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Analytical Study Based Optimal Placement of Energy Storage Devices in Distribution Systems to Support Voltage and Angle Stability

Yih Der Kuo
University of Wisconsin-Milwaukee

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ANALYTICAL STUDY BASED OPTIMAL PLACEMENT OF ENERGY STORAGE DEVICES IN DISTRIBUTION SYSTEMS TO SUPPORT VOLTAGE AND ANGLE STABILITY

by

Yih Der Kuo

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of

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ABSTRACT

ANALYTICAL STUDY BASED OPTIMAL PLACEMENT OF ENERGY STORAGE DEVICES IN DISTRIBUTION SYSTEMS TO SUPPORT VOLTAGE AND ANGLE STABILITY

by

Yih Der Kuo

The University of Wisconsin-Milwaukee, 2017
Under the Supervision of Professor Adel Nasiri

Larger penetration of Distributed Generations (DG) in the power system brings new flexibility and opportunity as well as new challenges due to the generally intermittent nature of DG. When these DG are installed in the medium voltage distribution systems as components of smart grid, further support is required to ensure smooth and controllable operation. To complement the uncontrollable output power of these resources, energy storage devices need to be incorporated to absorb excessive power and provide power shortage in time of need. They also can provide reactive power to dynamically help the voltage profile. Energy Storage Systems (ESS) can be expensive and only a limited number of them can
practically be installed in distribution systems. In addition to frequency regulation and energy
time shifting, ESS can support voltage and angle stability in power network. This thesis
applies a Jacobian matrix-based sensitivity analysis to determine the most appropriate node in
a grid to collectively improve the voltage magnitude and angle of all the nodes by
active/reactive power injection. IEEE 14-bus distribution system is selected to demonstrate
the performance of the proposed method due to its clear and simple configuration. The
developed technique is also applied to the IEEE 123-bus system to further evaluate the
effectiveness and demonstrate the performance for a more complicated system. As opposed to
most previous studies, this method does not require an iterative loop with convergence
problem nor a network-related complicated objective function.
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CHAPTER 1 OVERVIEW

Power systems nowadays are facing many problems, rising cost of fuel, emissions that causes climate change, stability issues and others. With availability, cost and environmental concerns of fossil fuel recognized, the need for new power sources is imminent. Distribution power sources are growing gradually to provide more energy to the increasing global electrical energy demand. Most of the distribution power sources provide clean energy compared to traditional fossil fuel and can also provide sufficient power for the load with a large enough distribution power plant. However, distributed energy could also cause problems. Distribution energy sources, such as solar and wind are not stable and highly dependent on weather condition, they cannot be scheduled to follow the demand change. The power generated from renewable energy sources can impact the distribution grid [1]. Either the grid frequency or the voltage magnitude. In this situation the active and reactive power support is necessary. The frequency regulation can be achieved through active power control, while the magnitude regulation is controlled by reactive power. Distribution energy can be better utilized in microgrid rather than in a traditional power grid.

Microgrid is a new type of power grid, providing an efficient and smart way to supply energy. A microgrid contains groups of local power sources, loads, and storage. It can connect to the conventional grid or operate as an independent system. Figure 1.1 shows a microgrid model, where the microgrid connected to the utility grid through a switch. The energy sources
can be wind, solar, energy storage system or other distributed generation. That means microgrid can be disconnected from the traditional grid and operate in islanded mode at the time of need. With distributed generation, a microgrid can provide energy to support the utility grid in connected mode.

Figure 1. 1 Microgrid model

Providing the power from a distributed energy usually leads to a better energy efficiency due to the reduction of transmission losses. The distributed energy can be connected quickly and not limited by transmission and distribution systems. Higher penetration of distributed energy means the microgrid is more likely to suffer from the instability of distributed energy sources. Keeping the power system stable is an important issue. Storage systems play vital roles in microgrids in maintaining the stability. Storage systems can be placed in a microgrid
to regulate voltage and frequency; and can absorb energy during low load demand and discharge while the load demand soars.

The growing penetration of wind turbines in the grid leads to the instability of frequency. The absence of synchronization torque for variable-speed wind generators causes the decreases in overall system inertia, as well as the reduction of frequency regulation capabilities [2]. The active power support from energy storage provides frequency regulation.

With high photovoltaic (PV) penetration rates, solar power generation will produce some negative impacts on the system, such as voltage rise and fluctuation, and reverse power flow [3]. The reactive power support can mitigate the negative impacts from the high PV penetration.

Storage systems are expensive, it is impossible to have a storage unit install everywhere in the grid. Connecting a storage system randomly to the grid would not have the best performance. It is necessary to identify a location that the storage system can impact and regulate the system efficiently.

The objective of this study is to find the optimal location for a storage system in a straightforward and fast way. To identify the optimal location of storages, the sensitivity indices are proposed. The sensitivity indices are based on the Jacobian matrix, obtained by the Newton-Raphson method. The sensitivity indices can be calculated from the inverse of Jacobian matrix. The effect of active and reactive powers at voltage magnitude and angle can
be studied through the sensitivity indices. The details of the Newton-Raphson method and
sensitivity indices will be discussed in chapter three.

The indices will reveal the information about how well a node can impact the whole
system with active and reactive power injection. The sensitivity indices considered the Nodes
with high indices values as a suitable location for the storage system. Nodes with lower
sensitivity indices have a lower impact on the voltage angle and magnitude in the system. The
evaluation of indices will be present in chapter four.
CHAPTER 2 LITERATURE REVIEW

The topics of interest in this study including the distributed generation and its impacts, voltage angle stability, sensitivity analysis and the optimal placement are presented here.

2.1 Distribute Generation

Distributed Generation (DG) refers to electric power generation within distribution networks or on the customer side of the network [4]. DG consists of many small energy sources connected to a distribution system. The energy sources in DG could be generator or energy storage system. DG can be categorized as below:

Conventional generator

- Gas or steam turbine based synchronous generator
- Diesel generator

Non-Conventional Generators

- Storage Devices
- Solar-based inverter
- Wind turbine based doubly fed induction generator
- Wind turbine based constant speed squirrel cage induction generator
- Wind turbine based permanent magnet synchronous generator
DG provides benefits for energy efficiency of distribution systems, such as environmental pollution reduction and enhanced system reliability [5]. The advantages of DGs in a power system are:

- Minimizing Power Losses
- Voltage Profile Improvement
- Voltage Stability
- Reliability
- Power Quality

The approach to determine the optimal DG-unit's size, power factor and location to minimize the total system real power loss is proposed in [6]. The reliability of DG in distribution systems is studied in [7]. Residential renewable energy generator is a common type of DG for power network support, due to its small scale and easy to install. Renewable DG can be installed in household, factories, offices and distributed system. Extra energy generation can send back to the distribution network and support demand in the grid. Energy storage system in DG can also mitigate the intermittency, provides system security, reliability, and energy arbitrage [8].

Energy storage systems enable higher penetration of renewable energy systems such as solar and wind energy, by mitigating their instability. Combining the wind power generation system with energy storage will reduce the fluctuation of wind power [9]. The storage system
can also be used to smooth out the fluctuation of PV system caused by cloud passing [10].

Another advantage of energy storage system is the ability of apply voltage and frequency control with the support of active and reactive power. Because of the price of storage systems are high, it is necessary to select a highly effective location for energy storage system. Choosing a random location will not necessarily lead to the best efficiency.

The demand for clear and substantial energy calls for an increasing penetration of renewable energy in power systems. A microgrid is a cluster of DGs place in the system to provide power to the local load [11], the sources of energy could be renewable. Contrary to the traditional grid, the microgrid can operate in islanded mode in case of grid power outage. Some microgrids could have a high penetration of renewable energy source for lower carbon emission, but high penetration of renewable energy source may cause power reliability and quality problems. The DG placement strategies for grid reinforcement is discussed in [12]. Selection of the best location and size of DGs provide power loss reduction and voltage profile enhancement. As an example of a typical microgrid, Figure 2.1 shows the configuration of the standard IEEE 34-node microgrid from the site of the Power Energy Society (PES) [13].
The IEEE 34-node system is an existing testing feeder located in Arizona with two regulators, a transformer and operates at voltage value 24.9 kV. Load types includes PQ loads and impedance loads in forms of three-phase, two-phase and single-phase.

2. 2 Voltage Angle Stability

The active and reactive power support from storage system can stabilize both of the angle and voltage profiles in the distribution system. In [14], the impacts of large-scale wind power penetration on the angle and voltage stability are discussed. The study analyzed the changing nature of power system’s behavior under large-scale wind turbine in some cases. Reference [15] focuses on the impact of PV installation on the short term voltage stability. PV severely impairs the short-term voltage stability when power is shutted off after a voltage sag. Operating the PV system at leading a power factor and deploying PV equipment with the
ability to inject reactive current by the PV inverters would improve the short-term voltage stability.

To enhance system stability, [16] presents a new reactive power loss index for identifying the weak buses in the system. The index can be used to select the optimal location for reactive compensation devices, which provides reactive power support to prevent the voltage collapse. The index computed from the reactive power support and loss allocation algorithm using the Y-bus method. The fuzzy logic approach is used to identify the most severe network line outages in the system. The benefit of this approach is that the index is obtained without iterations, the other existing methods need iteration to approach the weak buses.

A method to utilize the DG units to improve the voltage stability margin is proposed in [17]. It selected the candidate buses for the DG unit installation with voltage sensitivity analysis. A reduced Jacobian matrix is calculated to analyze the sensitivity of voltage magnitude variations to active power injection. The nodes with high sensitivity are considered as the candidate buses. The method only concerns the sensitivity of voltage magnitude to active power injection in the candidate buses selection process. The voltage angle has not been considered in this paper. Also, PV bus is not being considered in this reference.
A major advantage of the storage systems is the fast and accurate active power control. Reference [18] utilized a small-scale battery energy storage system to eliminate the secondary frequency drop issue during the rotor speed restoration of the permanent magnet synchronous generator wind turbine generator. The storage system provided frequency regulation on demand and discontinue when the system frequency is restored to the specified value.

2.3 Jacobian-Based Sensitivity Analysis

The Jacobian matrix is a natural by-product of the Newton-Raphson load flow algorithm [19]. It is obtained by solving the equation (2.1) iteratively.

