\alpha_1 t + 56\alpha_1 k t^2 \alpha_2 + 14\alpha_1 k t \alpha_2 + \alpha_2 k t + 2\alpha_2 t^2 - 6\alpha_1 t^2 + 12\alpha_1 t^3 \alpha_2 + 4\alpha_1 t^2 \alpha_2) / 8\alpha_2 \alpha_1 (2t^3 + 9kt^2 + 4k^3 + 11k^2 t) \quad (22)$$

Given (21) and (22), each owner maximizes profit given the incentive parameter of his rival. This generates two best response functions in terms of the incentive parameters that when solved simultaneously yield:

$$\alpha_1 = \alpha_2 = 2(8k^2 + 15kt + 6t^2) / (16k^2 + k^2 t + 4kt^2 + 30kt + 4t^3 + 12t^2) \quad (23)$$

Note that the incentive parameters with linear costs (9) are recovered when  $k=0$ .

Returning (23) to (21) and (22) yields:

$$L_1 = (k^2 + 3kt + 2t^2) / (8k^2 + 15kt + 6t^2) \quad (24)$$

$$L_2 = (7k^2 + 12kt + 4t^2)/(8k^2 + 15kt + 6t^2) \quad (25)$$

Again, (24) and (25) generalize the linear case such that if  $k=0$ , we recover the locations at the thirds, .333 and .667. Moreover, if we set the values of (24) and (25) to the efficient locations of .25 and .75 and solve for  $k$ , it emerges that  $k=.425t$ . Thus for this value of convexity, the locations chosen under delegation will be efficient indicating that there is clearly scope for delegation to increase welfare.

We present additional detail on the equilibrium in the following proposition.

**Proposition 5:** When both firms delegate under convex production cost, i)  $\frac{\partial(L_2-L_1)}{\partial k} > 0$  and ii)  $\frac{\partial \alpha_i}{\partial k} > 0$ .

**Proof:**

$$i) \frac{\partial(L_2-L_1)}{\partial k} = 2t(9k^2 + 20kt + 12t^2)/(8k^2 + 15kt + 6t^2)^2 > 0.$$

$$ii) \partial \alpha_i / \partial k = 2t^2(17k^2 + 52kt + 36t^2)/(k^2t + 16k^2 + 30kt + 4kt^2 + 12t^2 + 4t^3)^2 > 0.$$

Thus, for larger  $k$  the owners adopt less aggressive managers (ii) and their managers locate farther away from each other (i). This implies that owners with convex costs will place less weight on output than those with linear costs.<sup>12</sup> This reflects the continued desire to turn the marginal customer over to the rival as driven by the cost convexity. Nonetheless, the influence of delegation may be most important for large  $k$ , as this is when there is the largest incentive to move toward the corners in the absence of delegation (Gupta 1994).

To examine the influence of delegation we first identify the equilibrium profits and social welfare. Returning (24) and (25) to (19) and (20) yields the profits:

$$\pi_1 = \pi_2 = \frac{64k^5 + 360k^4t + 753k^3t^2 + 718k^2t^3 + 300kt^4 + 40t^5}{8(8k^2 + 15kt + 6t^2)^2} \quad (26)$$

Social welfare is the difference between consumers' willingness to pay and total social cost. The social cost is now the sum of transport cost and production cost:

$$SW = r - SC \quad \text{where}$$

$$SC = \frac{1}{2}t(L_1)^2 + \frac{1}{2}t(x^* - L_1)^2 + \frac{1}{2}t(L_2 - x^*)^2 + \frac{1}{2}t(1 - L_2)^2 + \frac{1}{2}k(x^*)^2 + \frac{1}{2}k(1 - x^*)^2 \quad (27)$$

First, we note that the profits in (26) can be used as part of confirming that the delegation equilibrium will be chosen. The proof is in Appendix 2. Second, we summarize the welfare effects by evaluating (27) by subtracting the value without delegation from that with delegation. This difference we label  $\Delta SW$ .

**Proposition 6:** With quadratic production cost and delegation: i) profits decline and ii) social welfare increases when the degree of convexity is larger than  $0.248t$  and decreases when the degree of convexity is less than  $0.248t$ .

**Proof:** i) See Appendix 2.

ii) Plugging (24) and (25) into equation (27) yields

$$SW = r - \frac{280tk^4 + 453k^3t^2 + 337k^2t^3 + 120kt^4 + 20t^5 + 64k^5}{4(8k^2 + 15kt + 6t^2)^2} \quad (28)$$

Plugging  $L_1 = \frac{2t+k}{8(k+t)}$  and  $L_2 = \frac{6t+7k}{8(k+t)}$  (Gupta 1994) into equation (27) yields

$$SW = r - \frac{4t^3 + 16kt^2 + 21tk^2 + 8k^3}{32(k+t)^2} \quad (29)$$

Subtracting equation (29) from equation (28) yields

$$\Delta SW = \frac{t^2(16k^5 + 93tk^4 + 188t^2k^3 + 144t^3k^2 + 16t^4k - 16t^5)}{32(8k^2 + 15kt + 6t^2)^2(k+t)^2} \quad (30)$$

Setting equation (30) equal to zero and solving yields a single real root at  $k=0.248t$ . It can be checked that (30) is positive (negative) for larger (smaller) values of  $k$ . ■

With linear production cost, firms locate efficiently without delegation and proposition 2 shows that they then move toward each other with delegation. Thus, when  $k$  is small (only slightly larger than zero), firm locations without delegation are close to efficient and their movements toward each other with delegation are similar to those when  $k=0$ . As a consequence, when  $k$  is small, the result in Proposition 6 ii) mimics that in Proposition 2 -- social welfare diminishes with delegation. Yet, when  $k$  is large the firms without delegation locate far away from the efficient locations (toward the corners). The movement toward each other caused by delegation improves efficiency in this case. Thus for the first time, we have found that delegation can improve social welfare. The requirement is that the cost function must be sufficiently convex.

### 5. Third Extension: $N$ Firms

In this subsection, we return to the basic model with linear costs (normalized to zero) and imagine  $n$  firms instead of two. Firm location is designated by  $L_i$ ,  $i = 1 \dots n$  with  $L_{i+1} \geq L_i$ . The equilibrium delivered price schedule is  $p^*(x, L_i, L_{i+1}) = t(L_2 - x)$  if

$x \leq L_1$ ;  $p^*(x, L_i, L_{i+1}) = \max\{t(x - L_i), t(L_{i+1} - x)\}$  if  $L_i \leq x \leq L_{i+1}$ ;

$p^*(x, L_i, L_{i+1}) = t(x - L_{n-1})$  if  $x \geq L_n$ .

Given the delivered price schedule, we identify firms' profits. The profit of firm 1 is in equation (1) and profit of firm  $n$  is:

$$\pi_n = \int_{\frac{L_{n-1}+L_n}{2}}^{L_n} t(x - L_{n-1}) - t(L_n - x) dx + \int_{L_n}^1 t(x - L_{n-1}) - t(x - L_n) dx \quad (31)$$

A representative interior firm has profit:

$$\pi_i = \frac{t}{4}(L_{i+1} - L_{i-1})^2 - \frac{t}{4}(L_i - L_{i-1})^2 - \frac{t}{4}(L_{i+1} - L_i)^2 \quad (32)$$

The manager of an interior firm maximizes  $I_i = \alpha_i \pi_i + (1 - \alpha_i) \left( \frac{L_{i+1} - L_{i-1}}{2} \right)$  with respect to  $L_i$  and the best response function is:

$$L_i = \frac{L_{i+1} + L_{i-1}}{2} \quad (33)$$

This quickly shows that the incentive parameter is irrelevant. In the two firm case, either firm can increase its output by moving toward the other. If an interior firm moves toward one rival to gain output, it gives up exactly the same output to its rival on the other side. Thus, an interior firm can do no better than locating in the middle of its rivals. As a consequence, the interior firms locate symmetrically between the corner firms:

$$L_i = \frac{(n-i)L_1 + (i-1)L_n}{n-1} \quad (34)$$

In contrast to an interior firm, a corner firm has no rival on one side and so by being more aggressive can gain output and force the remaining firms to retreat. The



obvious problem is that if it pushes too far, an interior firm will simply jump over the corner firm to establish itself in the corner. Thus, under delegation, the owners of firm 1 and  $n$  must set an incentive parameter that maximizes profit subject to the constraint that the manager locates so as to retain the corner position. To solve this constrained maximization we recognize the “no jump constraint” is binding for all  $n \geq 3$  and work back through location problem to identify the appropriate incentive parameter.

The manager of firm 1 continues to adopt its location to maximize the incentive contract and only cares about its rival firm 2. Thus, the location chosen by the manager depends on that of firm 2 exactly as in (5):  $L_1 = (1 + \alpha_1 t L_2 - \alpha_1) / 3\alpha_1 t$ . The owner of firm 1 can use this reaction function to set the associated incentive parameter once it knows the location implied by the no jump condition.

To determine the no jump location plugging (34) into (32) to yield the profit of firm 2 located in the interior:

$$\pi_2 = \frac{t(L_1 - L_n)^2}{2(n-1)^2} \quad (35)$$

If firm 2 jumps to the left corner it maximizes profit by locating at  $L_2 = \frac{1}{3}L_1$  and earns  $\pi_2 = \frac{t}{3}(L_1)^2$ . The farthest right location for firm 1 that prohibits jumping is determined by setting this profit equal to (35) and solving for  $L_1$  (recognizing  $L_n = 1 - L_1$ ).

$$L_1 = (\sqrt{6}n - \sqrt{6} - 6) / 2(n^2 - 2n - 5) \quad (36)$$

and analogously

$$L_n = (2n^2 - n(\sqrt{6} + 4) + \sqrt{6} - 4) / 2(n^2 - 2n - 5) \quad (37)$$

It can be checked that the corner firms move toward the edges as  $n$  increases. It also follows from (34) that:

$$L_i = (2i(n - 1 - \sqrt{6}) + n(\sqrt{6} - 2) + \sqrt{6} - 4)/2(n^2 - 2n - 5) \quad (38)$$

The owner of firm 1 returns (36) and (38) to (5) and solves for  $\alpha_1$ :

$$\alpha_1 = \alpha_n = \frac{n^2 - 2n - 5}{n^2 - tn - 2n + tn\sqrt{6} - 5t - 5} \quad (39)$$

This is the incentive parameter that maximizes profit given the constraint that interior firms will not jump.

Note that the owners place less emphasis on output as the number of firms increases,  $\frac{\partial \alpha_i}{\partial n} = \frac{t(\sqrt{6}-1)}{(-1+\sqrt{6}+n+t\sqrt{6}-t)^2} > 0$ . This follows from the incentive for interior firms to jump into the corner. As the total number of firms increases, a smaller corner distance will induce such jumping. Thus, the owners of the corner firms respond to an increase in the number of firms by decreasing their managers' incentives to move toward the middle.

Social welfare is the difference between total willingness to pay and total transport cost:

$$SW = r - TC \quad \text{where} \quad TC = \frac{1}{2}t(L_1)^2 + \sum_1^{n-1} \frac{1}{4}(L_{i+1} - L_i)^2 t + \frac{1}{2}t(1 - L_n)^2 \quad (40)$$

As before, we compute the difference between social welfare with delegation and without delegation,  $\Delta SW$ .

**Proposition 7:** With  $n$  firms and linear production cost, social welfare is lower with delegation.

**Proof:** Using the locations without delegation,  $L_i = \frac{2i-1}{2n}$ ,  $i = 1, 2, 3 \dots n$  (Hurter and Lederer, 1985), and the locations with delegation, (36), (37), and (38),  $\Delta SW =$

$$\left( r - \frac{t(n^3 + 3n^2 - 2\sqrt{6}n^2 - 8\sqrt{6}n - 3n + 10\sqrt{6} + 35)}{4(n^2 - 2n - 5)^2} \right) - \left( r - \frac{t}{4n} \right) = \frac{t(-7 + 2\sqrt{6})(n-1)(-n+1+\sqrt{6})^2}{4n(n^2 - 2n - 5)^2} < 0 \text{ for}$$

all integers  $n > 1$ . ■

The corner owners adopt delegation and set the incentive parameter to push managers as far toward the center as the no jump condition allows. As in the two firms case, this delegation harms welfare. Fundamentally, this extension stresses the importance of the assumption of a linear spatial market with distinct corners. Clearly, if the firms locate on a unit circle rather than line segment, there will be no advantage to using delegation. Any attempt to gain output in one direction is offset by an equal output loss in the other direction.

## 6. Fourth Extension: Considering Elastic Demand

We now follow Hamilton et al. (1989) to allow elastic linear demand at each location:  $q(x) = 1 - p(x)$ . The price at  $x$  remains the maximum of delivered costs,

$$p(x) = \max \{t(L_2 - x), t(x - L_1)\} \text{ and the indifferent consumer remains at } x^* = \frac{(L_1 + L_2)}{2}.$$

Also as in Hamilton et al. (1989), we assume that  $t < 0.9$  to rule out monopoly pricing and assure that the price schedule (the rival's delivered cost) binds. Thus, firm profits are:

$$\pi_1 = \int_0^{L_1} (t(L_2 - x) - t(L_1 - x))(1 - t(L_2 - x))dx + \int_{L_1}^{x^*} (t(L_2 - x) - t(x - L_1))(1 - t(L_2 - x))dx \quad (41)$$

$$\pi_2 = \int_{x^*}^{L_2} (t(x - L_1) - t(L_2 - x))(1 - t(x - L_1))dx + \int_{L_2}^1 (t(x - L_1) - t(x - L_2))(1 - t(x - L_1))dx \quad (42)$$

Owners delegate the location choice providing a contract that still rewards profit and quantity. With inelastic demand, rewarding quantity and the share of the unit market (market share) are identical but this equivalence vanishes with elastic demand. Anticipating the responses of the managers, the two owners simultaneously choose incentive parameters.

$$I_1 = \alpha_1 \pi_1 + (1 - \alpha_1)q_1 \quad \text{and} \quad I_2 = \alpha_2 \pi_2 + (1 - \alpha_2)q_2 \quad (43)$$

where  $q_1 = \int_0^{x^*} 1 - t(L_2 - x)dx$  and  $q_2 = \int_{x^*}^1 1 - t(x - L_1)dx$ .

Substituting (41) and (42) into (43), the managers of the two firms maximize with respect to the location choices yielding the two best response functions:

$$L_1 = \frac{7t\alpha_1 L_2 + 1 - 7\alpha_1 + \sqrt{28\alpha_1^2 t^2 L_2^2 - 56\alpha_1^2 t L_2 + 1 + 14\alpha_1 + 21\alpha_1^2}}{7t\alpha_1} \quad (44)$$

$$L_2 = \frac{7t\alpha_2 L_1 - 1 + 7\alpha_2 - \sqrt{28\alpha_2^2 t^2 L_1^2 + 56\alpha_2^2 t L_1 + 1 + 14\alpha_2 + 21\alpha_2^2 - 56\alpha_2^2 t + 28\alpha_2^2 t^2 - 56\alpha_2^2 t^2 L_1}}{7t\alpha_2} \quad (45)$$

At this point if we were to follow Hamilton et al. (1989 p.94), we would impose symmetry,  $L_2 = 1 - L_1$ . Yet, the structure of our game prohibits this as imposing symmetry requires that  $\alpha_1 = \alpha_2 = \alpha$  which implicitly removes the stage in which owners play a game in incentive parameters. At the same time, there is not a tractable analytic solution to jointly solving (44) and (45) to give locations as functions of

incentive parameters. As a consequence, we approach the owner and manager objective functions implicitly. We take all the appropriate derivatives from both stages and only impose symmetry afterward. While even this approach generates higher order terms in  $t$  that prohibit a closed form solution, we can solve the problem for any specific value of  $t$ .

As an illustration, set  $t = 1/2$  and return (41) and (42) to the contracts in (43).

