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Supply Chain Contracting in the Presence of Supply Uncertainty and Store Brand Competition

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by

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ABSTRACT

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by

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In today's complex business environment, manufacturers are striving to maintain a competitive advantage over their supply chain partners. Manufacturers' profitability is tightly linked to their strategic interactions with other entities in the supply chain. While numerous studies have been conducted to investigate such interactions in supply chains, certain issues remain unresolved. We apply a game-theoretic framework to analyze two distinct supply chain structures in the presence of supply uncertainty and store brand competition in two essays, respectively.

In the first chapter, we study a decentralized assembly supply chain under supply uncertainty. In a decentralized assembly supply chain, one assembler assembles a set of n components, each produced by a different supplier, into a final product to meet an uncertain market demand. Each supplier faces an uncertain production capacity such that only the lesser of the planned production quantity and the realized capacity can be delivered to the assembler. We assume that the suppliers' random capacities and the random demand can follow an arbitrary continuous multivariate distribution. We formulate the problem as a two-stage Stackelberg game. The assembler and the suppliers adopt a so-called Vendor-Managed-Consigned-Inventory (VMCI) contract. We analytically characterize the equilibrium of this game, based on which we obtain several managerial insights. Surprisingly, we show that when a supplier's production cost increases or when his component salvage value

decreases, it hurts all other members and the entire supply chain, but it might sometimes benefit this particular supplier. Similarly, when the suppliers do not have supply uncertainty, it benefits the assembler but it does not necessarily benefit the suppliers. Furthermore, we demonstrate that when the suppliers' capacities become more positively correlated, the assembler is always better off, but the suppliers might be better or worse off. Later in the chapter, we also solve the game under the conventional wholesale-price contract. We find that the assembler always prefers the VMCI contract, and the suppliers always prefer the wholesale price contract. In addition, we illustrate that the VMCI contract is more efficient than the wholesale price contract for this decentralized assembly supply chain.

In the second chapter, we consider a two-tier decentralized supply chain with a national brand supplier and a retailer. The national brand supplier (she) distributes her products to consumers through the retailer. Meanwhile, the retailer (he) intends to develop and produce his own store brand through a manufacturing source that is different from the national brand supplier. The retailer holds the store brand production unit cost as private information, for which the national brand supplier only has a subjective assessment. Given a supply contract offered by the national brand supplier, the retailer simultaneously decides whether to accept the contract and whether to produce the store brand. The national brand supplier aims to design an optimal menu of contracts to maximize her expected profit as well as extract the retailer's private cost information. We formulate the problem as a two-stage screening game to analyze the strategic interaction between the two players. Despite the inherent computational complexity, we are able to derive the optimal menu of contracts for the national brand supplier, of which the format depends on the national brand supplier's unit production cost. Furthermore, we investigate how the model parameters affect the value of information for each member in the supply chain. We show that the retailer's private cost information becomes less valuable to both the national brand supplier and the retailer when the national brand unit production cost increases. We also illustrate that when the gap between the two possible cost values increases, the private cost information becomes more valuable to the national brand supplier, however the value of information to the retailer himself can either increase or decrease. Finally, we demonstrate that when the perceived quality of the national brand increases, the value of information to the retailer first decreases then increases, but the impact on the value of information to the national brand supplier can be either positive or negative.

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Chapter 1

Component Procurement for an Assembly Supply Chain with Random Capacities and Random Demand

1.1 Introduction

In recent years, outsourcing has become a common practice in a number of industries, including automobiles, electronics, computers, and others, in which Original Equipment Manufacturers (OEMs) outsource the component production to independent suppliers (or contract manufacturers) and finish the assembly in-house. Kallstrom (2015) reported that the automotive suppliers' proportion of value added to worldwide automobile manufacture has been steadily increasing from 56% in 1985 to 82% in 2015. Rajaram (2015) pointed out that "The global market for electronics contract manufacturing (ECM) services should total $515.6 billion in 2015, and reach nearly $561.2 billion by 2016 and $845.8 billion by 2021, with a five-year compound annual growth rate (CAGR) of 8.6% from 2016-2021." These OEMs' operations heavily depend on the reliability of the component supply. Sodhi and Tang (2012) commented that "problems in one link of the supply chain have caused unmitigated disaster to another link, resulting in large financial and non-financial damage." In particular, when the upstream supply capacity is uncertain, it might affect downstream OEMs significantly. For example, due to supply problems, Sony had to delay the launch of Sony PlayStation 3 in

2006, which hurts not only Sony's short-term revenues but also its long-term market share (Sodhi and Tang, 2012).

There are various reasons for the supply capacity uncertainties, such as unforeseen production line breakdown, suspended delivery due to disagreement, and so on. In 2014, an eye-catching news story in the technology industry was the winding-down relationship between Apple and one of its suppliers, GT Advanced Technology (GTAT). Apple planned to use the sapphire crystal glass manufactured by GTAT on the iPhone 6 and two models of Apple Watch. However, GTAT failed to deliver the agreed amount and quality of glass to Apple, and ended up filing bankruptcy due to the financial pressure. Consequently, Apple had to turn to a different material for the screen (Gokey, 2015). A more recent case in 2016 was that two Volkswagen suppliers, one following the other, suspended contractually agreed deliveries of components. This shortage of components led to bottlenecks in production, causing severe disruptions at several Volkswagen factories (Bronst, 2016). Numerous cases have shown that the supply uncertainty has a significant impact on such an assembly supply chain since the supply chain performance is determined by the bottleneck agent (Gurnani and Gerchak, 2007). Taking account of the uncertainties from all suppliers and an often uncertain market demand, the supply chain risk could be even more significant. Such supply chains are further complicated by the fact that each component supplier and the manufacturer are independent entities which seek to maximize their own profits. Hence, in this chapter, we seek to study the strategic interactions among the upstream suppliers and the downstream manufacturer in a decentralized assembly system in the presence of random capacities from the suppliers and random demand from the market.

We consider a decentralized assembly supply chain with one manufacturer (referred as assembler hereafter) and n suppliers each producing a different component. The assembler assembles the n components into a final product to satisfy a single-period uncertain market demand with an exogenous fixed retail price. Each supplier's production capacity is uncertain as well. We assume the random demand and the suppliers' random capacities follow an arbitrary multivariate continuous distribution. In our base model, we assume that the assembler and the suppliers adopt a Vendor-Managed-Consigned-Inventory (VMCI) contract. That is, each supplier manages his own inventory and the assembler only pays to each supplier for the components actually used by the assembler according to the pre-agreed unit price. We formulate the problem as a two-stage Stackelberg game. First, the assembler proposes a unit price to each supplier. Second, the suppliers make their own production quantity decisions simultaneously. Then the suppliers' uncertain capacities are realized. Each supplier's final deliverable quantity is the lesser of its realized capacity and planned production quantity. The suppliers deliver their available components to the assembler who will then assemble the components into final products to satisfy its realized market demand, and each supplier collects a payment from the assembler.

As noted by Gümüş et al. (2008), VMI and consignment inventory are two separate supply chain strategies, i.e., VMI permits the vendor to initiate orders for the customer, while consignment inventory means that the vendor owns the goods until the customer uses them and the vendor is only paid for the goods used by the customer. Although VMI can be implemented with or without consignment inventory in practice, a recent survey by Gatorpoint Research across various industry sectors including wholesale, manufacturing, retailing, and telecom regarding their VMI strategies indicates that "49% of responders use consigned inventory at their buyers' VMI location "(Gatorpoint Research 2013). In our VMCI contract, we assume that the assembler and the suppliers adopt the VMI and consignment inventory together because without the consignment inventory agreement (i.e., the suppliers are only paid for the components delivered to the assembler), the suppliers would deliver as much inventory as possible to the assembler. Similar VMCI contracts are adopted in Gerchak and Wang (2004), Fang, et al. (2008), Bazan et. al. (2014), and Lee and Cho (2014). In a general VMI contract without consignment inventory, an upstream supplier may decide the inventory level and the delivery schedule for a downstream retailer according to predetermined minimum and maximum inventory levels set by the retailer, and the retailer owns the inventory upon receiving it. (see Fry et al. 2001). Bischescu and Fry (2009) assume that a continuous review (Q, R) inventory policy is used in a supply chain, where the supplier determines the replenishment quantity Q and the retailer decides the reorder point R. More discussions about various forms of VMI contracts can be found in a recent survey paper regarding VMI by Gorvidan (2013).

Despite the complexity, we are able to derive the analytical equilibrium prices and order/production quantities in closed-forms for the VMCI contract. Our results illustrate several important managerial insights for the decentralized assembly supply chain under supply uncertainties. Under the VMCI contract, we obtain the following results. First, when a supplier's production cost increases, we show that the profits of the assembler, all other suppliers, and the entire supply chain all decrease, but the profit of this particular supplier may sometimes improve. This result is quite surprising but it is consistent with similar results derived in Fang et al. (2008) and Granot and Yin (2008). Second, when a supplier's component salvage value increases, it benefits the assembler, all other suppliers, and the entire supply chain, but surprisingly it may hurt this supplier himself. This result is relatively new in the literature since the salvage values are assumed to be zero in Fang et al. (2008) and Granot and Yin (2008). Third, by comparing our results with those in Gerchak and Wang (2004), we find that when the suppliers display no capacity uncertainties, the assembler is always better off, but the suppliers may sometimes be worse off. These three results suggest that the assembler may need to offer some incentives to suppliers with certain cost structures to reduce his production cost or capacity uncertainty and to improve his salvage value since such actions may hurt the suppliers but always benefit the assembler. Fourth, we observe that when the final product retail price increases or the assembly cost decreases, every one is better off. This result validates our intuition, and it is indeed consistent with most literature. Last but not the least, we demonstrate that when the suppliers' random capacities become more positively correlated under the multivariate normal distribution, the assembler is always better off, but the suppliers may be better or worse off. We believe this result is new since it is often assumed in the literature that random yield/capacity distributions are independent. Based on this result, we suggest that under a VMCI contract, the assembler could select suppliers from the same regions such that the suppliers' random capacities may be positively correlated.

In addition to the base model under the VMCI contract, in section 1.6, we also analyze and solve the game under the traditional wholesale-price contract (i.e., without VMI or consignment inventory), i.e., the suppliers first choose their respective wholesale prices simultaneously, then the assembler orders from the suppliers. Comparing the respective results from the wholesale price contract and the VMCI contract, we show in section 1.7 that the assembler and entire supply chain would prefer the VMCI contract, whereas the suppliers would prefer the wholesale price contract. This result is consistent with Gerchak and Wang (2004) who study the same two contracts for a decentralized assembly system with perfectly reliable suppliers.

The remainder of this chapter is organized as follows. In section 1.2, we provide a literature review. In section 1.3, we present our model setup for the decentralized system under the VMCI contract, and then derive the optimal production plan for the corresponding centralized system as a benchmark. In section 1.4, we characterize the equilibrium behaviors for both the suppliers and the assembler in the decentralized system under the VMCI contract. We then derive our managerial insights regarding the VMCI contract in section 1.5. In section 1.6, we present the model and the equilibrium results for the traditional wholesale price contract. In section 1.7, we compare the system performance and individual firm's performance under the VMCI contract with those under the wholesale price contract. Finally, we conclude the chapter with major findings and further research directions in section 1.8.

All mathematical proofs are summarized in a separate Appendix.

1.2 Literature Review

Our work is most related to two streams of literature regarding supply uncertainties and assembly systems. Supply uncertainties can be attributed to different causes in the production process. In the literature, the supply uncertainties can be characterized into three categories: random capacity, random yield, and supply disruption. Random capacity denotes an uncertain exogenous upper bound on the actual production quantity (Bollapragada et al. 2004). Random yield refers to the situation where only a random fraction of products turn out to be usable due to production defects, machine breakdowns, and so on (Yao 1988). Supply disruption can be considered as a special case of random capacity with a Bernoulli distribution following which either the full order or nothing is delivered (Gurnani et al. 2000). On the other hand, the papers studying assembly systems can be grouped in two categories, i.e., decentralized and centralized assembly systems. In a centralized assembly system, the assembler aims to derive the optimal inventory policy for all the components and the optimal component allocation policy among various final products among which common components are shared (Song and Zipkin 2003). Due to outsourcing, the assembler now faces a decentralized assembly system. The research focus in decentralized assembly systems is to study the strategic interactions under contractual arrangements among independent component suppliers and the assembler who seek to optimize their own profits (Gerchak and Wang 2004). We summarize the relevant papers in Table 1.1.

There are abundant papers studying random yields in the area of supply chain and operations management. In Table 1.1, we only list the papers analyzing both random yields and assembly systems. In particular, Yao (1988), Gerchak et al. (1994), Gurnani et al.(2000), and Pan and So (2010) explore the inventory/production/pricing decisions for a centralized

Table 1.1: Papers related to supply uncertainties and assembly systems

	Decentralized Assembly System	Centralized Assembly System
Random Yield	Gurnani & Gerchak (2007) Güler & Bilgiç (2009) Güler (2015) Pan & So (2015)	Yao (1988) Gerchak et al. (1994) Gurnani et al. (2000) Pan & So (2010)
Supply Disruption	Li et al. (2017)	Gurnani et al. (1996) DeCroix (2013) Yin et al. (2017)
Random Capacity	This essay	Bollapragada et al. (2004) Xiao et al. (2010) Bollapragada et al. (2015) Ji et al. (2016)

assembly system subject to random supply yields and investigate how the random yields affect the system performance. A few papers study random yields in a decentralized assembly system. Gurnani and Gerchak (2007) propose a contract to coordinate a decentralized assembly system with two component suppliers both experiencing random yields and an assembler facing a deterministic market demand. Güler and Bilgiç (2009) extend their work by establishing two coordinating contracts with three parameters for a decentralized assembly system consisting of N suppliers with random yields and an assembler with random market demand. Under the same system structure as that in Güler and Bilgiç (2009), Güler (2015) is able to form a coordinating contract with two parameters only. Pan and So (2015) derive the equilibrium production and pricing decisions for a decentralized assembly system with two suppliers under a similar VMI contract to our VMCI contract, and one of the two suppliers is exposed to the random yield.

There are several papers investigating the optimal inventory/ordering policies for the centralized assembly system with supply disruptions, e.g., Gurnani et al. (1996), DeCroix (2013), and Yin et al. (2017). We found only one recently published paper, i.e., Li et al. (2017), studying a decentralized assembly system with two suppliers and one assembler, and they assume one of the two suppliers faces the supply disruption. In their work, they explore how a cost-sharing contract would affect the performance of individual firms and the entire system.

Compared with the vast amount of the random yield literature, Feng (2010) points out "There are relatively few papers analyzing random supply capacity." We here focus on the papers studying centralized assembly systems with random capacity. For example, Bollapragada et al. (2004) characterize the optimal base-stock inventory policy which minimizes the total inventory investment in a centralized assembly system. Xiao et al. (2010) study a single-product single-period assemble-to-order system with uncertain assembly capacity, and they identify conditions under which an assemble-in-advance strategy should be adopted to maximize the centralized system profit. Bollapragada et al. (2015) explore the component procurement and assembly decisions for a cost-minimizing assembly system. Ji et al. (2016) derive an optimal production planning decision for a centralized single-period cost-minimizing assembly system with random production and assembly capacity. All these papers study a centralized assembly system where the assembler decides the inventory decisions for the entire system. However, we study a decentralized assembly system (supply chain), where the focus is to apply game theory to explore the strategic interactions among the suppliers and the assembler.

To our best knowledge, our study is the first one that considers a decentralized assembly system with random supply capacities. We build a general model for the decentralized assembly system with n suppliers (each of whom experiences a random capacity) and an assembler facing a stochastic market demand. Furthermore, our model allows the random capacities of the n suppliers to follow any arbitrary multivariate continuous distribution with correlations, while all the papers for random yield and random capacity listed in Table 1.1 assume independent distributions. Incorporating correlated random capacities into our model, we are able to show that when the suppliers' capacities become more positively correlated under a general multivariate normal distribution, the assembler is always better off, but the suppliers may be better or worse off. Our work can be considered as an extension of Gerchak and Wang (2004). Our model setup is similar to theirs except that they assume all suppliers are perfectly reliable whereas we relax this assumption and consider that all suppliers have random capacities. Compared with their results, we demonstrate that when a component supplier faces an uncertain capacity, it always hurts the assembler, but it may sometimes benefit this specific supplier, which is quite surprising.

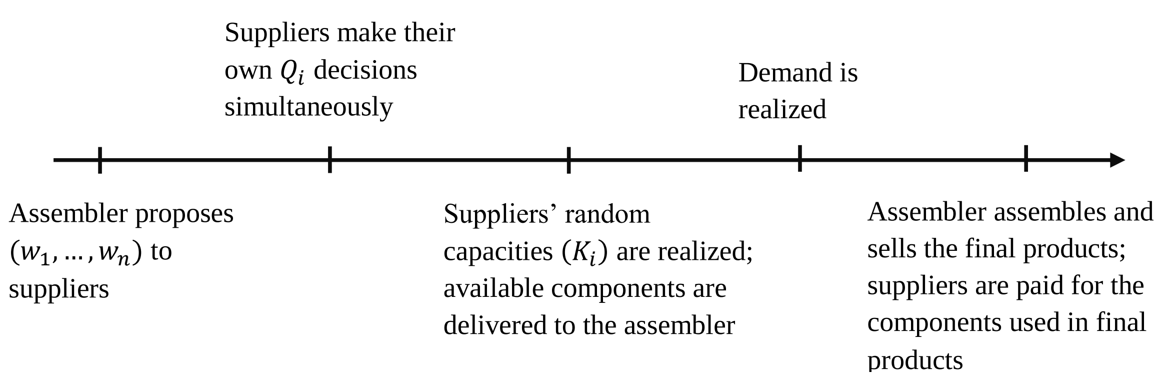
1.3 Model Setup

In this section, we first elaborate our model setup for the decentralized assembly system under the VMCI contract, then we solve the corresponding centralized system as a benchmark for the decentralized system.

1.3.1 Decentralized System under VMCI Contract

We consider a decentralized supply chain with n upstream suppliers and a downstream assembler. Without loss of generality, we assume one unit of the assembler's final product consists of n (sets of) components, each produced by a different supplier, i.e., supplier i (he) produces component i ($i = 1, ..., n$). The assembler (she) assembles the n components into the final product to satisfy a single-period uncertain market demand, D, with a fixed retail price of \$$p$ per unit. The assembler and the suppliers agree to adopt the VMCI contract, i.e., each supplier manages his own inventory and the assembler only pays to each supplier for the components that are actually used by the assembler based on the pre-agreed unit price. Before the final demand D is realized, the assembler initiates the procurement in advance by offering a unit price, w_i, to supplier i, $i = 1, ..., n$. Given the prices, the suppliers make their own production decisions simultaneously, i.e., supplier i decides his production quantity Q_i and incurs a total production cost of c_iQ_i, in which \$$c_i$ is the unit production cost for component i. Furthermore, each supplier's production capacity, K_i ($i = 1, ..., n$) is uncertain. The random vector $(K_1, ..., K_n, D)$ is assumed to follow an arbitrary multivariate continuous distribution. After each supplier's uncertain production capacity is realized, only $\min\{Q_i, K_i\}$ units of component i can be delivered to the assembler. In most cases, the assembler's assembly time is much shorter than the suppliers' lead time, so we assume that the assembler can start the final assembly after demand D is realized and the assembly cost is c_0 per unit. Thus, the assembler can sell $\min\{\min_{i=1,...,n}(Q_i, K_i), D\}$ units of the final product to the market. Each supplier collects the payment from the assembler for the used components. Unassembled components, if any, are salvaged by each supplier at a salvage value of $\$s_i$/unit for component i. Assume $c_i > s_i$ so that the suppliers will not produce unlimited components. s_i can also be negative to represent a disposal cost, if applicable. We assume all parties are risk neutral and that all information about the retail price, salvage value, costs, and distributions are common knowledge to all parties.

Figure 1.1: Sequence of events.