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = J \begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\]  \hspace{1cm} (2.1)

Where \( \Delta P \) and \( \Delta Q \) are the active and reactive power mismatch at each bus. \( J \) is the Jacobian matrix, represents the sensitivity measurement of the active and reactive power on the bus voltage angle and magnitude. \( \Delta \delta \) and \( \Delta V \) are the corrections of the voltage angle and magnitude. With the updated of voltage angle and magnitude in each step of the iteration, \( \Delta P \) and \( \Delta Q \) calculated at each iteration. The process of iteration will continue until \( \Delta P \) and \( \Delta Q \) are falling in certain tolerance threshold.

One problem of the Newton-Raphson power flow is that the iteration fails to converge when the smallest singular value of the Jacobian matrix becomes too small. A methodology of Jacobian matrix improvement by directly eliminate the singularity [12]. Continuation
power flow methods developed to solve the ill-condition in conventional Newton-Raphson power flow Jacobian matrix.

There are numbers of research study the sensitivity based on the Jacobian matrix. Reference [20] introduces a software ElecNetKit (Electric Network Toolkit) that simplifies the development of voltage management strategies with sensitivity analysis. The research focuses on the sensitivity of voltage magnitude to active and reactive power. Perturb-and-observe method is used when the Jacobian matrix is not available, the method makes small modifications of active and reactive powers and measures the impact on voltage magnitude. ElecNetKit can interface with OpenDSS, an open-source distribution systems simulator package developed by the Electric Power Research Institute (EPRI). It takes OpenDSS models as a starting point to fully develop the project. The perturb-and-observe network algorithm is applied to get sensitivities because the Jacobian matrix is unavailable in OpenDSS. The method generated sensitivities of the voltage magnitude at various buses by changes active and reactive powers. Afterward, the method is validated with Newton-Raphson load flow algorithm written in MATLAB. One major drawback of this approach is that the sensitivity data is obtained through the perturb-and-observe method and it is time-consuming, especially for the large system.

Reference [21] presents a reactive power control method for PV systems to mitigate the voltage fluctuations base on voltage sensitivity analysis. The variations in the voltage
magnitude of buses are concerned. The required reactive power adjustment to compensate the voltage fluctuations is calculated from the Jacobian matrix.

2. 4 Optimal Placement of Storage

The connection of DGs could create negative impact to the system. The key issues related to optimal placement and sizing of DG have been discussed in [22]. The key issues related to DG are:

- Distribution process and planning issues:

  Local protection systems need to be adopted for distributed generation connection.

- Distribution network issues:

  Distribution systems are usually not designed to connect to distributed generation.

- Power quality:

  Distributed generators connected to distribution systems might cause some power quality issues.

- Power reserve and balancing

  The Distribution System Operator must be able to manage fast reacting local power generation and in some cases procure the needed power reserve from the upstream transmission.

- Connection issues
There are problems in connecting various distributed generations to conventional centralized power generation.

The energy storage system has become one of the most viable solutions for facilitating increased penetration of renewable DG resources. Reference [23] - [25] are focused on increasing the penetration of renewable energy with the optimal energy storage systems placement. Reference [23] proposed a flywheel energy storage that provides primary frequency regulation. An optimal location for the deployment of energy storage system can be found with small-signal stability analysis and time-domain simulations. In reference [24], a methodology is proposed for optimally allocating energy storage systems in distribution systems under a high penetration of wind energy. The energy storage system in the system is to minimize the annual electricity cost. Reference [25] indicated that energy storage over 10 minutes using flywheel allows 10% more wind energy to be absorbed without grid reinforcement and 24 hours using redox flow cells allows up to 25% more wind energy to be absorbed. Together, these studies indicate that energy storage systems provide benefits to the system with renewable DG resources.

The optimal location for the storage system is to reach the best performance. There is some evidence to suggest that the location of the storage system influences the performance significantly. The impacts of different storage technologies on the economic dispatch of a distribution system is studied in reference [26]. The study suggested that the optimal location
of the storage has a significant impact on the minimization of the operational cost instant of increase in size and the type of storage technology. Reference [27] model a distribution network as a continuous tree with the linearized DistFlow model by solving the power flow equations. The approach demonstrates that storage devices should be placed near the leaves of the network and far from the substation and the storage will have better performance on loss reduction. These studies indicate that the performance of energy storage highly depends on the location.

The method to select the optimal placement of storage system has drawn particular attention in many studies. The method of optimal placement of DGs and storage devices for improving system transient stability has been discussed in [28] - [33]. Reference [28] proposes a novel approach for the optimal placement of energy storage devices based on microgrid structure preserving energy function. The purpose of the approach is to improve microgrid transient stability. Fault scan is using fault critical clearing time as the key index to find the most serious fault case and then calculating microgrid branch transient energy function to identify the optimal installation position for energy storage.

References [29] and [30] considered the optimal location to reducing the transmission network congestion. [29] proposed a program that determines the optimal location and the size of a storage device, which is placed to store extra energy from renewable sources for later usage and to reduce the transmission network congestion. The method focused on
reducing the system-wide operating cost and the cost of investments in energy storage. The minimum profit constraint influences the optimal storage placing and sizing. Reference [30] proposed a three-stage decomposition of the problem and considered both the economic and technical aspects. The location and distribution of storage are dependent on the distribution of wind resources.

Reference [31] formulated the optimal storage placement problem for load shifting at slow time scales. The study focused on minimizing the cost of conventional generation by load shifting with energy storage devices, the method in [31] smooths the generation curve rather than following the load variations.

The approach for optimal placement of the battery energy storage system in the distribution system to reduce the distribution system losses is proposed in [32]. The loss sensitivity index is generated through considering the changes in the power system performance and the battery energy storage system parameters. The Newton-Raphson iterative technique has been used to solve the power flow. Bus voltages, voltage phase angles, and the power flows are obtained for sensitivity index. Result shows that battery energy storage system can reduce losses in the system during charging mode.

Both [33] and [34] located the optimal location based on Genetic Algorithms (GA). Reference [33] is finding the optimal locations and capacities of energy storage systems for power system vulnerability mitigation. The vulnerability assessment is proposed to calculate
the impact factors for power systems based on generator and line outages. The optimal locations and capacities for energy storage systems placement are derived and solved by a genetic algorithm-based method. The buses with larger bus impact severity value are considered as better locations for energy storage systems. Reference [34] proposed a method based on GA for optimal placement of DG units. Maximize the loadability of reactive power capabilities. The algorithm obtained the reactive power injected and DG bus location for optimal placement. Due to the iterative process in the genetic algorithm-based method, more computational time is required for the larger system.

In the view of decreasing system cost with DGs connection, the optimal placement of energy storage systems also concerned. Reference [35] minimize the power system cost and improve the system voltage profiles with optimal location and sizing under the consideration of uncertainties in wind power production. The method searched the minimum cost and voltage deviation iteratively. [36] determining the best location for installation of the energy storage systems for daily operation. The process of iteration is done for every bus of the network to find the best location for installation of the storage system.

2.6 Conclusion

The process of iteration has been widely used for the optimal placement of storage system. In [32], iteration is used in the genetic algorithm-based method. Reference of [29]
formalize and construct the storage budget and demand problem for optimal location iteratively, sizing and control of storage devices in a power network. In references [36] and [37], iteration is used to approach minimum cost. Due to the iterative process, selecting optimal location for storage system in the larger system would be more challenging and time-consuming. Moreover, there would be some limitation in the method to reach the converged state in the iterative process.
CHAPTER 3 JACOBIAN MATRIX AS A SENSITIVITY ANALYSIS TOOL

3.1 Power flow analysis

Power flow analysis is one of the most common methods of planning and designing in power systems. The power flow study is to obtain the voltage magnitude and angle of the at each bus and the real and reactive power flow in each line [19]. In power flow analysis, there are three types of buses that can be classified.

- Load buses (PQ): A non-generator bus or load bus is often called a PQ bus. Voltage angle (δ) and magnitude (|V|) are unknown. Power generated (P_G, Q_G) are zero, power drawn by the load (P_L, Q_L) are known. Mismatches ΔP and ΔQ can be defined.

- Voltage-controlled buses (PV): Bus with generator connected that |V| is kept constant, P_G is specified. Q_G cannot be known in advance so mismatch ΔQ cannot be defined, δ is unknown as well.

- Slack buses (Swing): Bus which δ serves as reference for the angles of all other bus voltages, and |V| is specified.

The following table shows the characteristic of each bus types.
Table 3.1 Types of bus

<table>
<thead>
<tr>
<th>Bus type</th>
<th>Unknown</th>
<th>Known</th>
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<tbody>
<tr>
<td>PQ bus</td>
<td>$\delta,</td>
<td>V</td>
</tr>
<tr>
<td>PV bus</td>
<td>$\delta, Q$</td>
<td>$</td>
</tr>
<tr>
<td>Slack bus</td>
<td>$P, Q$</td>
<td>$\delta,</td>
</tr>
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Power flow analysis is similar to circuit analysis except that the power flow analysis is nonlinear and circuit analysis is linear. The equation of power flow complex matrix is:

$$I = Y_{bus}V$$  \hspace{1cm} (3.1)

Where $Y_{bus}$ is the bus admittance matrix, $V$ is the complex node voltage vector, $I$ is the nodal current injection vector. In $Y_{bus}$ matrix, the typical element $Y_{in}$ are:

$$Y_{ij} = |Y_{in}| \angle \theta_{in} = |Y_{in}| \cos \theta_{in} + j|Y_{in}| \sin \theta_{in} = G_{in} + jB_{in}$$  \hspace{1cm} (3.2)

The voltage at a bus $i$ can be obtained using:

$$V_i = |V_i| \angle \theta_i = |V_i| (\cos \delta_i + j \sin \delta_i)$$  \hspace{1cm} (3.3)

The net current injected into the network at bus $i$ in terms of the elements $Y_{in}$ in $Y_{bus}$ is given in:

$$I_i = Y_{i1}V_1 + Y_{i2}V_2 + \cdots + Y_{in}V_N = \sum_{n=1}^{N} Y_{in}V_n$$  \hspace{1cm} (3.4)

The complex conjugate of the power injected at bus $i$ is:
\[ S_i = V_i^* I_i = V_i^* \sum_{n=1}^{N} Y_{in} V_n \]  \hfill (3.5)

Substituting from equations (3.4) and (3.5) and obtain:

\[ S_i = \sum_{n=1}^{N} |Y_{in} V_i V_n| \angle (\theta_{in} + \delta_n - \delta_i) \]  \hfill (3.6)