Each manager maximizes (43) with respect to their location choice where the partial

derivatives  $F_1 = \frac{\partial I_1}{\partial L_1} = 0$  and  $F_2 = \frac{\partial I_2}{\partial L_2} = 0$  are:

$$F_1 = \frac{7}{16}\alpha_1 L_1 L_2 - \frac{7}{32}\alpha_1 L_1^2 + \frac{3}{8}\alpha_1 L_2 - \frac{3}{32}\alpha_1 L_2^2 - \frac{7}{8}\alpha_1 L_1 + \frac{1}{2} - \frac{1}{8}L_2 + \frac{1}{8}L_1 - \frac{1}{2}\alpha_1 \quad (46)$$

$$F_2 = -\frac{7}{16}\alpha_2 L_1 L_2 + \frac{7}{32}\alpha_2 L_2^2 + \frac{5}{8}\alpha_2 L_1 + \frac{3}{32}\alpha_2 L_1^2 - \frac{7}{8}\alpha_2 L_2 - \frac{1}{2} + \frac{1}{8}L_2 - \frac{1}{8}L_1 + \frac{7}{8}\alpha_2 \quad (47)$$

Keeping these in implicit form, differentiate  $F_1[\alpha_1, \alpha_2, L_1(\alpha_1, \alpha_2), L_2(\alpha_1, \alpha_2)]$  and

$F_2[\alpha_1, \alpha_2, L_1(\alpha_1, \alpha_2), L_2(\alpha_1, \alpha_2)]$  with respect to  $\alpha_1$ :

$$\frac{\partial F_1}{\partial \alpha_1} + \frac{\partial F_1}{\partial L_1} \frac{\partial L_1}{\partial \alpha_1} + \frac{\partial F_1}{\partial L_2} \frac{\partial L_2}{\partial \alpha_1} = 0 \quad (48)$$

$$\frac{\partial F_2}{\partial \alpha_1} + \frac{\partial F_2}{\partial L_1} \frac{\partial L_1}{\partial \alpha_1} + \frac{\partial F_2}{\partial L_2} \frac{\partial L_2}{\partial \alpha_1} = 0 \quad (49)$$

The implicit function theorem and Cramer's rule give:  $\frac{\partial L_1}{\partial \alpha_1} = -\frac{1}{J_F} \left( \det \begin{bmatrix} \frac{\partial F_1}{\partial \alpha_1} & \frac{\partial F_1}{\partial L_2} \\ \frac{\partial F_2}{\partial \alpha_1} & \frac{\partial F_2}{\partial L_2} \end{bmatrix} \right)$  and

$\frac{\partial L_2}{\partial \alpha_1} = -\frac{1}{J_F} \left( \det \begin{bmatrix} \frac{\partial F_1}{\partial L_1} & \frac{\partial F_1}{\partial \alpha_1} \\ \frac{\partial F_2}{\partial L_1} & \frac{\partial F_2}{\partial \alpha_1} \end{bmatrix} \right)$  where  $J_F = \det \begin{bmatrix} \frac{\partial F_1}{\partial L_1} & \frac{\partial F_1}{\partial L_2} \\ \frac{\partial F_2}{\partial L_1} & \frac{\partial F_2}{\partial L_2} \end{bmatrix}$ . With (46) and (47), these yield:

$$\frac{\partial L_1}{\partial \alpha_1} = \frac{(-14L_1 L_2 + 7L_1^2 - 12L_2 + 3L_2^2 + 28L_1 + 16)(7\alpha_2 L_2 + 2 - 14\alpha_2 - 7\alpha_2 L_1)}{8\alpha_1 \alpha_2 (7L_2^2 - 31L_2 + 7L_1^2 - 10L_1 L_2 + 34 + 27L_1) + 16\alpha_1 (L_2 - 2) - 16\alpha_2 (L_1 + 1)} \quad (50)$$

$$\frac{\partial L_2}{\partial \alpha_1} = \frac{(-14L_1L_2 + 7L_1^2 - 12L_2 + 3L_2^2 + 28L_1 + 16)(7\alpha_2L_2 + 2 - 10\alpha_2 - 3\alpha_2L_1)}{8\alpha_1\alpha_2(7L_2^2 - 31L_2 + 7L_1^2 - 10L_1L_2 + 34 + 27L_1) + 16\alpha_1(L_2 - 2) - 16\alpha_2(L_1 + 1)} \quad (51)$$

Differentiation of (41) now generates the owner's first order condition,  $\frac{\partial \pi_1}{\partial \alpha_1} = \frac{\partial \pi_1}{\partial L_1} \frac{\partial L_1}{\partial \alpha_1} +$

$\frac{\partial \pi_1}{\partial L_2} \frac{\partial L_2}{\partial \alpha_1}$ , as a function of terms we have just derived.

$$\begin{aligned} \frac{\partial \pi_1}{\partial \alpha_1} = & \frac{\partial L_1}{\partial \alpha_1} \left( \frac{7}{16} L_1 L_2 - \frac{7}{32} L_1^2 + \frac{1}{4} L_2 - \frac{3}{32} L_2^2 - \frac{3}{4} L_1 \right) + \\ & \frac{\partial L_2}{\partial \alpha_1} \left( -\frac{3}{16} L_1 L_2 + \frac{7}{32} L_1^2 + \frac{1}{4} L_1 - \frac{5}{32} L_2^2 + \frac{1}{4} L_2 \right) = 0 \end{aligned} \quad (52)$$

Having identified the first order conditions for all players, we now impose symmetry,

$L_2 = 1 - L_1$  and  $\alpha_1 = \alpha_2 = \alpha$ , which with (46) gives  $L_1$  in terms of  $\alpha$ :

$$L_1 = \frac{-5\alpha + 2 + \sqrt{-17\alpha^2 + 52\alpha + 4}}{12\alpha} \quad (53)$$

In combination with (50) and (51), returning (53) to (52) and solving for  $\alpha$  yields

$\alpha_1 = \alpha_2 = 0.86$  and in equilibrium,  $L_1 = 0.359$ ,  $L_2 = 0.641$ , and  $\pi_1 = \pi_2 = 0.048$ .

Social welfare remains the difference between total surplus and total transport cost,  $SW = TS - TC$ . Surplus at any point  $x$  is the sum of consumer surplus,  $\frac{1}{2}(q(x))^2$  and total revenue,  $q(x)(1 - q(x))$  yielding the following:

$$\begin{aligned} TS = & \int_0^{x^*} (1 - t(L_2 - x)) - \frac{1}{2}(1 - t(L_2 - x))^2 dx + \int_{x^*}^1 (1 - t(x - L_1)) - \\ & \frac{1}{2}(1 - t(x - L_1))^2 dx \end{aligned} \quad (54)$$

The cost at any point  $x$  is the transport cost to that point times the quantity sold at that point yielding the following:

$$TC = 2\left(\int_0^{L_1} t(L_1 - x)(1 - t(L_2 - x))dx + \int_{L_1}^{x^*} t(x - L_1)(1 - t(L_2 - x))dx\right) \quad (55)$$

The equilibrium locations imply that  $TS=0.478$ ,  $TC=0.056$ , and  $SW=0.422$ . This can be contrasted to the no delegation equilibrium ( $\alpha_1 = \alpha_2 = 1$ ):  $L_1 = 0.27$ ,  $L_2 = 0.73$ ,  $\pi_1 = \pi_2 = 0.065$ ,  $TS=0.468$ ,  $TC=0.047$ , and  $SW=0.421$ . Thus, delegation causes the firms to move toward each other with surplus increasing slightly more than costs,  $\Delta SW = 0.001$ .

Repeating the above procedure for  $t = 0.1, 0.2, \dots, 0.8$  allows the following characterization (these equilibria are shown in Table 2 and compared to those without delegation):

**Result 6.1:** With linear elastic demand, delegation i) moves the firms closer together, ii) lowers firm profits, iii) increases consumer surplus, and iv)  $\Delta SW < 0$  if  $t=0.1, 0.2, 0.3$ , and  $0.4$  and  $\Delta SW > 0$  if  $t = 0.5, 0.6, 0.7$ , and  $0.8$ .

The manager increases sales by moving toward the rival and capturing the lower price, high demand locations. As the firms move toward each other, the price falls at all locations increasing quantity, revenue and consumer surplus. At the same time, as the firms move toward each other total transport cost increases (Hamilton et al. 1989). Thus, a given movement of firms toward each other has offsetting consequences on social welfare. When  $t$  is small, delegation moves the firms toward each other generating a small reduction in price and so only modest gains in consumer surplus. These gains are outweighed by the increase in transport cost and the decline in profit. However, when  $t$  is large, the movement toward each other generates a large reduction in the price and large

gain in consumer surplus. In this case the gain in consumer surplus is larger than the decline in profit and increase in transport cost. Table 2 shows that delegation decreases social welfare for the cases of  $t=0.1, 0.2, 0.3$ , and  $0.4$ , but increases welfare for higher values of  $t$ .

While the ultimate influence on social welfare depends on the level of transport costs, many of the basic insights carry over from the inelastic demand case. When owners have an incentive to delegate, managers locate closer together. The consequence for owners remains reduced profitability relative to the case without delegation.

Comparison to the basic model is also valuable. The presence of elastic demand increases the incentive of managers to move toward each other. This follows because the elastic demand means that as the firms move toward each other and the discriminatory price schedule falls and so the quantity demand increases at all points. Thus, for a given movement toward the rival, the quantity increases more quickly. Owners recognize this and so under elastic demand the weight placed on profit is larger. The owners can create the same aggressiveness with less weight on quantity compared to inelastic demand.

## **7. Conclusion**

In both Cournot and Stackelberg competition delegating quantity choices to managers results in greater total output and greater social welfare. We model delegating location decisions to managers under spatial price discrimination and show this creates more aggressive managers who move toward each other. The impact of this on social welfare depends on the setting. With linear production costs and inelastic demand, welfare declines with both simultaneous and sequential location. With convex



production costs, welfare can increase but only if there is sufficient convexity. With elastic demand, welfare will increase for high transport cost but decrease for low transport cost. Thus, the beneficial effects of delegation identified in quantity games, do not routinely carry over. We also emphasize the importance of the corner firms in our linear markets showing that it is the market edge that allows incentive contracts on quantity to influence location.

These new findings could set the stage for further research. As we have shown that convexity reduces the weight put on quantity (and so market share), one might be able to test if market share contracts are less likely or less lucrative in industries characterized by cost convexity. This fits with a recent call in the management literature that empirical researchers should begin to examine the determinants of inter-industry heterogeneity in the use of delegation contracts (Sengul et al. 2012). The structure (convexity) of costs could be one such determinant.

A second avenue for research might follow recent work by Liu and Serfes (2004) and Colombo (2009) adopt the Hotelling type model with mill pricing but imagine the ability to imperfectly price discriminate (the firms know the segment of the market of the consumer but not the exact location). An issue would be whether delegation influences location in such a setting. Finally, one might imagine introducing a bargaining game between owners and managers into the model. Following van Witteloostuijn et al. (2007), this game would set the incentive parameters. Despite these potential variations, we emphasize our contribution. We uniquely examine spatial price discrimination and find that delegation will be adopted and often harms social welfare.

THREE ESSAYS ON THE ECONOMICS OF SPATIAL PRICE DISCRIMINATION:

II) CONSISTENT LOCATION CONJECTURES UNDER SPATIAL PRICE  
DISCRIMINATION

## 8. Introduction

Despite a long and durable history, the canonical model of spatial price discrimination has not been solved for an equilibrium generated by consistent location conjectures. We model such conjectures and present an equilibrium that differs sharply from that associated with Nash behavior. Instead of the well known symmetric duopoly locations at the quartiles that minimize transport cost (Hurter and Lederer, 1985), we confirm a principle of minimum differentiation with the two firms locating at the middle and maximizing total transport cost. We contrast this result associated with linear production cost with an otherwise similar equilibrium that assumes convex production cost. In this latter case, the extent of convexity determines whether the equilibrium locations improve or detract from the welfare associated with Nash behavior.

Conjectural variations generalize the Nash assumption and are seen as implicitly modeling the beliefs formation process of each player about the conduct of other. The solution concept of conjectures that match other players' actual responses has been identified as more general and superior to Nash. It represents a "consistency of beliefs" absent in Nash where players stubbornly refuse to learn about the behavior of other players (See Aliprantis and Chakrabarti, 2000 p. 138).

This generalization has been criticized as it remains fundamentally one-shot with no explicit maximization of a stream of payoffs and because it can generate out of equilibrium behavior that can make little sense lacking normal stability properties (Friedman, 1983 pp. 109–110). Despite this criticism, it remains popular as a substitute for complete dynamic modeling. Indeed, a substantial literature has embedded static Nash behavior in a fully dynamic model and isolated the conditions under which the outcomes

match that of consistent conjectures (see Cabral, 1995 for an early example). As Martin (2002, p. 51) emphasizes, “These results provide a formal justification for using the static conjectural variations model as a short cut to analyze inherently dynamic models.” More recently, Possajennikov (2009) demonstrates that consistent conjectures are evolutionarily stable for a wide range of well-behaved games and that their use often simplifies evolutionary analysis. In addition to being a shortcut to full dynamic modeling, the conjectural variations framework has been used widely in the literature on oligopolistic product markets (Bresnahan, 1981), the private provision of public goods (Sugden 1985, Itaya and Shimomura, 2001) and in modeling teamwork (Heywood and McGinty, 2012). Moreover, it has found practical empirical applications both in bidding strategies for electric power (Song et al., 2003) and in estimating market power for antitrust purposes (Perloff et al., 2007).

We bring this equilibrium to spatial models as they have proven especially useful in capturing aspects of competition in horizontally differentiated markets (Tirole, 1988). Specifically, they provide a general method for examining markets in which either actual location or an ordered product characteristic differentiates output.<sup>13</sup> A very limited literature has considered consistent conjectures in these models but the conjectures are either not about location itself or the setting is not one of spatial price discrimination. Thus, Capozza and Van Order (1989) consider a Chamberlin type model in which entry occurs in the spatial market until profits are zero. Each firm faces uniformly distributed consumers with downward sloping demand. The conjectures modeled are those of rivals' reactions to own price changes.<sup>14</sup> Mulligan and Fik (1994, 1995) consider both price and

location conjectures and do so in a typical mill pricing model in which the consumer pays transport costs thus eliminating the possibility of spatial discrimination.

Spatial price discrimination allows modeling circumstances in which the price includes delivery and so the price a firm is able to charge depends upon how "close" it is to its rivals. Thisse and Vives (1988) show that such pricing is the preferred alternative when firms are able to adopt it and Greenhut (1981) identifies it as "nearly ubiquitous" among actual products that have substantial freight costs. Thus, our inquiry into location conjectures in the context of spatial price discrimination maintains salience as well as being a sensible extension of the existing literature.

In what follows, we first study the case in which two firms spatially price discriminate with simultaneous choice of locations. With constant marginal cost, the two firms co-locate resulting in higher social cost than with Nash. When production cost is quadratic, the two firms do not co-locate but continue to locate closer to each other than with a Nash assumption. We show that the degree of the cost convexity determines whether the consistent conjectures equilibrium has higher or lower social cost than the Nash equilibrium. Next, the assumption of constant marginal costs is recovered and the number of firms is extended to  $n$ . We show that the consistent conjectures equilibrium continues to result in firms being too close together and so generates higher social cost than Nash. Finally, we draw conclusions.

## **9. The Basic Model with Linear Production Costs**

Consumers are uniformly distributed over a unit line segment. Each consumer has inelastic demand for one unit of the good, with reservation price  $r$ . We assume  $r$  is

sufficiently large that it is profitable to serve all customers. Two firms engage in spatial price discrimination and share identical constant marginal cost set to zero without loss of generality. The transport cost per unit of distance is  $t$  and the locations of firm 1 and firm 2 are  $L_1$  and  $L_2$  respectively,  $L_1 \leq L_2$ . In the first stage, firms simultaneously choose their locations and the delivered price schedules are announced in the second stage.

The delivered price schedule for the consumer at  $x$  is  $p^*(x, L_i) = \max(t|x - L_i|)$  and Figure 1 depicts the price schedules for the two firms as the outer envelope of the delivered costs. The market is divided by the indifferent customer who faces the same delivered price from each firm. That customer is located at  $x^* = \frac{(L_1 + L_2)}{2}$ . Given the delivered price schedule, the profits of the two firms are

$$\pi_1 = \int_0^{L_1} t(L_2 - x) - t(L_1 - x)dx + \int_{L_1}^{x^*} t(L_2 - x) - t(x - L_1)dx \quad (56)$$

$$\pi_2 = \int_{x^*}^{L_2} t(x - L_1) - t(L_2 - x)dx + \int_{L_2}^1 t(x - L_1) - t(x - L_2)dx \quad (57)$$

and are identified in Figure 1.

Each firm maximizes profit with respect to its own location generating best response functions

$$BR_1: L_1 = L_2(1 + \lambda_{21})/(3 - \lambda_{21}) \quad (58)$$

$$BR_2: L_2 = (L_1(1 + \lambda_{12}) - 2\lambda_{12} + 2)/(3 - \lambda_{12}) \quad (59)$$

Where  $\lambda_{i,j} = \partial L_i / \partial L_j$  is firm  $j$ 's conjecture about firm  $i$ . The consistent-conjectures equilibrium requires that the actual derivative along the best response function equal the conjecture. This requires that each firm correctly anticipates the location movements of

its rival in response to its own movement and this is built into the maximization. Defining

$$\rho_{ij} = \frac{\partial BR_i}{\partial L_j}, i \neq j \text{ and setting } \rho_{ij} = \lambda_{ij} \text{ yields } \lambda_{12} = \frac{1+\lambda_{21}}{3-\lambda_{21}} \text{ and } \lambda_{21} = \frac{1+\lambda_{12}}{3-\lambda_{12}}.$$

Simultaneous solution yields  $\lambda_{12}^* = \lambda_{21}^* = 1$  as the consistent conjectures and then (58)

and (59) imply that  $L_1 = L_2$ . Thus, the firms collocate and the symmetric location

equilibrium occurs at  $L_1^* = L_2^* = \frac{1}{2}$  as the symmetry of the problem implies that  $L_1^* = 1 -$

$L_2^*$ .<sup>15</sup> The critical point is that minimum differentiation occurs and both firms earn zero profits.