1.3.2 Centralized System

We now analyze the corresponding centralized system as a benchmark for the decentralized system. In a centralized assembly system, the assembler, as a central planner, decides the production quantity, $Q_i, \forall i = 1, ..., n$, for each component to maximize the total system profit, denoted as Π_C, as follows:

$$\begin{aligned}\Pi_C(Q_1,...,Q_n) &= E\{(p-c_0)\min_{i=1,...,n}(Q_i,K_i,D)-\sum_{i=1}^n c_iQ_i+\sum_{j=1}^n s_j[\min(Q_j,K_j)-\min_{i=1,...,n}(Q_i,K_i,D)]^+\}\\ &= E\{(p-c_0)\min_{i=1,...,n}(Q_i,M)-\sum_{i=1}^n c_iQ_i+\sum_{j=1}^n s_j[\min(Q_j,K_j)-\min_{i=1,...,n}(Q_i,M)]^+\}, \quad (1.1)\end{aligned}$$

where $M\stackrel{\text{def}}{=}\min(K_1,...,K_n,D)$. For notational convenience, let $f_M(.)$ and $F_M(.)$ be the respective PDF and CDF of M, which can be derived from the multivariate distribution of $(K_1,...,K_n,D)$ since $F_M(x)=1-Pr(K_1>x,...,K_n>x,D>x), \forall x$. The total system profit Π_C here simply represents the assembler's revenue from selling the final products to satisfy the uncertain market demand D minus the assembly cost and the component production costs plus the revenue from salvaging leftover components. Recall that without loss of generality, we assume that each final product is composed of one unit of the n components. Hence, it is straightforward to derive the following result:

Lemma 1.1. *The optimal solution to problem (1.1) satisfies* $Q_1=...=Q_n\stackrel{def}{=}Q_c$.

Note that the assembler has to match the n components to assemble the final product. It is best for the assembler to plan the same production quantity for all components, i.e., $Q_1=...=Q_n$, because for each additional leftover component, the assembler can only get the salvage value of the component, which is lower than the production cost of the component. Substituting $Q_1=...=Q_n=Q_c$ into (1.1), we have

$$\max_{Q_c}\Pi_C(Q_c)=E\{(p-c_0)\min_{i=1,...,n}(Q_c,M)-\sum_{i=1}^n c_iQ_c+\sum_{j=1}^n s_j[\min(Q_c,K_j)-\min_{i=1,...,n}(Q_c,M)]^+\}.$$

For notational convenience, let $f_D(\cdot)$ and $F_D(\cdot)$ denote the marginal probability density function (PDF) and cumulative distribution function (CDF) of D, respectively. Similarly, let $f_i(\cdot)$ and $F_i(\cdot)$ be the marginal PDF and CDF of $K_i, i=1,...,n$, respectively. We do not make any assumption on the relationship among the probability distributions of D and K_i, for example, K_i is stochastically larger(smaller) than D or $K_j, j \neq i$. As we will show in the remainder of the chapter, our result holds valid without any of such assumptions. After some algebra, we can show the following result:

Proposition 1.1. Π_C *is strictly concave in* Q_c. *Hence, the optimal solution to the centralized problem given by (1.1), denoted as* Q_c^*, *is uniquely determined by*

$$(p - c_0 - \sum_{i=1}^{n} s_i)\bar{F}_M(Q_c^*) - \sum_{i=1}^{n} c_i + \sum_{i=1}^{n} s_i \bar{F}_i(Q_c^*) = 0, \tag{1.2}$$

where $\bar{F}_i(x) \overset{def}{=} 1 - F_i(x)$ *for all* $i = 1, ..., n, D, M$.

1.4 VMCI CONTRACT

In this section, we apply the backward induction method to study the decentralized system under the VMCI contract, i.e., we first analyze the suppliers' problem in section 1.4.1, and then we investigate the assembler's optimal pricing scheme in section 1.4.2 based on the anticipated best responses from the suppliers.

1.4.1 Suppliers' Problem

Given any prices, $w_1, ..., w_n$, offered by the assembler, the n suppliers make their respective production decisions simultaneously to maximize their own expected profits. To encourage each supplier to produce a positive quantity of his component, the assembler must propose a feasible contract such that $w_i > c_i$ for all $i = 1, ..., n$. We aim to characterize the Nash equilibrium (or equilibria) for the suppliers' production decisions, $(Q_1, ..., Q_n)$. To do so, we first need to derive the best response production quantity for supplier i, $i = 1, \ldots, n$, given other suppliers' production quantities, defined as $Q_{-i} = \mathbf{Q_j}$ such that $j \in \{\{1, ..., n\} \setminus \{i\}\}$. For any given Q_{-i}, supplier i chooses Q_i to maximize his expected profit function as follows:

$$\Pi_i(Q_i|Q_{-i}) = E\{w_i \min_{j=1,\ldots,n}(Q_j, M) - c_i Q_i + s_i[\min(Q_i, K_i) - \min_{j=1,\ldots,n}(Q_j, M)]^+\},$$

In supplier i's profit function, the first term is the payment he gets from the assembler for the components used in the final product, the second term is the supplier's production cost, and the last term represents the supplier's revenue from salvaging his leftover components, if any.

For notational convenience, we define $\underline{Q}_{-i} = \min_{j=1,\ldots,n, j\neq i}(Q_j)$. Hence, supplier i's profit function can be further reduced to two forms based on two possible scenarios (I) $Q_i \leq \underline{Q}_{-i}$, or (II) $Q_i > \underline{Q}_{-i}$, as below.

$$\Pi_i(Q_i|Q_{-i}) = \begin{cases} \Pi_i^I(Q_i|Q_{-i}), & Q_i \leq \underline{Q}_{-i} \\ \Pi_i^{II}(Q_i|Q_{-i}), & Q_i > \underline{Q}_{-i} \end{cases}$$

where

$$\Pi_i^I(Q_i|Q_{-i}) \stackrel{\text{def}}{=} w_i E[\min(Q_i, M)] - c_i Q_i + s_i E[\min(Q_i, K_i) - \min(Q_i, M)]^+, \tag{1.3}$$

$$\Pi_i^{II}(Q_i|Q_{-i}) \stackrel{\text{def}}{=} w_i E[\min(\underline{Q}_{-i}, M)] - c_i Q_i + s_i E[\min(Q_i, K_i) - \min(\underline{Q}_{-i}, M)]^+. \tag{1.4}$$

Π_i^I and Π_i^{II} are continuous at $Q_i = \underline{Q}_{-i}$.

When supplier i plans a production quantity Q_i that is lower than any other supplier's production quantity, the assembled quantity of the assembler's final product becomes only dependent on Q_i but not Q_{-i}. We can derive the first order condition for $\Pi_i^I(Q_i|Q_{-i})$ as

$$\frac{\partial \Pi_i^I(Q_i|Q_{-i})}{\partial Q_i} = (w_i - s_i)\bar{F}_M(Q_i) - c_i + s_i \bar{F}_i(Q_i) = 0. \tag{1.5}$$

Since $w_i > c_i > s_i$ and $\bar{F}_i(\cdot), i = 1, \ldots, n, M$ are decreasing functions, Π_i^I is strictly concave in Q_i. Note that at $Q_i = 0$, $\frac{\partial \Pi_i^I(Q_i|Q_{-i})}{\partial Q_i} = w_i - c_i > 0$, and as Q_i approaches to ∞, $\frac{\partial \Pi_i^I(Q_i|Q_{-i})}{\partial Q_i} \to -c_i < 0$. There exists a unique solution to the first-order condition of $\Pi_i^I(Q_i|Q_{-i})$ given by equation (1.5).

However, if supplier i plans a higher production quantity than some other supplier, then Q_i will not constrain the final product quantity, but it still affects supplier i's own production cost. It is straightforward to show that the first derivative of $\Pi_i^{II}(Q_i|Q_{-i})$ is

$$\frac{\partial \Pi_i^{II}(Q_i|Q_{-i})}{\partial Q_i} = -c_i + s_i \bar{F}_i(Q_i) < 0,$$

which implies that $\Pi_i^{II}(Q_i|Q_{-i})$ is strictly decreasing for all $Q_i > \underline{Q}_{-i}$. Combining the properties of functions $\Pi_i^{I}(Q_i|Q_{-i})$ and $\Pi_i^{II}(Q_i|Q_{-i})$, we obtain the following result:

Lemma 1.2. *Given any Q_{-i} chosen by the other $n-1$ suppliers, supplier i's optimal production quantity, Q_i^*, is*

$$Q_i^* = \min(\tilde{Q}_i, \underline{Q}_{-i}), \forall i = 1, ..., n, \tag{1.6}$$

where $\tilde{Q}_i$ is the unique solution to (1.5).

Note from (1.5) that $\tilde{Q}_i$ is a function of w_i. Hence, we can analyze the relationship between $\tilde{Q}_i$ and w_i, which can be characterized in the following Lemma.

Lemma 1.3. *$\tilde{Q}_i$ is increasing in w_i.*

Lemma 1.3 echoes with our intuition that a higher unit price offered by the assembler tends to drive up the supplier's planned production level.

Solving the n best response functions given by (1.6) simultaneously, we are able to characterize the Nash equilibrium (N.E.) for the suppliers' problem in the Proposition below.

Proposition 1.2. *For any $(w_1, ..., w_n)$ chosen by the assembler, any $(Q_1^*, ..., Q_n^*)$ satisfying $Q_1^* = ... = Q_n^* \leq \min\limits_{j=1,...,n}(\tilde{Q}_j) \stackrel{def}{=} \tilde{Q}$ constitutes a N.E. for the suppliers, among which $Q_1^* = ... = Q_n^* = \tilde{Q}$ is the unique Pareto-optimal N.E.*

Proposition 1.2 indicates that all suppliers would produce the same quantity in equilibrium. This result is driven by the fact that the assembler needs to assemble the suppliers' components into final products to sell in her market, so unmatched components will not be used and paid by the assembler. Therefore, in equilibrium, every supplier shall produce the same quantity. Although there are numerous equilibria for the suppliers' problem, there always exists a unique Pareto-optimal N.E., i.e., $\tilde{Q}$, which maximizes the expected profit for every supplier among all the equilibria. As a result, we consider this equilibrium solution as the outcome of the suppliers' problem.

1.4.2 Assembler's Problem

Anticipating that all suppliers simultaneously choose their Pareto-optimal N.E., $\tilde{Q}$, given in Proposition 1.2, the assembler's decision problem is to choose the optimal pricing scheme to maximize her expected profit, i.e.,

$$\max_{(w_1,...,w_n)} \Pi_0(w_1, ..., w_n) = E[(p - c_0 - \sum_{i=1}^{n} w_i) \min(\tilde{Q}, M)]. \tag{1.7}$$

According to the VMCI contracting arrangement, the assembler pays the suppliers only for the components used in the final product. We first obtain the following property for the assembler's optimal pricing scheme:

Lemma 1.4. *The assembler's optimal pricing scheme, denoted as $(w_1^*, ..., w_n^*)$, satisfies*

$$w_i^* = s_i + \frac{c_i - s_i \bar{F}_i(\tilde{Q})}{\bar{F}_M(\tilde{Q})}, i = 1, ..., n, \tag{1.8}$$

under which we have $\tilde{Q}_1 = ... = \tilde{Q}_n = \tilde{Q}$.

The result of Lemma 1.4 can be explained as follows. From Lemma 1.3, we know that for all $i = 1, ..., n$, as w_i increases, $\tilde{Q}_i$ increases. Because $\tilde{Q} = \min_{i=1,...,n} \tilde{Q}_i$, it is best for the assembler to set up the prices such that the optimal prices would induce $\tilde{Q}_1 = ... = \tilde{Q}_n = \tilde{Q}$. Otherwise, there exists $\tilde{Q}_j$ such that $\tilde{Q}_j > \tilde{Q}$, then reducing w_j to lower $\tilde{Q}_j$ to equal $\tilde{Q}$ would simply improve the assembler's unit profit margin without reducing her expected sales volume.

Following Lemma 1.4, we can transform the assembler's problem given by (1.7) to the following problem:

$$\max_{\tilde{Q}} \Pi_0(\tilde{Q}) = \{p - c_0 - \sum_{i=1}^{n}[s_i + \triangle_i(\tilde{Q})]\}S(\tilde{Q}), \tag{1.9}$$

where

$$S(\tilde{Q}) \stackrel{\text{def}}{=} E[\min(\tilde{Q}, M)] \tag{1.10}$$

$$\triangle_i(\tilde{Q}) \stackrel{\text{def}}{=} \frac{c_i - s_i\bar{F}_i(\tilde{Q})}{\bar{F}_M(\tilde{Q})}, \forall i = 1, ..., n \tag{1.11}$$

$S(\tilde{Q})$ represents the expected sales quantity of the final product.

For the remainder of the chapter, we assume that (A1) K_i ($i = 1, ..., n$) and M have increasing failure rates (IFR) (i.e., $\frac{f_j(\tilde{Q})}{\bar{F}_j(\tilde{Q})}, \forall j = 1, ..., n, M$, is increasing in $\tilde{Q}$), and (A2) M is smaller than any K_i in the hazard rate order. As discussed in Gerchack and Wang (2004), commonly used distributions, including uniform, normal, and Weibull families subject to parameter restrictions, have IFRs. Gupta and Gupta (2001) prove that when $(K_1, ...K_n, D)$ follow a multivariate normal distribution, M retains the IFR property. Since $M = \min(K_1, ..., K_n, D)$, M can be considered as the system lifetime of a serial system with $n+1$ parts. (A2) simply means that at time t, the failure rate of this serial system is higher than the failure rate of any individual part in the system, i.e., $\frac{f_M(t)}{\bar{F}_M(t)} \geq \frac{f_j(t)}{\bar{F}_j(t)}, \forall j = 1, ..., n, D$ (see Boland et al. 1994). After some algebra manipulation, we can obtain a unique optimal solution to the assembler's problem, formally described in the Proposition below.

Proposition 1.3. *There exists a unique solution, $\tilde{Q}^*$, to the assembler's problem (1.9), which can be determined by solving*

$$-\sum_{i=1}^{n}\triangle_i'(\tilde{Q}^*)S(\tilde{Q}^*) + [p - c_0 - \sum_{i=1}^{n} s_i - \sum_{i=1}^{n}\triangle_i(\tilde{Q}^*)]\bar{F}_M(\tilde{Q}^*) = 0. \tag{1.12}$$

Note that without assumptions (A1)-(A2), the optimal $\tilde{Q}^*$ can also be found by solving (1.12), which means the existence of the N.E. is always guaranteed. Moreover, when

$(K_1, ..., K_n, D)$ follow a multivariate normal distribution, we do not need these assumptions to derive the managerial insights regarding correlations among random capacities and demand in section 1.5.4.

1.5 Managerial Insights

In this section, we investigate how variations in the model parameters influence the equilibrium as well as the expected profits of the suppliers, the assembler, and the entire supply chain. The results here could provide useful managerial insights for managing the decentralized assembly supply chain in the presence of supply uncertainty and random demand. To facilitate our discussions, we use Π_0^*, $\Pi_i^*(i = 1, ..., n)$, and $\Pi_D^* = \sum_{j=0}^{n} \Pi_j^*$ to denote the equilibrium expected profits for the assembler, supplier i, and the entire decentralized supply chain, respectively. Moreover, in the remainder of this chapter, we refer to "decrease" and "increase" in a weak sense, i.e., "decrease" means " non-increase," and "increase" means "non-decrease."

1.5.1 Component Cost

We first analyze how a change in the component costs can affect the suppliers' production decision $(\tilde{Q}^*)$ and the assembler's pricing scheme $(w_1^*, ..., w_n^*)$ in equilibrium and the subsequent profitability of every supply chain member. Suppose all other model parameters stay the same, except that one supplier's, say supplier i's, component cost c_i increases. The following result illustrates the impact of such an increase in c_i on the equilibrium and the profits.

Proposition 1.4. *Suppose supplier i's unit cost, c_i, increases. Then,*

(i) $\tilde{Q}^*$ decreases;

(ii) w_j^* for all $j = 1, ..., n, j \neq i$ decrease, but w_i^* increases;

(iii) Π_0^*, Π_D^*, *and* Π_j^* *decrease for all* $j = 1, ..., n, j \neq i$.

As one supplier's cost increases, this supplier tends to plan a lower production quantity. This tendency leads other suppliers to produce less as well since the suppliers are only paid for the components used in the assembler's final product and the assembler's final production quantity is limited by the supplier who delivers the lowest quantity. As illustrated by the numerical example in Table 1.2, when supplier 1's cost increases, the assembler would offer supplier 1 a higher price (w_1^*) to compensate him. However, anticipating that all suppliers shall plan a lower identical production quantity in equilibrium, the assembler has room to offer lower prices to all other suppliers (e.g., w_2^* decreases in Table 1.2) because these suppliers' costs remain unchanged. Given lower prices, all other suppliers' profits would decrease. Intuitively, the assembler, as the leader of the game, also suffers from any supplier's cost increase, and we prove that the entire supply chain suffers as well.

Table 1.2: $n = 2, p - c_0 = 200, s_1 = s_2 = 10$, $c_2 = 50$, K_1, K_2, D (are i.i.d) $\sim Exp(0.01)$

c_1	$\tilde{Q}^*$	w_1^*	w_2^*	Π_1^*	Π_2^*	Π_0^*	Π_D^*	Π_C^*	Π_D^*/Π_C^*
28	17.37	43.00	80.05	119.83	239.27	1041.94	1401.03	1761.2	79.55%
38	15.07	56.20	75.06	**127.36**	175.15	833.3	1135.81	1438.95	78.93%
48	13.04	67.99	70.95	122.17	128.01	658.75	908.93	1159.46	78.39%
58	11.22	78.69	67.79	109.77	92.83	512.66	715.26	918.05	77.91%
68	9.57	88.52	64.53	93.66	66.36	390.79	550.82	710.95	77.48%
78	8.08	97.63	61.95	76.15	46.41	289.86	412.42	535.04	77.08%

Interestingly, we observe that this particular supplier (i.e., supplier 1 in Table 1.2) may sometimes become better off when his production cost increases (see the entry as bolded). As explained earlier, the assembler would offer a higher unit price to the supplier whose production cost increases. Facing a higher price and a higher cost at the same time, this supplier's profit margin might increase or decrease, as indicated by our example in Table 1.2. In conjunction with a lower equilibrium production quantity which leads to a lower sales in the final product market, this supplier could be better or worse off. This result is surprising yet consistent with the result in section 6.2 of Fang et al. (2008) and Proposition 3 in Granot and Yin (2008). Under the VMCI contract, the suppliers are not always motivated to improve the efficiency of their production processes. Therefore, the assembler might need to provide additional incentives for suppliers to reduce their costs.

One can easily prove that the optimal decentralized system profit is strictly less than the optimal centralized system profit due to double-marginalization. In order to explore how the decentralized system efficiency is affected by each supplier's component cost, we quantify the decentralized system efficiency as Π_D^*/Π_C^*. Although the derivative of Π_D^*/Π_C^* with respect to c_i is analytically intractable, our numerical experiments demonstrate that the decentralized system efficiency decreases in the supplier's production cost, as illustrated by the last column of Table 1.2. This observation has been consistently confirmed by 1000 runs of simulations based on randomly generated data sets. Under the VMCI contract, each supplier (game follower) bears his own overstocking risk, which would be amplified when his production cost increases. The supplier, being self-interested, shall then plan his production more conservatively, which hurts the entire system.

1.5.2 Salvage Value

Second, we investigate how the variation in the component salvage values can affect the equilibrium decision and the resulting expected profit for each member. Recall that we assume $s_i < c_i, \forall i = 1, \ldots, n$, to guarantee that no supplier would produce an unlimited quantity.

Proposition 1.5. *Suppose supplier i's salvage value, s_i, increases. Then,*

(i) $\tilde{Q}^*$ *and* w_j^* *increase for all* $j = 1, ..., n,\ j \neq i$;

(ii) Π_0^*, Π_D^*, *and* Π_j^* *increase for all* $j = 1, ..., n,\ j \neq i$.

Proposition 1.5 (i) indicates that when supplier i's salvage value improves, the suppliers' equilibrium production quantity increases and the equilibrium prices for all suppliers except supplier i also increase. A higher salvage value for supplier i's component can, to some extent, mitigate this supplier's overstocking risk, which therefore drives up his own production quantity, and the assembler does not need to offer a higher price to supplier i. However, to induce all other suppliers to plan a higher production quantity in equilibrium, the assembler would have to offer higher prices to them since these suppliers' cost structures remain unchanged. As a result, all other suppliers' profit margins are greater given higher prices so that these suppliers are better off.

Surprisingly, when supplier i's own salvage value increases, supplier i may sometimes be worse off, as illustrated by the numerical example given in Table 1.3 (see the entry as bolded). This result is surprising but consistent with the similar result derived above for the component cost. As indicated in the example, when supplier 1's salvage value (s_1) increases which tends to benefit this supplier, but the assembler would offer supplier 1 a lower price (w_1) which tends to hurt supplier 1's profitability. Consequently, these two conflicting factors may drive supplier 1's profit either up or down. However, the assembler, acting as the leader of the game, always benefits from an increase in any supplier's component salvage value, and the entire supply chain profit improves as well.