Separating the equation into real and reactive parts and the equations obtain:

\[ P_i = \sum_{n=1}^{N} |Y_{in} V_i V_n| \cos(\theta_{in} + \delta_n - \delta_i) \]  \hfill (3.7)

\[ Q_i = -\sum_{n=1}^{N} |Y_{in} V_i V_n| \sin(\theta_{in} + \delta_n - \delta_i) \]  \hfill (3.8)

Then,

\[ P_{Lsch} = P_{ig} - P_{il} \]  \hfill (3.9)

\( P_{Lsch} \) is the net scheduled power being injected in to the network at bus \( i \), \( P_{Lctc} \) is the calculated value of \( P_i \). The definition of mismatch \( \Delta P \) is \( P_{Lsch} \) minus \( P_{Lctc} \).

\[ \Delta P = P_{Lsch} - P_{Lctc} \]  \hfill (3.10)

Therefore, the reactive power mismatch \( \Delta Q \) is:
\[ \Delta Q = Q_{i,sch} - Q_{i,clc} \] (3.11)

3.2 Newton-Raphson load flow

The Newton-Raphson method is to obtain the Jacobian matrix. To apply the method, set bus voltages and line admittances in polar form and get:

\[ P_i = |V_i|^2 G_{ii} + \sum_{n=1, n \neq i}^{N} |Y_{in} V_i V_n| \cos(\theta_{in} + \delta_n - \delta_i) \] (3.12)

\[ Q_i = -|V_i|^2 B_{ii} - \sum_{n=1, n \neq i}^{N} |Y_{in} V_i V_n| \sin(\theta_{in} + \delta_n - \delta_i) \] (3.13)

\( G_{ii} \) and \( B_{ii} \) come from the definition of \( Y_{ij} \) in equation (3.2). The real and reactive power mismatches for bus \( i \) is:

\[ \Delta P_i = P_{i,sch} - P_{i,clc} \] (3.14)

\[ \Delta Q_i = Q_{i,sch} - Q_{i,clc} \] (3.15)

Power flow calculations usually employ iterative techniques such as the Newton-Raphson procedures. At each interval, \( P_{i,sch} \) and \( Q_{i,sch} \) are being calculated based on the estimated \( \delta_i \) and \( |V_i| \) from equations (3.12) – (3.13). Then mismatches \( \Delta P_i \) and \( \Delta Q_i \) need to be calculated from equations (3.14) – (3.15). By Comparing the calculated values with the known values the errors are being generated and from the error the new estimations of values of \( \delta_i \) and \( |V_i| \) are being derived as the starting values for the next iteration. At each iteration, the real and reactive power have to be within specified tolerances.
The Jacobian sub-matrices are being calculated and triangularized in each iteration to update the Jacobian matrix. The Newton-Raphson method calculates mismatches until real and reactive power mismatches at all buses fall within specified tolerances. Figure 3.1 is the diagram of the Newton-Raphson iteration process.

![Diagram of the Newton-Raphson iteration process](image)

In the diagram, P and Q are bus real and reactive power. dP and dQ are real and reactive power mismatches for each bus. The tolerance value could be in the range of $10^{-3}$ to $10^{-7}$. 

---

22
The Jacobian matrix of n buses system has the structure below:

\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\
\frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\]

(3.16)

Where \( \Delta P \) and \( \Delta Q \) are the real and reactive power mismatch, the Jacobian matrix is the middle part of the equation which represents the sensitivity measurement of the real and reactive power on the bus voltage angle and magnitude. The Jacobian matrix for a N-bus system has shown in equation (3. 17).

\[
\begin{bmatrix}
\Delta P_2 \\
\Delta P_3 \\
\vdots \\
\Delta P_N
\end{bmatrix} =
\begin{bmatrix}
J_1 & J_2 \\
J_3 & J_4
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_2 \\
\Delta \delta_2 \\
\vdots \\
\Delta \delta_N
\end{bmatrix}
\]

(3.17)

\[
\begin{bmatrix}
\Delta Q_2 \\
\Delta Q_3 \\
\vdots \\
\Delta Q_N
\end{bmatrix} =
\begin{bmatrix}
J_1 & J_2 \\
J_3 & J_4
\end{bmatrix}
\begin{bmatrix}
\Delta |V_2| \\
\Delta |V_3| \\
\vdots \\
\Delta |V_N|
\end{bmatrix}
\]

The mismatches for the slack bus does not including, since \( \Delta P_1 \) and \( \Delta Q_1 \) are undefined with \( P_1 \) and \( Q_1 \) are not scheduled. \( \Delta \delta_1 \) and \( \Delta |V_1| \) are also not including, when corrections of angle and magnitude are both zero at the slack bus. The groups of \( J_1, J_2, J_3, \) and \( J_4 \) are:
\[ J_1 = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \cdots & \frac{\partial P_2}{\partial \delta_N} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & \frac{\partial P_3}{\partial \delta_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_N}{\partial \delta_2} & \frac{\partial P_N}{\partial \delta_3} & \cdots & \frac{\partial P_N}{\partial \delta_N} \end{bmatrix} \]

(3.18)

\[ J_2 = \begin{bmatrix} \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} & \cdots & \frac{\partial P_2}{\partial |V_N|} \\ \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} & \cdots & \frac{\partial P_3}{\partial |V_N|} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_N}{\partial |V_2|} & \frac{\partial P_N}{\partial |V_3|} & \cdots & \frac{\partial P_N}{\partial |V_N|} \end{bmatrix} \]

(3.19)

\[ J_3 = \begin{bmatrix} \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \cdots & \frac{\partial Q_2}{\partial \delta_N} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \cdots & \frac{\partial Q_3}{\partial \delta_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_N}{\partial \delta_2} & \frac{\partial Q_N}{\partial \delta_3} & \cdots & \frac{\partial Q_N}{\partial \delta_N} \end{bmatrix} \]

(3.20)

\[ J_4 = \begin{bmatrix} \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} & \cdots & \frac{\partial Q_2}{\partial |V_N|} \\ \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} & \cdots & \frac{\partial Q_3}{\partial |V_N|} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_N}{\partial |V_2|} & \frac{\partial Q_N}{\partial |V_3|} & \cdots & \frac{\partial Q_N}{\partial |V_N|} \end{bmatrix} \]

(3.21)

By collecting equations (3.17) – (3.20) and putting them back to (3.16), we obtain:
The dimension of the Jacobian matrix is decided by the number of PV bus and PQ bus.

At PQ buses, both angle and magnitude are existing. At PV bus, the magnitude is kept constant and only angle existing in the matrix. If bus $N$ is defined as PV bus, the Jacobian matrix will be reduced as:
\[
\begin{bmatrix}
\Delta P_2 \\
\Delta P_3 \\
\vdots \\
\Delta P_N \\
\Delta Q_2 \\
\Delta Q_3 \\
\vdots \\
\Delta Q_{N-1}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \cdots & \frac{\partial P_2}{\partial \delta_N} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} & \cdots & \frac{\partial P_2}{\partial |V_{N-1}|} \\
\frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \cdots & \frac{\partial P_3}{\partial \delta_N} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} & \cdots & \frac{\partial P_3}{\partial |V_{N-1}|} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial P_N}{\partial \delta_2} & \frac{\partial P_N}{\partial \delta_3} & \cdots & \frac{\partial P_N}{\partial \delta_N} & \frac{\partial P_N}{\partial |V_2|} & \frac{\partial P_N}{\partial |V_3|} & \cdots & \frac{\partial P_N}{\partial |V_{N-1}|} \\
\frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \cdots & \frac{\partial Q_2}{\partial \delta_N} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} & \cdots & \frac{\partial Q_2}{\partial |V_{N-1}|} \\
\frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \cdots & \frac{\partial Q_3}{\partial \delta_N} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} & \cdots & \frac{\partial Q_3}{\partial |V_{N-1}|} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial Q_{N-1}}{\partial \delta_2} & \frac{\partial Q_{N-1}}{\partial \delta_3} & \cdots & \frac{\partial Q_{N-1}}{\partial \delta_N} & \frac{\partial Q_{N-1}}{\partial |V_2|} & \frac{\partial Q_{N-1}}{\partial |V_3|} & \cdots & \frac{\partial Q_{N-1}}{\partial |V_{N-1}|}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_2 \\
\Delta \delta_3 \\
\vdots \\
\Delta \delta_N \\
\Delta |V_2| \\
\Delta |V_3| \\
\vdots \\
\Delta |V_{N-1}|
\end{bmatrix}
\]

The size of Jacobian matrix can be calculated as:

\[
J_{(npv+2npq)\times(npv+2npq)}
\]

(3.24)

Where \(npv\) is the number of PV bus and \(npq\) is number of PQ bus. The size of \(J_1, J_2, J_3, \) and \(J_4\) are:

\[
J_{1(npv+npq)\times(npv+npq)}
\]

(3.25)

\[
J_{2(npv+npq)\times(npq)}
\]

(3.26)

\[
J_{3(npq)\times(npv+npq)}
\]

(3.27)

\[
J_{4(npq)\times(npq)}
\]

(3.28)
3. 3 Sensitivity analysis

While the power flow equations are solved and Jacobian matrix obtained. The inverse of the Jacobian matrix can be calculated, and the matrix has the following structure:

\[
\begin{bmatrix}
\Delta \delta_2 \\
\Delta \delta_3 \\
\vdots \\
\Delta \delta_N \\
\end{bmatrix} = \begin{bmatrix}
\frac{\partial \delta_2}{\partial P_2} & \frac{\partial \delta_2}{\partial P_3} & \cdots & \frac{\partial \delta_2}{\partial P_n} & \frac{\partial \delta_2}{\partial Q_2} & \frac{\partial \delta_2}{\partial Q_3} & \cdots & \frac{\partial \delta_2}{\partial Q_N} \\
\frac{\partial \delta_3}{\partial P_2} & \frac{\partial \delta_3}{\partial P_3} & \cdots & \frac{\partial \delta_3}{\partial P_n} & \frac{\partial \delta_3}{\partial Q_2} & \frac{\partial \delta_3}{\partial Q_3} & \cdots & \frac{\partial \delta_3}{\partial Q_N} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{\partial \delta_N}{\partial P_2} & \frac{\partial \delta_N}{\partial P_3} & \cdots & \frac{\partial \delta_N}{\partial P_n} & \frac{\partial \delta_N}{\partial Q_2} & \frac{\partial \delta_N}{\partial Q_3} & \cdots & \frac{\partial \delta_N}{\partial Q_N} \\
\end{bmatrix} \begin{bmatrix}
\Delta P_2 \\
\Delta P_3 \\
\vdots \\
\Delta P_N \\
\end{bmatrix}
\]