The linear production and transport costs have generated an equilibrium that shares some characteristics with that outside the spatial context. Bresnahan (1981) shows that a duopoly engaged in quantity competition with linear production costs will adopt consistent conjectures that result in the competitive quantity being produced and no profit being earned. While our firms also earn zero profit, the social welfare implications are obviously very different. With inelastic demand the social welfare is

$$SW = r - TC \text{ where } TC = \frac{1}{2}t(L_1)^2 + \frac{t}{4}(L_2 - L_1)^2 + \frac{1}{2}t(1 - L_2)^2 \quad (60)$$

allowing us to summarize:

**Proposition 8:** In a simultaneous duopoly location game with constant marginal cost, i) the Nash conjecture is inconsistent, and ii) the consistent-conjectures equilibrium results in lower social welfare.

**Proof:**

i) With Nash,  $BR_1: L_1 = \frac{1}{3}L_2$  and  $BR_2: L_2 = \frac{1}{3}L_1 + \frac{2}{3} \Rightarrow \rho_{ij} = \frac{1}{3} \neq \lambda_{ij} = 0$ .

ii)  $SW\left(L_1 = \frac{1}{2}, L_2 = \frac{1}{2}\right) = r - \frac{1}{4}t < SW\left(L_1 = \frac{1}{4}, L_2 = \frac{3}{4}\right) = r - \frac{1}{8}t$ .

In spatial price discrimination with Nash, each firm locates in the middle of its own market thereby minimizing social cost. With consistent location conjectures, both firms locate in the middle of the overall market with their profits equal to zero and with social cost maximized among the set of symmetric locations.

It is interesting to consider the case where only one firm adopts a consistent conjecture as this also mimics results known outside the spatial context. Boyer and Moreaux (1983) show that in a quantity game the firm adopting consistent conjectures produces the same amount as a Stackelberg leader and that the firm retaining Nash conjectures produces the same amount as a Stackelberg follower. In our case, if firm 2 retains a Nash conjecture it has a best response function of  $L_2 = \frac{1}{3}L_1 + \frac{2}{3}$ . Given this firm 1 adopts a consistent conjecture of 1/3 regarding firm 2. This results in a best response function from (58) of  $L_1 = \frac{1}{2}L_2$ . The resulting locations are 2/5 and 4/5 exactly those that emerge from the two firms sequential location equilibrium as presented by Gupta (1992). We note that when only one firm adopts a consistent conjecture, the locations obviously result in lower welfare than with Nash.

## 10. Quadratic Production Cost

We now remove the assumption of constant marginal cost and assume that firm 1 and firm 2 share identical quadratic cost functions. Gupta (1994) shows that under spatial price discrimination with convex costs, duopolists locate inefficiently outside the quartiles. Thus, we want to examine whether the adoption of consistent conjectural variation can improve social welfare which it obviously cannot do with linear costs.



We assume quadratic production cost of the form  $C_i = \frac{1}{2}kq_i^2$  for  $i=1,2$ . The market retains a uniform distribution of customers which for convenience we normalize to one. We follow Gupta's assumption that the delivered cost is the sum of marginal production cost plus transport cost. The equilibrium delivered price schedule for any consumer located at  $x$  is  $p^*(x, L_1, L_2) = \max(\{t|x - L_1| + kx^*\}, \{t|L_2 - x| + k(1 - x^*)\})$  and is depicted in Figure 2. Here, the indifferent consumer located at  $x^*$  divides the market between the two firms into  $x^*$  and  $1 - x^*$ .<sup>16</sup> The profits of firm 1 and firm 2 can be expressed as

$$\pi_1 = \int_0^{L_1} t(L_2 - x) + k(1 - x^*) - t(L_1 - x) dx + \int_{L_1}^{x^*} t(L_2 - x) + k(1 - x^*) - t(x - L_1) dx - \frac{1}{2}k(x^*)^2 \quad (61)$$

$$\pi_2 = \int_{x^*}^{L_2} t(x - L_1) + kx^* - t(L_2 - x) dx + \int_{L_2}^1 t(x - L_1) + kx^* - t(x - L_2) dx - \frac{1}{2}k(1 - x^*)^2 \quad (62)$$

The best response functions are

$$BR_1: L_1 = -\frac{L_2(1+\lambda_{21})(2kt^2+4t^3)+(1+\lambda_{21})(2k^2t+4kt^2)}{2kt^2(\lambda_{21}-15)-16k^2t+4t^3(\lambda_{21}-3)} \quad (63)$$

$$BR_2: L_2 = -\frac{L_1(1+\lambda_{12})(2kt^2+4t^3)+2k^2t(7-\lambda_{12})+8kt^2(3-\lambda_{12})+8t^3(1-\lambda_{12})}{2kt^2(\lambda_{12}-15)-16k^2t+4t^3(\lambda_{12}-3)} \quad (64)$$

Setting  $\rho_{12} = \lambda_{12}$  and  $\rho_{21} = \lambda_{21}$  yields the consistent conjectures

$$\lambda_{12} = -\frac{(1+\lambda_{21})(2kt^2+4t^3)}{2kt^2(\lambda_{21}-15)-16k^2t+4t^3(\lambda_{21}-3)} \quad (65)$$

$$\lambda_{21} = -\frac{(1+\lambda_{12})(2kt^2+4t^3)}{2kt^2(\lambda_{12}-15)-16k^2t+4t^3(\lambda_{12}-3)} \quad (66)$$

Simultaneously solving (65) and (66) yields two sets of roots but ultimately only one generates positive profits and locations on the unit line segment:

$$\lambda_{12}^* = \lambda_{21}^* = \frac{4k^2 + 7kt + 2t^2 - 2\sqrt{4k^4 + 14k^3t + 16k^2t^2 + 6kt^3}}{t(2t+k)} > 0 \quad (67)$$

Returning (67) to (63) and (64) and simultaneously solving yields

$$L_1^* = \frac{2k^2 + 4kt + 2t^2 - \sqrt{4k^4 + 16k^2t^2 + 14k^3t + 6t^3k}}{4t(k+t)} \quad (68)$$

$$L_2^* = \frac{-2k^2 + 2t^2 + \sqrt{4k^4 + 16k^2t^2 + 14k^3t + 6t^3k}}{4t(k+t)} \quad (69)$$

Plugging (68) and (69) into (61) and (62) yields

$$\pi_1^* = \pi_2^* = \pi^* = \frac{-2k^2 - 3tk + \sqrt{2k(2k+3t)(t+k)^2}}{4t} \quad (70)$$

Note that if  $k=0$ , the equilibrium values above all revert to those from the previous section with each firm collocating at  $1/2$ .

The total social cost is

$$SC = \frac{1}{2}t(L_1)^2 + \frac{1}{2}t(x^* - L_1)^2 + \frac{1}{2}t(L_2 - x^*)^2 + \frac{1}{2}t(1 - L_2)^2 + \frac{1}{2}k(x^*)^2 + \frac{1}{2}k(1 - x^*)^2 \quad (71)$$

This allows us to identify the consequences of the consistent-conjectures equilibrium.

**Proposition 9:** With quadratic production cost and two firms, i) the Nash conjecture is inconsistent, ii) firms locate closer to each other than in Nash; iii) firm profits are lower than with Nash and iv) social welfare is greater or less than in Nash as  $k \geq 0.313t$ .

**Proof:**

$$i) \text{ With Nash, } BR_1: L_1 = \frac{2t^2L_2 + tL_2k + 2kt + k^2}{15kt + 8k^2 + 6t^2} \text{ and } BR_2: L_2 = \frac{2t^2L_1 + tL_1k + 12kt + 7k^2 + 4t^2}{15kt + 8k^2 + 6t^2} \Rightarrow$$

$$\rho_{12} = \frac{2t^2 + tk}{15kt + 8k^2 + 6t^2} \neq \lambda_{12} = 0; \rho_{21} = \frac{2t^2 + tk}{15kt + 8k^2 + 6t^2} \neq \lambda_{21} = 0.$$

$$ii) \text{ In Nash } L_1 = \frac{2t+k}{8(k+t)} \text{ and } L_2 = \frac{6t+7k}{8(k+t)}. \text{ Thus,}$$

$$L_1^* - L_1 = \frac{7kt + 4k^2 + 2t^2 - 2\sqrt{4k^4 + 16k^2t^2 + 14k^3t + 6t^3k}}{8t(t+k)} > 0 \text{ and}$$

$$L_2^* - L_2 = \frac{-7kt - 4k^2 - 2t^2 + 2\sqrt{4k^4 + 16k^2t^2 + 14k^3t + 6t^3k}}{8t(t+k)} < 0.$$

$$iii) \text{ In Nash, } \pi_1 = \pi_2 = \pi = \frac{8k^3 + 31tk^2 + 36t^2k + 12t^3}{64(t+k)^2}. \text{ Thus,}$$

$$\pi^* - \pi = \frac{\sqrt{2k(2k+3t)(t+k)^2(16t^2 + 32tk + 16k^2)} - (159k^2t^2 + 120k^3t + 32k^4 + 84t^3k + 12t^4)}{64(t+k)^2} < 0.$$

$$iv) \Delta = SC(L_1^*, L_2^*) - SC(L_1, L_2) =$$

$$\frac{-\sqrt{2k(2k+3t)(t+k)^2(8t^2 + 24tk + 16k^2)} + (115k^2t^2 + 104k^3t + 32k^4 + 48t^3k + 4t^4)}{32(t+k)^2}$$

Solving  $\Delta = 0$  yields  $k=0.313t$  (given  $k>0$  and  $t>0$ ). And  $\Delta SC \geq 0$  as  $k \leq 0.313t$ .

Under consistent conjectures, each firm knows that its rival will move toward the endpoint as it moves toward the middle and builds this into its optimal location. As a result, the firms have a greater incentive to move toward the middle and locate closer to each other under consistent conjectures.

Under Nash conjectures, Gupta (1994) shows that the two firms locate inefficiently far apart. Yet, when  $k$  is small, this deviation from the first best locations remains small and the movement toward each other caused by introducing consistent conjectures lowers welfare relative to Nash. This mimics the result with linear costs. However, when  $k$  is large, the deviation from first best locations under Nash is large and

the movement toward each other caused by consistent conjectures results in welfare greater than that associated with Nash.

## 11. Linear Costs with $n$ Firms

We now return to the case of linear costs as in Section 2 but consider  $n$  firms engaging in spatial price discrimination. We show that the Nash equilibrium remains inconsistent. We consider the cases for  $n=3$  and  $n=4$  showing that very different patterns of consistent conjectures and locations emerge for odd and even numbers of firms. Finally, we present an illustrative generalization to  $n$  firms for the odd and even number of firms. The basic conclusion from section 2 remains. The consistent conjecture equilibria are socially inefficient. It is recognized that this section remains largely an elaborate simulation and that the conclusions are based largely on induction. A tractable general analytic solution for any  $n$  has proven elusive.

### 11.1. Nash is Inconsistent

We order the location of the  $n$  firms,  $L_i, i=1..n$ , such that  $L_{i+1} > L_i$ . The equilibrium delivered price schedule is  $p^*(x, L_2) = t(L_2 - x)$  if  $x \leq L_1$ ;  $p^*(x, L_i, L_{i+1}) = \max\{t(x - L_i), t(L_{i+1} - x)\}$  if  $L_i \leq x \leq L_{i+1}$ ;  $p^*(x, L_{n-1}) = t(x - L_{n-1})$  if  $x \geq L_n$ . In Figure 3, the bold envelope depicts the price schedule.

Given the delivered price schedule profits can be identified. Firm 1's profit is shown in equation (56) and the profit of firm  $n$  is

$$\pi_n = \int_{\frac{L_{n-1}+L_n}{2}}^{L_n} t(x - L_{n-1}) - t(L_n - x)dx + \int_{L_n}^1 t(x - L_{n-1}) - t(x - L_n)dx \quad (72)$$

The profit of any representative interior firm is

$$\pi_i = \frac{t}{4}(L_{i+1} - L_{i-1})^2 - \frac{t}{4}(L_i - L_{i-1})^2 - \frac{t}{4}(L_{i+1} - L_i)^2 \quad (73)$$

Under Nash, firms 1,  $n$  and  $i$  maximize (56), (72), and (73) with respect to their own locations assuming the other locations do not change, a zero conjecture. This generates three best response functions:  $BR_1: L_1 = \frac{1}{3}L_2$ ,  $BR_n: L_n = \frac{2}{3} + \frac{1}{3}L_{n-1}$  and  $BR_i: L_i = \frac{L_{i+1} + L_{i-1}}{2}$ .

**Proposition 10:** With  $n$  firms, the Nash equilibrium is never consistent.

**Proof:** For interior firms,  $\rho_{i,i+1} = \frac{1}{2} \neq \lambda_{i,i+1} = 0$  and  $\rho_{i,i-1} = \frac{1}{2} \neq \lambda_{i,i-1} = 0$ ; for corner firms,  $\rho_{1,2} = \frac{1}{3} \neq \lambda_{1,2} = 0$  and  $\rho_{n,n-1} = \frac{1}{3} \neq \lambda_{n,n-1} = 0$ .

Inconsistency emerges as each firm assumes that its rivals' locations are fixed even as the slopes of each best response functions are not zero. The assumptions that firms make about how their rivals respond to their location changes do not match how their rivals actually respond.

## 11.2. Equilibrium when $n = 3$

Firm 1's profit is shown in (56). The profits of firm 2 and firm 3 follow from (72) and (73). Each firm maximizes its own profits given its conjectures about its rivals. Firm 1's best response function is unchanged from (58) and firm 2's and firm 3's best response functions are

$$BR_2: L_2 = \frac{L_1(1-\lambda_{32})+L_3(1-\lambda_{12})}{2-\lambda_{32}-\lambda_{12}} \quad (74)$$

$$BR_3: L_3 = \frac{L_2(1+\lambda_{23})+2(1-\lambda_{23})}{3-\lambda_{23}} \quad (75)$$

The actual responses of firm 2 to the changes in firm 1's and firm 3's locations are

$$\rho_{21} = \frac{(1-\lambda_{32})}{2-\lambda_{32}-\lambda_{12}} \text{ and } \rho_{23} = \frac{(1-\lambda_{12})}{2-\lambda_{32}-\lambda_{12}}.$$

Likewise, firm 1's and firm 3's actual responses to the changes in firm 2's location are  $\rho_{12} = \frac{1+\lambda_{21}}{3-\lambda_{21}}$  and  $\rho_{32} = \frac{1+\lambda_{23}}{3-\lambda_{23}}$ . Simultaneously

solving  $\lambda_{12} = \rho_{12}$ ,  $\lambda_{21} = \rho_{21}$ ,  $\lambda_{23} = \rho_{23}$  and  $\lambda_{32} = \rho_{32}$  yields three sets of roots. Two sets of roots are indeterminate in that they imply relationships between the locations but no specific locations. Moreover, they involve two firms in a single location. The third set of roots has the first and third firm making identical conjectures about the middle firm and the middle firm making two identical conjectures about its two rivals.

$$\lambda_{21}^* = \frac{1}{2}, \lambda_{23}^* = \frac{1}{2}, \lambda_{12}^* = \frac{3}{5}, \lambda_{32}^* = \frac{3}{5} \quad (76)$$

Returning (76) into (58), (74) and (75) and solving simultaneously yields

$$L_1^* = \frac{3}{10}, L_2^* = \frac{1}{2}, L_3^* = \frac{7}{10} \quad (77)$$

This provides a symmetric equilibrium that has the lowest social cost of any of the three sets of roots but even it remains too concentrated around the center.<sup>17</sup> Social cost is

$$SC = \frac{1}{2}L_1^2t + \frac{1}{4}(L_2 - L_1)^2t + \frac{1}{4}(L_3 - L_2)^2t + \frac{1}{2}(1 - L_3)^2t \quad (78)$$

Plugging (77) into (78) yields  $SC^* = 0.11t$ . In the Nash equilibrium, the market is equally divided among these three firms and social cost is equal to  $0.0833t$ .