Table 1.3: $n = 2, p - c_0 = 120, s_2 = 29,\ c_1 = 50,\ c_2 = 30,\ K_1, K_2, D$ (are i.i.d) $\sim Exp(0.005)$

s_1	$\tilde{Q}^*$	w_1^*	w_2^*	Π_1^*	Π_2^*	Π_0^*	Π_D^*	Π_C^*	Π_D^*/Π_C^*
16	18.97	63.12	33.82	118.96	35.28	380.83	535.07	689.83	77.57%
26	20.78	62.27	34.27	121.87	43.12	418.54	583.53	749.44	77.86%
36	22.92	61.24	34.84	**123.11**	53.63	463.93	640.67	819.06	78.22%
46	25.47	59.92	35.55	121.29	68.02	519.34	708.66	900.98	78.65%

Similar to section 1.5.1, we again compute Π_D^*/Π_C^* to evaluate the effect of one component's salvage value on the decentralized system efficiency. The example provided in Table 1.3 demonstrates that when component 1's salvage value s_1 increases from 16 to 46, the decentralized system efficiency increases from 77.57% to 78.65%. All our numerical results (1000 simulations) show that when one component's salvage value increases, the decentralized system efficiency improves as well. As one component's salvage value increases, this component supplier potentially faces less inventory risk of overstocking. Therefore, this supplier tends to produce more of his components, which benefits the overall decentralized system.

1.5.3 Retail Price and Assembly Cost

Third, we explore how a change in the retail price or assembly cost of the final product impacts the equilibrium production quantity, pricing decisions, and each party's profitability. With some derivations, we summarize the results in the following Proposition.

Proposition 1.6. *Suppose the final product's retail price p increases (or the assembly cost c_0 decreases). Then,*

(i) $\tilde{Q}^*$ and w_j increase for all $j = 1, ..., n$;

(ii) Π_0^*, Π_D^*, and Π_j^* increase for all $j = 1, ..., n$.

When the retail price of the assembler's final product increases or when the assembler's assembly cost decreases, the assembler has motivations to induce all suppliers to produce more by paying higher prices to the suppliers. Consequently, every supplier's profit improves. As the leader of the game, the assembler always benefits when she gets a higher retail price for her product or when she incurs a lower assembly cost. Since every one is better off, the entire supply chain is better off as well.

1.5.4 Correlations among Random Capacities and Demand

Assume $(K_1, ..., K_n, D) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, i.e., the random vector follows a multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma} = [\sigma_{ij}]_{i,j=1,...,n+1}$, where the correlation coefficient between i and $j, j \neq i$, $\rho_{ij} = \sigma_{ij}/\sqrt{\sigma_{ii}\sigma_{jj}}$.

Proposition 1.7. *If σ_{ab} for some $a \neq b; a, b = 1, ..., n+1$ increases, then the random variable M stochastically increases, and Π_0^* increases as well.*

Note that the proof of Proposition 1.7 is based on the theory of supermodularity in Topkis (1998), hence assumptions (A1)-(A2) are not required to prove this result. As σ_{ab} increases, the correlation coefficient, ρ_{ab}, increases. When any two of the random variables in $(K_1, ..., K_n, D)$ become more positively correlated, M becomes stochastically larger. Recall that M is defined as $\min(K_1, ..., K_n, D)$ which represents the maximum quantity the assembler can sell to the market, and the assembler's final sales is actually the minimum of M and $\tilde{Q}^*$. When M increases, even if the assembler induces the suppliers to produce the same quantity ($\tilde{Q}^*$), the assembler's profit would still increase because she can sell more products to the market without increasing her costs. As the leader of the game, the assembler is able to optimally change the prices offered to the suppliers such that the equilibrium production quantity ($\tilde{Q}^*$) changes optimally for her. Hence, the assembler is always better off when any two random variables in $(K_1, ..., K_n, D)$ become more positively correlated.

Table 1.4: $n = 2, (\mu_1, \mu_2, \mu_D) = (500, 500, 600), (\sigma_1, \sigma_2, \sigma_D) = (180, 180, 30), \rho_{1D} = \rho_{2D} = 0, p - c_0 = 5112, c_1 = 100, s_1 = 66, c_2 = 150, s_2 = 80$

ρ_{12}	$\tilde{Q}^*$	w_1^*	w_2^*	Π_0^*	Π_1^*	Π_2^*	Π_D^*
-0.99	412.50	212.58	334.83	1552120	33426	54878	1640423
-0.96	412.51	212.59	334.86	1552124	33431	54886	1640440
-0.9	413.13	213.38	336.10	1552539	33699	55314	1641552
-0.5	430.39	226.52	355.55	1578571	38522	62522	1679615
0	458.30	240.16	373.36	1643060	44550	70850	1758460
0.5	490.76	251.46	386.12	1742330	51100	79478	1872908
0.9	524.17	254.91	386.10	1886973	**56338**	**85744**	2029055
0.96	530.80	252.41	381.03	1930979	**56511**	**85586**	2073076
0.99	535.50	249.41	375.49	1968850	**56278**	**84907**	2110035

On the contrary, the suppliers may be better or worse off as verified by our numerical experiments. In Table 1.4, we report the equilibrium results of a randomly generated numerical example. In this example, as the correlation coefficient between K_1 and K_2 (ρ_{12}) increases from -0.99 to 0.99, we observe that the equilibrium production quantity ($\tilde{Q}^*$) and the assembler's profit always increase, but the prices and profits for both suppliers could either increase or decrease. This result can be explained as follows. When ρ_{12} increases, M increases. Therefore, even if the assembler does not pay each supplier a higher price, the suppliers still have a tendency to produce more in equilibrium because under the consignment contract, each suppliers are only paid for the quantity sold to the market which is $\min(M, \tilde{Q}^*)$. Hence, the assembler does not necessarily need to offer higher prices to the suppliers to induce a higher production quantity. Since each supplier may obtain a higher or lower price from the assembler, the suppliers' profits might increase or decrease.

1.5.5 Random Capacity vs. Reliable Supply

In this study, random capacities are the key factors influencing every member's strategy and profit in equilibrium. Note that Gerchak and Wang (2004) analyze a model under the revenue-sharing contract which is similar to our model under the VMCI contract, and the only difference is that they assume all suppliers are perfectly reliable and the suppliers do not have any capacity constraints. By comparing our results under random supply capacities with their results under reliable supplies, we can investigate the impact of supply uncertainties on the system performance as well as each individual firm's profitability.

Following the similar algebra procedure, we can derive $\tilde{Q}_i^{r*}, w_i^{r*}, \Pi_i^{r*}, \Pi_0^{r*}$, and Π_D^{r*} which denote the equilibrium production quantity and unit price for supplier i, and the respective equilibrium profits for supplier i, the assembler, and the system under reliable supplies. It is straightforward to show the following results: the assembler will choose a pricing scheme $(w_1^{r*}, ..., w_n^{r*})$ such that $w_i^{r*} = s_i + \frac{c_i - s_i}{\bar{F}_D(\tilde{Q}^{r*})}, \forall i = 1, ..., n$, to induce the same production quantity, namely $\tilde{Q}^{r*}$, from all suppliers; then if $\frac{f_D(Q)}{[\bar{F}_D(Q)]^2} \int_0^Q \bar{F}_D(x)dx$ is increasing in Q, there exists a unique solution $\tilde{Q}^{r*}$ to the suppliers' problem that satisfies

$$[p - c_0 - \sum_{i=1}^{n} s_i]\bar{F}_D(\tilde{Q}^{r*}) - \sum_{i=1}^{n}(c_i - s_i)[1 + \frac{f_D(\tilde{Q}^{r*})}{[\bar{F}_D(\tilde{Q}^{r*})]^2}\int_0^{\tilde{Q}^{r*}} \bar{F}_D(x)dx] = 0.$$

As noted in Gerchak and Wang (2004), the assumption (i.e., $\frac{f_D(Q)}{[\bar{F}_D(Q)]^2}\int_0^Q \bar{F}_D(x)dx$ is increasing in Q) is very weak, and any demand distribution with IFR satisfies this assumption. In conjunction with Lemma 1.4 and Proposition 1.3, we compare Π_0^* and Π_0^{r*} to establish the following Proposition.

Proposition 1.8. $\Pi_0^* < \Pi_0^{r*}$.

Proposition 1.8 indicates that the assembler's expected profit under random capacities is strictly less than her expected profit under reliable supplies.

Table 1.5: $n = 2, p - c_0 = 254, c_1 = 30, s_1 = 28, c_2 = 45, s_2 = 20, D \sim Exp(0.008)$, under random capacity: $K_1 \sim Exp(0.004), K_2 \sim Exp(0.003), (D, K_1, K_2)$ are mutually independent.

	$\tilde{Q}^{r*}, \tilde{Q}^*$	w_1^{r*}, w_1^*	w_2^{r*}, w_2^*	Π_0^{r*}, Π_0^*	Π_1^{r*}, Π_1^*	Π_2^{r*}, Π_2^*	Π_D^{r*}, Π_D^*
Reliable Supply	128.59	33.52	89.05	10610.9	188.87	2360.81	13160.6
Random Capacity	55.28	44.31	83.39	4779.06	356.8	927.85	6063.71

When the suppliers are perfectly reliable, all suppliers are able to deliver the same quantity in equilibrium. With random capacities, the suppliers cannot always deliver the same quantity although their planned production quantities are identical in equilibrium. Hence, there exists additional efficiency loss due to mismatched components in the assembly system. As a result, the assembler, as the leader of the game, is always worse off with supply uncertainties.

Interestingly, we find that the suppliers may sometimes be better off with random capacities, as illustrated by our numerical example in Table 1.5. In the presence of random capacities, the suppliers tend to be more conservative on their production decisions thus might potentially reduce the final product sales quantity. As the follower of the game, the supplier may use her random capacity as a leverage to negotiate with the assembler. Hence, the assembler might need to offer a higher unit price to the supplier to induce a larger production quantity. Consequently, the supplier with certain cost/risk structures might obtain a higher profit.

1.6 Wholesale Price Contract

In this section, we apply the traditional wholesale price contract in this decentralized assembly supply chain. As in Gerchak and Wang (2004), we assume that under the wholesale price contract, the assembler bears the inventory risks, and the suppliers are the game leaders while the assembler is the follower. That is, the component suppliers first offer their own wholesale prices $w_i, i = 1, \dots, n$, to the assembler simultaneously, then the assembler decides the production quantity for each supplier, i.e., Q_i for supplier i, $i = 1, ..., n$. After the random capacities $K_i, i = 1, ..., n$ are realized, supplier i delivers $\min(Q_i, K_i)$ to the assembler and receives w_i for each unit delivered. After the demand D is realized, the assembler assembles the available components and sells $\min_{i=1,\dots,n}(Q_i, K_i, D)$ to the market. The leftover components, if any, are salvaged by the assembler at \$$s_i$/unit for component $i, i = 1, ..., n$. Based on the backward induction, we first solve the assembler's problem and then solve the suppliers' problem.

Assembler's Problem

Given any prices $(w_1, \dots, w_n)$ offered by the suppliers, the assembler's problem is to choose $(Q_1, ..., Q_n)$ to maximize her expected profit below

$$\Pi_0 = E\{(p - c_0) \min_{i=1,\dots,n}(Q_i, M) - \sum_{i=1}^{n} w_i \min(Q_i, K_i) + \sum_{j=1}^{n} s_j[\min(Q_j, K_j) - \min_{i=1,\dots,n}(Q_i, M)]^+\}, \tag{1.13}$$

where the first term is the assembler's revenue from the final product market, the second term is her payments to the suppliers, and the last term is her revenue from salvaging the leftover components.

Suppose, WLOG, supplier k receives the lowest order quantity from the assembler, i.e., $Q_k = \min(Q_1, ..., Q_n)$. Since $M = \min(K_1, ..., K_n, D) \leq K_i$ and for all $j = 1, ..., n$, $\min(Q_j, K_j) \geq \min(Q_k, M)$ always holds, the assembler's expected profit function can be reduced to

$$\Pi_0(Q_1, ..., Q_n | Q_k = \min(Q_1, ..., Q_n)) = E\{(p - c_0 - \sum_{i=1}^{n} s_i) \min(Q_k, M) - \sum_{i=1}^{n} [(w_i - s_i) \min(Q_i, K_i)]\}$$

Therefore, it is best for the assembler to order the same quantity from all suppliers because $w_i > s_i$ for all the components. Define $Q_1 = ... = Q_n = \utilde{Q}$. We obtain the following result.

Proposition 1.9. Π_0 *is quasi-concave in* $\utilde{Q}$ *and has a unique solution* $\utilde{Q}^*$ *to the assembler's problem which is determined by the first-order condition*

$$(p - c_0 - \sum_{i=1}^{n} s_i) \bar{F}_M(\utilde{Q}^*) = \sum_{i=1}^{n} (w_i - s_i) \bar{F}_i(\utilde{Q}^*). \tag{1.14}$$

Suppliers' Problem

Anticipating an identical order quantity of $\utilde{Q}^*$ from the assembler, the suppliers simultaneously decide their respective wholesale prices to maximize their own expected profits. Specifically, for any wholesale prices chosen by the other $n-1$ suppliers, supplier i's problem can be written as

$$\max \Pi_i(w_i | w_j, j \neq i) = E\{w_i \min(\utilde{Q}^*, K_i) - c_i \utilde{Q}^*\}. \tag{1.15}$$

Following the same procedure in section 1.4.2, we can establish a one-to-one relationship between $\utilde{Q}^*$ and w_i for any given wholesale prices chosen by the other suppliers. Hence, we can equivalently optimize supplier i's expected profit Π_i over $\utilde{Q}^*$ instead of w_i. For notational convenience, let $L(Q) \stackrel{\text{def}}{=} \left[\sum_{i=1}^{n}(c_i - s_i\bar{F}_i(Q)) - (p - c_0 - \sum_{i=1}^{n} s_i)\bar{F}_M(Q)\right] / \sum_{i=1}^{n} \frac{\int_0^Q \bar{F}_i(x)dx}{\bar{F}_i(Q)}, \forall Q.$ After solving each supplier's best response, we are able to obtain the N.E. for this game, formally presented in the following Proposition:

Proposition 1.10. *There exists a pure-strategy N.E. for the game. The suppliers' equilibrium prices are*

$$w_i^*(\underaccent{\tilde}{Q}^*) = \frac{1}{\bar{F}_i(\underaccent{\tilde}{Q}^*)}[c_i - L(\underaccent{\tilde}{Q}^*)\frac{\int_0^{\underaccent{\tilde}{Q}^*} \bar{F}_i(x)dx}{\bar{F}_i(\underaccent{\tilde}{Q}^*)}], \ \forall i = 1, ...n,$$

where the assembler's equilibrium order quantity $\underaccent{\tilde}{Q}^*$ can be determined by solving

$$\sum_{i=1}^{n} \frac{f_i(\underaccent{\tilde}{Q}^*)}{\bar{F}_i(\underaccent{\tilde}{Q}^*)}(c_i - s_i\bar{F}_i(\underaccent{\tilde}{Q}^*)) - f_M(\underaccent{\tilde}{Q}^*)(p - c_0 - \sum_{i=1}^{n} s_i) - L(\underaccent{\tilde}{Q}^*)\sum_{i=1}^{n} \frac{f_i(\underaccent{\tilde}{Q}^*)\int_0^{\underaccent{\tilde}{Q}^*} \bar{F}_i(x)dx}{[\bar{F}_i(\underaccent{\tilde}{Q}^*)]^2} - L(\underaccent{\tilde}{Q}^*) = 0. \tag{1.16}$$

Although we could not prove the uniqueness of $\underaccent{\tilde}{Q}^*$, our result is still one step further than the result obtained in Gerchak and Wang (2004) since under the wholesale price contract, they only obtain the equation for the equilibrium quantity Q, i.e., (24) in their paper, for a special case in which all suppliers are identical. Under the same assumption, we are able to prove the uniqueness of $\underaccent{\tilde}{Q}^*$ as well.

1.7 VMCI vs. Wholesale Price

Finally, we compare the VMCI contract with the wholesale price contract in this section. Using the centralized system as the baseline, we first compare the system performance under the VMCI contract with that under the wholesale price contract. We use Mathematica 11.2 to conduct an extensive numerical experiment based on 2000 randomly generated data sets. The model parameters are randomly generated as follows: $n = 2, D \sim Exp(\lambda_0), K_1 \sim Exp(\lambda_1), K_2 \sim Exp(\lambda_2), (\lambda_0, \lambda_1, \lambda_2) \sim$ Uniform[0.001, 0.01], D, K_1, K_2 are mutually independent, $(c_1, c_2) \sim$ Uniform[30, 50], $(s_1, s_2) \sim$ Uniform[10,29], $(p - c_0) \sim$ Uniform[200, 300].

All our results (from 2000 simulations) consistently indicate that the equilibrium production quantity under the wholesale price is lower than that under the VMCI contract, which is strictly lower than the optimal centralized production quantity. Due to the double-marginalization effect, the assembler/suppliers in a decentralized system become more conservative, i.e., they would plan an equilibrium production quantity strictly lower than the optimal centralized production quantity. As a result, there always exists efficiency loss in the decentralized system. Recall that under the VMCI contract, the assembler is the game leader and the suppliers bear all the inventory overstocking risks, whereas under a wholesale price contract, the assembler is the game follower and the assembler bears the inventory risks. Hence, under the VMCI contract, the assembler tends to induce a higher production quantity in equilibrium.

To distinguish between the two contracts, we now use Π_W^* and Π_V^* to denote the respective decentralized system profits under the wholesale-price contract and the VMCI contract. Since the centralized system profit function is strictly concave in the production quantity (as demonstrated in Proposition 1.1), $\Pi_W^* < \Pi_V^* < \Pi_C^*$ always holds, as shown in Figure 1.2. To enhance the clarity of Figure 1.2, we sort the results from the 2000 iterations by the centralized system profits in ascending order. Figure 1.2 indicates that the VMCI contract is more efficient than the wholesale price contract for the supply chain. This result is consistent with the results from the supply chain contracting literature including Gerchak and Wang (2004), Granot and Yin (2008), and others. We also observe that the decentralized system efficiencies under the wholesale price contract are about 40-70% whereas the decentralized system efficiencies under the VMCI contract are about 50-80%.

Figure 1.2: System profits comparison

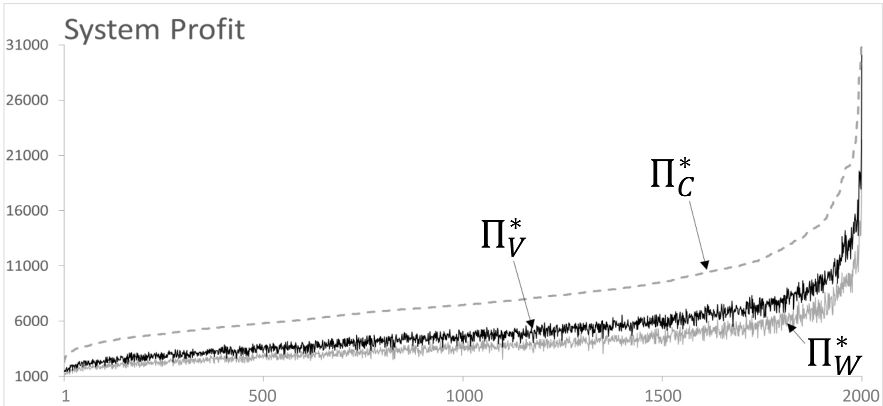
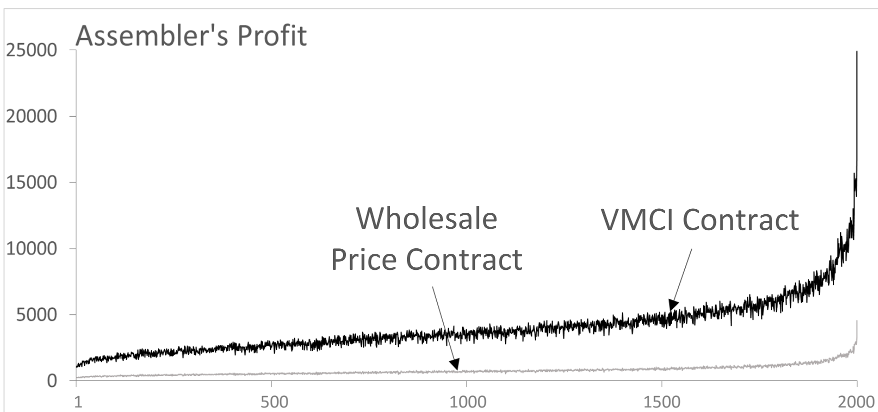


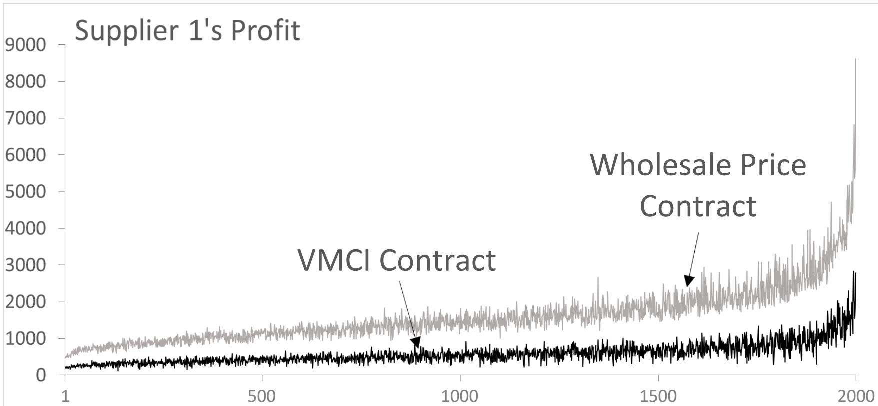
Figure 1.3: Assembler's optimal profit comparison



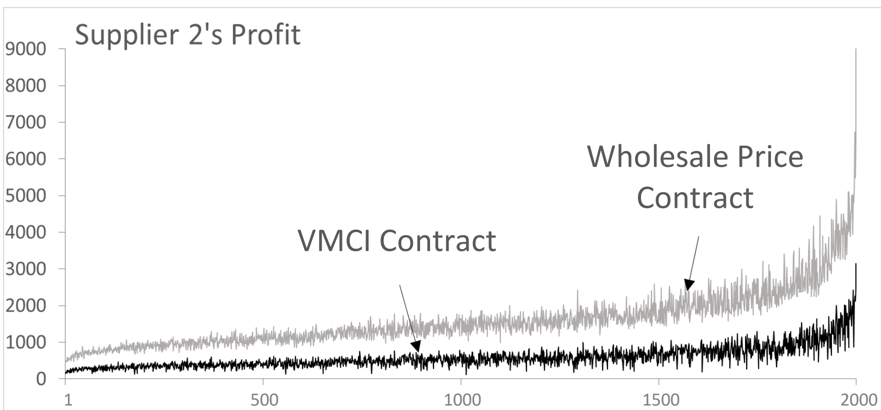
Our results also remark that the assembler is more profitable under the VMCI contract (see Figure 1.3) whereas the suppliers would earn a higher profit under the wholesale price contract (see Figure 1.4). Such observations can be explained by the first-mover advantage since the assembler is the game leader under the VMCI contract while the suppliers are the game leaders under the wholesale price contract.