The inverse Jacobian matrix can also separate into four sub-matrices, \( J_1^{-1} \), \( J_2^{-1} \), \( J_3^{-1} \), and \( J_4^{-1} \). The structure of four sub-matrices in inverse Jacobian Matrix has shown in equation (3. 30).

\[
[\Delta \delta] = \begin{bmatrix}
J_1^{-1} & J_2^{-1} \\
J_3^{-1} & J_4^{-1}
\end{bmatrix} [\Delta P]
\]

(3. 30)

In the case that only one slack bus and PQ buses existing in the system, the size of four sub-matrices are equal. However, if PV bus exists in the system, the sizes are not equal. For
example, there is a system with four buses that is one slack bus, one PV bus, and two PQ buses. From equation (3.25) – (3.28) the size of the Jacobian matrix is:

\[ J_{(5) \times (5)} \]

For the sub-matrices of \( J_1, J_2, J_3, \) and \( J_4 \), the size are:

\[ J_{1(3) \times (3)} \]
\[ J_{2(3) \times (2)} \]
\[ J_{3(2) \times (3)} \]
\[ J_{4(2) \times (2)} \]

Similarly, the inverse Jacobian matrix, the size of four sub-matrices are different. For the optimal location for storage, PV buses are not considered as the potential candidates. Values in the matrices that content PV buses elements need to be removed from the matrices. While only PQ elements exist in \( J_4^{-1} \), the rest of them, \( J_1^{-1}, J_2^{-1} \) and \( J_3^{-1} \), all include PV bus elements in their matrices.

The example of remove PV bus elements is demonstrated, the PV bus is set at bus 4. The inverse of Jacobian matrix is shown in Table 3.2 where color distinguishes four sub-matrices.
Table 3. 2 Inverse Jacobian matrix of 4 buses

<table>
<thead>
<tr>
<th>$\frac{\partial \delta_2}{\partial P_2}$</th>
<th>$\frac{\partial \delta_2}{\partial P_3}$</th>
<th>$\frac{\partial \delta_2}{\partial P_4}$</th>
<th>$\frac{\partial \delta_2}{\partial Q_2}$</th>
<th>$\frac{\partial \delta_2}{\partial Q_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial \delta_3}{\partial P_2}$</td>
<td>$\frac{\partial \delta_3}{\partial P_3}$</td>
<td>$\frac{\partial \delta_3}{\partial P_4}$</td>
<td>$\frac{\partial \delta_3}{\partial Q_2}$</td>
<td>$\frac{\partial \delta_3}{\partial Q_3}$</td>
</tr>
<tr>
<td>$\frac{\partial \delta_4}{\partial P_2}$</td>
<td>$\frac{\partial \delta_4}{\partial P_3}$</td>
<td>$\frac{\partial \delta_4}{\partial P_4}$</td>
<td>$\frac{\partial \delta_4}{\partial Q_2}$</td>
<td>$\frac{\partial \delta_4}{\partial Q_3}$</td>
</tr>
<tr>
<td>$\frac{\partial</td>
<td>V_2</td>
<td>}{\partial P_2}$</td>
<td>$\frac{\partial</td>
<td>V_2</td>
</tr>
<tr>
<td>$\frac{\partial</td>
<td>V_3</td>
<td>}{\partial P_2}$</td>
<td>$\frac{\partial</td>
<td>V_3</td>
</tr>
</tbody>
</table>

Values in the inverse of Jacobian matrix include PV bus elements are marked in Table 3. 3 and removed in Table 3. 4

Table 3. 3 Marking PV bus elements in the inverse Jacobian matrix of 4 buses

<table>
<thead>
<tr>
<th>$\frac{\partial \delta_2}{\partial P_2}$</th>
<th>$\frac{\partial \delta_2}{\partial P_3}$</th>
<th>$\frac{\partial \delta_2}{\partial P_4}$</th>
<th>$\frac{\partial \delta_2}{\partial Q_2}$</th>
<th>$\frac{\partial \delta_2}{\partial Q_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial \delta_3}{\partial P_2}$</td>
<td>$\frac{\partial \delta_3}{\partial P_3}$</td>
<td>$\frac{\partial \delta_3}{\partial P_4}$</td>
<td>$\frac{\partial \delta_3}{\partial Q_2}$</td>
<td>$\frac{\partial \delta_3}{\partial Q_3}$</td>
</tr>
<tr>
<td>$\frac{\partial \delta_4}{\partial P_2}$</td>
<td>$\frac{\partial \delta_4}{\partial P_3}$</td>
<td>$\frac{\partial \delta_4}{\partial P_4}$</td>
<td>$\frac{\partial \delta_4}{\partial Q_2}$</td>
<td>$\frac{\partial \delta_4}{\partial Q_3}$</td>
</tr>
<tr>
<td>$\frac{\partial</td>
<td>V_2</td>
<td>}{\partial P_2}$</td>
<td>$\frac{\partial</td>
<td>V_2</td>
</tr>
<tr>
<td>$\frac{\partial</td>
<td>V_3</td>
<td>}{\partial P_2}$</td>
<td>$\frac{\partial</td>
<td>V_3</td>
</tr>
</tbody>
</table>
Table 3. 4 removed PV bus elements in the inverse Jacobian matrix of 4 buses

| \( \frac{\partial \delta_2}{\partial P_2} \) | \( \frac{\partial \delta_2}{\partial P_3} \) | \( \frac{\partial \delta_2}{\partial Q_2} \) | \( \frac{\partial \delta_2}{\partial Q_3} \) |
| \( \frac{\partial \delta_3}{\partial P_2} \) | \( \frac{\partial \delta_3}{\partial P_3} \) | \( \frac{\partial \delta_3}{\partial Q_2} \) | \( \frac{\partial \delta_3}{\partial Q_3} \) |
| \( \frac{\partial |V_2|}{\partial P_2} \) | \( \frac{\partial |V_2|}{\partial P_3} \) | \( \frac{\partial |V_2|}{\partial Q_2} \) | \( \frac{\partial |V_2|}{\partial Q_3} \) |
| \( \frac{\partial |V_3|}{\partial P_2} \) | \( \frac{\partial |V_3|}{\partial P_3} \) | \( \frac{\partial |V_3|}{\partial Q_2} \) | \( \frac{\partial |V_3|}{\partial Q_3} \) |

Now the four sub-matrices of the inverse Jacobian matrix are all in the same size, PV bus elements are removed.

3.4 The definition of indices

Four basic sensitivity indices have been proposed in [35]. Four basic indices are shown in equation (3.31) – (3.34), the indices are the summation of the columns of the inverse Jacobian matrix and divided by the number of PQ buses. The sensitivity of voltage angle and magnitude to the active and reactive power can be displayed as the mathematical weighted form.

\[
\text{Index}_{\text{APT}} = \frac{\sum_{k=1}^{n} \left| \frac{\partial \delta_k}{\partial P_t} \right|}{npq}
\]  

(3.31)
Index_{VPi} = \frac{\sum_{k=1}^{n} \left| \frac{\partial |V_k|}{\partial P_i} \right|}{npq} \quad (3.32)

Index_{AQi} = \frac{\sum_{k=1}^{n} \left| \frac{\partial \delta_k}{\partial Q_i} \right|}{npq} \quad (3.33)

Index_{VQi} = \frac{\sum_{k=1}^{n} \left| \frac{\partial |V_k|}{\partial Q_i} \right|}{npq} \quad (3.34)

Where \( i \) and \( npq \) are the number of PQ bus in the system. The summation of the columns of the inverse of Jacobian matrix meaning the sensitivity of the voltage angle and magnitude to the active and reactive power injection.

The high value means the large impact will make to the voltage angle and magnitude. It should be mentioned that nodes with higher sensitivity index value can influence voltage angle and magnitude at other nodes better, it is more effective to place a storage system at the node with the high sensitivity index value.

Index_{API} describes the influence of injected active power at bus \( i \) on the collective angle improvement in all of the network. Index_{VPi} means the sensitivity of voltage magnitude to the active power injection. Index_{AQi} is the influence of injected reactive power at bus \( i \) on the collective angle and Index_{VQi} is the influence of the injected reactive power to the voltage magnitude.

Since voltage angle is more sensitive to the active power compared to the reactive power and the magnitude is more under the influence of the reactive power rather than the active
power, $\text{Index}_{API}$ and $\text{Index}_{VQi}$ are dominant indices. The values of sensitivity indices in dominant indices are greater than the non-dominant indices which are $\text{Index}_{AQi}$ and $\text{Index}_{VPi}$.

The following compound indices are defined to describe the system in more meaningful ways. $\text{Index}_{API}$ minus $\text{Index}_{VPi}$ is defined as $\text{Index}_{pi}$, which means how much active power can influences the angle but not the magnitude.

$\text{Index}_{AQi}$ is subtracted from $\text{Index}_{VQi}$ to obtain the influence of reactive power on the magnitude rather than the angle, $\text{Index}_{Qi}$ and $\text{Index}_{Ai}$ represents how much can the voltage angle be influenced by the active power injection but not reactive power. Similarly, $\text{Index}_{vi}$ shows how much the voltage magnitude can be influenced by the reactive power injection.

The equations of four new defined indices are:

\[
\text{Index}_{pi} = \text{Index}_{API} - \text{Index}_{VPi} \tag{3. 35}
\]

\[
\text{Index}_{Qi} = \text{Index}_{VQi} - \text{Index}_{AQi} \tag{3. 36}
\]

\[
\text{Index}_{Ai} = \text{Index}_{API} - \text{Index}_{AQi} \tag{3. 37}
\]

\[
\text{Index}_{vi} = \text{Index}_{VQi} - \text{Index}_{VPi} \tag{3. 38}
\]

The summation of $\text{Index}_{pi}$ and $\text{Index}_{Qi}$ is equal to the summation of $\text{Index}_{Ai}$ and $\text{Index}_{vi}$ and it represents the mathematical weighted index to show how much the collective voltage angle and magnitude of a network can be influenced by the injected active and reactive powers.
\[ \text{Index}_{Ti} = \text{Index}_{Pi} + \text{Index}_{Qi} \]  

(3.39)

The value of \( \text{Index}_{Ti} \) represent how much that voltage angle and magnitude will be influenced by the injection of apparent power \( S \) at bus \( i \). The higher value means the voltage angle and magnitude at the rest of the bus will be influenced by the injection of apparent power \( S \) at bus \( i \) significantly.