### 11.3. Equilibrium when $n = 4$

The profit expressions of firm 1 and firm 2 remain as above. The profit for firm 3 and 4 come from (72) and (73). Again, each firm maximizes its own profits given its conjectures about its immediate rivals. The best response functions of firm 1 and firm 2 are shown in (58) and (74). Those for firm 3 and 4 are

$$BR_3: L_3 = \frac{L_2(1-\lambda_{43})+L_4(1-\lambda_{23})}{2-\lambda_{23}-\lambda_{43}} \quad (79)$$

$$BR_4: L_4 = \frac{L_3(1+\lambda_{34})+2(1-\lambda_{34})}{3-\lambda_{34}} \quad (80)$$

Simultaneously solving  $\lambda_{12} = \rho_{12}$ ,  $\lambda_{21} = \rho_{21}$ ,  $\lambda_{23} = \rho_{23}$ ,  $\lambda_{32} = \rho_{32}$ ,  $\lambda_{34} = \rho_{34}$  and  $\lambda_{43} = \rho_{43}$  yields two sets of roots neither of which result in definitive locations. As one set of roots yields two sets of two firms collocating, we adopt that which requires only one set of two firms collocating. Those roots are

$$\lambda_{12}^* = \frac{1}{3}, \lambda_{21}^* = 0, \lambda_{23}^* = 1, \lambda_{32}^* = 1, \lambda_{34}^* = 0, \lambda_{43}^* = \frac{1}{3} \quad (81)$$

Returning (81) into (58), (74), (79), and (80) yields:  $L_1 = \frac{1}{3}L_2$ ,  $L_2 = L_3$ ,  $L_4 = \frac{1}{3}L_3 + \frac{2}{3}$ .

We impose symmetry on these locations which yields locations with the lowest social cost:

$$L_1^* = \frac{1}{6}, L_2^* = L_3^* = \frac{1}{2}, L_4^* = \frac{5}{6} \quad (82)$$

When the number of firms is equal to 4, the social cost can be shown as

$$SC = \frac{1}{2}L_1^2t + \frac{1}{4}(L_2 - L_1)^2t + \frac{1}{4}(L_3 - L_2)^2t + \frac{1}{4}(L_4 - L_3)^2t + \frac{1}{2}(1 - L_4)^2t \quad (83)$$

Plugging (82) into (83) yields  $SC^* = 0.0833t$ . The critical point is that using our assumption of symmetry to try to lower the social cost, it remains clear that consistent conjectures are inefficient relative to Nash. Under Nash, the efficient locations are  $1/8$ ,  $3/8$ ,  $5/8$  and  $7/8$ . And the minimized social cost is equal to  $0.0625t$ .<sup>18</sup>

#### 11.4. $n$ firms

We now follow the results derived above focusing first on the odd number of firms and seeking the only set of roots that yield numerical values of each firm's location. When the number of firms is even, this remains impossible as it was for 2 and 4 firms. Every set of roots yields only relationships among locations but no unique locations. Moreover, there is always at least one pair of firms that collocate. Thus, for even numbers of firms we limit the set of roots in those in which only a single pair of firms collocate. If there are more than one such set, we simply adopt that in which the first two interior firms co-locate and impose symmetry to identify the resulting social cost minimizing locations.

In examining an odd number of firms, we found no general deductive approach to show the pattern of resulting roots and locations. Yet, these patterns do emerge in a straightforward fashion when repeating the consistent conjectures exercise with increasingly larger number of firms. Thus, we examined the equilibrium with  $n = 1, 3, 5, 7, 9$  and  $11$  and generalized.<sup>19</sup> When the number of firms is odd, the conjectures that generate specific locations are always of the form:  $\lambda_{i,i-1} = \frac{3+i}{n+7}$  and  $\lambda_{i,i+1} = \frac{n+4-i}{n+7}$  when



$i$  is odd and always of the form  $\lambda_{i,i-1} = \frac{n+3-i}{n+5}$  and  $\lambda_{i,i+1} = \frac{2+i}{n+5}$  when  $i$  is even. In turn, these forms result in locations for the firms  $i=1 \dots n$  identified as follows:

$$L_i = \frac{2(n+3)+(i-1)(n+5)}{(n+1)(n+7)} \text{ when } i \text{ is odd}$$

$$L_i = \frac{i}{n+1} \text{ when } i \text{ is even}$$

The equilibrium locations under consistent conjectures as identified above are shown for  $n=1, 3, 5, 7, 9$  and  $11$  in Table 3. The pattern confirms that the locations under consistent conjectures are too concentrated around the center. The social cost associated with consistent conjectures is always above that for Nash. The percentage gap between the two is also shown in Table 3. This gap remains relatively large throughout our examination. Moreover, while social cost decreases with the number of firms, the decline with Nash appears faster at least with the range we examine.

We now consider an even number of firms. As described, all potential roots result in at least one pair of firms co-locating (as we saw with  $n=2$  and  $n=4$ ). Moreover, none of the roots define specific locations. Thus, to generalize we continue to limit our attention to the set of roots that generate only a single case of co-location and we assume that co-location to be by first two interior firms. We then assume symmetry given the resulting set of roots so as to minimize social cost.<sup>20</sup> Again, we build up the pattern by computing the equilibrium when  $n = 2, 4, 6, 8, 10$  and  $12$ .<sup>21</sup>

When the number of firms is even and larger than 2,  $\lambda_{12} = \frac{1}{3}$ ;  $\lambda_{i,i-1} = \frac{n-i+3}{n}$  and  $\lambda_{i,i+1} = \frac{i-3}{n}$  when  $i$  is odd;  $\lambda_{i,i-1} = \frac{i-2}{n+2}$  and  $\lambda_{i,i+1} = \frac{n-i+4}{n+2}$  when  $i$  is even. And the

locations are of the forms:

$$L_i = \frac{2}{n+8} \text{ when } i \text{ is equal to } 1$$

$$L_i = \frac{(i+3)n+2i-18}{(n-2)(n+8)} \text{ when } i \text{ is odd and larger than } 1$$

$$L_i = \frac{(i+4)n-12}{(n-2)(n+8)} \text{ when } i \text{ is even}$$

Table 4 illustrates the locations with consistent conjectures. It also isolates the percentage difference in cost between consistent conjectures and Nash. Again, firm locations are too concentrated around the center with consistent conjectures. Social costs are routinely higher than under Nash and, as a percentage, the gap remains large. Again, social cost declines with the number of firms but within our range (beyond  $n=2$ ) it declines faster with Nash.

## 12. Conclusion

This paper confirms that Nash location conjectures are inconsistent in the canonical model of spatial price discrimination. The consistent conjectural variation under duopoly with constant marginal cost equals one and the firms collocate with minimal differentiation. As a result, both firms earn zero profits. This mimics to some extent results from a basic quantity game with linear costs but the welfare implications are reversed. Unlike that basic quality game, consistent conjectures result in lower social welfare in the location game. We extend the basic model by increasing the number of firms in the case of linear costs. Firms remain too concentrated around the center and social welfare continues to be smaller.

When production costs are convex, the original Nash locations are not optimal and we show that the consistent conjecture equilibrium can improve the social welfare with sufficient cost convexity. While this model becomes complex for more than two firms, the basic point is the contrast between it and the linear cost model. We recognize that generalizing both the original Nash model and the conjectures model to more than two firms remains for future work.

Finally, we note that introducing downward sloping demand at each point on the unit line segment has been shown to generate duopoly locations away from the quartiles under Nash (Hamilton et al. 1989) even with linear production cost. An interesting future exploration might examine how consistent locations conjectures would change these locations.

THREE ESSAYS ON THE ECONOMICS OF SPATIAL PRICE DISCRIMINATION:

III) HOW TO LICENSE A TRANSPORT INNOVATION

### 13. Introduction

Economists have long studied how to license a patent for a process innovation that lowers production cost. Yet, this choice has rarely been cast in a spatial context and no attention has been paid to innovations that lower transport cost. We change this by explicitly adopting a model of spatial price discrimination and by determining how to charge for an innovation that lowers transport costs. Under spatial price discrimination, transport cost is an essential element in the ability of firms to earn profit as they charge a price that includes delivery of the product. A firm that reduces its transport cost gains both customers and profit. Yet, there has been no study of whether to license or how to charge for an innovation that lowers transport cost.

Improving distribution and lowering transport costs command enormous attention from businesses. Recently consumer firms Unilever and Evian North American took internal and reformed their transport management in efforts to reduce costs (Unilever, 2011 and Harps, 2006). More generally, innovations that reduce transport cost include, but are not limited to, efficient route planning and load tendering, advanced transit management and tracking, improvements in actual transit technology (be it by any mode or modes) and reducing time and costs associated with loading/unloading (Stefansson and Lumsden, 2009).

When transportation and distribution costs comprise a substantial share of overall costs, economists know there are strong incentives to engage in spatial price discrimination. Thisse and Vives (1988) show that discriminatory pricing is the preferred alternative when firms are able to adopt it and Greenhut (1981) identifies it as "nearly ubiquitous" among actual markets in which the products have substantial freight costs.

Thus, rather than allow customers to arrange their own delivery, the price quoted for goods includes delivery. This internalizes transport cost for the firm generating incentives to lower that cost through innovation. As we will show, this brings to the forefront a licensing strategy that explicitly includes distance. Such licensing has received little attention by economists but there exist famous historical examples. Congressional hearings describe how in 1852 Thomas Sayles licensed the patent of double-acting brakes to many US railroad companies with a distance fee of \$10 per mile (US Senate, 1878). Earlier in 1843 Charles Wheatstone licensed the patent for the five-needle telegraph also on a per mile basis. The royalty was £ 20 per mile for any first 10 miles laid with the per mile charge declining for additional increments (Bowers, 2001).

Outside the spatial context, economists focusing on process innovations often consider fixed fee licenses or royalty licenses. In basic Cournot competition models, the fixed fee license is typically superior to per unit royalty licensing when the innovator is not also producing the product (Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien et al., 1992). Yet, if the innovating firm also produces the product, a royalty license will be chosen because it provides both licensing fees and a competitive advantage (Wang, 1998; Kamien and Tauman, 2002).

In the spatial framework, nearly all previous research has been set in Hotelling type models of price competition. Taking location choices as exogenous both Poddar and Sinha (2004) and Kabiraj and Lee (2011) emphasize that a royalty will be chosen to license a production cost reducing innovation. Allowing location to be endogenous but on a circle, Caballero-Sanz et al (2002) show an outside innovator will adopt a royalty rather than an auction or a fixed fee. Matsumura et al. (2009) assume a royalty will be the

licensing method by an inside innovator and show the resulting locations on a line will be at the end points to maximize the product differentiation. None of these studies considers spatial price discrimination or a transport cost reducing innovation. Heywood and Ye (2010) do adopt a spatial price discrimination model but focus on a process innovation that lowers production costs and so they assume transport cost is constant and unaffected by the innovation.

In this paper, we first model a duopoly in which one firm innovates to distribute more cheaply and contemplates licensing this innovation to its rival. We show that under spatial price discrimination both traditional choices, a fixed fee or a royalty, are *unprofitable*. The innovation will simply not be shared. This finding is important in its own right as they are the traditional licensing mechanisms considered. We go on to show that when the innovator charges a license fee based on distance, the innovation will be licensed. We present an initially counter-intuitive result that the innovator will be able to charge a distance fee that exceeds the cost reduction of the innovation. Yet, we emphasize that this makes sense as even fees above the cost reduction allow the rival to earn additional profit because of an accommodating location choice of the innovator. We describe the licensing equilibrium showing that it increases social welfare relative to not licensing but that the equilibrium resulting licensing fee is too large to maximize social welfare.

We next contrast this choice of the distance fee with an outsider who innovates and decides among the same three licensing methods. Here the innovator earns profit from any method of licensing but earns the most by charging a fixed fee. Critically, there is none of the accommodation in location that makes the distance fee so profitable to the

inside innovator. Depending upon the size of the innovation (the extent to which it lowers transport costs), the innovator may find it profitable to license to one or both duopolists. We isolate the unique role played by the prisoner's dilemma. Jointly the duopolists would prefer not to accept the innovation as reduced transport cost reduces the profitability of spatial price discrimination. It is the ability of the outside innovator to craft a dominate strategy that results in both duopolists purchasing the license and in an improvement in social welfare.

Thus, by uniquely combining spatial price discrimination and an innovation that lowers transportation costs, we provide a series of new insights. In the final section we summarize and suggest future avenues of research.

#### **14. An Inside Innovator**

The market is normalized to a unit line segment and consumers are uniformly distributed along the market. Each consumer can buy one unit with reservation price  $\Gamma$ . We assume that  $\Gamma$  is everywhere above the delivered cost of the two competing firms. The transport costs per unit of distance for the firms are  $t_1$  and  $t_2$  and the locations are  $L_1$  and  $L_2$  where  $L_1 \leq L_2$ . We arbitrarily set a constant per unit production cost to zero and imagine that firm 1 creates a new technology that lowers its transport cost by  $c$ , such that  $t_1 = t - c$ ,  $t_2 = t$ , and  $c < t$ .

The game consists of three stages. In the first stage, firm 1 decides whether or not to license to firm 2 and if so, whether to do so by means of a fixed fee  $F$ , an output royalty  $r$  or a per unit of distance fee,  $d$ . Given the licensing decision made in the first



stage, both firms simultaneously choose their locations in second stage. The spatial price schedule is announced in the third stage.

The model is solved by backward induction. We illustrate by first examining the case of no licensing. The price schedule under discrimination is the outer envelope of the rivals' delivered costs as shown in Figure 4. For any consumer located at  $x$  the price is  $p(x, L_1, L_2) = \max \{t(L_2 - x), (t - c)(x - L_1)\}$ . Customers buy from the firm with lower delivered cost and so the consumer located at  $x^* = \frac{-L_1 t + L_1 c - t L_2}{-2t + c}$  is indifferent between firms.<sup>22</sup> The general expressions for profits (before paying or receiving licensing fees) are

$$\pi_1 = \int_0^{L_1} t_2(L_2 - x) - t_1(L_1 - x) dx + \int_{L_1}^{x^*} t_2(L_2 - x) - t_1(x - L_1) dx \quad (84)$$

$$\pi_2 = \int_{x^*}^{L_2} t_1(x - L_1) - t_2(L_2 - x) dx + \int_{L_2}^1 t_1(x - L_1) - t_2(x - L_2) dx \quad (85)$$

as identified in Figure 4.

In the second stage, each firm maximizes its own profit with respect to its location given  $t_1 = t - c$ ,  $t_2 = t$ , and  $x^*$ . Solving the resulting best response functions simultaneously yields  $L_1^{NL} = \frac{t}{4t - 2c}$  and  $L_2^{NL} = \frac{3t - c}{4t - 2c}$ . Returning these to (84) and (85) yields

$$\pi_1^{NL} = \frac{t^2(3t - c)}{4(2t - c)^2} \quad (86)$$

$$\pi_2^{NL} = \frac{7tc^2 - 8ct^2 + 3t^3 - 2c^3}{4(2t - c)^2} \quad (87)$$

Note that when  $c=0$  (the firms have the same transport cost), the firms locate at the quartiles,  $L_1 = \frac{1}{4}$  and  $L_2 = \frac{3}{4}$  (Hurter and Lederer, 1985) but move increasingly right as  $c$  increases. We now consider the three possible licensing schemes and ultimately compare  $\pi_1^{NL}$  and firm 1's profit under the alternative licensing schemes to determine the final equilibrium.

#### 14.1. Licensing with a Fixed Fee $F$

If firm 1 licenses with fixed fee  $F$ ,  $t_2 = t - c$ . The resulting price schedule becomes  $p(x, L_1, L_2) = \max \{(t - c)(L_2 - x), (t - c)(x - L_1)\}$ . The total profits are  $\pi_1^F = \pi_1 + F$  and  $\pi_2^F = \pi_2 - F$ , where  $\pi_1$  and  $\pi_2$  are (84) and (85) after recognizing that  $t_1 = t_2 = t - c$  and  $x^* = \frac{L_1 + L_2}{2}$ .

Firm 1 and firm 2 have identical transport costs, locate at the quartiles and  $\pi_1^F = \frac{3}{16}(t - c) + F$  and  $\pi_2^F = \frac{3}{16}(t - c) - F$ . Firm 1 charges the largest fixed fee acceptable to firm 2 by making it indifferent to licensing:  $F = \frac{3}{16}(t - c) - \pi_2^{NL} = \frac{c(8t^2 - 13tc + 5c^2)}{16(2t - c)^2}$  and so

$$\pi_1^F = \frac{6t^3 - 8ct^2 + tc^2 + c^3}{8(2t - c)^2} \quad (88)$$

#### 14.2. Licensing with a Per Unit of Output Royalty $r$

Firm 1 charges a royalty  $r$  for each unit sold regardless of distance shipped. Now  $t_2 = t - c$  but the price schedule includes  $r$ :  $p(x, L_1, L_2) = \max \{r + (t - c)(L_2 - x), (t - c)(x - L_1)\}$ . The resulting profits are  $\pi_1^R = \pi_1 + r$  and  $\pi_2^R = \pi_2 - r(1 - x^*)$

where  $t_1 = t_2 = t - c$  and  $x^* = \frac{L_1 t - L_1 c + r + t L_2 - c L_2}{2(t-c)}$ . The expression for the firm 2's profit shows it paying the royalty on each unit it sells but that firm 1 includes  $r$  on each unit sold by either firm. Obviously, it receives  $r$  as the payment from firm 2 on the units firm 2 sells but the fact that firm 2's delivered cost is uniformly increased by  $r$  means that it implicitly receives  $r$  on all of its own units sold as well.