Figure 1.4: Suppliers' profits comparison under two contracts.

(a) Supplier 1's optimal profit comparison



(b) Supplier 2's optimal profit comparison



1.8 Conclusion

We studied a decentralized assembly supply chain with an assembler and n component suppliers each producing a different component. Due to exogenous factors, each supplier's capacity is uncertain such that only the lesser of planned production quantity and actual realized capacity can be delivered to the assembler. In the first part of the chapter, we focused on the VMCI contract. Under the VMCI arrangement, the assembler offers a unit price to each supplier who subsequently decides the production plan. After the demand is realized, the assembly process begins and the suppliers are paid by the assembler for their used components only. Unused components, if any, are salvaged by each supplier. Using a game-theoretic framework, we derived the equilibrium solution for this problem. Our results revealed several interesting insights for the decentralized assembly system with supply uncertainties. Our analysis illustrated that while a reduction in one supplier's component cost, capacity uncertainty, or an increase on the component salvage value would improve the assembler's and other suppliers' profitability, it does not necessarily benefit this specific supplier. Hence, it is critical for the assembler to offer proper incentives to induce the suppliers to reduce their costs or capacity uncertainties, and to increase their salvage values. In addition, we showed that when the suppliers' random capacities become more positively correlated, it always benefits the assembler but not necessarily the suppliers.

Since simple wholesale price contracts are often used in practice and discussed in the literature, we analyzed a wholesale price contract in the second part of the chapter. Under the wholesale price contract, the suppliers are the leaders of the game, and the assembler is the follower. That is, the suppliers first decide their respective wholesale prices, then the assembler decides the order quantities. After obtaining the analytical equilibrium solution for the wholesale price contract, we were able to compare the wholesale price contract with the VMCI contract. First, we found that the assembler prefers the VMCI contract and the suppliers prefer the wholesale price contract, which can be explained by the first-mover advantage. Second, by comparing the decentralized system profit under the VMCI or the wholesale price contracts with the centralized system profit, we revealed that the VMCI contract is more efficient than the wholesale price contract for the entire supply chain. This result further confirms that the prior result in the decentralized assembly supply chain literature (see, e.g., Gerchak and Wang 2004, Grant and Yin 2008) still holds when the component suppliers have random capacity constraints.

There are several limitations of our model. First of all, our study, similar to many other related papers, assumed that each of the random demand and random capacities follows a continuous probability distribution for the derivation simplicity. However, in many real operations environments, it is important to precisely use discrete distributions to describe D, K_i, and Q, especially for those critical products which only come in small discrete quantities. Hence, future studies may adopt discrete distributions to represent a more realistic case. It will also be worthy to study a scenario with both supply disruption and random capacity. That is, there is a discrete probability under which a supplier cannot deliver any component due to disruptive events, and if the supplier's production is not disrupted in the very beginning, the production capacity is a random variable following a continuous distribution. Second, we assumed that each component is provided by a single supplier. One might also consider dual-sourcing or multi-sourcing for the same component to examine whether the horizontal competition could mitigate the supply risk and improve the supply chain efficiency. The model used in Jiang and Wang (2010) might be helpful in this direction. Third, we assumed that there is only one final assembled product. In a general assembly system, same components may be used in several final products. Therefore, it might be fruitful to follow the model in Bernstein et al. (2007) to analyze how supply uncertainties could affect the performance of a multi-product assembly system. Finally, in our model we assumed all the suppliers and the assembler are independent. One might extend our work to allow the suppliers and the assembler to form coalitions, as in Granot and Yin (2008), to reduce the supply chain risk due to supply uncertainties.

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1.A Appendix

Proof. Lemma 1.1. In the centralized system, without loss of generality, suppose that $Q_k = \min_{i=1,...,n} Q_i$. The component k can be referred as the "bottleneck". Since min, $[\ ]^+$, and the composition of such functions are Lipschitz functions, the expectation and the derivatives can be interchanged (Glasserman, 1994), i.e., $\frac{\partial E\Pi}{\partial Q} = E[\frac{\partial \Pi}{\partial Q}]$. Taking the differentiation of Π_C w.r.t. Q_k, we obtain

$$\begin{aligned}\frac{\partial \Pi_C}{\partial Q_k} &= (p - c_0)Pr(M \geq Q_k) - c_k - \sum_{i=1, i\neq k}^{n} s_i Pr(M \geq Q_k) + \\ &\quad s_k Pr(Q_k \leq K_k) Pr(\min(Q_k, K_1, ..., K_{k-1}, K_{k+1}, ..., K_n, D) \neq Q_k \mid Q_k \leq K_k) \\ &= (p - c_0)\bar{F}_M(Q_k) - c_k - \bar{F}_M(Q_k) \sum_{i=1, i\neq k}^{n} s_i + s_k(\bar{F}_k(Q_k) - \bar{F}_M(Q_k)) \\ &= (p - c_0 - \sum_{i=1}^{n} s_i)\bar{F}_M(Q_k) - c_k + s_k \bar{F}_k(Q_k).\end{aligned} \tag{1.A.1}$$

$\bar{F}_M(\cdot)$ and $\bar{F}_i(\cdot)$, $i = 1, ..., n$, are decreasing functions, therefore Π_C is strictly concave.

For any other component i, $i \neq k$, direct differentiation of Π_C w.r.t. Q_i yields that

$$\frac{\partial \Pi_C}{\partial Q_i} = -c_i + s_i \bar{F}_i(Q_i) < 0$$

Since Π_C is strictly decreasing in Q_i, the central planner will choose the same production quantity for all components to match with Q_k, i.e., $Q_1 = \ldots = Q_k = \ldots = Q_n$. □

Proof. Proposition 1.1. The proof is embedded in the main body of the chapter, hence we do not repeat it here. □

Proof. Lemma 1.2. The proof is embedded in the main body of the chapter, hence we do not repeat it here. □

Proof. Lemma 1.3. Recall that $\tilde{Q}_i$ is the solution to (5). From $\frac{\partial \Pi_i^I(Q_i)}{\partial Q_i}$, it can be easily derived that $\frac{\partial^2 \Pi_i^I}{\partial Q_i \partial w_i} = \bar{F}_M(Q_i) > 0$, and, $\frac{\partial^2 \Pi_i^I}{\partial Q_i^2} = -f_M(Q_i)(w_i - s_i) - s_i f_i(Q_i) < 0$. Hence,

$\frac{\partial \tilde{Q}_i(w_i)}{\partial w_i} = -\frac{\partial^2 \Pi_i^I}{\partial Q_i \partial w_i} / \frac{\partial^2 \Pi_i^I}{\partial Q_i^2} > 0$, that is, $\tilde{Q}_i(w_i)$ is increasing in w_i. □

Proof. Proposition 1.2. Solving the n best response functions given by (1.6) simultaneously, it is straightforward to show that $Q_1^* = ... = Q_n^* \leq \tilde{Q}$ is a N.E. for the suppliers' problem. Because $\Pi_i^I(Q_i|Q_{-i})$ is increasing in Q_i for all $Q_i \leq \tilde{Q}$, $Q_1^* = ... = Q_n^* = \tilde{Q}$ is the unique Pareto-optimal N.E. □

Proof. Lemma 1.4. Without loss of generality, suppose $\tilde{Q}_1(w_1^*) < \tilde{Q}_2(w_2^*) < \ldots < \tilde{Q}_n(w_n^*)$. Thus, $\min\{\tilde{Q}_1(w_1^*), \tilde{Q}_2(w_2^*), ..., \tilde{Q}_n(w_n^*)\} = \tilde{Q}_1(w_1^*)$. Recall from Lemma 1.3 that $\tilde{Q}_i, i = 1, \ldots, n$, is an increasing function in w_i. Then, for any $i = 2, ..., n$, there exists a $\bar{w}_i < w_i^*$ such that $\tilde{Q}_1(w_1^*) = \tilde{Q}_i(\bar{w}_i) < \tilde{Q}_i(w_i^*)$. Therefore,

$$\min\{\tilde{Q}_1(w_1^*), \tilde{Q}_2(w_2^*), ..., \tilde{Q}_n(w_n^*)\} = \min\{\tilde{Q}_1(w_1^*), \tilde{Q}_2(\bar{w}_2), ..., \tilde{Q}_n(\bar{w}_n)\} = \tilde{Q}_1(w_1^*).$$

By offering such a pricing scheme $(w_1^*, \bar{w}_2, ..., \bar{w}_n)$, the assembler's expected profit becomes

$$\Pi_0(w_1^*, \bar{w}_2, ..., \bar{w}_n) = (p - c_0 - w_1^* - \sum_{i=2}^{n} \bar{w}_i) E[\min_{i=1,\ldots,n} (\tilde{Q}_1(w_1^*), M)].$$

Since $w_1^* + \sum_{i=2}^n \bar{w}_i < \sum_{i=1}^n w_i^*$, $\Pi_0(w_1^*, \bar{w}_2, ..., \bar{w}_n) > \Pi_0(w_1^*, w_2^*, ..., w_n^*)$ which results in contradiction against the optimality. Therefore, the optimal pricing scheme should be designed such that

$$\tilde{Q}_1(w_1^*) = \tilde{Q}_2(w_2^*) = ... = \tilde{Q}_n(w_n^*) = \tilde{Q}. \tag{1.A.2}$$

Building upon such one-to-one relationship between $\tilde{Q}$ and each $w_i^*, i = 1, \ldots, n$, we can express each w_i^* as a function of $\tilde{Q}$ by transforming equation (1.5). □

Proof. Proposition 1.3. Direct differentiation of (1.9) gives

$$\frac{d\Pi_0}{d\tilde{Q}} = -\sum_{i=1}^{n} \triangle_i'(\tilde{Q}) S(\tilde{Q}) + (p - c_0 - \sum_{i=1}^{n} s_i - \sum_{i=1}^{n} \triangle_i(\tilde{Q})) \bar{F}_M(\tilde{Q}) = 0, \tag{1.A.3}$$

and

$$\frac{d^2\Pi_0}{d\tilde{Q}^2} = -\sum_{i=1}^{n} \triangle_i''(\tilde{Q})S(\tilde{Q}) - 2\sum_{i=1}^{n} \triangle_i'(\tilde{Q})\bar{F}_M(\tilde{Q}) - (p - c_0 - \sum_{i=1}^{n} s_i - \sum_{i=1}^{n} \triangle_i(\tilde{Q}))f_M(\tilde{Q})).$$

Similar to Theorem 2 in Cachon and Lariviere (2001), an obvious condition for ensuring a concave profit function of the assembler is that $\sum_{i=1}^{n} \triangle_i''(\tilde{Q}) \geq 0$. Checking the first derivative of $\triangle_i(\tilde{Q})$, we have

$$\triangle_i'(\tilde{Q}) = \frac{s_i f_i(\tilde{Q})}{\bar{F}_M(\tilde{Q})} + \frac{(c_i - s_i\bar{F}_i(\tilde{Q}))f_M(\tilde{Q})}{(\bar{F}_M(\tilde{Q}))^2} = s_i \frac{f_i(\tilde{Q})}{\bar{F}_i(\tilde{Q})}\frac{\bar{F}_i(\tilde{Q})}{\bar{F}_M(\tilde{Q})} + \triangle_i(\tilde{Q})\frac{f_M(\tilde{Q})}{\bar{F}_M(\tilde{Q})}$$

As a result of the IFR property and increasing $\frac{\bar{F}_i(\tilde{Q})}{\bar{F}_M(\tilde{Q})}$, $\triangle_i'(\tilde{Q})$ is increasing in $\tilde{Q}$. At $\tilde{Q} = 0$, $\frac{d\Pi_0}{d\tilde{Q}} = p - \sum_{i=0}^{n} c_i$. In conjunction with the concavity of Π_0, there must exist a unique solution, namely $\tilde{Q}^*$, to equation $(1.A.3)$. □

Proof. Proposition 1.4. (i) Since $\tilde{Q}^*$ is obtained from solving $\frac{d\Pi_0}{d\tilde{Q}} = 0$, $\frac{\partial \tilde{Q}^*}{\partial c_i} = -\frac{\partial^2\Pi_0}{\partial\tilde{Q}\partial c_i} / \frac{\partial^2\Pi_0}{\partial\tilde{Q}^2}$. From equation $(1.A.3)$, we can derive that

$$\begin{aligned}
\frac{\partial^2\Pi_0}{\partial\tilde{Q}\partial c_i} &= -S(\tilde{Q})\sum_{j=1}^{n}\frac{\partial\triangle_j'(\tilde{Q})}{\partial c_j} - \bar{F}_M(\tilde{Q})\sum_{j=1}^{n}\frac{\partial\triangle_j(\tilde{Q})}{\partial c_j} \\
&= -S(\tilde{Q})\frac{\partial\triangle_i'(\tilde{Q})}{\partial c_i} - \bar{F}_M(\tilde{Q})\frac{\partial\triangle_i(\tilde{Q})}{\partial c_i} \\
&= -\frac{S(\tilde{Q})f_M(\tilde{Q})}{[\bar{F}_M(\tilde{Q})]^2} - 1 < 0,
\end{aligned}$$

In conjunction with the property of a concave Π_0, $\frac{\partial\tilde{Q}^*}{\partial c_i} < 0$.

(ii) For supplier j, $\forall j \neq i$, since $w_j(\tilde{Q}^*)$ is independent of c_i, the result follows directly from $\frac{dw_j^*}{dc_i} = \frac{\partial\triangle_j^*}{\partial\tilde{Q}^*}\frac{\partial\tilde{Q}^*}{\partial c_i} < 0$.

For supplier i, we first substitute $\triangle_i'(\tilde{Q})$ into the first order condition from Proposition

1.3, and expand the function such that

$$-\sum_{j=1,j\neq i}^{n}\triangle_j'(\tilde{Q}^*)S(\tilde{Q}^*)-\frac{s_i f_i(\tilde{Q}^*)}{\bar{F}_M(\tilde{Q}^*)}S(\tilde{Q}^*)-\triangle_i(\tilde{Q}^*)S(\tilde{Q}^*)\frac{f_M(\tilde{Q}^*)}{\bar{F}_M(\tilde{Q}^*)}$$
$$+(p-c_0-\sum_{i=1}^{n}s_i-\sum_{j=1,j\neq i}^{n}\triangle_j(\tilde{Q}^*))\bar{F}_M(\tilde{Q}^*)-\triangle_i(\tilde{Q}^*)\bar{F}_M(\tilde{Q}^*)=0,$$

where $h_0(\cdot)\overset{\text{def}}{=}\frac{g(\cdot)}{\bar{G}(\cdot)}$ and $h_i(\cdot)\overset{\text{def}}{=}\frac{f_i(\cdot)}{\bar{F}_i(\cdot)}, i=1,...,n$ for notational convenience. It implies that the optimal quantity $\tilde{Q}^*$ satisfies

$$\triangle_i(\tilde{Q}^*)=\frac{(p-c_0-\sum_{i=1}^{n}s_i-\sum_{j=1,j\neq i}^{n}\triangle_j(\tilde{Q}^*))\bar{F}_M(\tilde{Q}^*)-\sum_{j=1,j\neq i}^{n}\triangle_j'(\tilde{Q}^*)S(\tilde{Q}^*)-\frac{s_i f_i(\tilde{Q}^*)}{\bar{F}_M(\tilde{Q}^*)}S(\tilde{Q}^*)}{\sum_{i=0}^{n}h_i(\tilde{Q}^*)S(\tilde{Q}^*)+\bar{F}_M(\tilde{Q}^*)},$$

following which $w_i(\tilde{Q}^*)=s_i+\triangle_i(\tilde{Q}^*)$ is independent of c_i. Together with the IFR property, we recognize that the right-hand side of the above function decreases in $\tilde{Q}^*$, i.e., $\frac{\partial\triangle_i^*}{\partial\tilde{Q}^*}<0$. Hence, $\frac{dw_i^*}{dc_i}\equiv\frac{d\triangle_i^*}{dc_i}=\frac{\partial\triangle_i^*}{\partial\tilde{Q}^*}\frac{\partial\tilde{Q}^*}{\partial c_i}>0$.

(iii) For analyzing the assembler's expected profit, suppose c_i decreases to $\hat{c}_i$ under which the optimal quantity and the assembler's profit function are denoted by $\hat{\tilde{Q}}^*$ and $\hat{\Pi}_0$, respectively. Due to $c_i>\hat{c}_i$, it follows from (9) that $\Pi_0(\tilde{Q}^*)<\hat{\Pi}_0(\tilde{Q}^*)$. Since $\hat{\tilde{Q}}^*$ is the optimal solution to maximizing $\hat{\Pi}_0$, $\Pi_0(\tilde{Q}^*)<\hat{\Pi}_0(\tilde{Q}^*)<\hat{\Pi}_0(\hat{\tilde{Q}}^*)$, which implies that Π_0^* decreases in c_i.

Given the optimal quantity $\tilde{Q}^*$, the decentralized system profit, Π_D^*, is written as

$$\Pi_D^*=(p-c_0)E[\min_{i=1,\ldots,n}(\tilde{Q}^*,M)]-\sum_{i=1}^{n}c_i\tilde{Q}^*+\sum_{i=1}^{n}s_iE[\min(\tilde{Q}^*,K_i)-\min_{j=1,\ldots,n}(\tilde{Q}^*,M)]^+\tag{1.A.4}$$

Direct differentiation of Π_D^* w.r.t. c_i yields

$$\frac{d\Pi_D^*}{dc_i}=[p-c_0-\sum_{i=1}^{n}s_i-\sum_{i=1}^{n}\triangle_i(\tilde{Q}^*)]\bar{F}_M(\tilde{Q}^*)\frac{\partial\tilde{Q}^*}{\partial c_i}-\tilde{Q}^*<0.$$

For supplier j, $\forall j\neq i$, substituting (1.8) into Π_j provides

$$\Pi_j^*=(s_j+\triangle_j(\tilde{Q}^*))E[\min_{i=1,\ldots,n}(\tilde{Q}^*,K_i,D)]-c_j\tilde{Q}^*+s_jE[\min(\tilde{Q}^*,K_j)-\min_{i=1,\ldots,n}(\tilde{Q}^*,K_i,D)]^+,$$

which is independent of c_i. Then differentiation of Π_j^* w.r.t. c_i yields $\frac{d\Pi_j^*}{dc_i} = \frac{\partial \Pi_j^*}{\partial \tilde{Q}^*}\frac{\partial \tilde{Q}^*}{\partial c_i}$. Together with $\frac{\partial \Pi_j^*}{\partial \tilde{Q}^*} = \left(\frac{s_j f_j(\tilde{Q}^*)}{\bar{F}_M(\tilde{Q}^*)} + \frac{(c_j - s_j \bar{F}_j(\tilde{Q}^*))f_M(\tilde{Q}^*)}{\bar{F}_M(\tilde{Q}^*)^2}\right) S(\tilde{Q}^*) > 0$, the result follows from part (i). □

Proof. Proposition 1.5. (i) Following (1.A.3), it can be derived that

$$\frac{\partial^2 \Pi_0}{\partial \tilde{Q} \partial s_i} = \frac{\bar{F}_i(\tilde{Q})S(\tilde{Q})}{\bar{F}_M(\tilde{Q})}\left(\frac{f_M(\tilde{Q})}{\bar{F}_M(\tilde{Q})} - \frac{f_i(\tilde{Q})}{\bar{F}_i(\tilde{Q})}\right) + \bar{F}_i(\tilde{Q}) - \bar{F}_M(\tilde{Q}) > 0.$$

Hence, we obtain that $\frac{\partial \tilde{Q}^*}{\partial s_i} = -\frac{\partial^2 \Pi_0}{\partial \tilde{Q} \partial s_i} / \frac{\partial^2 \Pi_0}{\partial \tilde{Q}^2} > 0$.