3.5 IEEE 14-node test feeder as a case study for sensitivity indices

To demonstrate how to apply the Newton-Raphson load flow and obtain the Jacobian matrix, the IEEE 14-node test feeder was chosen as an example. The test feeder includes 14 buses, 5 generators, and 11 loads. The system is shown in Figure 3.2.
The system, including one slack bus, four PV buses and nine PQ buses. The testing model is constructed in Matlab. After solving power flow with the Newton-Raphson method, the Jacobian matrix is obtained and shown in Table 3.5.
The size of the Jacobian matrix can be calculated in (3.24), the number of PV bus is four, PQ bus is nine and one slack bus.

\[
J_{(npv+2\times npq)\times(npv+2\times npq)}
\]

(3.24)

Where \( npv \) is four and \( npq \) is nine, the size of Jacobian matrix is \( J_{(22)\times(22)} \). Then inverse the Jacobian matrix and get inverse Jacobian matrix shows in Table 3.6. Matlab software is used to implement the method. Gray boxes show the corresponding code in the text throughout this thesis.

\[
\text{Jinv} = \text{abs}(100*\text{inv}(J));
\]
Where Jinv is the inverse of Jacobian matrix. Since the numerical values of Jinv are small, it is multiplied by 100 for the sake of presentation. The absolute value of Jinv is considered so that negative and positive values do not cancel out each other. Table 3.6 shows Jinv that are generated from the above codes.

<table>
<thead>
<tr>
<th>J1</th>
<th>J2</th>
<th>J3</th>
<th>J4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.097</td>
<td>4.918</td>
<td>4.312</td>
<td>3.894</td>
</tr>
<tr>
<td>4.579</td>
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<td>3.393</td>
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</tr>
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</tr>
<tr>
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<td>3.743</td>
<td>3.331</td>
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</tr>
<tr>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
</tr>
</tbody>
</table>

In the system, there are four PV buses. PV buses are already connected to a local generator or utility and hence they do not need a storage device to help the angle or voltage profile. Nodes with PV buses need to be removed from the inverse Jacobian matrix. To do that, J−1 1 , J−1 2 , J−1 3 , and J−1 4 have to be identified from the matrix.

From equation (3.25) – (3.28), the size of four sub-matrices can be calculated. After the processing, J−1 1 , J−1 2 , J−1 3 , and J−1 4 can be identified and shown in Table 3.7.
Table 3.7 The four groups of inverse Jacobian matrix

<table>
<thead>
<tr>
<th>5.097</th>
<th>4.958</th>
<th>4.322</th>
<th>3.753</th>
<th>3.191</th>
<th>2.661</th>
<th>2.140</th>
<th>1.649</th>
<th>1.202</th>
<th>0.801</th>
<th>0.487</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.008</td>
<td>0.156</td>
<td>1.699</td>
<td>1.008</td>
<td>1.279</td>
<td>1.974</td>
<td>1.974</td>
<td>2.070</td>
<td>1.972</td>
<td>1.648</td>
<td>1.344</td>
</tr>
<tr>
<td>0.038</td>
<td>0.219</td>
<td>1.229</td>
<td>1.555</td>
<td>1.071</td>
<td>1.594</td>
<td>1.594</td>
<td>1.769</td>
<td>1.769</td>
<td>1.485</td>
<td>1.485</td>
</tr>
<tr>
<td>0.011</td>
<td>0.099</td>
<td>0.797</td>
<td>0.428</td>
<td>0.119</td>
<td>0.471</td>
<td>0.471</td>
<td>0.872</td>
<td>1.716</td>
<td>0.974</td>
<td>0.212</td>
</tr>
<tr>
<td>0.022</td>
<td>0.141</td>
<td>0.851</td>
<td>0.306</td>
<td>0.957</td>
<td>1.054</td>
<td>1.054</td>
<td>2.665</td>
<td>2.665</td>
<td>2.822</td>
<td>1.814</td>
</tr>
<tr>
<td>0.020</td>
<td>0.125</td>
<td>0.699</td>
<td>0.300</td>
<td>0.597</td>
<td>1.063</td>
<td>1.063</td>
<td>2.286</td>
<td>4.322</td>
<td>2.195</td>
<td>0.462</td>
</tr>
<tr>
<td>0.012</td>
<td>0.070</td>
<td>0.399</td>
<td>0.148</td>
<td>0.014</td>
<td>0.090</td>
<td>0.090</td>
<td>1.271</td>
<td>2.531</td>
<td>5.606</td>
<td>5.606</td>
</tr>
<tr>
<td>0.006</td>
<td>0.025</td>
<td>0.099</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.217</td>
<td>2.531</td>
<td>5.606</td>
<td>5.606</td>
</tr>
<tr>
<td>0.005</td>
<td>0.027</td>
<td>0.133</td>
<td>0.049</td>
<td>0.116</td>
<td>0.176</td>
<td>0.376</td>
<td>0.491</td>
<td>0.398</td>
<td>1.390</td>
<td>1.341</td>
</tr>
<tr>
<td>0.015</td>
<td>0.084</td>
<td>0.538</td>
<td>0.233</td>
<td>0.463</td>
<td>1.226</td>
<td>1.256</td>
<td>1.750</td>
<td>1.407</td>
<td>0.961</td>
<td>0.512</td>
</tr>
</tbody>
</table>

Once the four sub-matrices of the inverse of Jacobian matrix are identified, the PV bus elements can be identified and removed. The following code identifies values that contain PV bus elements, the sub-matrix $J^{-1}$ contain PV bus elements in both column and row as it is shown in Table 3.8. In the following code, type is number in one of the three types of nodes, slack bus is set as type 1, PV type 1 and PVQ is type 3.
Table 3.8 PV bus elements in $J_1^{-1}$ matrix

|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|

```matlab
Jinv1=zeros(npq+npv,npq+npv);
for i=1:nbus-1
    for j=1:nbus-1
        if type(i+1)==3 && type(j+1)==3
            Jinv1(j,i)=Jinv(j,i)
        end
    end
end
```

After identifying and removing all of the columns and rows that contain PV bus elements, the program finds the new $J_1^{-1}$ matrix and it has shown in Table 3.9.

```matlab
npv=npv-1;
Jinv1(find(Jinv1==0))=[];
Jinv1=reshape(Jinv1,npq,npq);
```

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Table 3.9 New $J_1^{-1}$ matrix

|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|

In the sub-matrix $J_2^{-1}$, only PV rows exist. The process that finding PV bus elements in matrix is shown below and results are shown in Table 3.10.

```matlab
Jinv2=zeros(npq+npv,npq);
for i=npq+npv+1:npq+npv+npq
    for j=1:npq+npv
        if type(j+1)==3
            Jinv2(j,i-npq-npv)=Jinv(j,i)
        end
    end
end
```

39
Table 3.10 PV bus element in $J_2^{-1}$ matrix

<p>| | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.110</td>
<td>0.170</td>
<td>0.053</td>
<td>0.061</td>
<td>0.066</td>
<td>0.041</td>
<td>0.024</td>
<td>0.042</td>
<td>0.085</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.505</td>
<td>0.506</td>
<td>0.202</td>
<td>0.181</td>
<td>0.174</td>
<td>0.098</td>
<td>0.051</td>
<td>0.079</td>
<td>0.189</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.888</td>
<td>0.278</td>
<td>0.423</td>
<td>0.436</td>
<td>0.331</td>
<td>0.153</td>
<td>0.017</td>
<td>0.007</td>
<td>0.181</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.485</td>
<td>0.807</td>
<td>0.131</td>
<td>0.038</td>
<td>0.007</td>
<td>0.025</td>
<td>0.027</td>
<td>0.067</td>
<td>0.075</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.059</td>
<td>0.125</td>
<td>0.864</td>
<td>1.752</td>
<td>1.603</td>
<td>0.913</td>
<td>0.102</td>
<td>0.467</td>
<td>1.400</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.655</td>
<td>0.224</td>
<td>0.440</td>
<td>0.850</td>
<td>0.687</td>
<td>0.353</td>
<td>0.073</td>
<td>0.031</td>
<td>0.407</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.655</td>
<td>0.224</td>
<td>0.440</td>
<td>0.850</td>
<td>0.687</td>
<td>0.353</td>
<td>0.073</td>
<td>0.031</td>
<td>0.407</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.597</td>
<td>0.235</td>
<td>0.352</td>
<td>0.826</td>
<td>0.675</td>
<td>0.358</td>
<td>0.121</td>
<td>0.016</td>
<td>0.374</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.504</td>
<td>0.217</td>
<td>0.146</td>
<td>0.388</td>
<td>2.791</td>
<td>1.451</td>
<td>0.116</td>
<td>0.067</td>
<td>0.071</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.298</td>
<td>0.181</td>
<td>0.309</td>
<td>0.582</td>
<td>0.757</td>
<td>4.351</td>
<td>0.102</td>
<td>0.253</td>
<td>0.600</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.072</td>
<td>0.118</td>
<td>0.855</td>
<td>1.723</td>
<td>1.569</td>
<td>0.888</td>
<td>7.874</td>
<td>0.764</td>
<td>0.870</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>0.136</td>
<td>0.142</td>
<td>0.688</td>
<td>1.379</td>
<td>1.275</td>
<td>0.731</td>
<td>1.116</td>
<td>3.799</td>
<td>0.649</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.359</td>
<td>0.171</td>
<td>0.215</td>
<td>0.363</td>
<td>0.364</td>
<td>0.212</td>
<td>0.327</td>
<td>1.499</td>
<td>6.920</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The new $J_2^{-1}$ matrix has shown in Table 3.11. The size of the sub-matrix now becomes $9 \times 9$. The red blocks in Table 3.10 have been removed.

```matlab
Jinv2(find(Jinv2==0))=[];
Jinv2=reshape(Jinv2,npq,npq);
```
Table 3.11 New \( J_2^{-1} \) matrix