In the second stage the locations again follow from each firm maximizes profit with respect to location and simultaneously solving the resulting best response functions.

These locations are functions of  $t$ ,  $c$  and  $r$  and when returned to the profit expressions

$$\text{yield } \pi_1^R = \frac{12c^2 - 76rc - 24tc + 76rt - 13r^2 + 12t^2}{64(t-c)} \text{ and } \pi_2^R = \frac{3(4c^2 + 12rc - 8tc - 12rt + 9r^2 + 4t^2)}{64(t-c)}. \text{ Firm 1}$$

maximizes  $\pi_1^R$  with respect to  $r$  subject to the constraint that  $\pi_2^R = \pi_2^{NL}$ . The constraint

ensures that the rival will accept the license and it is easy to show that  $\frac{\partial \pi_1^R}{\partial r} > 0$  for  $r$  less

than that implied by the constraint.<sup>23</sup> The solution implied by the constraint is then

$$r = \frac{2(-3c + 6t - 2\sqrt{6c^2 - 15tc + 9t^2})(t-c)}{9(2t-c)} \text{ which generates equilibrium locations } L_1^R =$$

$$\frac{6c - 12t + \sqrt{6c^2 - 15tc + 9t^2}}{18(-2t+c)} \text{ and } L_2^R = \frac{6c - 12t + \sqrt{6c^2 - 15tc + 9t^2}}{6(-2t+c)}. \text{ These locations imply that}$$

$$\pi_1^R = \frac{345t^3 + 389tc^2 - 664t^2c - 70c^3 + (132tc - 44c^2 - 88t^2)\sqrt{6c^2 - 15tc + 9t^2}}{108(2t-c)^2} \quad (89)$$

### 14.3. Licensing with a Per Unit of Distance Fee $d$

Firm 1 licenses with a fixed rate of  $d$  per unit of distance such that  $t_2 = t - c + d$ .

The equilibrium pricing schedule is  $p(x, L_1, L_2) = \max \{(t - c + d)(L_2 - x), (t -$

$c)(x - L_1)\}$ . Substituting  $t_1 = t - c$ ,  $t_2 = t - c + d$ , and  $x^* = \frac{-L_1 t + L_1 c - L_2 t + L_2 c - L_2 d}{-2t + 2c - d}$  into (84) and (85) yields  $\pi_1^D = \pi_1 + \int_{x^*}^{L_2} d(L_2 - x)dx + \int_{L_2}^1 d(x - L_2)dx$  and  $\pi_2^D = \pi_2$ .

In the second stage the locations again follow from each firm maximizing profit with respect to location and simultaneously solving the best response functions:  $L_1^D = \frac{2c^2 - 2cd - 4tc + 2t^2 + 2td + d^2}{8c^2 - 7cd - 16tc + 2d^2 + 8t^2 + 7td}$  and  $L_2^D = \frac{2(3c^2 - 3cd - 6tc + 3t^2 + 3td + d^2)}{8c^2 - 7cd - 16tc + 2d^2 + 8t^2 + 7td}$ . These values are returned to the profit expressions to yield  $\pi_1^D$  and  $\pi_2^D$  as shown in Appendix 3. Firm 1 now maximizes  $\pi_1^D$  with respect to  $d$  subject to the constraint that  $\pi_2^D = \pi_2^{NL}$ . The constraint again insures that the rival will accept the license and it can be checked that  $\frac{\partial \pi_1^D}{\partial d} > 0$  for all values of  $d$ . Thus, the constraint binds and has a single positive real root,  $d'(c, t)$ , that can be solved but is a messy higher order function of  $c$  and  $t$  (although available upon request). Critically, the optimal value  $d$  that is always above  $c$ . We show this by normalizing  $t=1$  so that  $c$  becomes the share of the transport cost eliminated by innovation and  $d$  becomes the share of transport cost being charged as a distance fee. In this case,  $d'(c, t = 1) - c = 0$  only when  $c$  itself equals zero and is otherwise positive.<sup>24</sup> For example when the innovation reduces transport cost to half of its previous size,  $c=0.5$ , the profit maximizing distance fee is  $d'=0.657$ .

We will show that the optimal choice is to license with the distance fee  $d$  but first we explain the effect of  $d$  on locations and why the optimal  $d$  is greater than  $c$ . We do this in a proposition that highlights the accommodating response of the innovator to a large distance fee.

**Proposition 11:** i)  $\frac{\partial L_1^D}{\partial d} > 0$  and  $\frac{\partial L_2^D}{\partial d} > 0$ ; ii) if  $d=c$ ,  $L_1^D < L_1^{NL}$  and  $L_2^D < L_2^{NL}$  and  $\pi_2^D - \pi_2^{NL} > 0$ .

**Proof:** i)  $\frac{\partial L_1^D}{\partial d} = \frac{(t-c)(3d^2+8d(t-c)+2(t-c)^2)}{(-16tc-7cd+2d^2+8c^2+8t^2+7td)^2} > 0$ ;

$$\frac{\partial L_2^D}{\partial d} = \frac{2(t-c)(d^2+4d(t-c)+3(t-c)^2)}{(-16tc-7cd+2d^2+8c^2+8t^2+7td)^2} > 0.$$

ii)  $L_1^D(d=c) - L_1^{NL} = \frac{c(c-t)(2c-3t)}{2(-2t+c)(3(t-c)^2+3t(t-c)+2t^2)} < 0$ ;  $L_2^D(d=c) - L_2^{NL} =$

$$\frac{c(c-t)^2}{2(-2t+c)(3(t-c)^2+3t(t-c)+2t^2)} < 0;$$

$$\pi_2^D(d=c) - \pi_2^{NL} = \frac{c(-3t+2c)(-t+c)^3(5(t-c)^2+7t(t-c)+4t^2)}{4(-9tc+3c^2+8t^2)^2(-2t+c)^2} > 0.$$

Given that firm 2 has the new transport technology, as  $d$  increases firm 2 has increasingly higher transport costs than firm 1 and, as a consequence, the optimal locations of both firms move right. Critically, when  $d=c$ , each firm has identical transport costs to those without licensing but the locations differ from those without licensing. Firm 1 locates less aggressively, closer to the left as shown by ii), as it earns licensing revenue that grows with the market share of firm 2. The more accommodating location of firm 1 implies that when  $d=c$ , firm 2 also moves left and so earns profits above what it earned without the new technology despite having the same transport costs. Thus, firm 1 can actually increase the value of  $d$  above  $c$  moving both firms right until firm 2 earns profit identical to its no licensing case.<sup>25</sup> This ability of firm 1 to dramatically increase licensing revenue through an accommodating location sets up the final equilibrium.

#### 14.4. The Equilibrium

The choice of licensing scheme follows from comparing the profit of firm 1 in the four cases outlined.

**Proposition 12:** i) Fixed fee and royalty licensing are never profitable; ii) The only profitable license uses a unit distance fee.

**Proof:**

i) Subtracting (86) from (88) yields  $\pi_1^F - \pi_1^{NL} = -\frac{c(c+3t)}{8(2t-c)} < 0$ . Subtracting (86) from (89)

yields  $\pi_1^R - \pi_1^{NL} = -\frac{70c^3 - 389tc^2 + 637t^2c - 264t^3 + (44c^2 - 132tc + 88t^2)\sqrt{6c^2 - 15tc + 9t^2}}{108(2t-c)^2} < 0$ .

ii)  $\pi_1^D(d=c) - \pi_1^{NL} = \frac{c(c-t)^3(6(c^3-t^3) - 9tc^2 - 25t(t-c)^2 - 17(t-c)t^2)}{4(8t^2+3c^2-9tc)^2(2t-c)^2} > 0$ ;  $\frac{\partial \pi_1^D}{\partial d} =$

$\frac{8d^6 + 84(t-c)d^5 + 374(t-c)^2d^4 + 901(t-c)^3d^3 + 1272(t-c)^4d^2 + 1016d(t-c)^5 + 368(t-c)^6}{2(2d^2 + 7d(t-c) + 8(t-c)^2)^3} > 0$ . From the

above analysis, we have already known that  $d' > c$  so  $\pi_1^M(d=d') - \pi_1^{NL} > 0$ .

The best fixed fee is not sufficient to outweigh the disadvantage of facing a rival with equal transport costs. Similarly, the royalties earned per unit of output and the resulting increased production cost of firm 2 do not outweigh the disadvantage of facing a rival with equal transport costs. Only when firm 1 can not only maintain but expand its transport cost advantage will it make sense to license and the licensing scheme that can accomplish this is dependent upon distance.

### 15. Illustrating the Consequences of the Licensing Equilibrium

We now illustrate the equilibrium that uses the distance fee. We first present a simulation of the locations for different size innovations and contrast those with the

locations that happen without licensing. We then show that the equilibrium with licensing improves social welfare relative to that without licensing but that it generates a licensing fee that is too large to maximize social welfare.

To get a sense of the impact of the distance fee licensing on firms' locations we consider a series of cost reductions associated with innovation,  $c = 0.1t, \dots, 0.9t$ . For each of the nine values from a 10 percent reduction in costs to a 90 percent reduction in costs, we calculate the equilibrium locations associated with no licensing and with the licensing equilibrium we have derived. These are presented in Table 5 and illustrate that the two firms move to the right in either equilibrium as  $c$  increases. Critically, for any given value of  $c$ , the licensee locates further to the right under the licensing equilibrium than in the case of no licensing. At the same time, the innovator locates further to the left under the licensing equilibrium than under the case of no licensing. Thus, the availability of licensing pushes the firms toward the corners. The movement right by the licensee is generally far smaller than the accommodating movement of the innovator left.

The consequence of licensing on social welfare depends, in part, on these movements. Any movement toward symmetry improves welfare as does the fact that fewer real resources are used to transport the goods of the licensee.

Now, we compare the social welfare with and without licensing. The social welfare ( $SW$ ) follows as the difference between total willingness to pay and the real transport cost ( $TC$ ):  $SW = \Gamma - TC$ . The transport cost under no licensing is:  $TC^{NL} = \frac{1}{2}(t - c)(L_1)^2 + \frac{1}{2}(t - c)(x^* - L_1)^2 + \frac{1}{2}t(L_2 - x^*)^2 + \frac{1}{2}t(1 - L_2)^2$  where  $x^* = \frac{-L_1t + L_1c - tL_2}{-2t + c}$ . The transport cost under distance fee licensing is:  $TC^D = \frac{1}{2}(t - c)(L_1)^2 + \frac{1}{2}(t - c)(x^* - L_1)^2 + \frac{1}{2}(t - c)(L_2 - x^*)^2 + \frac{1}{2}(t - c)(1 - L_2)^2$ . Note that the real

transport cost per unit for firm 2 does not include the licensing transfer to firm 1 yet that transfer influences the location of the indifferent consumer so that  $x^* =$

$$\frac{-L_1 t + L_1 c - L_2 t + L_2 c - L_2 d}{-2t + 2c - d}. \text{ The social welfare under no licensing is: } SW^{NL} = \Gamma - \frac{t(t-c)}{4(2t-c)} \text{ but}$$

the social welfare under distance fee licensing remains a higher order function of  $c$  and  $t$ .

By again normalizing  $t=1$ , both social welfare functions can be easily graphed over the range of  $c$ . These are shown in Figure 5 and make clear that  $SW^{d'} - SW^{NL} > 0$  for all  $0 < c < t$ . Licensing improves welfare.

While it is clear that a governmental authority concerned with welfare should allow licensing with a distance fee, it may wish to control the size of that fee. On the one hand, a fee of zero would result in efficient locations at the quartiles and minimize the real resources spent on transportation.<sup>26</sup> The problem with setting such a fee is that it lowers the profit of firm 1 and so licensing will not happen. Thus, we explore whether there exists a governmentally set fee that maximizes the increase in welfare associated with licensing subject to the constraint that the innovator has an incentive to license.

We have already shown that the profit of firm 1 increases in  $d$  and it can be shown that social welfare decreases in  $d$ . Thus, the welfare maximizing fee is that which makes firm 1 indifferent to licensing and that this fee is lower than the equilibrium level. When one solves the implied constraint that  $\pi_1^D$  in the appendix equals  $\pi_1^{NL}$  it yields a higher order polynomial root that can again be easily graphed. Figure 6 shows  $d^* < d'$  and while  $d' > c$ ,  $d^* < c$  for all  $0 < c < t$ . The distances associated with the socially optimal fee are illustrated in Table 5. They show that both firms locate to the left of either the no-licensing case or the licensing equilibrium. The smaller fee promotes the leftward accommodation of the innovator but does not allow it to be fully exploited. Indeed, it can



be confirmed that the profitability of the rival increases as a result of the licensee even as that of innovator remains the same as without licensing. The total transport costs are illustrated in Table 6 showing the welfare maximizing costs to be lowest but closer to those in the licensing equilibrium than the costs in that equilibrium are to those in the case without licensing.

We recognize that we have illustrated only the solution to the static problem of how to price an existing innovation. We have not modeled the R&D process and a governmental authority concerned with providing optimal dynamic incentives for innovation might adopt an alternative fee. This would presumably be higher and provide a licensing return to the innovator raising the traditional issues of dynamic vs. static efficiency common in studying patents.

## 16. An Outside Innovator

We now consider a transport cost-reducing innovation from a market outsider. The basics of the framework remain identical but now without licensing  $t_1 = t_2 = t$  and with licensing  $t_i = t - c$ . The game again consists of three stages. In the first stage, the outsider now decides whether to license and if so, by which method and to how many firms. We consider the same three potential licensing fees. In the second stage the firms decide whether or not to accept the license and then simultaneously locate. In the third stage, the price schedule is announced.

We again solve by backward induction and first review the no licensing case. The consumer located at  $x$  is charged  $p(x, L_1, L_2) = \max \{t(L_2 - x), t(x - L_1)\}$ . Firm profits are  $\pi_1^{NL} = \pi_1$  and  $\pi_2^{NL} = \pi_2$  where  $\pi_1$  and  $\pi_2$  are (84) and (85) after substituting

$t_1 = t_2 = t$  and  $x^* = \frac{L_1 + L_2}{2}$ . With identical transport cost per unit of distance, firms locate at the quartiles and

$$\pi_1^{NL} = \pi_2^{NL} = \frac{3}{16}t \quad (90)$$

Obviously, the outside innovator earns no profit in this case,  $L=0$  (where  $L$  denotes the outside innovator's licensing revenue).

### 16.1. Licensing with a Fixed Fee $F$

If the outside innovator licenses to both firms with a fixed fee  $F$ ,  $L=2F$  and  $t_1 = t_2 = t - c$ . The price schedule is  $p(x, L_1, L_2) = \max \{(t - c)(L_2 - x), (t - c)(x - L_1)\}$ . The total profits are  $\pi_1^F = \pi_1 - F$  and  $\pi_2^F = \pi_2 - F$  where  $\pi_1$  and  $\pi_2$  are (84) and (85) with  $t_1 = t_2 = t - c$  and  $x^* = \frac{L_1 + L_2}{2}$ . The firms locate at quartiles and  $\pi_1^F = \pi_2^F = \frac{3}{16}(t - c) - F$ . Importantly, if the firms were to cooperatively decide, they would not accept the license as both the fixed fee and the lower transport cost reduce profits. However, in a non-cooperative equilibrium, there exists the fear that the rival will accept the innovation if one firm unilaterally declines. We describe the resulting prisoner's dilemma.