For supplier j, $\forall j \neq i$, since $w_j(\tilde{Q}^*)$ is independent of s_i, the result follows directly from $\frac{dw_j^*}{ds_i} = \frac{\partial \triangle_j^*}{\partial \tilde{Q}^*}\frac{\partial \tilde{Q}^*}{\partial s_i} > 0$.

(ii) Suppose the parameter s_i increases to $\hat{s}_i$ under which the corresponding optimal quantity and the assembler's optimal expected profit are denoted by $\hat{\tilde{Q}}^*$ and $\hat{\Pi}_0$. From (1.9), it follows that $\Pi_0(\tilde{Q}^*) < \hat{\Pi}_0(\tilde{Q}^*)$. Since $\hat{\tilde{Q}}^*$ is the optimal solution for the system with parameter $\hat{s}_i$, $\Pi_0(\tilde{Q}^*) < \hat{\Pi}_0(\tilde{Q}^*) < \hat{\Pi}_0(\hat{\tilde{Q}}^*)$, i.e., the assembler's expected profit increases in s_i. Direct differentiation of Π_D^* w.r.t. s_i yields that

$$\frac{d\Pi_D^*}{ds_i} = \frac{\partial \Pi_D^*}{\partial \tilde{Q}^*}\frac{\partial \tilde{Q}^*}{\partial s_i} + E[\min(\tilde{Q}^*, K_i) - \min_{j=1,\ldots,n}(\tilde{Q}^*, M)]^+ > 0.$$

Furthermore, since the expected profit for supplier $j, j \neq i$, is not affected by s_i and increasing on $\tilde{Q}^*$. It directly follows from part (i) that Π_j^* increases as s_i increases. □

Proof. Proposition 1.6. (i) Applying the same method as in the proof of Proposition 4, it is straightforward to obtain $\frac{\partial \tilde{Q}^*}{\partial p} = -\frac{\partial^2 \Pi_0}{\partial \tilde{Q} \partial p} / \frac{\partial^2 \Pi_0}{\partial \tilde{Q}^2} > 0$. Since $w_i(\tilde{Q}^*), i = 1, \ldots, n$, is not affected by p, the result follows directly.

(ii) Since $\Pi_j(\tilde{Q}^*)$, $\forall j = 1, \ldots, n$ is independent of p, the result follows directly from (i).

For the assembler's profit, the proof is analogous to the proof of Proposition 1.5(i) and thus omitted here. As a consequence of increasing Π_0^* and $\Pi_j^*, \forall j = 1, \ldots, n$, the total system profit increases as well.

The analysis for the impact of decreasing c_0 on the system is analogous to above, thus we omit the details here. $\square$

Proof. Proposition 1.7. Let $\mathbf{Y} \overset{\text{def}}{=} (K_1, ..., K_n, D)$, $\mathbf{Y}^L = (K_1^L, ..., K_n^L, D^L) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\mathbf{Y}^H = (K_1^H, ..., K_n^H, D^H) \sim N(\boldsymbol{\mu'}, \boldsymbol{\Sigma'})$. Assume $\boldsymbol{\mu} = \boldsymbol{\mu'}$, $\sigma_{ab} \leq \sigma'_{ab}$ for some $a \neq b; a, b = 1, ..., n+1$ and $\sigma_{ij} = \sigma'_{ij}, \forall (i,j) \notin \{(a,b),(b,a)\}$. Then, following Theorem 4.2 of Müller and Scarsini (2000), we have $Y^L \leq_{sm} Y^H$, i.e., Y^L is smaller than Y^H in the supermodular order which implies that $Ef(Y^L) \leq Ef(Y^H)$ for all supermodular functions f such that the expectations exist by Definition 2.3 of Müller and Scarsini (2000). It follows from Example 2.6.2 (f) on page 46 of Topkis (1998) that function $\min(K_1, ..., K_n, D)$ is a supermodular function in $(K_1, ..., K_n, D)$. Therefore, we have $S(\tilde{Q}|Y^L) \leq S(\tilde{Q}|Y^H), \forall \tilde{Q}$ since $S(\tilde{Q}) \overset{(1.10)}{=} E[\min(\tilde{Q}, M)] = E[\min(\tilde{Q}, K_1, ..., K_n, D)]$. Definition 2.4 and remarks on page 110 of Müller and Scarsini (2000) immediately imply that $Y^L \leq_{uo} Y^H$, i.e., $\bar{F}_{Y^L}(\mathbf{t}) \leq \bar{F}_{Y^H}(\mathbf{t}), \forall \mathbf{t} \in R^{n+1}$. Let $\mathbf{t} = (t, ..., t) \in R^{n+1}, \forall t \in R$, then we have $Pr(K_1^L > t, ..., K_n^L > t, D^L > t) \leq Pr(K_1^H > t, ..., K_n^H > t, D^H > t)$ which implies that $Pr(\min(K_1^L, ..., K_n^L, D^L) > t) = \bar{F}_M(t|Y^L) \leq Pr(\min(K_1^H, ..., K_n^H, D^H) > t) = \bar{F}_M(t|Y^H)$ due to $M = \min(K_1, ...K_n, D)$. In conjunction with (1.11), we have $\Delta_i(\tilde{Q}|Y^L) \geq \Delta_i(\tilde{Q}|Y^H), \forall \tilde{Q}, i = 1, ..., n$. Let $\tilde{Q}^{*i}, i = L, H$ denote the optimal $\tilde{Q}$ which maximizes $\Pi_0(\tilde{Q}|Y^i)$ in (1.9). Hence, we have $\Pi_0(\tilde{Q}^{*H}|Y^H) \geq \Pi_0(\tilde{Q}^{*L}|Y^H) \geq \Pi_0(\tilde{Q}^{*L}|Y^L)$. $\square$

Proof. Proposition 1.8. Recall that $\Pi_0(\tilde{Q}) = E[(p - c_0 - \sum_{i=1}^n w_i(\tilde{Q})) \min(\tilde{Q}, M)]$ and $\Pi_0^r(\tilde{Q}^r) = E[(p - c_0 - \sum_{i=1}^n w_i^r(\tilde{Q}^r)) \min(\tilde{Q}^r, D)]$. We first establish that $\sum_{i=1}^n w_i(\tilde{Q}^*) - \sum_{i=1}^n w_i^r(\tilde{Q}^*) = \sum_{i=1}^n \frac{s_i F_i(\tilde{Q}^*)}{\bar{F}_M(\tilde{Q}^*)} + \sum_{i=1}^n \frac{c_i - s_i}{\bar{F}_M(\tilde{Q}^*)} - \sum_{i=1}^n \frac{c_i - s_i}{\bar{F}_D(\tilde{Q}^*)}$. Since $\bar{F}_D(\tilde{Q}^*) - \bar{F}_M(\tilde{Q}^*) = Pr(D > \tilde{Q}^*) - Pr(M > \tilde{Q}^*) = Pr(D > \tilde{Q}^*) - Pr(K_1 > \tilde{Q}^*, ..., K_n > \tilde{Q}^*|D > \tilde{Q}^*)Pr(D > \tilde{Q}^*) > 0$, $\sum_{i=1}^n w_i(\tilde{Q}^*) > \sum_{i=1}^n w_i^r(\tilde{Q}^*)$. In conjunction with $\min(\tilde{Q}^*, M) \leq \min(\tilde{Q}^*, D)$, we have $\Pi_0^*(\tilde{Q}^*) < \Pi_0^r(\tilde{Q}^*)$. Since $\tilde{Q}^{r*}$ is the optimal solution to maximizing Π_0^r, it follows that $\Pi_0^*(\tilde{Q}^*) < \Pi_0^r(\tilde{Q}^*) < \Pi_0^{r*}(\tilde{Q}^{r*})$. $\square$

Proof. Proposition 1.9. Taking the derivative of $\Pi_0(\underset{\sim}{Q})$ w.r.t. $\underset{\sim}{Q}$, we obtain

$$\frac{\partial \Pi_0(\underset{\sim}{Q})}{\partial \underset{\sim}{Q}} = \bar{F}_M(\underset{\sim}{Q}) \left[(p - c_0 - \sum_{i=1}^{n} s_i) - \sum_{i=1}^{n} \frac{(w_i - s_i)\bar{F}_i(\underset{\sim}{Q})}{\bar{F}_M(\underset{\sim}{Q})} \right].$$

It follows from the assumption (A2), there exists a unique solution, namely $\underset{\sim}{Q}^* \in [0, \infty)$, to the assembler's problem (1.13) that is uniquely determined by $\frac{\partial \Pi_0(\underset{\sim}{Q})}{\partial \underset{\sim}{Q}}|_{\underset{\sim}{Q}=\underset{\sim}{Q}^*} = 0$. □

Proof. Proposition 1.10. Given any wholesale prices $w_j, j \neq i$, offered by all other suppliers, supplier i will set his wholesale price w_i according to $\frac{\partial \Pi_0(\underset{\sim}{Q})}{\partial \underset{\sim}{Q}}|_{\underset{\sim}{Q}=\underset{\sim}{Q}^*} = 0$ such that

$$w_i = s_i + \frac{1}{\bar{F}_i(\underset{\sim}{Q}^*)} [\bar{F}_M(\underset{\sim}{Q}^*)(p - c_0 - \sum_{i=1}^{n} s_i) - \sum_{j=1, j\neq i}^{n} (w_j - s_i)\bar{F}_j(\underset{\sim}{Q}^*)] \tag{1.A.5}$$

Following Proposition 1.9, we can derive $\frac{\partial \underset{\sim}{Q}^*}{\partial w_i} = -\frac{\partial^2 \Pi_0 / \partial \underset{\sim}{Q} \partial w_i}{\partial^2 \Pi_0 / \partial \underset{\sim}{Q}^2} = \frac{\bar{F}_i(\underset{\sim}{Q})}{\partial^2 \Pi_0 / \partial \underset{\sim}{Q}^2} < 0$, and $\frac{\partial^2 \underset{\sim}{Q}^*}{\partial w_i^2} = -\frac{\bar{F}_i(\underset{\sim}{Q}) f_i(\underset{\sim}{Q})}{(\partial^2 \Pi_0 / \partial \underset{\sim}{Q}^2)^2} < 0$. That is, $\underset{\sim}{Q}^*(w_i)$ is decreasing concave in $w_i, \forall i = 1, \ldots, n$. It then follows that the inverse function $w_i(\underset{\sim}{Q}^*)$ is decreasing and concave in $\underset{\sim}{Q}^*$. Based on the one-to-one relationship between w_i and $\underset{\sim}{Q}^*$, we can equivalently optimize supplier i's expected profit Π_i over $\underset{\sim}{Q}$ instead of w_i. Substituting (1.A.5) into problem (1.15) and checking the second derivative w.r.t. $\underset{\sim}{Q}$, we have

$$\frac{\partial^2 \Pi_i(\underset{\sim}{Q})}{\partial \underset{\sim}{Q}^2} = 2\frac{\partial w_i(\underset{\sim}{Q})}{\partial \underset{\sim}{Q}} \bar{F}_i(\underset{\sim}{Q}) + \frac{\partial^2 w_i(\underset{\sim}{Q})}{\partial \underset{\sim}{Q}^2} \int_0^{\underset{\sim}{Q}} \bar{F}_i(x)dx - w_i(\underset{\sim}{Q}) f_i(\underset{\sim}{Q}) < 0.$$

That is, $\Pi_i(\underset{\sim}{Q})$ is continuous and concave with respect to supplier i's own strategy. Moreover, each w_i is strictly constrained to be in $[c_i, p - c_0]$ so that each supplier's strategy space is compact and convex. Therefore, there exists a pure-strategy N.E. for the game (Debreu, 1952).

Substituting (1.A.5) into (1.15) then taking derivative w.r.t. $\underset{\sim}{Q}$, we have

$$\begin{aligned}\frac{\partial \Pi_i(\underset{\sim}{Q})}{\partial \underset{\sim}{Q}} = &\int_0^{\underset{\sim}{Q}} \bar{F}_i(x)dx\{\frac{f_i(\underset{\sim}{Q})}{[\bar{F}_i(\underset{\sim}{Q})]^2}[\bar{F}_M(\underset{\sim}{Q})(p-c_0-\sum_{i=1}^{n} s_i) - \sum_{j=1,j\neq i}^{n}(w_j - s_j)\bar{F}_j(\underset{\sim}{Q})] \\ &+ \frac{1}{\bar{F}_i(\underset{\sim}{Q})}[-f_M(\underset{\sim}{Q})(p-c_0-\sum_{i=1}^{n} s_i) + \sum_{j=1,j\neq i}^{n}(w_j - s_j)f_j(\underset{\sim}{Q})]\} \qquad (1.A.6) \\ &+ s_i\bar{F}_i(\underset{\sim}{Q}) + \bar{F}_M(\underset{\sim}{Q})(p-c_0-\sum_{i=1}^{n} s_i) - \sum_{j=1,j\neq i}^{n}(w_j - s_j)\bar{F}_j(\underset{\sim}{Q}) - c_i,\end{aligned}$$

in which all other w_j, $j \neq i$, are embedded.

For any other supplier j, $j \neq i$, direct differentiation of $\Pi_j(w_j)$ (i.e., problem (1.15)) w.r.t. w_j yields

$$\frac{\partial \Pi_j(w_j)}{\partial w_j} = \int_0^{\underset{\sim}{Q}^*} \bar{F}_j(x)dx + (w_j\bar{F}_j(\underset{\sim}{Q}^*) - c_j)\frac{\partial \underset{\sim}{Q}^*}{\partial w_j}, \qquad (1.A.7)$$

and

$$\frac{\partial^2 \Pi_j(w_j)}{\partial w_j^2} = \bar{F}_j(\underset{\sim}{Q}^*)\frac{\partial \underset{\sim}{Q}^*}{\partial w_j} - w_j f_j(\underset{\sim}{Q}^*)(\frac{\partial \underset{\sim}{Q}^*}{\partial w_j})^2 + (w_j\bar{F}_j(\underset{\sim}{Q}^*) - c_j)\frac{\partial^2 \underset{\sim}{Q}^*}{\partial w_j^2}.$$

The first-order condition (1.A.7) should be discussed in two cases: (1) $w_j\bar{F}_j(\underset{\sim}{Q}^*) - c_j \geq 0$; and (2) $w_j\bar{F}_j(\underset{\sim}{Q}^*) - c_j < 0$. If $w_j\bar{F}_j(\underset{\sim}{Q}^*) - c_j \geq 0$ (referred as case 1), the first order condition is non-monotonic and Π_j is concave in w_j. Then there is a unique optimal solution, namely w_j^*, to $\frac{\partial \Pi_j(w_j)}{\partial w_j}|_{w_j = w_j^*} = 0$. However, if $w_j\bar{F}_j(\underset{\sim}{Q}^*) - c_j < 0$ (referred as case 2), the first-order condition is monotonically increasing in w_j, implying that the optimal w_j^* will be ∞ which will lead to an order quantity of 0. Case 2 is clearly dominated by case 1 thus not in the equilibrium. Therefore, the unique optimal solution w_j^* is the solution to $\frac{\partial \Pi_j(w_j)}{\partial w_j}|_{w_j = w_j^*} = 0$, satisfying $w_j^*\bar{F}_j(\underset{\sim}{Q}^*) = c_j - \frac{\partial^2 \Pi_0}{\partial \underset{\sim}{Q}^{*2}}\frac{\int_0^{\underset{\sim}{Q}^*}\bar{F}_j(x)dx}{\bar{F}_j(\underset{\sim}{Q}^*)}$. Summing up all $w_j^*\bar{F}_j(\underset{\sim}{Q}^*)$, $\forall j = 1, \ldots, n$, and then subtracting equation (1.14), we can obtain an equivalent expression for $\frac{\partial^2 \Pi_0}{\partial \underset{\sim}{Q}^{*2}}$, which is denoted by $L(\underset{\sim}{Q})$ and presented in the main body of the chapter. Subsequently, w_j^* can now become a function of $\underset{\sim}{Q}$ only, i.e., $w_j^*(\underset{\sim}{Q}) = \frac{1}{\bar{F}_j(\underset{\sim}{Q})}[c_j - L(\underset{\sim}{Q})\frac{\int_0^{\underset{\sim}{Q}}\bar{F}_j(x)dx}{\bar{F}_j(\underset{\sim}{Q})}]$.

By substituting $w_j^*(\utilde{Q})$ into (1.A.6) and setting it equal to 0, the equilibrium production quantity $\utilde{Q}^*$ can be determined by solving equation (1.16). $\square$

Chapter 2

Procurement Design for a National Brand Supplier in the Presence of Store Brand Competition

2.1 Introduction

Store brands, also known as private labels or generic brands, are a line of products branded and managed solely by a retailer for sales. In recent years, having realized that store brands are an efficient instrument to improve gross margin, many retailers have been gradually shifting their emphasis to the development of store brands. Store brand products have been steadily earning trust among consumers and are becoming recognized as good alternatives to national brands. Achieving significant success in Europe, store brands hold market shares of 40% or higher in seven countries, including UK, Germany, and Switzerland (Private Label Manufacturers Association International, 2017). ACNielsen reported that in 2014, the dollar shares of private label products had achieved 17.5% of the total sales revenue in the U.S. Marchat (2018) stated that "during 2017, store brand sales across all outlets measured by Nielsen came in at $122.3 billion, up from $119.1 billion, while units moved up to 44.6 billion from 43.9 billion." Private Label Manufacturers Association (PLMA, 2018) further commented that "since Nielsen's statistics do not include some of the biggest and best store brands retailers in the country, such as Costco, Aldi, and Trader Joe's, estimates of their private label sales and other major operators not counted, such as Amazon, you can conservatively add another \$35 billion to private label sales for a total of \$160 billion." In addition, unit sales of store brands in the mass channel increased by 9.1% in 2017 while unit volume of national brands declined by 1%. Since retailers present a serious threat to national brand suppliers by expanding their private labels, the rules of the game have changed. Retailers who used to be simply a distribution channel for national brands have now also become competitors to the national brand suppliers. The traditional advantages of national brands are no longer as strong and are beginning to lose relevance. Hence, the national brands recognize the need to strategically defend against retailer brands (Alliance Consulting Group). Since the recession ended, national brands have stepped up both promotional activity and innovation efforts to protect share positions and drive growth (ACNielsen 2014). However, precisely how a national brand supplier should adjust the supply contracting strategy to cope with the rising store brand competition has not been completely addressed.

We first examine the various store brand supply structures. PLMA International Council identifies three general categories of store brand manufacturers, which are (1) large manufacturers who produce their own brands and utilize the excess capacity to produce store brand products, (2) smaller-size and regional manufacturers that specialize in particular product lines and concentrate on producing store brands almost exclusively, (3) manufacturing facilities run by major retailers and wholesalers to provide store brands for themselves and other retail chains. (See PLMA International: Private Label Today). In the latter two cases, store brands do not share the same manufacturing source with national brands. Given the potential competition with the national brand suppliers, the retailers do not have any incentive to share the core information about store brand. Under such circumstances, a set of research questions naturally arises: How should a national brand supplier design the supply contract? What impact does the private information have on the supply chain participants? To our best knowledge, such questions have not been fully answered in the literature.