<table>
<thead>
<tr>
<th>1.669</th>
<th>1.008</th>
<th>1.974</th>
<th>2.070</th>
<th>1.972</th>
<th>1.648</th>
<th>1.344</th>
<th>1.442</th>
<th>1.888</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.229</td>
<td>1.555</td>
<td>1.594</td>
<td>1.749</td>
<td>1.766</td>
<td>1.734</td>
<td>1.695</td>
<td>1.731</td>
<td>1.814</td>
</tr>
<tr>
<td>0.797</td>
<td>0.428</td>
<td>1.471</td>
<td>1.927</td>
<td>1.716</td>
<td>0.974</td>
<td>0.212</td>
<td>0.458</td>
<td>1.488</td>
</tr>
<tr>
<td>0.831</td>
<td>0.369</td>
<td>1.854</td>
<td>2.645</td>
<td>2.282</td>
<td>0.951</td>
<td>0.448</td>
<td>0.012</td>
<td>1.875</td>
</tr>
<tr>
<td>0.699</td>
<td>0.300</td>
<td>1.603</td>
<td>2.288</td>
<td>4.632</td>
<td>2.195</td>
<td>0.462</td>
<td>0.066</td>
<td>1.575</td>
</tr>
<tr>
<td>0.369</td>
<td>0.148</td>
<td>0.890</td>
<td>1.271</td>
<td>2.531</td>
<td>5.404</td>
<td>0.332</td>
<td>0.115</td>
<td>0.827</td>
</tr>
<tr>
<td>0.055</td>
<td>0.033</td>
<td>0.087</td>
<td>0.122</td>
<td>0.124</td>
<td>0.090</td>
<td>8.826</td>
<td>1.578</td>
<td>0.864</td>
</tr>
<tr>
<td>0.133</td>
<td>0.049</td>
<td>0.336</td>
<td>0.481</td>
<td>0.394</td>
<td>0.120</td>
<td>1.341</td>
<td>4.632</td>
<td>2.514</td>
</tr>
<tr>
<td>0.538</td>
<td>0.233</td>
<td>1.226</td>
<td>1.750</td>
<td>1.497</td>
<td>0.596</td>
<td>0.312</td>
<td>2.030</td>
<td>9.749</td>
</tr>
</tbody>
</table>

In \( J_3^{-1} \) matrix, PV bus elements need to be removed in columns. The result of identifying PV bus elements shown in Table 3.11 as red blocks.

```matlab
Jinv3=zeros(npq,npq+npv);
for i=1:npq+npv
    for j=npq+npv+1:npq+npv+npq
        if type(i+1)==3
            Jinv3(j-npq-npv,i)=Jinv(j,i)
        end
    end
end
```
Table 3.12 PV bus element identified in $J_3^{-1}$

<table>
<thead>
<tr>
<th>Element</th>
<th>$1.669$</th>
<th>$1.008$</th>
<th>$1.279$</th>
<th>$1.974$</th>
<th>$1.974$</th>
<th>$2.070$</th>
<th>$1.972$</th>
<th>$1.648$</th>
<th>$1.344$</th>
<th>$1.442$</th>
<th>$1.888$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.008</td>
<td>0.156</td>
<td>1.008</td>
<td>1.279</td>
<td>1.974</td>
<td>1.974</td>
<td>2.070</td>
<td>1.972</td>
<td>1.648</td>
<td>1.344</td>
<td>1.442</td>
<td>1.888</td>
</tr>
<tr>
<td>0.038</td>
<td>0.219</td>
<td>1.229</td>
<td>1.555</td>
<td>1.671</td>
<td>1.594</td>
<td>1.749</td>
<td>1.766</td>
<td>1.734</td>
<td>1.695</td>
<td>1.731</td>
<td>1.814</td>
</tr>
<tr>
<td>0.011</td>
<td>0.099</td>
<td>0.797</td>
<td>0.428</td>
<td>0.119</td>
<td>1.471</td>
<td>1.471</td>
<td>1.927</td>
<td>1.716</td>
<td>0.974</td>
<td>0.212</td>
<td>0.458</td>
</tr>
<tr>
<td>0.022</td>
<td>0.141</td>
<td>0.831</td>
<td>0.369</td>
<td>0.597</td>
<td>1.854</td>
<td>1.854</td>
<td>2.645</td>
<td>2.282</td>
<td>0.951</td>
<td>0.448</td>
<td>0.012</td>
</tr>
<tr>
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<td>0.123</td>
<td>0.699</td>
<td>0.300</td>
<td>0.597</td>
<td>1.603</td>
<td>1.603</td>
<td>2.288</td>
<td>4.632</td>
<td>2.195</td>
<td>0.462</td>
<td>0.066</td>
</tr>
<tr>
<td>0.012</td>
<td>0.070</td>
<td>0.369</td>
<td>0.148</td>
<td>0.414</td>
<td>0.890</td>
<td>0.890</td>
<td>1.271</td>
<td>2.531</td>
<td>5.404</td>
<td>0.332</td>
<td>0.115</td>
</tr>
<tr>
<td>0.000</td>
<td>0.005</td>
<td>0.055</td>
<td>0.033</td>
<td>0.040</td>
<td>0.087</td>
<td>0.087</td>
<td>0.122</td>
<td>0.124</td>
<td>0.090</td>
<td>8.826</td>
<td>1.578</td>
</tr>
<tr>
<td>0.005</td>
<td>0.027</td>
<td>0.133</td>
<td>0.049</td>
<td>0.186</td>
<td>0.336</td>
<td>0.336</td>
<td>0.481</td>
<td>0.394</td>
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<td>1.341</td>
<td>4.632</td>
</tr>
<tr>
<td>0.015</td>
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<td>0.538</td>
<td>0.233</td>
<td>0.443</td>
<td>1.226</td>
<td>1.226</td>
<td>1.750</td>
<td>1.497</td>
<td>0.596</td>
<td>0.312</td>
<td>2.020</td>
</tr>
</tbody>
</table>

New $J_3^{-1}$ matrix after removing PV bus elements has been shown in Table 3.13.

```matlab
Jinv3(find(Jinv3==0))=[];
Jinv3=reshape(Jinv3,npq,npq);
```

Table 3.13 The new $J_3^{-1}$ matrix

<table>
<thead>
<tr>
<th>Element</th>
<th>$1.669$</th>
<th>$1.008$</th>
<th>$1.974$</th>
<th>$2.070$</th>
<th>$1.972$</th>
<th>$1.648$</th>
<th>$1.344$</th>
<th>$1.442$</th>
<th>$1.888$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.229</td>
<td>1.555</td>
<td>1.594</td>
<td>1.749</td>
<td>1.766</td>
<td>1.734</td>
<td>1.695</td>
<td>1.731</td>
<td>1.814</td>
<td></td>
</tr>
<tr>
<td>0.797</td>
<td>0.428</td>
<td>1.471</td>
<td>1.927</td>
<td>1.716</td>
<td>0.974</td>
<td>0.212</td>
<td>0.458</td>
<td>1.488</td>
<td></td>
</tr>
<tr>
<td>0.831</td>
<td>0.369</td>
<td>1.854</td>
<td>2.645</td>
<td>2.282</td>
<td>0.951</td>
<td>0.448</td>
<td>0.012</td>
<td>1.875</td>
<td></td>
</tr>
<tr>
<td>0.699</td>
<td>0.300</td>
<td>1.603</td>
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<td>4.632</td>
<td>2.195</td>
<td>0.462</td>
<td>0.066</td>
<td>1.575</td>
<td></td>
</tr>
<tr>
<td>0.369</td>
<td>0.148</td>
<td>0.890</td>
<td>1.271</td>
<td>2.531</td>
<td>5.404</td>
<td>0.332</td>
<td>0.115</td>
<td>0.827</td>
<td></td>
</tr>
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<td>0.055</td>
<td>0.033</td>
<td>0.087</td>
<td>0.122</td>
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</tr>
<tr>
<td>0.138</td>
<td>0.049</td>
<td>0.336</td>
<td>0.481</td>
<td>0.394</td>
<td>0.120</td>
<td>1.341</td>
<td>4.632</td>
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<td></td>
</tr>
<tr>
<td>0.538</td>
<td>0.233</td>
<td>1.226</td>
<td>1.750</td>
<td>1.497</td>
<td>0.596</td>
<td>0.312</td>
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<td>9.749</td>
<td></td>
</tr>
</tbody>
</table>
In $J^{-1}$ matrix, only PQ bus elements existing, no change need to be done in $J^{-1}$. Table 3.14 shows the $J^{-1}$ matrix.

```matlab
Jinv4=zeros(npq,npq);
for i=npq+npv+1:npq+npv+npq
    for j=npq+npv+1:npq+npv+npq
        Jinv4(j-npq-npv,i-npv-npq)=Jinv(j,i)
    end
end
```

Table 3.14 $J^{-1}$ matrix

<table>
<thead>
<tr>
<th></th>
<th>4.043</th>
<th>2.520</th>
<th>1.750</th>
<th>1.673</th>
<th>1.386</th>
<th>0.699</th>
<th>0.133</th>
<th>0.247</th>
<th>1.073</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.493</td>
<td>4.128</td>
<td>1.119</td>
<td>1.110</td>
<td>0.926</td>
<td>0.472</td>
<td>0.086</td>
<td>0.172</td>
<td>0.723</td>
<td></td>
</tr>
<tr>
<td>1.812</td>
<td>1.141</td>
<td>7.825</td>
<td>5.414</td>
<td>4.474</td>
<td>2.252</td>
<td>0.404</td>
<td>0.768</td>
<td>3.428</td>
<td></td>
</tr>
<tr>
<td>1.718</td>
<td>1.099</td>
<td>5.368</td>
<td>10.760</td>
<td>8.891</td>
<td>4.475</td>
<td>0.796</td>
<td>1.521</td>
<td>6.803</td>
<td></td>
</tr>
<tr>
<td>1.423</td>
<td>0.913</td>
<td>4.452</td>
<td>8.924</td>
<td>14.156</td>
<td>7.122</td>
<td>0.662</td>
<td>1.260</td>
<td>5.641</td>
<td></td>
</tr>
<tr>
<td>0.727</td>
<td>0.469</td>
<td>2.279</td>
<td>4.566</td>
<td>7.230</td>
<td>12.998</td>
<td>0.341</td>
<td>0.643</td>
<td>2.885</td>
<td></td>
</tr>
<tr>
<td>0.132</td>
<td>0.082</td>
<td>0.407</td>
<td>0.817</td>
<td>0.677</td>
<td>0.342</td>
<td>13.749</td>
<td>4.682</td>
<td>2.569</td>
<td></td>
</tr>
<tr>
<td>0.252</td>
<td>0.164</td>
<td>0.793</td>
<td>1.588</td>
<td>1.311</td>
<td>0.658</td>
<td>4.663</td>
<td>8.661</td>
<td>4.811</td>
<td></td>
</tr>
<tr>
<td>1.099</td>
<td>0.704</td>
<td>3.437</td>
<td>6.890</td>
<td>5.692</td>
<td>2.864</td>
<td>2.527</td>
<td>4.722</td>
<td>21.122</td>
<td></td>
</tr>
</tbody>
</table>

The process that identifies and removes PV bus elements in the whole matrix is shown in Table 3.15 and Table 3.16. The red blocks in Table 3.15 are values that contain PV bus elements.
The inverse of Jacobian matrix now is a group of four equal size sub-matrices. The indices for sensitivity analysis can be calculated. The indices from equation (3.28) – (3.31) can be obtained through the following code.