Imagine only firm 1 purchases the license, then  $\pi_1^{F1} = \pi_1 - F$  and  $\pi_2^{F1} = \pi_2$  where  $t_1 = t - c$ ,  $t_2 = t$ , and  $x^* = \frac{-L_1 t + L_1 c - t L_2}{-2t + c}$ . In the second stage each firm maximizes its profit with respect to its location and solving the resulting best response functions simultaneously yields the equilibrium locations. These locations yield  $\pi_1^{F1} =$

$\frac{t^2(3t-c)}{4(2t-c)^2} - FI$  and  $\pi_2^{F1} = \frac{7tc^2 - 8ct^2 + 3t^3 - 2c^3}{4(2t-c)^2}$ . The fixed fee  $FI$  makes firm 1 indifferent

about buying the innovation and is determined by subtracting (90) from  $\frac{t^2(3t-c)}{4(2t-c)^2}$ . Thus

$$L = FI = \frac{tc(8t-3c)}{16(2t-c)^2} \quad (91)$$

Thus, for any fixed fee less than or equal to  $FI$  firm 1 purchases the license when its rival doesn't. If firm 1, indeed, purchases the license, firm 2 could be better off by also purchasing the license. The profit for each firm if both firms purchase the license is

$\frac{3}{16}(t-c) - F$  which can be compared to  $\pi_2^{F1}$ . Firm 2 thus purchases the license

whenever the fixed fee is less than or equal to  $\frac{3}{16}(t-c) - \pi_2^{F1} = \frac{c(8t^2 - 13tc + 5c^2)}{16(2t-c)^2} = F^*$ .

It can be shown that  $F^* < FI$  and thus it is the largest fee that will make purchasing the license a dominant strategy for each firm. The outsider's licensing revenue is

$$L = 2F^* = \frac{c(8t^2 - 13tc + 5c^2)}{8(2t-c)^2} \quad (92)$$

By comparing the licensing revenue in (91) to that in (92), the outside innovator decides whether to license to one firm or two firms.

**Proposition 13:** The outside innovator licenses by a fixed fee to both firms when

$0 < c < .427t$  and to only one firm when  $.427t < c < t$ .

**Proof:** Subtracting (91) from (92) yields  $\frac{8t^2c - 23tc^2 + 10c^3}{16(2t-c)^2}$  which when set equal to zero

and solved yields  $c = .427t$  and the sign can be checked either side of this critical value.

When two licenses are sold, equilibrium locations remain at the quartiles.

Moreover, because of the need to make licensing a dominant strategy for both firms, the profit of each firm increases as a result of licensing. If only a single firm is licensed, the full value of the cost reduction is extracted by the innovator. Which strategy is chosen depends on the relative size of  $c$  to  $t$ . When  $c$  is small relative to  $t$ , a single firm with access to the technology gains relatively little market share and as a consequence it is less profitable to sell only one license. When the cost reduction is large, extracting the full value can be more profitable as the single licensed firm gains substantial market share.

## 16.2. Licensing with an Output Royalty $r$

If the outside innovator licenses to both firm 1 and firm 2 with an output royalty  $r$  for each unit sold then  $t_1 = t_2 = t - c$  and the price schedule is  $p(x, L_1, L_2) = \max \{r + (t - c)(L_2 - x), r + (t - c)(x - L_1)\}$ . As both firms pay  $r$  per unit, the royalty influences neither locations nor the resulting profits. Thus,  $\pi_1^r = \pi_1$  and  $\pi_2^r = \pi_2$ , where  $\pi_1$  and  $\pi_2$  are (84) and (85) with  $t_1 = t_2 = t - c$  and  $x^* = \frac{L_1 + L_2}{2}$ . As  $\Gamma$  continues to exceed the delivered costs by assumption,  $L = r$  and  $\pi_1^r = \pi_2^r = \frac{3}{16}(t - c)$ . While the firms no longer lose profit to a fixed fee, they continue to earn less than without the technology and the prisoner's dilemma remains.

If only firm 1 purchases the patent,  $t_1 = t - c$ ,  $t_2 = t$ , and  $L = (r1)x^*$ . The price schedule becomes  $p(x, L_1, L_2) = \max \{t(L_2 - x), (r1) + (t - c)(x - L_1)\}$ . Firm profits are  $\pi_1^{r1} = \pi_1 - (r1)x^*$  and  $\pi_2^{r1} = \pi_2 + (r1)(1 - x^*)$ , where  $\pi_1$  and  $\pi_2$  are (84) and (85) with  $t_1 = t - c$ ,  $t_2 = t$ , and  $x^* = \frac{tL_1 + tL_2 - (r1) - cL_1}{2t - c}$ . In the second stage, each firm maximizes its profit over its own location and simultaneously solving the resulting best

response functions yields equilibrium locations. Returning these locations to outside innovator's licensing revenue and firms' profits yields  $L = \frac{(r1)(t-2(r1))}{(2t-c)}$  and

$$\pi_1^{r1} = \frac{(2(r1)-t)^2(3t-c)}{4(2t-c)^2} \quad (93)$$

$$\pi_2^{r1} = \frac{(t+2(r1)-c)^2(3t-2c)}{4(2t-c)^2} \quad (94)$$

Firm 1 gains the market advantage of lower transport cost but must pay the royalty. The outsider maximizes royalty income and it can be shown that  $\frac{\partial L}{\partial r1} > 0$  for all royalties that would induce firm 1 to purchase the license. Thus, the outside innovator chooses  $r1$  so that (93) equals the profit without licensing in (90):  $r1 = \frac{-6t^2+2tc+\sqrt{36t^4-48t^3c+21t^2c^2-3c^3t}}{4(c-3t)}$ .

The resulting licensing revenue is

$$L=(r1)x^* = \frac{t(-3c^2+12tc-12t^2+2\sqrt{3t(3t-c)(2t-c)^2})}{8(3t-c)(2t-c)} \quad (95)$$

If firm 1 purchases the license, firm 2 might also be better off purchasing the license.

When two licenses are sold, the profit of each firm is  $\frac{3}{16}(t-c)$  which equals (94) when

$$r^* = \frac{-4c^2+10tc-6t^2+\sqrt{6c^4-39tc^3+93c^2t^2-96t^3c+36t^4}}{4(3t-2c)} \quad (96)$$

It can be shown  $r^* < r1$  and thus both firms will purchase the license as a dominant strategy for any royalty less than or equal to  $r^*$ . Thus, when selling to two firms  $L=r^*$ .

The optimal number of firms to be licensed can be determined.

**Proposition 14:** The outside innovator licenses by an output royalty to both firms when  $0 < c < .847t$  and to only one firm when  $.847t < c < t$ .

**Proof:** Subtracting (95) from (96) yields

$$\frac{54tc^3 - 8c^4 - 127t^2c^2 + 120t^3c - 36t^4 - (10ct - 2c^2 - 12t^2)\sqrt{3(2c^2 - 5ct + 3t^2)(2t - c)^2} + (4tc - 6t^2)\sqrt{3t(3t - c)(2t - c)^2}}{8(3t - 2c)(3t - c)(2t - c)}$$

which when set equal to zero and solved yields  $c = .847t$  and the sign can be checked either side of this critical value.

Again, if both firms are licensed the locations remain at the quartiles. The outside innovator collects the smaller royalty but for all units sold in the market. With one license, it collects the larger royalty but only on the units of one firm. The share of the market for that one firm is larger when  $c$  is larger. Thus, only for large values of  $c$ , will the higher royalty fee cause the outsider to sell only to one firm. This largely mimics the result with the fixed fee.

### 16.3. Licensing with a Per Unit of Distance Fee $d$

If the outside innovator licenses to both firms with a distance fee  $d$ , then  $t_1 = t_2 = t - c + d$ . The price schedule is  $p(x, L_1, L_2) = \max \{(t - c + d)(L_2 - x), (t - c + d)(x - L_1)\}$ . Substituting  $t_1 = t_2 = t - c + d$  and  $x^* = \frac{L_1 + L_2}{2}$  into (84) and (85) yields  $\pi_1^d = \pi_1$  and  $\pi_2^d = \pi_2$ . The outside innovator's total income is  $L = \int_0^{L_1} d(L_1 - x)dx + \int_{L_1}^{x^*} d(x - L_1)dx + \int_{x^*}^{L_2} d(L_2 - x)dx + \int_{L_2}^1 d(x - L_2)dx$ . Each firm earns  $\pi_1^d = \pi_2^d = \frac{3}{16}(t - c + d)$ .

If only firm 1 purchases the patent then  $t_1 = t - c + d1$  and  $t_2 = t$ . The price schedule is  $p(x, L_1, L_2) = \max \{t(L_2 - x), (t - c + d1)(x - L_1)\}$ . Firms' profits are  $\pi_1^{d1} = \pi_1$  and  $\pi_2^{d1} = \pi_2$  where  $\pi_1$  and  $\pi_2$  are (84) and (85) with  $t_1 = t - c + d1$ ,  $t_2 = t$ , and  $x^* = \frac{-L_1 t + L_1 c - L_1(d1) - t L_2}{-2t + c - (d1)}$ . The outside innovator's revenue is  $L = \int_0^{L_1} (d1)(L_1 - x)dx + \int_{L_1}^{x^*} (d1)(x - L_1)dx$ . In stage 2, each firm maximizes its profit with respect to its own location and simultaneously solving the resulting best response functions yields equilibrium locations. Returning the equilibrium locations to the outside innovator's licensing revenue and each firm's profit yields  $L = \frac{(d1)t^2}{4(2t - c + (d1))^2}$  and

$$\pi_1^{d1} = \frac{t^2(-c + (d1) + 3t)}{4(2t - c + (d1))^2} \quad (97)$$

$$\pi_2^{d1} = \frac{(3t - 2c + 2(d1))(t - c + (d1))^2}{4(2t - c + (d1))^2} \quad (98)$$

The outside innovator maximizes  $L$  with respect to  $d1$  subject to the constraint that (97) is larger than or equal to (90). It can be checked that  $\frac{\partial L}{\partial d1} > 0$  for values of  $d1$  that meet the constraint. Thus,  $d1 = c$  and  $L = \frac{1}{16}c$ .

Thus, for any distance fee less than or equal to  $d1$  firm 1 will purchase the license when its rival doesn't. If firm 1 has the innovation, firm 2 could be better off by also purchasing the license. The maximum amount the second firm will pay is given by setting  $\pi_2^d = \frac{3}{16}(t - c + d)$  equal to (98) and solving. Thus,  $d^* = c$  and the outside innovator will charge  $c$  no matter how many firms are licensed and so will license both firms. With two licenses sold, the licensing revenue is

$$L = \frac{1}{8}c \quad (99)$$

In this case there is no tradeoff between a higher licensing fee and the number of firms licensed.

#### 16.4. The Equilibrium

The final equilibrium reflects the outsider's choice of licensing scheme.

**Proposition 15:** For an outside innovator all three forms of licensing are profitable but charging a fixed fee is the most profitable.

**Proof:** A piecewise comparison of  $L$  across all values of  $c$  from 0 to  $t$  for each licensing scheme shows the fixed fee dominates. This comparison is presented in Figure 7.

The optimal distance fee is charged to both firms and as a consequence, the licensing revenue is simply linear in  $c$  from (99). For either a fixed fee or the royalty a single firm is licensed with large enough  $c$ . The inflection points from propositions (13) and (14) are illustrated in Figure 7. Because paying a fixed fee gives the single licensed firm a bigger cost advantage and market share, the inflection comes at smaller  $c$  than for the royalty. As a consequence of this larger advantage, the value of the license is greatest to a single firm when paying a fixed fee making it the optimal single license for the outsider. Importantly, this large advantage for the single licensed firm means the greatest competitive harm to the excluded rival. As a consequence, the excluded rival will pay the most to receive a second license under the fixed fee. Thus, regardless of whether licensing to one or two firms the fixed fee will be chosen



As in the case of an insider innovating, licensing improves social welfare. Social welfare remains the difference between total willingness to pay and real transport cost. With two licenses sold, the reduction in unit transport cost improves social welfare without changing firms' locations relative to the no licensing case. Moreover, these locations are first best. When only one firm is licensed the fact that asymmetric locations emerge is outweighed by the fact that transport costs are lower for most of the market.<sup>27</sup> Yet, the locations are not first best and a social planner would prefer licensing to both firms.

## 17. Conclusion

This paper is unique in studying the licensing of transport cost-reducing innovation. We recognize that such an innovation can be critical under spatial price discrimination in which the consumer pays a delivered price. While the Cournot quantity model argues that royalties are preferable to fixed fees for an insider, we show that both of these fee structures are impossible for the innovation we study. Neither generates profit and it is simply better for the innovator to enjoy lower transport cost. We show that only a per distance fee can provide the innovator with both licensing revenue and a superior competitive position.

When innovator is outside the market, we show that the fixed fee license always generates the highest licensing revenue. This result in some ways mimics that from simply Cournot competition. Like that case the innovator can extract the most revenue but the mechanism differs. Specifically, we demonstrate the importance of the prisoner's dilemma as jointly the firms would prefer *not* to have the innovation that inherently limits

the profitability of spatial price discrimination. It is only the fear that the rival will purchase the license that allows the innovator to establish a dominant strategy in which each firm pays for the license. This differs from the case of process innovation in the Cournot quantity model. We show that as for high levels of  $c$ , the outsider will prefer to sell to only one firm.

There are a number of possible extensions of this work. One of our basic assumptions has been that regardless of the licensing decisions, the full market is served. This could be modified. First, when the reservation price is low enough, purchasing the license could lower transport cost so that the firms could acquire new customers. Second, when the innovator is an insider, the location advantage associated with lower transport cost becomes muted. This may imply that fixed fee licensing and the royalty licensing could be profitable and even change the relative ordering. Finally, one might consider downward sloping demand curves at each point in the market suggesting another gain to the firms from purchasing the license.

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## Appendix A: Proposition 1

*Case 1: Both firms use delegation*

Equation (11) gives the payoffs with delegation.

*Case 2: Both firms do not use delegation*

Plugging  $L_1 = 1/4$  and  $L_2 = 3/4$  (Hurter and Lederer 1985) into equations (1) and (2) yields  $\pi_1 = \pi_2 = .188t$ .

*Case 3: Only Firm 1 adopts delegation (symmetric to only Firm 2 adopting delegation)*

Imaging that owner 1 adopts delegation and owner 2 does not. With backward induction, we know the pricing equilibrium and first solve the location game. Firm 1's manager maximizes equation (3) and Firm 2's owner maximizes equation (2) as  $\alpha_2 = 1$ .

Solving the best response functions yields the locations:

$$L_1 = (2\alpha_1 t + 3 - 3\alpha_1)/8\alpha_1 t$$

$$L_2 = (6\alpha_1 t + 1 - \alpha_1)/8\alpha_1 t$$

Returning these to equation (1), the incentive parameter chosen by the owner of Firm 1 is:

$$\alpha_1 = 5/(2t + 5)$$

Returning this value to the locations above and to equations (1) and (2) yields:

$$L_1 = .4 \text{ and } L_2 = .8, \pi_1 = .2t, \text{ and } \pi_2 = .12t$$

The payoffs are all shown in Table 1 revealing delegation as a dominant strategy for each owner.

## Appendix B: Delegation with Convex Costs and Proposition 6

*Case 1: Both firms use delegation*

Equation (26) gives the payoffs with delegation.

*Case 2: Both firms do not use delegation*

Plugging  $L_1 = (2t + k)/8(k + t)$  and  $L_2 = (6t + 7k)/8(k + t)$  (Gupta 1994) into equations (19) and (20) yields:

$$\pi_1 = \pi_2 = \frac{31k^2t + 36kt^2 + 12t^3 + 8k^3}{64(k+t)^2}$$

*Case 3: Only Firm 1 adopts delegation (symmetric to only Firm 2 adopting delegation)*

In the location game Firm 1's manager maximizes equation (3) and Firm 2's owner maximizes equation (20) to generate best response functions and locations as a function of Firm 1's delegation parameter. In stage 2,  $\alpha_1$  is chosen by Firm 1's owner to maximize his profit. Returning  $\alpha_1$  to locations yields:

$$L_1 = \frac{2(4k^4 + 19tk^3 + 31k^2t^2 + 20kt^3 + 4t^4)}{232tk^3 + 289k^2t^2 + 140kt^3 + 64k^4 + 20t^4}$$

$$L_2 = \frac{56k^4 + 195tk^3 + 232k^2t^2 + 108kt^3 + 16t^4}{232tk^3 + 289k^2t^2 + 140kt^3 + 64k^4 + 20t^4}$$

Given above locations, firms' profits are:

$$\pi_1 = \frac{16k^5 + 88tk^4 + 177k^3t^2 + 158k^2t^3 + 60kt^4 + 8t^5}{2(232tk^3 + 289k^2t^2 + 140kt^3 + 64k^4 + 20t^4)}$$

$\pi_2 =$

$$\frac{512k^9 + 4672tk^8 + 18168k^7t^2 + 39271k^6t^3 + 51542k^5t^4 + 42063t^5k^4 + 20960t^6k^3 + 5992t^7k^2 + 864t^8k + 48t^9}{(232tk^3 + 289k^2t^2 + 140kt^3 + 64k^4 + 20t^4)^2}$$

Subtracting Firm 2's profit in case 3 from Firm 2's profit in case 1 yields:

$$\frac{t^3(474516k^5t^5+6976k^9t+41856k^8t^2+512k^{10}+145281k^7t^3+322062k^6t^4+467864k^4t^6+302128k^3t^7+120480k^2t^8+26048kt^9+2176t^{10})}{8(8k^2+15tk+6t^2)^2(289k^2t^2+140kt^3+64k^4+232k^3t+20t^4)^2}$$

Thus, the owner of Firm 2 will delegate if Firm 1 delegates and symmetry implies that the owner of Firm 1 will delegate if Firm 2 delegates. Subtracting Firm 1's profit in case 2 from Firm 1's profit in case 3 yields:

$$\frac{t^3(k^4+8k^3t+24k^2t^2+32kt^3+16t^4)}{64(289k^2t^2+140kt^3+64k^4+232k^3t+20t^4)(k+t)^2} > 0$$

Thus, Firm 1's owner will delegate if Firm 2 does not delegate and symmetry implies that the owner of Firm 2 will delegate if Firm 1 does not delegate. Thus, each firm has a dominant strategy of delegation.