We consider a decentralized supply chain with a national brand supplier (she) and a retailer (he). The national brand supplier distributes her products to consumers via the retailer through a two-part tariff supply contract, which includes a unit wholesale price and a lump-sum payment. Meanwhile, the retailer intends to develop and produce his own store brand. Similar to Yano (2017), we assume that the retailer has a manufacturing source that is different from the national brand supplier. The unit production cost of the store brand is privately discovered by the retailer. The national brand supplier only has a subjective assessment about the store brand production cost and believes that it has two possible values, high type or low type. Given the supply contract offered by the national brand supplier, the retailer can simultaneously decide whether to accept the supply contract and whether to introduce the store brand. We show that if the wholesale price is low and the lump-sum payment is lower than the threshold value, then the retailer will carry the national brand only. When the wholesale price is in the medium range and the lump-sum payment is lower than a certain threshold, then the retailer will carry both the national brand and store brand. However, if the wholesale price is too high or the lump-sum payment is above the threshold, then the retailer will prefer to carry the store brand only. Subsequently, the retailer decides the retail price(s) for the carried product(s) to sell to the market. The demand of the carried product(s) is determined by a quality-utility framework.

We first investigate a benchmark case assuming that the store brand cost information is common knowledge to both members in the supply chain. In this case, the national brand supplier will always offer a contract to achieve her first best profit and leave the retailer nothing but his reservation profit. We then analyze the case in which the store brand unit production cost is private information to the retailer only. Following the theory of incentives, we formulate the problem as a two-stage screening game to analyze the strategic interaction between the two players. The national brand supplier offers a menu of two contracts to the retailer to maximize her expected profit and induce the retailer to truthfully reveal his store brand cost information. The national brand supplier's contractual design problem presents inherent complexity since there exists eight possible parameter settings, each representing a distinct relationship among the national brand's and store brand's product characteristics (i.e., cost and quality), and under each parameter setting there exists up to five possible contract forms. Despite the computational complexity, we are able to derive the optimal menu of contracts that maximizes the national brand supplier's expected profit. We analytically demonstrate that the format of the optimal contract depends on the national brand supplier's own production cost. Moreover, we explore how the model parameters affect the value of information for each member of the supply chain. We show that when the national brand unit production cost increases, retailer's private cost information becomes less valuable to both the national brand supplier and the retailer. Our results also indicate that when the gap between the two possible cost values increases, the private cost information becomes more valuable to the national brand supplier, but the value of information to the retailer himself can increase or decrease. Finally, we illustrate that when the perceived quality of national brand increases, the value of information to the retailer first decreases then increases, but the impact on the value of information to the national brand supplier is not definitive. These observation are somewhat counter-intuitive and provide us with interesting managerial insights.

The rest of the chapter precedes as follows. In section 2.2, we review the related literature. We then introduce the modeling framework in section 2.3. Section 2.4 solves the retailer's problem. In section 2.5, we analyze the supplier's problem with complete information as a benchmark case and also design the optimal menu of contracts under asymmetric information. In section 2.6, we investigate the value of private information to both entities and derive analytical results to illustrate the impact of model parameters. In section 2.7, we will conclude the chapter with major results and further research directions.

2.2 Literature Review

Our study stems from the store brand literature and a stream of asymmetric information studies in operations management. The analytical research on store brands has been growing since 1990s. Mills (1995) demonstrated that store brand can be leveraged to eliminate double marginalization problem in the entire supply chain. Groznik and Heese (2010) studied interaction between one supplier and one retailer and analyzed the impact of the store-brand introduction on the supply chain. Wu and Wang (2005) investigated a more complicated setting with two national brand manufacturers and one common retailer. Dhar and Hoch (1997) conducted empirical analysis to investigate why the store brand performance varies among major grocery retailers in the US. Sethuraman (2009) provided a comprehensive summarization of the analytical results on the national brand and store brand marketing as wells as the external validity of those results. Groznik, and Heese (2010) investigated the interaction between retailers under the introduction of store brand. Yano et al. (2017) studied a case in which a retailer, such as a major grocery chain, manufactures store brand products in one's own factory and faces a decision whether to sell the factory. Fang, et al. (2013) investigated a two-stage Stackelberg game between national brand and store brand and analyzed wholesale contract design under both centralized and decentralized systems. To our best knowledge, no study has incorporated the private store brand cost information in the interaction between the national brand supplier and the retailer.

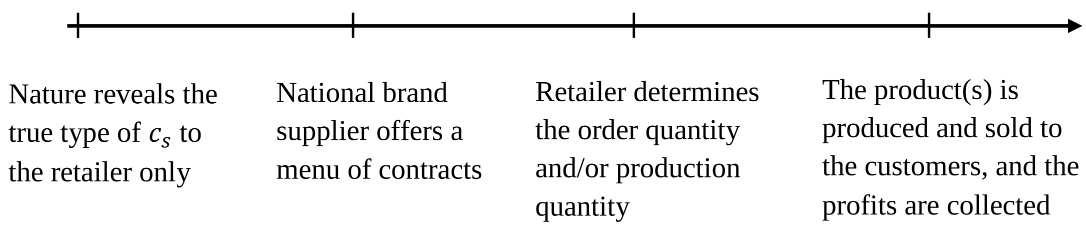
In supply chain literature, the principle-agent model (Laffont and Martimort 2002) has been utilized as a powerful instrument to cope with the information asymmetry. A principle (he) offers contracts to an agent (her), who holds the private information, to extract information revelation. The induction of private information is assured by setting up participation constraints and incentive compatibility constraints on the agent's profit. The participation constraints (also called individual rationality constraints) ensure the agent to at least achieve his reservation profit. The incentive constraints imply that the information rent will be given up to the agent if his type is inefficient so that she will not mimic the efficient type. Consequently, the incentive of conflict arising from cost information asymmetry will be successfully eliminated, preventing the national brand supplier from over-discounting his selling price (Fang 2012).

There was a large body of literature that applied principle-agent model in analyzing the operations management problems. Corbett and deGroote (2000) derived the optimal quantity discount policy for a single supplier single buyer supply chain under asymmetric information. Corbett (2001) investigated stochastic inventory systems in a supply chain where the supplier has private information about setup cost. Corbett et al. (1999) studied how a supplier is impacted by the obtaining better information about the buyer's cost structure. Xiao and Xu (2012) investigated R&D alliance strategy in the presence of asymmetric effort information from both participants. Ha (2001) studied a contract to maximize the supplier's profit in a one-supplier-one-buyer relationship for a short-life-cycle product. In his study, the reservation profit is fixed regardless of the agent's type, whereas in our study the agent's reservation profit deviates based on his type, which brings in more computational complexity.

2.3 Model Setup

We consider a two-tier decentralized supply chain with a national brand supplier and a retailer. The national brand supplier produces a product at a cost of c_n per unit with a quality value q_n. The national brand supplier distributes her product to consumers via the retailer. In addition to selling the national brand product from the supplier, the retailer is also able to produce and sell his store brand product at a cost of c_s per unit with quality value q_s. As pointed out by Bergs-Sennou et al. (2004), consumers usually perceive the store brand products having lower quality than the national brand products. Chung and Lee (2017) also commented that in reality, NBs often times demonstrate superior brand equity to the SBs, which leads to higher perceptional quality. Therefore, we make a reasonable assumption that $q_s < q_n$ to be consistent with Mills (1995), Chen et al. (2011), etc., and for the analytical simplicity. We shall discuss a contrary case (i.e., $q_s > q_n$) in the conclusion section. The unit production cost of store brand, c_s, is private information for the retailer only. The national brand supplier only has a subjective assessment about the retailer's store brand unit production cost. In details, the NB supplier believes that the retailer's unit production cost follows a discrete probability distribution and c_s has two possible values, denoted by c_s^H and c_s^L with respective probabilities v and $1 - v$. For convenience, for the remainder of this chapter, we call these two cost types "high cost" and "low cost."

Figure 2.1: Sequence of events.



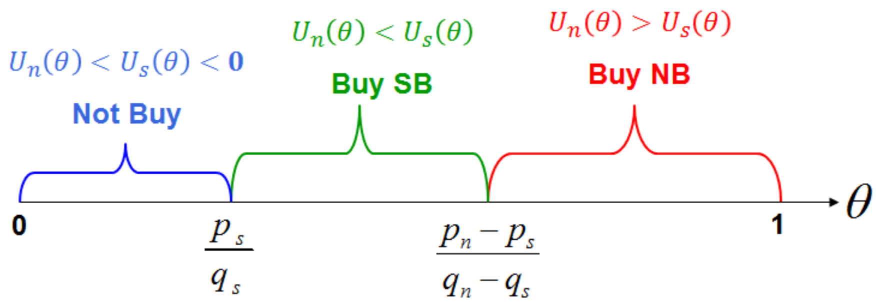
The sequence of events is illustrated in Figure 2.1. (1) Nature reveals the true type of c_s to the retailer only. (2) The national brand supplier offers a menu of two-part tariff contract to the retailer. That is, the retailer pays the national brand supplier a wholesale price w and a lump-sum payment T. (3) The retailer subsequently decides whether to accept the contract and which contract to accept. The retailer also simultaneously decides whether to produce his own store brand. If the retailer decides to carry the national brand (referred as NB hereafter) product and/or the store brand (referred as SB hereafter) product, he shall decide the NB and/or SB retail price p_n and/or p_s and their quantities Q_n and/or Q_s , respectively.

Then the product(s) is (are) sold to consumers, and payments are collected. Following the revelation principle, the supplier only needs to provide two contracts (w^H, T^H) and (w^L, T^L) corresponding to the retailer's two cost types in the menu to maximize her expected profit.

Next, we shall describe our demands for the national brand and/or SB products. Following Mills (1995), Chen, et al. (2011) , and Fang, et al. (2013), we assume that consumers who are interested in purchasing NB or SB are vertically heterogeneous with respect to their evaluations (θ) on the product quality. From now on, we refer a consumer with a quality evaluation parameter of θ as "consumer θ" for brevity. Consumers make purchasing decisions based on their evaluations of the product quality and price. When a product with quality level q is sold for \$$p$ per unit, consumer θ derives a utility function, $U(\theta) = \theta q - p$. If there is only one product available on the market, the consumer would purchase the product if and only if his/her utility $U(\theta)$ for the product is positive. If there are multiple products available on the market, he/she would purchase only the product which gives him/her the highest positive utility. Without loss of generality, we assume the entire population of potential consumers is normalized to 1. We further assume θ is uniformly distributed between 0 and 1.

In our model setup, the retailer has three options: (I) to accept a contract from the supplier and carry the NB only (named as case I for the remainder of this chapter); (II) to accept a contract from the supplier, carry the NB product and also introduce his own SB (named as case II); (III) to reject the supplier's contract and sell his SB only (named as case III). In the two extreme cases, i.e., in case I or III, the retailer chooses to carry the NB only or the SB only. Then only one product will be available on the market. Under such circumstances, we define $U_i(\theta) = \theta q_i - p_i$ is the utility that the consumer θ derives for product i, where $i = s, n$ representing the SB($i = s$) and NB($i = n$) respectively. Following $U_i(\theta) > 0$ and the uniform distribution of θ from 0 to 1, we directly obtain the demand for product i ($i = s, n$) is $1 - p_i/q_i$.

Figure 2.2: Market segmentation for NB and SB when $\frac{p_s}{q_s} < \frac{p_n - p_s}{q_n - q_s} < 1$.



Now we discuss the most interesting case, i.e., case II, in which the retailer decides to carry both the national brand and store brand products. In this case, the consumer θ derives the utility $U_i(\theta) = \theta q_i - p_i$ for product $i = s, n$ simultaneously, and he/she will purchase the one with a higher positive utility. We show that in order to ensure both products receive positive demands, the retailer has to set the retail prices such that $\frac{p_s}{q_s} < \frac{p_n - p_s}{q_n - q_s} < 1$, or equivalently, $\frac{p_s}{q_s} < \frac{p_n}{q_n} < 1$ (due to $q_s < q_n$). For the consumers who purchase the national brand product, it follows directly from $U_n(\theta) > U_s(\theta)$ that $\theta > \frac{p_n - p_s}{q_n - q_s}$. Figure 2.2 further shows the market segmentation for NB and SB under such conditions.

When the condition of $\frac{p_s}{q_s} < \frac{p_n - p_s}{q_n - q_s} < 1$ is not satisfied, only one brand (either NB or SB) has positive demand. Table 2.1 summarizes the demands for NB and SB as a function of the SB retail price.

In sum, we provide the list of all mathematical notations used in this chapter in Table 2.2. As a benchmark for the asymmetric information case, we also investigate the symmetric

Table 2.1: Demands for NB and SB as a function of p_s

Ranges for p_s	Demand for NB	Demand for SB
$p_s > \frac{p_n q_s}{q_n}$	$1 - \frac{p_n}{q_n}$	0
$p_n - q_n + q_s \leq p_s \leq \frac{p_n q_s}{q_n}$	$1 - \frac{p_n - p_s}{q_n - q_s}$	$\frac{p_n - p_s}{q_n - q_s} - \frac{p_s}{q_s}$
$p_s < p_n - q_n + q_s$	0	$1 - \frac{p_s}{q_s}$

Table 2.2: List of mathematical notations used.

Notation	**Definition**
Π_n	NB supplier's profit function
Π_r	Retailer's profit function
$Q_n(Q_s)$	NB (SB) production quantity
$p_n(p_s)$	NB (SB) product retail price
$q_n(q_s)$	NB (SB) product quality
θ	Consumer's quality evaluation parameter
$c_n(c_s)$	NB (SB) unit production cost
$c_s^H(c_s^L)$	Estimated high-type (low-type) SB unit production cost
w	Unit wholesale price paid to the NB supplier
T	Fixed lump-sum payment paid to the NB supplier
$w^H(w^L)$	Unit wholesale price for high-type (low-type) retailer
$T^H(T^L)$	Fixed lump-sum payment for high-type (low-type) retailer

information case, i.e., the supplier knows about the retailer's store brand cost, c_s.

2.4 Retailer's problem

Following the backward induction, we analyze the retailer's problem first. In this section, without loss of generality, we can suppress the superscript i $(i = L, H)$ which denotes the retailer's type from the parameter c_s^i and variables w^i and T^i for notational convenience. Furthermore, since the retailer knows his own type, the retailer's problem under asymmetric information is the same as that under symmetric information. Hence, the retailer's problem is to figure out his best response for any given contract (w, T) offered by the supplier. As specified in the previous section, the retailer's reaction falls into one of the following three cases: (I) the retailer accepts the supplier's contract and only sells the NB; (II) the retailer accepts the supplier's contract and sells both NB and SB; (III) the retailer rejects the supplier's contract and only sells his own SB. In summary, the retailer's profit function can be written as follows:

$$\Pi_r = \begin{cases} \Pi_r^I(p_n, Q_n) = (p_n - w)Q_n - T \\ \Pi_r^{II}(p_n, Q_n, p_s, Q_s) = (p_n - w)Q_n - T + (p_s - c_s)Q_s \\ \Pi_r^{III}(p_s, Q_s) = (p_s - c_s)Q_s \end{cases} \tag{2.4.1}$$

For the two extreme cases, case (I) and (III), it is straightforward to derive the following result:

Lemma 2.1. *For any given (w, T) contract offered by the supplier, if the retailer accepts the contract and only carries NB, then his optimal decision and profit are $p_n^{I*} = \frac{w+q_n}{2}, Q_n^{I*} = \frac{1}{2} - \frac{w}{2q_n}$, and $\Pi_r^{I*} = \frac{(q_n-w)^2}{4q_n} - T$; if the retailer rejects the contract and chooses to carry SB only, then his optimal decision and profit are $p_s^{III*} = \frac{c_s+q_s}{2}, Q_n^{III*} = \frac{1}{2} - \frac{c_s}{2q_s}$, and $\Pi_r^{III*} = \frac{(q_s-c_s)^2}{4q_s}$.*

Note that Π_r^{III*} is the retailer's optimal profit when he carries his SB only. Therefore, we can consider Π_r^{III*} as the retailer's reservation profit. The retailer would only do business with the NB supplier when he can obtain a higher profit than his reservation profit.

In the case II, the retailer decides to carry both NB and SB. Following the demand functions specified in Table 2.1, the retailer has to set the retail prices such that $p_n - q_n + q_s \leq p_s \leq \frac{p_n q_s}{q_n}$, under which both products have positive demands and we have

$$Q_n = D_n = 1 - \frac{p_n - p_s}{q_n - q_s}, Q_s = D_s = \frac{p_n - p_s}{q_n - q_s} - \frac{p_s}{q_s}.$$

Subsequently, the retailer's profit function reduces to

$$\Pi_r^{II}(p_n, p_s) = (p_n - w)(1 - \frac{p_n - p_s}{q_n - q_s}) + (p_s - c_s)(\frac{p_n - p_s}{q_n - q_s} - \frac{p_s}{q_s}) - T.$$

Taking derivatives of Π_r^{II} with respect to p_n and p_s, we obtain

$$\begin{aligned}\frac{\partial \Pi_r^{II}}{\partial p_n} &= 1-\frac{2(p_n-p_s)-w+c_s}{q_n-q_s}, \frac{\partial \Pi_r^{II}}{\partial p_s}=\frac{2(p_n-p_s)-w+c_s}{q_n-q_s}-\frac{2p_s-c_s}{q_s},\\ \frac{\partial^2 \Pi_r^{II}}{\partial p_n^2} &= \frac{-2}{q_n-q_s}<0, \frac{\partial^2 \Pi_r^{II}}{\partial p_n^2}\frac{\partial^2 \Pi_r^{II}}{\partial p_s^2}-(\frac{\partial^2 \Pi_r^{II}}{\partial p_n \partial p_s})^2=\frac{4}{q_s(q_n-q_s)}>0.\end{aligned}$$

Hence, we show the Hessian matrix of the retailer's profit function Π_r^{II} is negative definite. Then, by setting $\frac{\partial \Pi_r^{II}}{\partial p_n}=0$ and $\frac{\partial \Pi_r^{II}}{\partial p_s}=0$, we get the interior optimal solution

$$p_n^{II*}=\frac{w+q_n}{2}, p_s^{II*}=\frac{c_s+q_s}{2} \tag{2.4.2}$$

Furthermore, we need to derive the conditions under which these retail prices satisfy the aforementioned condition, $p_n-q_n+q_s \le p_s \le \frac{p_n q_s}{q_n}$. When p_n^{II*} and p_s^{II*} do not satisfy this condition, the retailer's maximum profit under case II would be achieved at a corner solution. By carefully comparing the retailer's maximum profits under cases I, II, and III, we can characterize the retailer's best response for any given contract (w, T) offered by the supplier as follows:

Proposition 2.1.

(I) If $0 \le w \le \frac{c_s q_n}{q_s}$ and $T \le \bar{T}^I(w)$ where

$$\bar{T}^I(w) \overset{def}{=} \frac{(q_n-w)^2}{4q_n}-\frac{(q_s-c_s)^2}{4q_s}, \tag{2.4.3}$$

then the retailer shall carry NB only, and $p_n^*=p_n^{I*}, Q_n^*=Q_n^{I*}$ given in Lemma 2.1,

(II) If $\frac{c_s q_n}{q_s}<w \le c_s+q_n-q_s$ and $T \le \bar{T}^{II}(w)$ where

$$\bar{T}^{II}(w) \overset{def}{=} \frac{(c_s-w+q_n-q_s)^2}{4(q_n-q_s)}, \tag{2.4.4}$$

then the retailer will carry both NB and SB, and $p_n^*=p_n^{II*}, p_s^*=p_s^{II*}$ given by (2.4.2), $Q_n^*=1-\frac{w-c_s}{2(q_n-q_s)}, Q_s^*=\frac{w-c_s}{2(q_n-q_s)}-\frac{c_s+q_n}{2q_s}$.

(III) If $w \leq \frac{c_s q_n}{q_s}$ and $T > \bar{T}^I(w)$ or $\frac{c_s q_n}{q_s} < w \leq c_s + q_n - q_s$ and $T > \bar{T}^{II}(w)$, or $w > c_s + q_n - q_s$ and $T \geq 0$, the retailer will carry SB only, and $p_s^* = p_s^{III*}, Q_s^* = Q_s^{III*}$ given in Lemma 2.1.

It is straightforward to show that $\bar{T}^I(w) = \bar{T}^{II}(w) = \frac{(q_s - c_s)^2(q_n - q_s)}{4q_s^2}$ at $w = \frac{c_s q_n}{q_s}$ and $\bar{T}^{II}(w) = 0$ at $w = c_s + q_n - q_s$. Since $\frac{c_s q_n}{q_s} < c_s + q_n - q_s$ due to $q_n > q_s$, Proposition 2.1 completely characterize the retailer's optimal response for any given (w, T) with $w \geq 0, T \geq 0$ chosen by the NB supplier. Proposition 2.1 can be further illustrated by the following Figure 2.3.

Figure 2.3: Retailer's Best Response given any (w, T).