44
Index\textsubscript{AP} is the summation of columns in $J_1^{-1}$ matrix and divide by the number of PQ bus ($npq$).

\begin{verbatim}
IndexAP=zeros(npq,1);
for i=1:npq
    for j=1:npq
        IndexAP(i)=IndexAP(i)+Jinv(j,i);
    end
end
IndexAP=IndexAP/npq;
\end{verbatim}

Index\textsubscript{VP} is the summation of columns in $J_2^{-1}$ matrix and divide by the number of PQ bus.

\begin{verbatim}
IndexVP=zeros(npq,1);
for i=1:npq
    for j=npq+1:npq*2
        IndexVP(i)=IndexVP(i)+Jinv(j,i);
    end
end
IndexVP=IndexVP/npq;
\end{verbatim}

Index\textsubscript{AQ} is the summation of columns in $J_3^{-1}$ matrix and divide by the number of PQ bus.

\begin{verbatim}
IndexAQ=zeros(npq,1);
\end{verbatim}
IndexVQ is the summation of columns in $J_{4^{-1}}$ matrix and divide by the number of PQ bus.

The five compound indices can be calculated from the four basic indices with equations (3.35) - (3.38).
All four basic indices are calculated, and the results are shown in Figure 3.

6. The Y axis represents the mathematical weigh in percentage, all indices are on the same scale. The X axis is the node’s number.

![Index$_{AP}$ for IEEE 14-node test feeder](image1)

**Figure 3.3 Index$_{AP}$ of 14-node system**

![Index$_{VP}$ for IEEE 14-node test feeder](image2)

**Figure 3.4 Index$_{VP}$ of 14-node system**
The results of $\text{Index}_P, \text{Index}_Q, \text{Index}_A,$ and $\text{Index}_V$ are shown in Figure 3. 7 - Figure 3. 10.
Figure 3.7 $\text{Index}_p$ of 14-node system

Figure 3.8 $\text{Index}_Q$ of 14-node system
Result of $\text{Index}_T$ is shown in Figure 3.11. The $\text{Index}_T$ represents the summation of $\text{Index}_P$ and $\text{Index}_Q$. 

Figure 3.9 $\text{Index}_A$ of 14-node system

Figure 3.10 $\text{Index}_V$ of 14-node system
Looking at $\text{Index}_P$ and $\text{Index}_A$, nodes 12, 13, and 14 have the highest values. $\text{Index}_P$ and $\text{Index}_A$ have a similar pattern. $\text{Index}_Q$ and $\text{Index}_V$ are similar as well, the highest nodes from these indices are 9, 10, and 14. The common top node between them is node 14 which is obtained from the result of $\text{Index}_T$. In other words, $\text{Index}_T$ suggested node 14 as the optimal node for storage placement. The storage system that is connected to the node would have the highest impact of regulating voltage angles and magnitudes.
CHAPTER 4 EVALUATION OF INDICES

From the previous discussions in chapter 3, the four basic indices can be obtained through the inverse of Jacobian matrix from Newton-Raphson method. Once the four basic indices have been calculated, five compound indices can also be obtained.

To verify the indices calculated for IEEE 14-node test feeder in chapter 3, an intensive case study has been conducted. A storage device is added to the system and it is moved from node to node. The default system without any storage is considered as the reference. For each installment of energy storage, the changed values of phase angle and voltage magnitude compared to the default case, have been recorded. Then the variation is compared with the indices to see if the variations of angle and magnitude have the same pattern of the indices.

The storage device is interfaced with grid through an inverter, both active and reactive power control can be done. That is to say that both active and reactive power can be injected from the point where the storage device is connected.

In this case, a storage device has moved along the nodes in the system. The output of storage device has changed from 100MW to 10MW and the injected of reactive power has changed in the range of 100MVar to 10MVar as well. To show how much the angle and magnitude have been affected by the active and reactive powers, the average variation in the
voltage angle and magnitude in every node has been summarized. The equations are shown in (4.1) and (4.2).

\[
\text{Variation}_{|v|} = \frac{\sum_{i=1}^{N}(|V|_{i_{\text{org}}} - |V|_{i_{\text{new}}})}{N} \quad (4.1)
\]

\[
\text{Variation}_{\delta} = \frac{\sum_{i=1}^{N}(\delta_{i_{\text{org}}} - \delta_{i_{\text{new}}})}{N} \quad (4.2)
\]

Where \( N \) is the number of nodes in system \(|V|_{i_{\text{org}}} \) and \( \delta_{i_{\text{org}}} \) are default angle and default magnitude at node \( i \) before adding any kind of energy storage system. \(|V|_{i_{\text{new}}} \) and \( \delta_{i_{\text{new}}} \) are angle and magnitude at node \( i \) after storage device connected. The summation of the variations in the angle and magnitude in every node can illustrate how much active and reactive power injection of a node can influence other nodes’ angle and magnitude. The summation of variation in angle degree at every node is shown in Figure 4.1. Each line represents the cumulative impact of its corresponding node on other nodes for a wide range of injected active power. Although active power does not change the magnitude by much, the variation of voltage magnitude has shown in figure 4.2. The cumulative impact of voltage magnitude with active power injected in different nodes shown as lines.
Figure 4.1 The cumulative variation of angles for a range of active power.

Figure 4.2 The cumulative variation of magnitudes for a range of active power.

The variation in voltage angle and magnitude while Q injection has shown in figure 4.3 and figure 4.4.
The variation of voltage angle and magnitude while apparent power injection has shown in Figure 4. 5 and Figure 4. 6.
From Figure 4.1 and Figure 4.3, the angle is more affected by the change on active power than reactive power. Figure 4.2 and Figure 4.4 show that the voltage magnitude is more sensitive to reactive power than active power.
To evaluate that the indices calculated in chapter 3 are reliable and able to reflect the sensitivity of angle and magnitude, a comparison is made. The summation of variation in angle and magnitude in every node when 20 MVA is injected at various nodes, is compared with the indices that proposed in chapter 3.

\[ \text{Index}_A \text{ is defined as } \text{Index}_{AP} \text{ subtract Index}_{AQ}, \text{ which represent the influences of the active power on the angle}. \text{ Index}_P \text{ is the angle’s sensitivity to the active power. The Index}_A \text{ and Index}_P \text{ compared with the variation on voltage angle is shown in Figure 4.7. The active power injection is considered as 20MW. The values of variation of actual angle and indices are scaled with the reference of node 14 to see if they are in the same pattern.} \]

![Index A and Index P with the variation on voltage angle of 20MW injection](image)

**Figure 4.7** Index\(_A\) and Index\(_P\) compared with the variation of voltage angle
Index\(V\) is defined as \(\text{Index}_{VQ} \) subtract \(\text{Index}_{VP}\), that represent the influences of reactive power injection on voltage magnitude. The comparison of \(\text{Index}_Q\), \(\text{Index}_V\) and the variation in voltage magnitude is shown in Figure 4.8.

![Index\(Q\) and Index\(V\) and the variation on V of 20MVar](image)

**Figure 4.8** The comparison of \(\text{Index}_V\), \(\text{Index}_P\) and the variation on voltage magnitude

In Figure 4.9, the variation of both of the angle and magnitude of 20 MVA with apparent power injected is compared with \(\text{Index}_T\). Again, the node 14 is selected as the base, values of variation and index are in the same scale. Since \(\text{Index}_T\) is the summation of two dominant indices and subtraction of non-dominant indices, the variation of angle and magnitude are summarized and compared with \(\text{Index}_T\).
With the increasing of injection, the variations of the angle and magnitude become less related to the Index, the variations of the angle and magnitude in some of the nodes increase rapidly rather than following the Index. The variations increase, especially happen to the voltage magnitude. In Figure 4. 6, the voltage magnitude in node 9 raised rapidly while the injection excess 60 MVA. In Figure 4.7 – 4.9, the variations still following the similar pattern with the proposed indices though 20 MVA is a significant power injection for the IEEE 14-node test feeder.

Figure 4. 9 $\text{Index}_T$ compared with the total variation of angle and magnitude
CHAPTER 5 INDICES OF IEEE 123-NODE TEST FEEDER

5. 1 IEEE 123-node test feeder

IEEE PES Distribution System Analysis Subcommittee's Distribution Test Feeder Working Group has released several test feeders. The IEEE 123-node test feeder is selected as an expansion testing system for sensitivity analysis. The system is shown in Figure 5. 1.

The IEEE 123-node test feeder operates at a nominal voltage of 4.16 kV. The test feeder has different types of conductors including overhead and underground lines, the main feeder is overhead three-phase.

Figure 5. 1 IEEE 123-node test feeder
Some of the nodes are renumbered to keep the total node numbers within 123. This modification is done for the sake of computer programing and the original numbers are still visible in Figure 5. 2 with a gray color.

The slack bus is node 115. Switches in the system are all closed, but the switches connected to node 195, 250, 350, and 451 which connect IEEE 123 network to other possible networks are open. All changes have done to nodes are listed in Table 5. 1, there are 122 nodes remain after the renaming.