Note that the proof of proposition 6 i) follows from subtracting Firm 1's profit in case 2 from Firm 1's profit in case 1:

$$-\frac{t^2(16k^5+127k^4t+392k^3t^2+584k^2t^3+416kt^4+112t^5)}{64(8k^2+15kt+6t^2)^2(k+t)^2} < 0$$

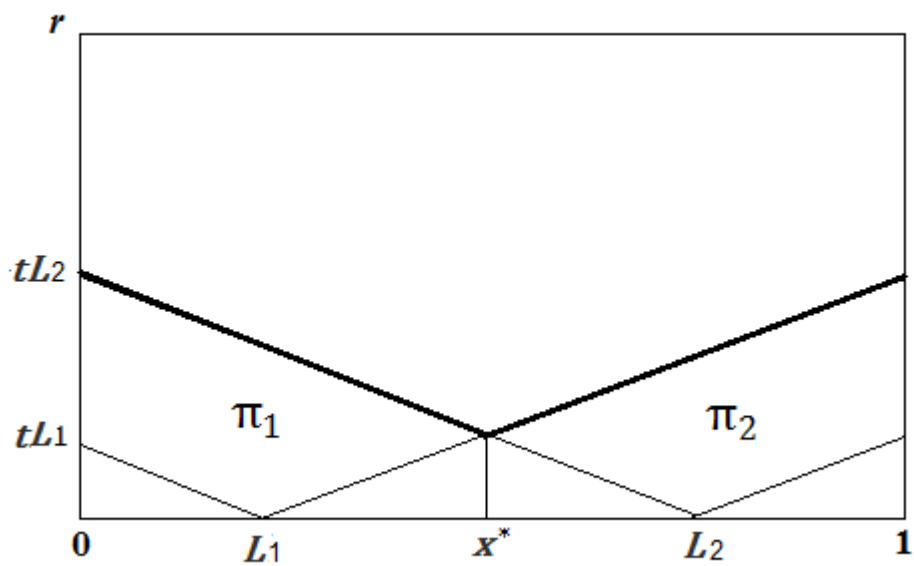
Thus, the dominant strategy reduces profit.

**Appendix C:**

$$\pi_1^D = -\frac{1}{2(8c^2-7cd-16tc+2d^2+8t^2+7td)^2} (24c^5 - 120tc^4 - 88c^4d + 352c^3dt + 114c^3d^2 + 240t^2c^3 - 528t^2c^2d - 342tc^2d^2 - 240t^3c^2 - 75c^2d^3 + 26cd^4 + 150ctd^3 + 352cdt^3 + 342cd^2t^2 + 120t^4c - 4d^5 - 75t^2d^3 - 26td^4 - 114t^3d^2 - 88t^4d - 24t^5)$$

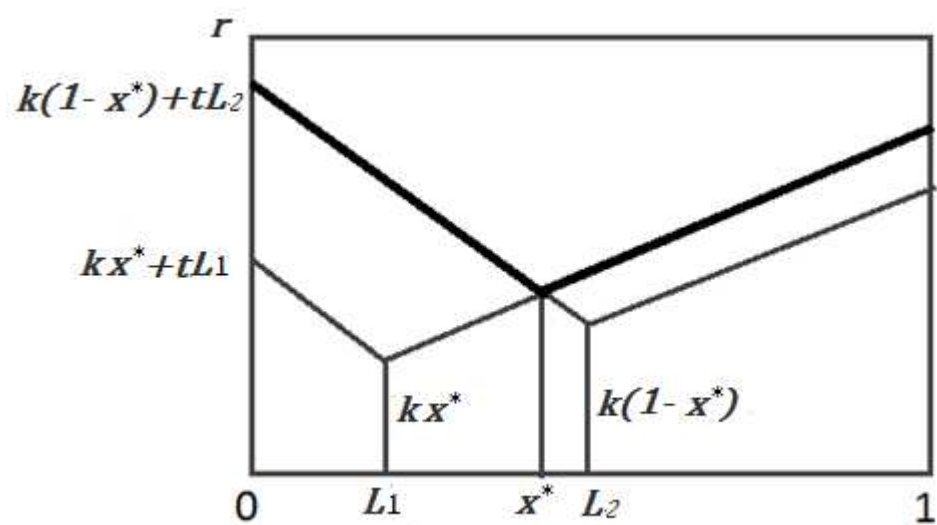
$$\pi_2^D = -\frac{1}{(8c^2-7cd-16tc+2d^2+8t^2+7td)^2} (12c^5 - 60tc^4 - 16c^4d + 64c^3dt + 7c^3d^2 + 120t^2c^3 - 96t^2c^2d - 21tc^2d^2 - 120t^3c^2 - c^2d^3 + 2ctd^3 + 64cdt^3 + 21cd^2t^2 + 60t^4c - 12t^5 - t^2d^3 - 7t^3d^2 - 16t^4d)$$

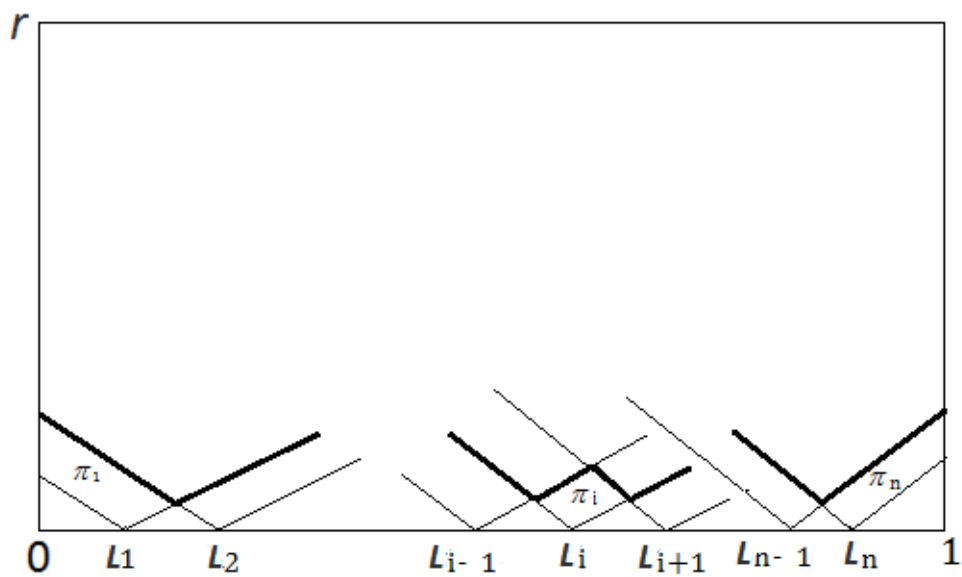
**Fig.1.** Spatial price discrimination in duopoly competition



*Note:* The thick line is the delivered price schedule with  $\pi_i$  the profit earned on the difference between the schedule and the delivered cost.

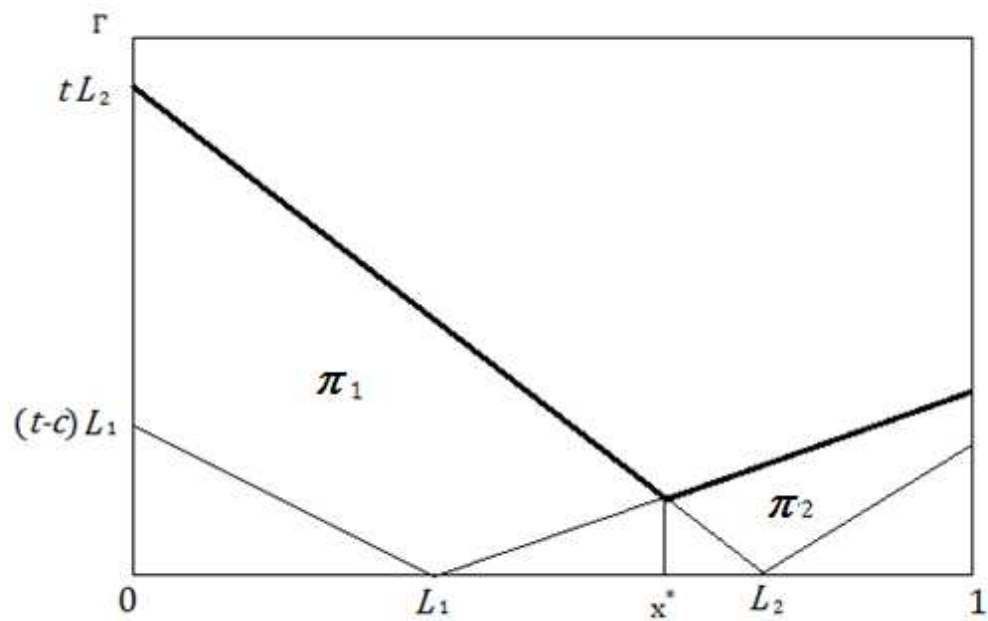
**Fig.2.** Spatial price discrimination with quadratic production cost



**Fig.3.** Spatial price discrimination with  $n$  firms

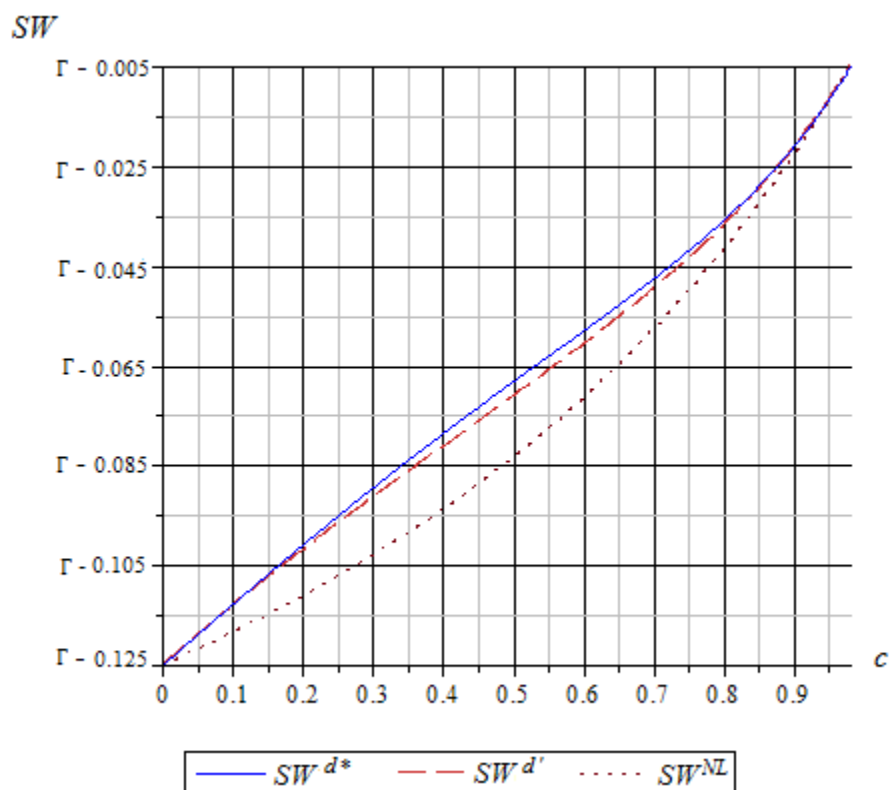


**Fig.4.** Spatial price discrimination in duopoly competition

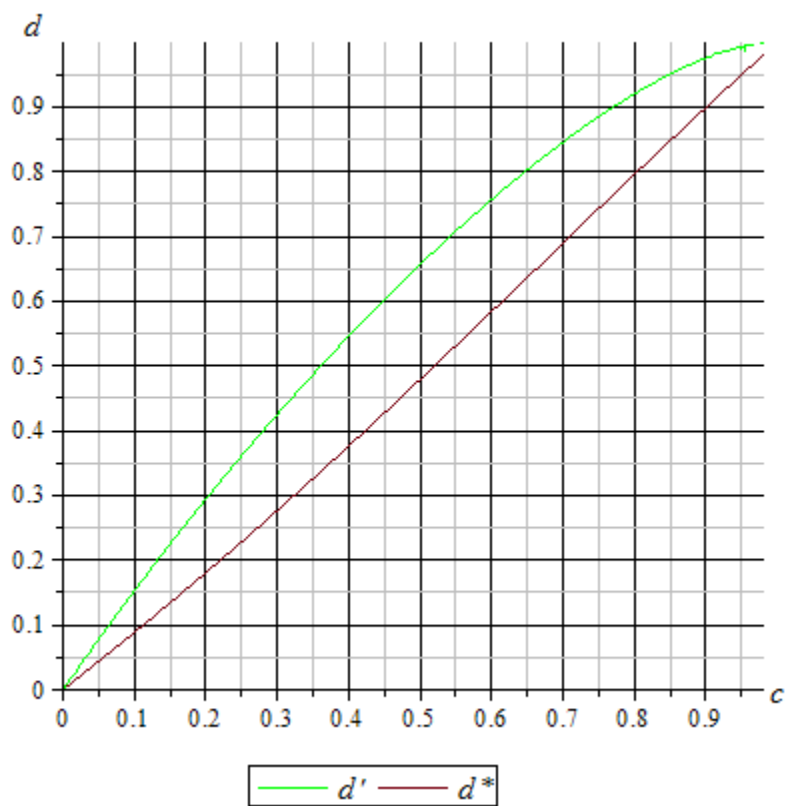


*Note:* The delivered price schedule is depicted by the thick lines and  $\pi_i$  is Firm  $i$ 's profit earned on the difference between the price schedule and the delivered cost.

**Fig.5.** Social welfare and the distance fee licensing

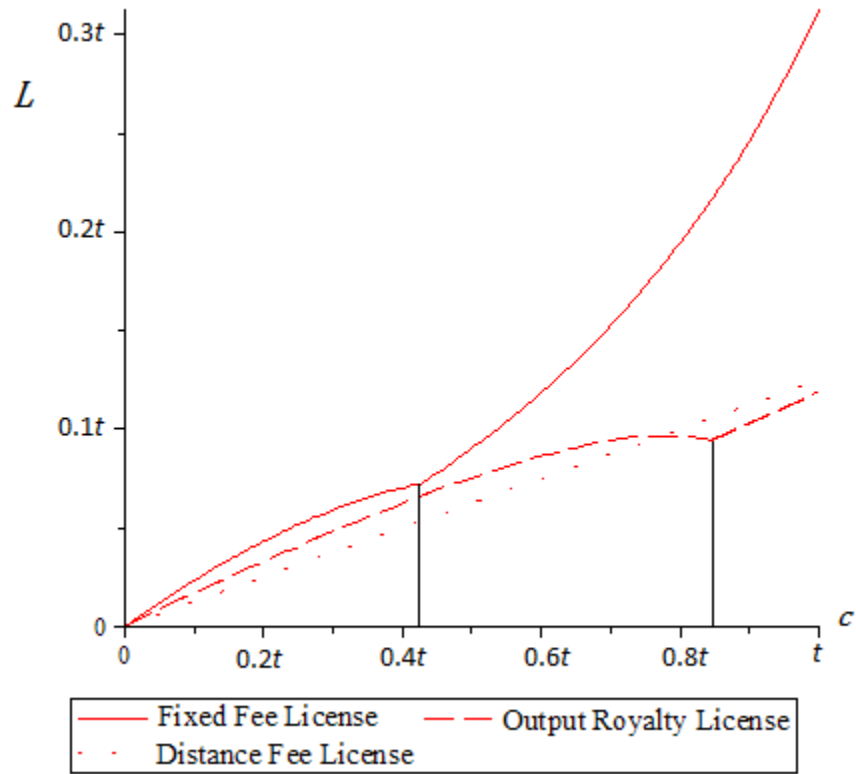


Notes: i)  $SW^{d^*}$  represents social welfare with the socially optimal distance fee,  $SW^{d'}$  represents social welfare with the equilibrium distance fee,  $SW^{NL}$  represents social welfare without licensing. ii) Inelastic demand implies that social welfare is the willingness to pay  $\Gamma$  minus the real resources spent on transportation.