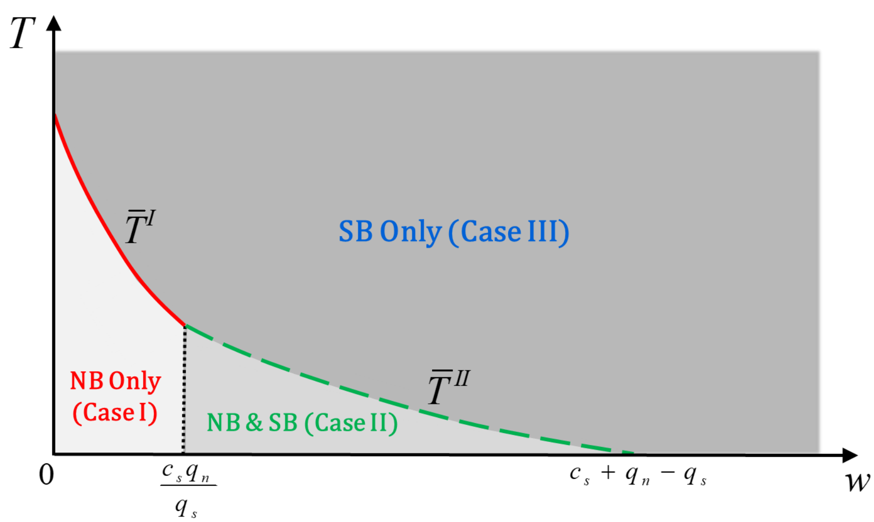


Figure 2.3 shows that for any wholesale price, $w \in [0, c_s + q_n - q_s]$, if the supplier charges a fixed payment T below the threshold $\bar{T}^I(w)$ or $\bar{T}^{II}(w)$, the retailer would carry the NB, otherwise, the retailer would not carry the NB. Furthermore, Figure 3 shows that when the supplier charges a wholesale price low enough, i.e., $w \leq \frac{c_s q_n}{q_s}$ and a fixed payment below the threshold $\bar{T}^I(w)$, then it is best for the retailer to carry NB only; when the wholesale price is in the medium range, (i.e., $\frac{c_s q_n}{q_s} < w \leq c_s + q_n - q_s$) and the fixed payment is below the threshold $\bar{T}^{II}(w)$, the retailer would carry both NB and SB; when the wholesale price is too

high (i.e., $w > c_s + q_n - q_s$) or the fixed payment is above the threshold, the retailer would carry SB only.

2.5 Supplier's problem

Unlike the retailer's problem, the supplier's problem under asymmetric information differs from that under symmetric information. In this section, we first solve the supplier's problem under symmetric information as the benchmark for the asymmetric information case, then we solve the supplier's problem under asymmetric information.

2.5.1 Symmetric Information

Under symmetric information, the retailer's type is known to the national brand supplier. Hence, based on the retailer's best response summarized in Proposition 2.1, we need to figure out the supplier's optimal contract, (w^*, T^*), which maximizes her profit function:

$$\Pi_n(w,T) = \begin{cases} \Pi_n^I = (w - c_n)(\frac{1}{2} - \frac{w}{2q_n}) + T, & \text{if } w \le \frac{c_s q_n}{q_s} \;\&\; T \le \bar{T}^I(w) \\ \Pi_n^{II} = (w - c_n)[1 - \frac{w - c_s}{2(q_n - q_s)}] + T, & \text{if } \frac{c_s q_n}{q_s} < w \le c_s + q_n - q_s \;\&\; T \le \bar{T}^{II}(w) \\ 0, & \text{otherwise.} \end{cases} \quad (2.5.1)$$

After some algebra, we derive the following result:

Lemma 2.2. *Under symmetric information,*

(I) If $c_n \le \frac{c_s q_n}{q_s}$, then the supplier's optimal contract is $w^* = c_n, T^* = \bar{T}^I(c_n) = \frac{(q_n - c_n)^2}{4q_n} - \frac{(q_s - c_s)^2}{4q_s}$, under which the retailer only carries NB, $\Pi_n^* = \frac{(q_n - c_n)^2}{4q_n} - \frac{(q_s - c_s)^2}{4q_s}$, and $\Pi_r^* = \Pi_r^{III*}$ given in Lemma 2.1.

(II) If $\frac{c_s q_n}{q_s} < c_n \le c_s + q_n - q_s$, the supplier's optimal contract is $w^* = c_n$, $T^* = \bar{T}^{II}(c_n) = \frac{(q_n - c_n - q_s + c_s)^2}{4(q_n - q_s)}$, under which the retailer carries both NB and SB, $\Pi_n^* = \frac{(q_n - c_n - q_s + c_s)^2}{4(q_n - q_s)}$, and $\Pi_r^* = \Pi_r^{III*}$.

(III) If $c_n > c_s + q_n - q_s$, it is not profitable for the national brand supplier to sell the NB product to the retailer and the retailer only carries SB, i.e., $\Pi_n^* = 0$ and $\Pi_r^* = \Pi_r^{III*}$.

Lemma 2.2 suggests that under symmetric information, the national brand supplier will charge a wholesale price equal to her production cost and a fixed payment as high as the threshold to earn her first-best profit, and the retailer always earns her reservation profit, i.e., Π_r^{III*}, which is his optimal profit when he sells his SB only. In addition, NB production cost plays a critical role in the supplier's problem. When the NB cost is low, the supplier is able to offer a low wholesale price and fixed payment to prevent the retailer from introducing his own SB; when the NB cost is medium, the supplier is not able to lower his wholesale price and fixed payment to prevent the retailer from introducing his SB, and the retailer would carry both NB and SB; when the NB cost is too high, it is best for the supplier not to do business with the retailer and the retailer would carry his SB only.

2.5.2 Asymmetric Information

When the retailer holds private information on his store brand cost, the national brand supplier can no longer achieve her first-best profit. Instead, she confronts the challenge of designing a menu of incentive compatible contracts to extract the retailer's private information by giving up the information rent to the retailer. Based on the revelation principle, the supplier needs to offer a menu of two contracts, i.e., $(w^L, T^L), (w^H, T^H)$, corresponding to the two cost types of the SB, to maximize her expected profit subject to the incentive compatibility (IC) and the individual rationality (IR) constraints. Hence, the supplier's problem can be described as as follows:

$$\max_{(w^H,T^H),(w^L,T^L)} \Pi_n = v\Pi_n(w^H,T^H,c_s^H) + (1-v)\Pi_n(w^L,T^L,c_s^L). \tag{2.5.2a}$$

subject to

$$\text{(I.C.H)} \quad \Pi_r(w^H,T^H,c_s^H) \geq \Pi_r(w^L,T^L,c_s^H) \tag{2.5.2b}$$

$$\text{(I.C.L)} \quad \Pi_r(w^L,T^L,c_s^L) \geq \Pi_r(w^H,T^H,c_s^L) \tag{2.5.2c}$$

$$\text{(I.R.H)} \quad \Pi_r(w^H,T^H,c_s^H) \geq \Pi_r^{III*}(c_s^H) \tag{2.5.2d}$$

$$\text{(I.R.L)} \quad \Pi_r(w^L,T^L,c_s^L) \geq \Pi_r^{III*}(c_s^L), \tag{2.5.2e}$$

in which $\Pi_r(w^i,T^i,c_s^j)$ $(i,j = L,H)$ denotes the type-j retailer's resulting profit when he chooses contract (w^i,T^i), and $\Pi_n(w^i,T^i,c_s^i)$ $(i = L,H)$ represents the supplier's profit when the type-i retailer chooses contract (w^i,T^i). Constraints (I.R.H.) and (I.R.L.) ensure that both types of the retailer earn at least their respective reservation profits (i.e., Π_r^{III*}) therefore do business with the supplier. It is worth mentioning that the IR constraints here are type-dependent because the retailer's reservation profits vary between the two types, which brings additional complexity to this problem. Moreover, it follows from the equation of Π_r^{III*} in Lemma 2.1 that $\Pi_r^{III}(c_s^L) > \Pi_r^{III}(c_s^H)$ due to $c_s^L < c_s^H$, which indicates that the low-cost retailer has a higher reservation profit. Constraints (I.C.H.) and (I.C.L.) guarantee that each type of the retailer would be more profitable by choosing the right contract consistent with his true type than by choosing the wrong contract.

To reduce the complexity of the optimality search, we further examine the four constraints and recognize a redundant constraint.

Lemma 2.3. *Following (I.C.H.), (I.R.L.), and $c_s^L < c_s^H$, (I.R.H.) is a redundant constraint.*

Lemma 2.3 implies that (I.R.H) will not be binding for any feasible solution to the supplier's problem, which means that the high-type retailer is guaranteed to earn a profit strictly higher than his reservation profit. This result can be explained intuitively as follows:

Recall that the low-type retailer is more cost effective in producing the SB than the high-type retailer (due to $c_s^L < c_s^H$), and the low-type retailer has a higher reservation profit than the high-type retailer. Hence, the high-type retailer has an incentive to mimic the low-type retailer, but the low-type retailer has no incentive to mimic the high-type retailer. As a result, the supplier has to offer the high-type retailer an information rent, i.e., the additional profit above his reservation profit, to prevent the high-type retailer from mimicking the low-type retailer.

As specified in Proposition 2.1, for any given contract (w, T) offered by the NB supplier, the retailer's best response falls into one of the three cases: (I) the retailer carries NB only; (II) the retailer carries NB and SB, and (III) the retailer carries SB only. Although constraints (I.R.H) and (I.R.L) ensure that both types of the retailer would do business with the NB supplier, i.e., the retailer would react according to case (I) or (II), the retailer's profit function Π_r given in (2.4.1) and the supplier's profit function Π_n given in (2.5.1) have different function forms under case (I) and case (II). Therefore, in order to derive the optimal menu of contracts for the supplier, we have to break down the supplier's optimization problem (2.5.2) into four subproblems corresponding to four possible options of the supplier's decision variables, denoted by (H^I, L^I), (H^{II}, L^I), (H^I, L^{II}), and (H^{II}, L^{II}), where we define (H^i, L^j) $(i, j = I, II)$ as the subset of the supplier's menu of contracts which would induce the high-type retailer to select case i and the low-type retailer to choose case j. Further study on these four contract options reveals the following result:

Lemma 2.4. *(H^{II}, L^I) contract is not a feasible option for the supplier.*

The definition of (H^{II}, L^I) contract implies that this subset of (w^L, T^L) and (w^H, T^H) would induce the the low-type retailer to carry SB only and the high-type retailer to carry both NB and SB. If (w^L, T^L) is good enough for the low-type retailer to carry SB only, then it must be good enough for the high-type retailer to carry SB only because Proposition

2.1 and $c_s^L < c_s^H$ imply that if $w^L \leq c_s^L$ & $T^L \leq \bar{T}^I(w^L|c_s = c_s^L)$, then $w^L \leq c_s^H$ & $T^L \leq \bar{T}^I(w^L|c_s = c_s^H)$. Moreover, constraint (I.C.H) ensures that (w^H, T^H) is more attractive than (w^L, T^L) to the high-type retailer. Hence, (w^H, T^H) must also be good enough to induce the high-type retailer to carry NB only. Consequently, (H^{II}, L^I) is never a feasible option for the supplier.

Lemma 2.4 slims down the range of our optimality search. Thereafter, we only need to analyze three contract options, (H^I, L^I), (H^I, L^{II}), and (H^{II}, L^{II}).

Recall from Lemma 2.3 that (I.R.H) is a redundant constraint. In fact, constraint (I.C.L) is redundant as well. First, we shall obtain the optimal solution to the supplier's problem in (2.5.2) by considering (I.C.H) and (I.R.L) constraints only, and later we would show that this optimal solution indeed satisfies constraint (I.C.L). Second, we would demonstrate constraints (I.C.H.) and (I.R.L.) must be binding at optimality. Lemma 2.1 and Proposition 2.1 imply that for any (w, T) offered by the supplier, the retailer's optimal decisions in p_n, p_s, Q_n, Q_s are not directly affected by T. Moreover, (2.4.1) and (2.5.1) show that the retailer's profit is decreasing in the fixed payment T, and the supplier's profit is increasing in T. Therefore, if (I.C.H) or (I.R.L) is not binding at optimality, then the national brand supplier could further improve her profit by simply increasing T^L or T^H until both (I.C.H) and (I.R.L) are binding.

Using the binding constrains (I.R.L) and (I.C.H), we can derive T^L and T^H as functions of w^L and w^H. In this way, we can convert the supplier's problem in (2.5.2) into an unconstrained optimization problem, i.e., $\max \Pi_n(w^L, w^H)$.

Before proceeding the analysis, for notational convenience, we summarize the $\bar{T}$ functions given in (2.4.3)-(2.4.4) as follows: for all $i = L, H$,

$$\bar{T}^i(w) \overset{\text{def}}{=} \begin{cases} \bar{T}^{Ii}(w) = \dfrac{(q_n - w)^2}{4q_n} - \dfrac{(q_s - c_s^i)^2}{4q_s}, & \text{if } 0 \le w \le \dfrac{c_s^i q_n}{q_s} \\ \bar{T}^{IIi}(w) = \dfrac{(q_n - w + c_s^i - q_s)^2}{4q_s}, & \text{if } \dfrac{c_s^i q_n}{q_s} < w \le c_s^i + q_n - q_s \\ 0, & \text{if } w > c_s^i + q_n - q_s. \end{cases} \tag{2.5.3}$$

The definition of $\bar{T}^i$ here unifies $\bar{T}^I$ and $\bar{T}^{II}$ given in Proposition 2.1 for both types of the retailer. Additionally, we establish the following relationship between $\bar{T}^H(w)$ and $\bar{T}^L(w)$.

Lemma 2.5. *For all $w \ge 0$, $\bar{T}^H(w) > \bar{T}^L(w)$.*

Consistent with our intuition, Lemma 2.5 simply indicates that when the store brand cost increases, the retailer becomes less competitive, then the supplier is able to charge a higher fixed payment, i.e., the threshold of the fixed payment $\bar{T}$ increases as well.

2.5.2.1 (H^I, L^I) Contract

According to Proposition 2.1, (H^I, L^I) contract refers to a subset of the supplier's menu of contracts such that $(w^H, T^H), (w^L, T^L)$ satisfying the conditions of $w^H \le \frac{c_s^H q_n}{q_s}$, $T^H \le \bar{T}^H(w^H) = \bar{T}^{IH}(w^H)$, $w^L \le \frac{c_s^L q_n}{q_s}$, $T^L \le \bar{T}^L(w^L) = \bar{T}^{IL}(w^L)$. Under (H^I, L^I) contract, the retailer would carry the NB only regardless of his type, i.e.,

$$\Pi_r(w^i, T^i, c_s^i) = \Pi_r^{I*}(w^i, T^i, c_s^i) \overset{\text{lemma 2.1}}{=} \frac{(q_n - w^i)^2}{4q_n} - T^i, \forall i = L, H.$$

In this case, following (2.5.1), the supplier's profit function is expressed as

$$\Pi_n = v\left[(w^H - c_n)(\frac{1}{2} - \frac{w^H}{2q_n}) + T^H\right] + (1 - v)\left[(w^L - c_n)(\frac{1}{2} - \frac{w^L}{2q_n}) + T^L\right]$$

In conjunction with $c_s^L < c_s^H$, we have $w^L \le \frac{c_s^L}{q_n} < \frac{c_s^H}{q_n}$ and $T^L \le \bar{T}^{IL}(w^L) \overset{(2.5.3)}{\le} \bar{T}^{IH}(w^L)$. Therefore, if the high-type retailer chooses (w^L, T^L), he will still carry NB only (case I), i.e., $\Pi_r(w^L, T^L, c_s^H) = \Pi_r^{I*}(w^L, T^L, c_s^H)$. From the binding constraints (I.C.H.) and (I.R.L.), we can directly derive that T^L and T^H as functions of w_L and w_H below.

$$T^L = \bar{T}^{IL}(w^L) = \frac{(q_n - w^L)^2}{4q_n} - \frac{(q_s - c_s^L)^2}{4q_s},$$

and

$$T^H = \frac{(q_n - w^H)^2}{4q_n} - \frac{(q_n - w^L)^2}{4q_n} + T^L = \frac{(q_n - w^H)^2}{4q_n} - \frac{(q_s - c_s^L)^2}{4q_s}. \tag{2.5.4}$$

2.5.2.2 (H^I, L^{II}) Contract

According to Proposition 2.1, (H^I, L^{II}) contract refers to a subset of $(w^H, T^H), (w^L, T^L)$ which satisfies the conditions of $w^H \leq \frac{c_s^H q_n}{q_s}, T^H \leq \bar{T}^{IH}(w^H), \frac{c_s^L q_n}{q_s} < w^L \leq c_s^L + q_n - q_s,\ T^L \leq \bar{T}^{IIL}(w^L)$. Under (H^I, L^{II}) contract, the high-type retailer would carry the NB only and the low-type retailer would carry both NB and SB. Consequently, the supplier's expected profit function becomes

$$\Pi_n = v[(w^H - c_n)(\frac{1}{2} - \frac{w^H}{2q_n}) + T^H] + (1-v)[(w^L - c_n)(\frac{1}{2} - \frac{w^L - c_s^L}{2(q_n - q_s)}) + T^L]$$

Setting (I.R.L.) to be binding, we have $\Pi_r(w^L, T^L, c_s^L) = \Pi_r^{II}(w^L, T^L, c_s^L) = \Pi_r^{III*}(c_s^L)$, which implies

$$T^L = \bar{T}^{IIL}(w^L) \overset{(2.5.3)}{=} \frac{(c_s^L - w^L + q_n - q_s)^2}{4(q_n - q_s)}.$$

Unlike the previous (H^I, L^I) case, under (H^I, L^{II}) contract, the high-type retailer's response to the low-type contract is not definite, i.e., the function form of $\Pi_r(w^L, T^L, c_s^H)$ varies depending on the model parameters. Due to $q_n > q_s$, we have $\frac{c_s^i q_n}{q_s} < c_s^i + q_n - q_s$ for all $i = L, H$. In conjunction with $c_s^L < c_s^H$, we have $\frac{c_s^L q_n}{q_s} < \frac{c_s^H q_n}{q_s} < c_s^H + q_n - q_s$. However, the relationship between $\frac{c_s^H q_n}{q_s}$ and $c_s^L + q_n - q_s$ is indefinite. Therefore, we need to consider the following two parameter settings for the cost-quality relationship, i.e., $c_s^L + q_n - q_s \leq \frac{c_s^H q_n}{q_s}$ and $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$.

(i) $c_s^L + q_n - q_s \leq \frac{c_s^H q_n}{q_s}$

In this parameter setting, we have $w^L \le c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s}$, $T^L = \bar{T}^{IIL}(w^L) = \bar{T}^L(w^L) < \bar{T}^H(w^L) = \bar{T}^{IH}(w^L)$ (by Lemma 2.5). Proposition 2.1 implies that $\Pi_r(w^L, T^L, c_s^H) = \Pi_r^I(w^L, T^L, c_s^H)$. As a result, setting (I.C.H.) to be binding, we obtain T^H as a function of w_L and w_H, i.e.,

$$T^H = \frac{(c_s^L - w^L + q_n - q_s)^2}{4(q_n - q_s)} + \frac{(q_n - w^H)^2}{4q_n} - \frac{(q_n - w^L)^2}{4q_n} \tag{2.5.5}$$

(ii) $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$

Under this parameter setting, we need to distinguish two possible subcases that differentiate on w^L: $w^L \le \frac{c_s^H q_n}{q_s}$, and $w^L > \frac{c_s^H q_n}{q_s}$. These two subcontracts are notated as $(H^I, L^{II} : \frac{c_s^L q_n}{q_s} < w^L \le \frac{c_s^H q_n}{q_s})$ and $(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$.

(ii.1) $(H^I, L^{II} : \frac{c_s^L q_n}{q_s} < w^L \le \frac{c_s^H q_n}{q_s})$

Since $w^L \le \frac{c_s^H q_n}{q_s}$, $T^L = \bar{T}^L(w^L) < \bar{T}^H(w^L)$ as in case (i), we still have $\Pi_r(w^L, T^L, c_s^H) = \Pi_r^I(w^L, T^L, c_s^H)$. Setting (I.C.H.) to be binding yields that T^H follows the same equation given in (2.5.7).