Figure 5. 2 IEEE 123-node test feeder after modify
Table 5. 1 Node changed from IEEE 123-node test feeder

<table>
<thead>
<tr>
<th>Node</th>
<th>Change</th>
<th>Node</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>Removed</td>
<td>195</td>
<td>Removed</td>
</tr>
<tr>
<td>149</td>
<td>115</td>
<td>160</td>
<td>118</td>
</tr>
<tr>
<td>135</td>
<td>122</td>
<td>610</td>
<td>Removed</td>
</tr>
<tr>
<td>250</td>
<td>120</td>
<td>300</td>
<td>121</td>
</tr>
<tr>
<td>251</td>
<td>Removed</td>
<td>151</td>
<td>116</td>
</tr>
<tr>
<td>152</td>
<td>117</td>
<td>450</td>
<td>Removed</td>
</tr>
<tr>
<td>451</td>
<td>Removed</td>
<td>197</td>
<td>119</td>
</tr>
</tbody>
</table>

Some of loads in the network are single phase. For the sake of this study, they are replaced with equivalent three-phase loads.
5.2 Sensitivity indices of IEEE 123-node test feeder

In the case of IEEE 123-node test feeder, Newton-Raphson iteration is processed and obtained the Jacobian matrix of IEEE 123-nod. The four basic indices are calculated from the inverse of Jacobian matrix. Since there is no PV bus exist in IEEE 123-node test feeder, every node is considered as the potential candidate, there is no need to reduce the size of sub-matrices. Every index is presented in percentage. The greater value means the node is a suitable location for a storage device according to indices. The four basic proposed indices are shown in Figure 5.3 – 5.7.

Figure 5.3 $\text{Index}_{AP}$ of IEEE 123-node test feeder
Figure 5. 4 Index$_{AQ}$ of IEEE 123-node test feeder

Figure 5. 5 Index$_{VP}$ of IEEE 123-node test feeder
The results of $\text{Index}_P$, $\text{Index}_Q$, $\text{Index}_A$, and $\text{Index}_V$ are shown in Figure 5. 7- Figure 5. 10.
Figure 5.8 $\text{Index}_Q$ of IEEE 123-node test feeder

Figure 5.9 $\text{Index}_A$ of IEEE 123-node test feeder
The total index, $\text{Index}_T$ is shown in Figure 5.11. In $\text{Index}_T$, the node with the highest sensitivity index value is node 83.
As the evaluation in chapter 4, the 20MW active power injected and moved all over the nodes. The variation of voltage angle in all nodes are summarized and compared with Index\textsubscript{p} and Index\textsubscript{A}. The variation of voltage angle, Index\textsubscript{p}, and Index\textsubscript{A} are all set as the same scale as the last node in the system. The result is shown in Figure 5.12.

![Figure 5.12 The variation of voltage angle, Index\textsubscript{p} and Index\textsubscript{A}](image)

The 20 MVar reactive power injected is compared with Index\textsubscript{Q} and Index\textsubscript{V} on voltage magnitude variation is shown in Figure 5.17.
The $\text{Index}_T$ is compared with the summation of voltage angle and voltage magnitude variation and shown in Figure 5. 14.
The testing is done and the values of variation of voltage magnitude and angle are obtained. The results show that the variation of voltage magnitude and angle both have the highest value in node 83 just as Index\(r\) has suggested. The second highest is node 114, the location of node 83 and node 114 are shown in Figure 5.15.

![Figure 5.15 Nodes with high variation in IEEE 123-node](image)

Both node 83 and node 114 could be a suitable location for storage placement; both nodes have high index value. A storage device that connected to these nodes could regulate voltage angle and magnitude with the highest impact. If the node with the highest index value
is not able to install a storage device, the nearby nodes with similar index value can also be considered as suitable locations and will have close performance.

In order to show that nodes suggested by the index are suitable locations for storage placement, node 83, 114, 67, and 13 have been selected. Since node 83 and 114 are relatively far from the utility feeder, a node that is closer to slack bus and a node at the middle distance from the slack bus are selected and compared with each other for different power input. The result of variation in voltage angle with active power injection is shown in Figure 5. 16. Node 83 and 114 have very close index value so they may look likes one line in the following figure.

![Variation of voltage angle with P injection](image)

*Figure 5. 16 Variations of voltage angle with active power injection on IEEE 123-node test feeder*
Variation of voltage magnitude with active power injection on IEEE 123-node test feeder is shown in Figure 5. 17.

Figure 5. 17 Variation of voltage magnitude with active power injection on IEEE 123-nodes test feeder

Variation of voltage angle and magnitude with reactive power injection are shown in Figure 5. 18 and Figure 5. 19.
Variation of voltage angle and magnitude with apparent power injection are shown in Figure 5. 20 and Figure 5. 21.
Figure 5. 20 Variation of voltage angle with apparent power injection on IEEE 123-node test feeder

Figure 5. 21 Variation of voltage magnitude with apparent power injection on IEEE 123-node test feeder

In Figure 5. 16, Figure 5. 18, and Figure 5. 20 which are corresponding to the $\text{Index}_A$ and $\text{Index}_P$, active power makes more impact to the cumulated variation of the angle than reactive power. Node 114 and 83 can make the greatest impact on the voltage angle and the
following nodes are node 67 and 13. In both of the figure, node 114 and 83 have similar results on the impact of voltage angle. The impact that node 13 can make is lower than node 114, 83, and 67. The impact of node 114 on the angle compared to node 83 is slightly higher in Figure 5. 18, lower in Figure 5.20 and about the same in Figure 5.16.

In Figure 5. 17, Figure 5. 19, and Figure 5.21, reactive power makes more impact to voltage magnitude compared to the active power. The results are also corresponding to the Index\textsubscript{Q} and Index\textsubscript{V}. Node 114 and 83 have almost the same and the highest impact on voltage magnitude compared to node 67 and 13. The impact of node 13 also far behind from node 114, 83, and 67. The suggested nodes by the sensitivity indices have a higher impact than any other random nodes.
CHAPTER 6 CONCLUSIONS AND FUTURE WORKS

6.1 Conclusion

Storage systems are critical to the distribution system, since they provide active and reactive power to improve both of the angle and voltage magnitude, absorb excessive power and provide power support in time of need. However, storage devices are expensive and cannot be installed on every node in the system. Finding the optimal location for a storage system that can impact the whole system is necessary. In this study, sensitivity indices are proposed for analyzing the optimal location for the storage system. The indices are obtained from the inverse of Jacobian matrix which can be calculated through the Newton-Raphson power flow methodology. Once the Newton-Raphson iteration is converged, no other iteration is required for the suggested method for finding the optimal locations. This is different from other approach in references [29], [32], [36], and [37]. Having the inverse of Jacobian matrix, PV nodes are removed from the matrix and reach four equal-sizing sub-matrices. The four basic indices can be calculated from these four sub-matrices.

The sensitivity indices can be generated in a simple and fast manner. These indices quantify the impact of active and reactive power on any node of the system on the overall angle and magnitude of the network. Therefore the optimal storage location is corresponding to the node with the highest index. The sensitivity of voltage angle and magnitude to active...
power and reactive power is a key for this method. The four basic indices are listed in Table 6.

1.

Table 6. 1 Four basic indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Equation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index_{AP}</td>
<td>$\text{Index}<em>{AP} = \frac{\sum</em>{k=1}^{n}</td>
<td>\frac{\partial \delta_{k}}{\partial P_i}</td>
</tr>
<tr>
<td>Index_{AQ}</td>
<td>$\text{Index}<em>{AQ} = \frac{\sum</em>{k=1}^{n}</td>
<td>\frac{\partial \delta_{k}}{\partial Q_i}</td>
</tr>
<tr>
<td>Index_{VP}</td>
<td>$\text{Index}<em>{VP} = \frac{\sum</em>{k=1}^{n}</td>
<td>\frac{\partial</td>
</tr>
<tr>
<td>Index_{VQ}</td>
<td>$\text{Index}<em>{VQ} = \frac{\sum</em>{k=1}^{n}</td>
<td>\frac{\partial</td>
</tr>
</tbody>
</table>

The five compound indices are shown in Table 6. 2. Nodes with higher index value can be chosen as the location for storage placement for different concerns.

Table 6. 2 Five compound indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Equation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index_p</td>
<td>Index_{AP} – Index_{VP}</td>
<td>Active power to voltage angle, not voltage magnitude</td>
</tr>
<tr>
<td>Index_Q</td>
<td>Index_{VQ} – Index_{AQ}</td>
<td>Reactive power to voltage magnitude, not voltage angle</td>
</tr>
<tr>
<td>Index_A</td>
<td>Index_{AP} – Index_{AQ}</td>
<td>Active power to voltage angle, not reactive power</td>
</tr>
<tr>
<td>Index_v</td>
<td>Index_{VQ} – Index_{vp}</td>
<td>Reactive power to voltage magnitude, not active power</td>
</tr>
<tr>
<td>Index_T</td>
<td>Index_{A} – Index_{V}</td>
<td>Combine of active power to voltage angle and reactive power to magnitude</td>
</tr>
</tbody>
</table>
The indices are first tested on IEEE 14-node test feeder mainly because it has a simple configuration. The method is also applied to IEEE 123-node test feeder to show the indices work on the larger system as well. The indices proposed in this study can be used for selecting the optimal location for the storage system.

The suggested indices are validated by injecting active and reactive power on various nodes and observe the cumulative effect on the angle and magnitude. The trends have the same pattern as the sensitivity indices have predicted. The result shows that nodes with higher index value make more influence on voltage angle or magnitude.

The model of IEEE 123-node test feeder and IEEE 14-node test feeder are constructed with Matlab, cases are simulated by adding a storage device to PQ loads and moving the storage device all along PQ bus in the system to see the influence on the voltage angle and magnitude. The results suggested that the variations of voltage angle and magnitude are following the proposed indices.

The proposed indices can be used as a tool for planning the placement of storage system. Different indices can be chosen depending on the needed. In the proposed indices, Indexₚ or Indexₐ can be used for selecting the best location of a capacitor bank since they only show the influence of injected reactive power over the voltage. Indexₐ or Indexₚ can be used to find the optimal place of a battery system. They only show the relation of the
active power on the angle. In general case of a storage device with an inverter that supposed to stabilize both of the angle and magnitude, $\text{Index}_T$ is a suitable tool.

One of the advantages of the suggested method over the other iteration-based methods is that this approach does not require a loop and hence does not have convergence problems. The optimal location can be found straightforwardly.

6. 2 Future works

The test was successful as it was able to identify the node with the best performance on voltage angle and magnitude regulation. In this study, only the optimal location has been discussed. In future, the capacity of the energy storage device can also be optimized as well. Since the cost of a storage system depends on its size, it is important to find the optimal size for the storage system. The larger the storage system is, the more power can be injected into the system, but the cost would also increase. Optimal size and location of the storage device could make the system reach the best performance at the lowest cost.
REFERENCES


[11] Q. Fu et al., "Generation capacity design for a microgrid for measurable power