**Fig.6.** Equilibrium distance fee and socially optimal distance fee

*Note:*  $d'$  represents the equilibrium distance fee and  $d^*$  represents the socially optimal distance fee.

**Fig.7.** Representation of proposition 15



*Note:*  $L$  is the licensing revenue of outside innovator.

**Table 1.** Payoffs from the basic model

		Firm 2	
		Delegation	No Delegation
Firm 1	Delegation	$.139t, .139t$	$.2t, .12t$
	No Delegation	$.12t, .2t$	$.188t, .188t$

**Table 2.** Equilibrium results with and without delegation under elastic demand

$t$	$\alpha_1 = \alpha_2$		$L_1 = 1 - L_2$		$\pi_1 = \pi_2$		$SW$	
	$D$	$ND$	$D$	$ND$	$D$	$ND$	$D$	$ND$
0.1	0.968	1.0	0.338	0.253	0.013	0.018	0.486	0.487
0.2	0.938	1.0	0.342	0.257	0.024	0.033	0.471	0.472
0.3	0.911	1.0	0.348	0.261	0.034	0.046	0.455	0.456
0.4	0.885	1.0	0.353	0.265	0.041	0.056	0.438	0.439
0.5	0.861	1.0	0.359	0.270	0.048	0.065	0.422	0.421
0.6	0.838	1.0	0.365	0.276	0.052	0.071	0.404	0.403
0.7	0.817	1.0	0.372	0.282	0.056	0.076	0.387	0.383
0.8	0.797	1.0	0.379	0.289	0.058	0.079	0.370	0.364

*Note:*  $D$  stands for “delegation” and  $ND$  stands for “no delegation”.

**Table 3.** Equilibrium locations for odd numbers of firms

<i>n</i>	<i>Locations</i>	<i>Social Cost Gap</i>
3	$L_1 = \frac{3}{10}, L_2 = \frac{1}{2}, L_3 = \frac{7}{10}$	0.2427
5	$L_1 = \frac{2}{9}, L_2 = \frac{1}{3}, L_3 = \frac{1}{2}, L_4 = \frac{2}{3}, L_5 = \frac{7}{9}$	0.2795
7	$L_1 = \frac{5}{28}, L_2 = \frac{1}{4}, L_3 = \frac{11}{28}, L_4 = \frac{1}{2}, L_5 = \frac{17}{28}, L_6 = \frac{3}{4}, L_7 = \frac{23}{28}$	0.2912
9	$L_1 = \frac{3}{20}, L_2 = \frac{1}{5}, L_3 = \frac{13}{40}, L_4 = \frac{2}{5}, L_5 = \frac{1}{2}, L_6 = \frac{3}{5}, L_7 = \frac{27}{40}, L_8 = \frac{4}{5}, L_9 = \frac{17}{20}$	0.2945
11	$L_1 = \frac{7}{54}, L_2 = \frac{1}{6}, L_3 = \frac{5}{18}, L_4 = \frac{1}{3}, L_5 = \frac{23}{54}, L_6 = \frac{1}{2}, L_7 = \frac{31}{54}, L_8 = \frac{2}{3}, L_9 = \frac{13}{18}, L_{10} = \frac{5}{6}, L_{11} = \frac{47}{54}$	0.2950

*Note:* The Social Cost Gap is the difference in social cost between the consistent conjecture equilibrium and the Nash equilibrium divided by the social cost of the consistent conjecture equilibrium. *n* – number of firms

**Table 4.** Equilibrium locations for even numbers of firms

<i>n</i>	<i>Locations</i>	<i>Social Cost Gap</i>
2	$L_1 = \frac{1}{2}, L_2 = \frac{1}{2}$	0.5
4	$L_1 = \frac{1}{6}, L_2 = L_3 = \frac{1}{2}, L_4 = \frac{5}{6}$	0.2497
6	$L_1 = \frac{1}{7}, L_2 = L_3 = \frac{3}{7}, L_4 = \frac{9}{14}, L_5 = \frac{5}{7}, L_6 = \frac{6}{7}$	0.2898
8	$L_1 = \frac{1}{8}, L_2 = L_3 = \frac{3}{8}, L_4 = \frac{13}{24}, L_5 = \frac{7}{12}, L_6 = \frac{17}{24}, L_7 = \frac{19}{24}, L_8 = \frac{7}{8}$	0.3208
10	$L_1 = \frac{1}{9}, L_2 = L_3 = \frac{1}{3}, L_4 = \frac{17}{36}, L_5 = \frac{1}{2}, L_6 = \frac{11}{18}, L_7 = \frac{2}{3}, L_8 = \frac{3}{4}, L_9 = \frac{5}{6}, L_{10} = \frac{8}{9}$	0.3388
12	$L_1 = \frac{1}{10}, L_2 = L_3 = \frac{3}{10}, L_4 = \frac{21}{50}, L_5 = \frac{11}{25}, L_6 = \frac{27}{50}, L_7 = \frac{29}{50}, L_8 = \frac{33}{50}, L_9 = \frac{18}{25}, L_{10} = \frac{39}{50}, L_{11} = \frac{43}{50}, L_{12} = \frac{9}{10}$	0.3491

*Note:* The Social Cost Gap is the difference in social cost between the consistent conjecture equilibrium and the Nash equilibrium divided by the social cost of the consistent conjecture equilibrium. *n* – number of firms



**Table 5.** Locations and distance fee licensing

$c$	Locations		
	$(L_1^{NL}, L_2^{NL})$	$(L_1^{d'}, L_2^{d'})$	$(L_1^{d*}, L_2^{d*})$
$0.1t$	(0.263, 0.763)	(0.256, 0.765)	(0.253, 0.759)
$0.2t$	(0.278, 0.778)	(0.265, 0.782)	(0.258, 0.770)
$0.3t$	(0.294, 0.794)	(0.276, 0.799)	(0.266, 0.784)
$0.4t$	(0.313, 0.813)	(0.290, 0.818)	(0.277, 0.801)
$0.5t$	(0.333, 0.833)	(0.308, 0.840)	(0.292, 0.821)
$0.6t$	(0.357, 0.857)	(0.330, 0.863)	(0.314, 0.846)
$0.7t$	(0.385, 0.885)	(0.357, 0.890)	(0.343, 0.876)
$0.8t$	(0.417, 0.917)	(0.392, 0.920)	(0.382, 0.911)
$0.9t$	(0.455, 0.955)	(0.438, 0.956)	(0.433, 0.953)

*Note:* i)  $L_i^{NL}$  is Firm  $i$ 's location associated without licensing,  $L_i^{d'}$  is Firm  $i$ 's location with the equilibrium distance fee, and  $L_i^{d*}$  is Firm  $i$ 's location given the socially optimal distance fee. ii) The illustration makes clear that  $L_1^{NL} > L_1^{d'} > L_1^{d*}$  and  $L_2^{d'} > L_2^{NL} > L_2^{d*}$ .

**Table 6.** Transport costs with and without distance fee licensing

$c$	$TC^{NL}$	$TC^{d'}$	$TC^{d^*}$
0.1t	0.118t	0.113t	0.112t
0.2t	0.111t	0.102t	0.101t
0.3t	0.103t	0.091t	0.089t
0.4t	0.094t	0.081t	0.078t
0.5t	0.083t	0.071t	0.068t
0.6t	0.071t	0.061t	0.058t
0.7t	0.058t	0.049t	0.047t
0.8t	0.042t	0.037t	0.036t
0.9t	0.023t	0.021t	0.020t

*Note:* i)  $TC^{NL}$  is the transport cost without licensing,  $TC^{d'}$  is the transport cost with the equilibrium distance fee, and  $TC^{d^*}$  is the transport cost given the socially optimal distance fee. ii) Inelastic demand implies that social welfare is the willingness to pay  $\Gamma$  minus the real resources spent on transportation.

## Endnotes

<sup>1</sup> Applications include mergers (González-Maestre and López-Cuñat 2001), mixed oligopolies (Barros 1995) and capital investment decisions (Baiman and Rajan 1995).

<sup>2</sup> We note that an uncovered market occurs in spatial price discrimination only when the firms' transport costs no longer constrain pricing and so there is no competition or need to discriminate.

<sup>3</sup> Outside the spatial context, researchers have considered partial delegation at both earlier and at later stages of multi-stage games (Gautier and Paolini 2007; Moner – Colonques et al. 2004; Tomaru et al. 2011).

<sup>4</sup> The only way a manager could increase output in the pricing stage is to price below its own delivered cost to the rival's customers. Regardless of the pricing response of the rival, this can never imply a profit gain to the original owner. Moreover, managers will not deviate from spatial price discrimination if delegated the ability to reduce by  $\varepsilon$  the delivered pricing schedule in the final stage. This follows because managers can always gain market share with a smaller profit loss by changing location rather than by lowering the price schedule. Contact the authors for this demonstration.

<sup>5</sup> This point is easily seen from either manager's first order condition. For example,

$$\frac{\partial I_1}{\partial L_1} = \alpha_1 \frac{\partial \pi_1}{\partial L_1} + (1 - \alpha_1) \frac{\partial q_1}{\partial L_1} = \alpha_1 \left( -\frac{3tL_1}{2} + \frac{tL_2}{2} \right) + \frac{(1-\alpha_1)}{2} = 0. \text{ Here the profit loss}$$

associated with moving toward the center grows with  $t$  for any given increase in output as

$$L_2 = 1 - L_1 \text{ and } \frac{1}{4} < L_1 < \frac{1}{2}.$$

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<sup>6</sup> One of the advantages of spatial models is that by providing an additional dimension to problems they can modify or reverse predictions. For a recent example using spatial price discrimination see Heywood and Ye (2011).

<sup>7</sup> In general the first-mover advantage can be lost if there are high costs for observing the leading firm's choice (Vardy 2004), or if the actions of the leader and the follower are strategic complements (Gal-Or 1985).

<sup>8</sup> Mathematically, returning (14) and (15) to (1) yields the leader's profit,  $\pi_1$ , as a function of the incentive parameters:  $\frac{\partial \pi_1}{\partial \alpha_1} = \frac{2(1-\alpha_1)}{5\alpha_1^3 t} \geq 0 \Rightarrow \alpha_1 = 1$ .

<sup>9</sup> From (15) it can be checked that  $\frac{\partial L_2}{\partial t} > 0$  for  $\alpha_2 < \alpha_1$ .

<sup>10</sup> A proof analogous to that for Proposition 1 is available from the authors.

<sup>11</sup> Equating the delivered prices from the two firms:  $kx^* + t(x^* - L_1) = k(1 - x^*) + t(L_2 - x^*) \Rightarrow x^* = \frac{k+t(L_1+L_2)}{2(k+t)}$ .

<sup>12</sup> Subtracting (9) from (23) yields  $\frac{kt(13k+18t)}{(tk^2+16k^2+30kt+4t^2k+12t^2+4t^3)(t+3)} \geq 0$  for  $k \geq 0$ .

<sup>13</sup> Thus airline flights between city pairs differ by departure time from early morning to late evening, the editorial policies of newspapers differ from liberal left to conservative right, and breakfast cereals differ in their sugar content.

<sup>14</sup> A somewhat different approach to establishing price conjectures is outlined by Norman (1989).

<sup>15</sup> Indeed, even without invoking the symmetric location assumption we will show that when the linear case is thought of as the limit of decreasing convexity, the two firms can be shown to collocate at the middle.

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<sup>16</sup> Equating the delivered prices from the two firms allows solving for  $x^*$  as follows:

$$kx^* + t(x^* - L_1) = k(1 - x^*) + t(L_2 - x^*) \Rightarrow x^* = \frac{k+t(L_1+L_2)}{2(k+t)}.$$

<sup>17</sup> The other two roots yield consistent conjectures that imply two of the three firms locate at the same place and even if we locate the three firms optimally (given the relationship between them), they have higher social cost than that implied by the locations in (77).

<sup>18</sup> Firms 2 and 3 co-locate and earn no profit.

<sup>19</sup> The derivations of the consistent conjectures location equilibria for the larger odd values of  $n$  are available upon request.

<sup>20</sup> We recognize that we are presenting a case with small deviations from Nash but our objective is to identify how substantial those deviations remain even in such a case. The full set of roots and potential locations becomes unwieldy as  $n$  increases. Yet, the fact that they all involve at least one case of collocation demonstrates that they are all less efficient than Nash.

<sup>21</sup> The derivations of the consistent conjectures location equilibria for the larger even values of  $n$  are available upon request.

<sup>22</sup> Equating the delivered prices from the two firms:  $t(L_2 - x^*) = (t - c)(x^* - L_1) \Rightarrow$

$$x^* = \frac{-L_1 t + L_1 c - t L_2}{-2t + c}.$$

<sup>23</sup> While solving the constraint for  $r$  yields two roots, only one returns locations within the unit market.

<sup>24</sup> While the expressions are complicated, this demonstration is available upon request.

$$\frac{\partial \pi_2^D}{\partial d} = -\frac{2(t-c)^2 d^4 + 21(t-c)^3 d^3 + 72(t-c)^4 d^2 + 96d(t-c)^5 + 40(t-c)^6}{(2d^2 + 7d(t-c) + 8(t-c)^2)^3} < 0.$$

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<sup>26</sup> Returning  $L_1^D = \frac{2c^2 - 2cd - 4tc + 2t^2 + 2td + d^2}{8c^2 - 7cd - 16tc + 2d^2 + 8t^2 + 7td}$  and  $L_2^D = \frac{2(3c^2 - 3cd - 6tc + 3t^2 + 3td + d^2)}{8c^2 - 7cd - 16tc + 2d^2 + 8t^2 + 7td}$  to  $TC^D$

and minimizing with respect to  $d$  yields  $d=0$ .

<sup>27</sup>  $TC^{NL} - TC^{d1} = \frac{1}{8}t - \frac{t(t-c)}{4(2t-c)} = \frac{tc}{8(2t-c)} > 0$  where  $TC^{NL}$  represents the total transport

cost without licensing and  $TC^{d1}$  represents the total transport cost with one license sold.

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## CURRICULUM VITAE

### Zheng Wang

#### **Fields**

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Primary: Industrial Organization  
Secondary: Labor Economics

#### **Education**

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Ph.D., Economics, University of Wisconsin-Milwaukee, May 2014 (expected)  
“Three Essays in the Economics of Spatial Price Discrimination”  
Advisor: Distinguished Professor John S. Heywood  
M.A., Economics, University of Windsor, Ontario, Canada, 2009  
B.A., International Trade and Economics, Capital University of Economics and  
Business, Beijing, China, 2007

#### **Research Papers**

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"Spatial Price Discrimination and Mergers with Convex Production Costs," *Letters in Spatial and Resource Sciences*, Vol.7 No.1, March (2014) pp.1-8. (with J.S. Heywood).

"Strategic Delegation under Spatial Price Discrimination," *Papers in Regional Science*, Forthcoming (with J.S. Heywood).

#### **Teaching Experience**

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##### **University of Wisconsin – Milwaukee**

###### *Undergraduate Instructor*

- Principles of Microeconomics, Fall 2013
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###### *Undergraduate Teaching Assistant*

- Principles of Macroeconomics (3 sections), Fall 2012

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- Principles of Microeconomics (3 sections), Spring 2012
  - Introductory Economics (5 sections), Fall 2011

### **University of Windsor**

*Graduate Assistant* -- Macroeconomics, September 2008 – April 2009

### **Professional Activities**

Presentation: University of Wisconsin – Milwaukee, Economics Department Seminar, “How to License a Transport Innovation,” November 2013  
Referee: *Journal of Industry, Competition and Trade*

### **University Service**

- UWM Open House, Economics Department Coordinator & Assistant, 2012
- UWM Open House, Economics Department Representative, 2011

### **Computer Skills**

- MAPLE, STATA, EVIEWS, SAS, SPSS

### **Academic Awards**

- International Graduate Student Scholarship, University of Windsor, 2008
- RENMIN Scholarship, Capital University of Economics and Business, 2004 – 2007
- Excellent Student Designation, Capital University of Economics and Business, 2004 – 2007

### **Employment & Internships**

- Research Assistant for the Malawi HIV Project under the supervision of Loren Galvao, Senior Scientist, College of Nursing, UW – Milwaukee, Dec 2010 – Sep 2011
- Customer Manager, GF Securities, Beijing, China, Oct 2009 – Jul 2010
- Intern, China Council for the Promotion of International Trade and International Chamber of Commerce, Beijing, China, Jun 2006 – Aug 2006
- Intern, China National Textiles I/E Corporation, Jan 2006 – Mar 2006