(ii.2) $(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$

For any $w^L \in (\frac{c_s^H q_n}{q_s}, c_s^L + q_n - q_s]$ and $T^L = \bar{T}^{IIL}(w^L)$, we have $\frac{c_s^H q_n}{q_s} < w^L < c_s^H + q_n - q_s$ and $T^L = \bar{T}^L(w^L) < \bar{T}^H(w^L) = \bar{T}^{IIH}(w^L)$. Therefore, Proposition 2.1 implies that if a high-type retailer chooses (w^L, T^L), he shall carry both NB and SB, i.e., $\Pi_r(w^L, T^L, c_s^H) = \Pi_r^{II}(w^L, T^L, c_s^H)$. Setting (I.C.H.) to be binding yields that

$$T^H = \frac{(q_n - w^H)^2}{4q_n} - \frac{q_n - w^L}{2}\left(\frac{1}{2} - \frac{w^L - c_s^H}{2(q_n - q_s)}\right) + \frac{q_s - c_s^H}{2}\left(\frac{w^L - c_s^H}{2(q_n - q_s)} - \frac{c_s^H}{2q_s}\right) + \frac{(c_s^L - w^L + q_n - q_s)^2}{4(q_n - q_s)}. \tag{2.5.6}$$

2.5.2.3 (H^{II}, L^{II}) Contract

Proposition 2.1 implies that (H^{II}, L^{II}) contract refers to a subset of $(w^H, T^H), (w^L, T^L)$ satisfying the conditions of $\frac{c_s^H q_n}{q_s} < w^H \leq c_s^H + q_n - q_s$, $T^H \leq \bar{T}^{IIH}(w^H)$ and $\frac{c_s^L q_n}{q_s} < w^L \leq c_s^L + q_n - q_s$, $T^L \leq \bar{T}^{IIL}(w^L)$. Under (H^{II}, L^{II}) contract, the retailer shall carry both brands regardless of his type. Consequently, the supplier's expected profit function becomes

$$\Pi_n = v[(w^H - c_n)(\frac{1}{2} - \frac{w^H - c_s^H}{2(q_n - q_s)}) + T^H] + (1-v)[(w^L - c_n)(\frac{1}{2} - \frac{w^L - c_s^L}{2(q_n - q_s)}) + T^L].$$

The binding (I.R.L) constraint implies that $T^L = \bar{T}^{IIL}(w^L)$. Similar to (H^I, L^{II}) contract, it is complex to predict the high-type retailer's response if he chooses the low-type contract. Following the similar steps, we can derive the T^H as a function of w^L, w^H as below:

(i) If $c_s^L + q_n - q_s \leq \frac{c_s^H q_n}{q_s}$ or $\{\frac{c_s^H q_n}{q_s} > c_s^L + q_n - q_s$ & $\frac{c_s^L q_n}{q_s} < w^L \leq \frac{c_s^H q_n}{q_s}\}$, then

$$\begin{aligned} T^H &= \frac{q_n - w^H}{2}(\frac{1}{2} - \frac{w^H - c_s^H}{2(q_n - q_s)}) + \frac{q_s - c_s^H}{2}(\frac{w^H - c_s^H}{2(q_n - q_s)} - \frac{c_s^H}{2q_s}) + \\ & \quad \frac{(c_s^L - w^L + q_n - q_s)^2}{4(q_n - q_s)} - \frac{(q_n - w^L)^2}{4q_n}. \end{aligned} \tag{2.5.7}$$

(ii) If $\frac{c_s^H q_n}{q_s} < w^L \leq c_s^L + q_n - q_s$, then

$$T^H = \frac{{c_s^L}^2 + (w^H - q_n + q_s)^2 + 2c_s^L(q_n - q_s - w^L) - 2c_s^H(w^H - w^L)}{4(q_n - q_s)}. \tag{2.5.8}$$

2.5.2.4 Optimal Contract

Substituting the T^L and T^H functions into each corresponding objective function respectively, the resulting Π_n becomes a two-dimensional function of w^L and w^H. To solve for the optimal w^{L*} and w^{H*} for the supplier's problem, we make two technical assumptions for the remainder of the chapter as follows:

1. We assume $(1-v)q_n - vq_s > 0$ to ensure the concavity of the objective function on w^L for any $w^L \in (\frac{c_s^L q_n}{q_s}, c_s^L + q_n - q_s]$. Otherwise, if Π_n is convex on w^L, the case II contract

Table 2.3: Optimal $(w^{H*},\, w^{L*})$ under Parameter Setting $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$

c_n interval:		$c_n \le \frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s} < c_n \le c_n^{(1)}$	$c_n^{(1)} < c_n \le c_s^L + q_n - q_s$
(H^I, L^I)	w^{H*}	c_n	c_n	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	c_n	$\frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s}$
(H^I, L^{II})	w^{H*}	c_n	c_n	c_n
	w^{L*}	$\frac{c_s^L q_n}{q_s}$	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$c_s^L + q_n - q_s$
(H^{II}, L^{II})	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{c_s^L q_n}{q_s}$	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$c_s^L + q_n - q_s$

for low type, L^{II}, will always be dominated by L^I or be better off to not offer any contract (case III).

2. We only discuss under $c_n \le c_s^L + q_n - q_s$ such that both types are willing to carry NB.

With some algebra, we obtain the local optimal whole prices under each contract type, highlight in blue in Table 2.3 and Table 2.4. Note that $c_n^{(1)} \overset{\text{def}}{=} \frac{c_s^L(q_n - vq_s) + (q_n - q_s)((1-v)q_n - vq_s)}{(1-v)q_n}$, $c_n^{(2)} \overset{\text{def}}{=} \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$, and $c_n^{(3)} \overset{\text{def}}{=} c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$.

In order to search for the global optimality, we compare the derived optimal solutions among all the contract types and parameter settings. As presented in Table 2.3 and 2.4, we need to analyze the national brand supplier's expected profit function under the two parallel parameter settings, i.e., $c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s}$ and $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$. If $c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s}$, there are three possible contract options (H^I, L^I), (H^I, L^{II}), (H^{II}, L^{II}); if $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, there are five possible contract options (H^I, L^I), $(H^I, L^{II} : \frac{c_s^L q_n}{q_s} < w^L \le \frac{c_s^H q_n}{q_s})$, $(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$, $(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L \le \frac{c_s^H q_n}{q_s})$ and $(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$. We compare the supplier's optimal profit across these contracts for each parameter setting. The results are summarized in the Proposition below.

Proposition 2.2. *Under asymmetric information, the national brand supplier's optimal*

Table 2.4: Optimal (w^{H*}, w^{L*}) under Parameter Setting $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$

(a) Scenario 1: $c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v} < \frac{c_s^H q_n}{q_s}$

c_n interval:		$c_n \le \frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s} < c_n \le c_n^{(2)}$	$c_n^{(2)} < c_n \le c_n^{(3)}$	$c_n^{(3)} < c_n < \frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s} < c_n \le c_s^L + q_n - q_s$
(H^I, L^I)	w^{H*}	c_n	c_n	c_n	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	c_n	$\frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s}$
$(H^I, L^{II} : \frac{c_s^L q_n}{q_s} < w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	c_n	c_n	c_n	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{c_s^L q_n}{q_s}$	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	c_n	c_n	c_n	c_n	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_s^L + q_n - q_s$	$c_s^L + q_n - q_s$
$(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	c_n	c_n
	w^{L*}	$\frac{c_s^L q_n}{q_s}$	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	c_n
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_s^L + q_n - q_s$	$c_s^L + q_n - q_s$

(b) Scenario 2: $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$

c_n interval:		$c_n \le \frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s} < c_n \le c_n^{(2)}$	$c_n^{(2)} < c_n \le \frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s} < c_n < c_n^{(3)}$	$c_n^{(3)} < c_n \le c_s^L + q_n - q_s$
(H^I, L^I)	w^{H*}	c_n	c_n	c_n	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	c_n	$\frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s}$	$\frac{c_s^L q_n}{q_s}$
$(H^I, L^{II} : \frac{c_s^L q_n}{q_s} < w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	c_n	c_n	c_n	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{c_s^L q_n}{q_s}$	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	c_n	c_n	c_n	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_s^L + q_n - q_s$
$(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L \le \frac{c_s^H q_n}{q_s})$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	c_n	c_n
	w^{L*}	$\frac{c_s^L q_n}{q_s}$	$\frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$
$(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L \le c_s^L + q_n - q_s)$	w^{H*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	c_n	c_n
	w^{L*}	$\frac{c_s^H q_n}{q_s}$	$\frac{c_s^H q_n}{q_s}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_n + \frac{v(c_s^H - c_s^L)}{1-v}$	$c_s^L + q_n - q_s$

menu of contracts is as follows:

(i) If $c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s}$*, then the national brand supplier's optimal contract* (H^*, L^*) *is given by*

$$(H^*, L^*) \quad = \quad \begin{cases} (H^I, L^I), & \text{if } \ c_n \le \frac{c_s^L q_n}{q_s} \\ (H^I, L^{II}), & \text{if } \ \frac{c_s^L q_n}{q_s} < c_n \le c_s^L + q_n - q_s. \end{cases}$$

(ii) If $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$*, then the national brand supplier's optimal contract* (H^*, L^*) *is given by*

$$(H^*, L^*) \quad = \quad \begin{cases} (H^I, L^I), & \text{if } \ c_n \le \frac{c_s^L q_n}{q_s} \\ (H^I, L^{II}), & \text{if } \ \frac{c_s^L q_n}{q_s} < c_n \le \frac{c_s^H q_n}{q_s} \\ (H^{II}, L^{II}), & \text{if } \ \frac{c_s^H q_n}{q_s} < c_n \le c_s^L + q_n - q_s. \end{cases}$$

Proposition 2.2 verifies that under asymmetric information the national brand's production cost still has a significant impact on the supplier's optimal contract design. Low-type retailer is more competitive than the high-type retailer. As the NB production cost increases, the supplier firstly loses the capability to prevent the introduction of SB for the low-type retailer, and then that of the high-type retailer.

The detailed optimal decisions for the supplier are summarized below.

Corollary 2.1. *The optimal menu of contracts* $(w^{H*}, T^{H*}), (w^{L*}, T^{L*})$*, i.e., the solution to* (2.5.2)*, is*

(i) If $c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s}$*, then*

$$w^{L*} \quad = \quad \begin{cases} c_n, & \text{if } \ c_n \le \frac{c_s^L q_n}{q_s} \\ \frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}, & \text{if } \ \frac{c_s^L q_n}{q_s} < c_n \le c_n^{(1)} \\ c_s^L + q_n - q_s, & \text{if } \ c_n^{(1)} < c_n \le c_s^L + q_n - q_s. \end{cases}$$

(ii) If $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$*, then*

$$w^{L*} = \begin{cases} c_n, & \text{if } c_n \le \frac{c_s^L q_n}{q_s} \\ \frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}, & \text{if } \frac{c_s^L q_n}{q_s} < c_n \le c_n^{(2)} \\ c_n + \frac{v(c_s^H - c_s^L)}{1-v}, & \text{if } c_n^{(2)} < c_n \le c_n^{(3)} \\ c_s^L + q_n - q_s, & \text{if } c_n^{(3)} < c_n \le c_s^L + q_n - q_s. \end{cases}$$

$T^{L*} = \bar{T}^L(w^{L*})$ *given in (2.5.3),* $w^{H*} = c_n$*, and* T^{H*} *can be found by substituting* w^{H*} *and* w^{L*} *into the respective* T^H *function given in (2.5.4)-(2.5.8) under the optimal contract* (H^*, L^*)*.*

After substituting w^{H*} and w^{L*} into T^{H*}, we do have $T^{H*} \ge \bar{T}^L(w^{H*})$. In conjunction with Proposition 2.1 and the binding (I.R.L) constraint, it is straightforward to obtain $\Pi_r(w^{L*}, T^{L*}, c_s^L) = \Pi_r^{III*}(c_s^L) \ge \Pi_r(w^{H*}, T^{H*}, c_s^L)$, which implies that constraint (I.C.L) is indeed satisfied by the optimal solution.

2.6 Managerial Implication

In this section, we investigate how the model parameters influence the value of information for the national brand supplier and the retailer. Proposition 2.2 has shown that the optimal contract varies based on the parameters such as NB and SB production cost and product quality. In-depth analysis on these parameters shall provide useful managerial insights for both the supplier and the retailer.

The value of information for each entity in the supply chain is quantified as the difference between his (her) optimal expected profits under symmetric information (denoted as $\Pi_i^{Sym*}, i = n, r$) and asymmetric information (denoted as $\Pi_i^{Aym*}, i = n, r$). Under symmetric information, the supplier, as the leader of the Stackelberg game, will extract the entire supply chain profit and leave the retailer only his reservation profit regardless of his type. However, in the presence of information asymmetry, the supplier would give up some information rent to the retailer so the retailer is induced to truthfully reveal his type. Our results show that the high-type retailer earns a positive information rent in addition to his reservation profit and the low-type retailer still obtains the reservation profit only. That is, the retailer's overall expected profit under asymmetric information is higher than that under symmetric information. Consequently, the retailer's valuation of his own SB cost information is $V_r = \Pi_r^{Sym*} - \Pi_r^{Asy*} = -vU$, where U denotes the information rent given up by the supplier to the high-type retailer. To simplify the interpretations, we use the absolute value of V_r, i.e., $|V_r|$, to reference the value of information to the retailer in the following discussions. On the contrary, the NB supplier's profit under symmetric information is higher than that under asymmetric information, the value of information to the supplier, i.e., $V_n = \Pi_n^{Sym*} - \Pi_n^{Asy*}$, is positive. We first compare the values of information to each entity and obtain the following Proposition.

Proposition 2.3. $V_n \geq |V_r|$.

Proposition 2.3 implies that the value of information to the NB supplier is higher than that to the retailer. Under symmetric information, as indicated in Lemma 2.2, the NB supplier's optimal wholesale price is always equal to her production cost regardless of the retailer's type. Hence, there exists no efficiency loss for both types of the retailer. However, under asymmetric information, there is efficiency loss for the low-type retailer due to the double marginalization, as indicated in Corollary 2.1 that $w^{L*} \geq c_n = w^{H^*}$. As a result, under asymmetric information, the NB supplier has to not only surrender the information rent to the high-type retailer, but also suffer from the efficiency loss for the low-type retailer. Recall that $|V_r|$ represents the retailer's expected information rent only. Hence, the value of information to the NB supplier is greater than that to the retailer.

In the following subsections, we explore the impact of a change in the NB/SB cost or quality on the value of information to the supplier or the retailer. To facilitate the discussions, we refer to "decrease" or "increase" in a weak sense, i.e., "decrease" means "non-increase", and "increase" means "non-decrease".

2.6.1 Production Cost

We first explore how a change in the unit production cost of the NB affects the value of information to each entity while keeping other parameters constant.

Proposition 2.4. *V_n and $|V_r|$ are decreasing in c_n.*

In the presence of asymmetric information, the national brand supplier's expected profit is compromised due to the information rent offered to the retailer. When the NB production cost increases and other model parameters remain unchanged, the supplier tends to charge higher wholesale prices to sustain her profitability. In other words, the supplier has the tendency to offer lower information rent to the retailer, i.e., $|V_r|$ decreases. When the NB production cost increases, the NB supplier's profits under symmetric and asymmetric information decrease simultaneously. However, the decrease under asymmetric information is less than that under symmetric information because the supplier offers lower information rent to the retailer under asymmetric information. Therefore, the value of information to the supplier decreases as well.

We next analyze the impact of SB costs, c_s^H and c_s^L, on the value of information to the supplier and the retailer, respectively.

Proposition 2.5. *V_n is increasing in c_s^H and decreasing in c_s^L.*

While SB cost is held as private information by the retailer, the high-type retailer has the tendency to mimic the low-type retailer. When the high-type SB cost (c_s^H) increases or the low-type SB cost (c_s^L) decreases unilaterally, the gap between the two possible values of the SB cost is enlarged. In other words, the NB supplier is more uncertain about the retailer's SB cost. Hence, the value of information to the supplier becomes higher.

We next examine the impact of c_s^H and c_s^L on the value of information to the retailer. Our analytical results indicate that V_r could either increase or decrease on $c_s^i, \forall i = L, H$. We show an examples for c_s^H and c_s^L, respectively.

EXAMPLE 1: $c_n = 509, q_n = 665, q_s = 208, c_s^L = 140, v = 0.659$. As c_s^H increases from 161 to 171 then to 181, $|V_r|$ increases from 0.8773 to 0.9756 then decreases to 0.8678.

EXAMPLE 2: $c_n = 228, q_n = 334, q_s = 251, c_s^H = 173, v = 0.259$. As c_s^L increases from 149 to 159 then to 169, $|V_r|$ increases from 0.1623 to 0.3231 then decreases to 0.1546.

Note that the information rent,$|V_r|$, is paid to the high-type retailer and weighted by the probability of high type v. The capability of paying the information rent depends on the supplier's overall expected profit and, furthermore, the competition situation between NB and SB, which is essentially influenced by the relationship among c_s^H, c_s^L and c_n. Intuitively, the supplier's relative competitive advantage could vary depending on the gap between the two possible types and the probability of the each type. Although holding other parameter constant, different combinations of c_s^H, c_s^L, v and c_n could present various partnership/competition situations between the supplier and retailer. In addition, Proposition 4 illustrates that the information rent tents to decreases on c_n whereas Proposition 5 shows a positive relationship between the value of information to the supplier and the gap between c_s^H and c_s^L. With such complication, the information rent $|V_r|$ could either increases or decrease on c_s^H and c_s^L.

2.6.2 Product Quality

We proceed to investigate how the product quality affects the value of private information for the supplier and the retailer. Recall that the product quality is a perceptional value evaluated by the customer. That is, the parameters q_n and q_s are exogenous factors in the model.

Proposition 2.6. *For the retailer, under $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$, $|V_r|$ is decreasing on q_n $\forall q_n \in$*

$(c_n - c_s^L + q_s, \frac{(1-v)c_n - c_s^L + q_s + \sqrt{((1-v)c_n - c_s^L + q_s)^2 - 4(1-v)vq_s(q_s - c_s^L)}}{2(1-v)})$ *and then increasing on* q_n $\forall q_n \geq \frac{(1-v)c_n - c_s^L + q_s + \sqrt{((1-v)c_n - c_s^L + q_s)^2 - 4(1-v)vq_s(q_s - c_s^L)}}{2(1-v)}$*; under* $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$*,* $|V_r|$ *is decreasing on* q_n $\forall q_n \in (c_n - c_s^L + q_s, c_n - c_s^L + q_s + \frac{v(c_s^H - c_s^L)}{1-v})$*, and then increasing on* q_n $\forall q_n \geq c_n - c_s^L + q_s + \frac{v(c_s^H - c_s^L)}{1-v}$.

When NB quality is very high such that $q_n > \frac{c_n q_s}{c_s^L}$, the supplier is fully capable of offer a menu of contract following which the retailer will carry NB only regardless of his type. Thus in this case the information rent is insensitive to q_n. If the NB quality is in a medium range, he starts to lose the ability of preventing the introduction of SB. Recall that the fundamental paradox in this supply chain is that if the retailer carries both products, he earns profit from selling the national brand, which also competes with his own brand. Section 4.1 has provided an intuition that when both product are in the market, a higher q_n would lead to a larger segment of demand for the national brand. As NB becomes more competitive, the retailer potentially becomes more profitable from selling NB, but demand for SB might be compromised. Meanwhile, it is easy to observe that when q_n decreases, the retailer tends to be charged with lower purchasing prices $(w^{H*}, T^{H*}, w^{L*}, T^{L*})$, which certainly diminishes the profit earned from selling NB. Hence, as q_n increases, the retailer's benefit of holding the private information could be enlarged or shrunk. Hence, the value of information to the retailer could increase or decrease.

Note that the consumers evaluate a perceptional value for each product and make the purchasing decision based on the combination of the product's price and quality. It implies that the exogenous parameters q_n and q_s have significant impacts on the demand and the resulting profits for the supplier and the retailer. However, various combinations of c_n, q_n, q_s, c_s^H and c_s^L represent different vertical partnership and horizontal competition situation between the supplier and retailer. In addition, we have analytically derived that q_n and q_s appear in both the numerators and denominators of the profit functions and the value of

Table 2.5: $c_n = 42, q_n = 110, c_s^H = 51, c_s^L = 30, v = 0.228$.

q_s	V_n	$\lvert V_r \rvert$
67	0.894	0.883
77	1.132	1.132
87	1.236	1.223
97	1.060	0.944

information functions which certainly brings in the complexity in the sensitivity analysis. Our numerical results show that as q_n increases, the value of information for the supplier could either increases or decrease, which can be demonstrated by an experiment with $c_n = 361, q_s = 482, c_s^H = 343, c_s^L = 336, v = 0.309$. As q_n increases from 493 to 513 then to 533, V_n firstly decreases from 3.42 to 0.32 then increases to 2.36. We also conduct numerical experiments to observe the impact of the SB quality on the value of information to the supplier and the retailer. Again, the results suggest that as q_s increases, V_n and $|V_r|$ could either increases or decrease. The following experiment is with $c_n = 42, q_n = 110, c_s^H = 51, c_s^L = 30, v = 0.228$. Table 2.5 summarizes the values of information for different values of q_s varying from 67 to 97.

2.7 Conclusion

We analyzed a two-tier supply chain with a national brand supplier and a retailer who is capable of developing his own SB to compete with the NB. The NB supplier offers a menu of two-part tariff contracts to the retailer to distribute the NB products to consumers. Given this supply contract, the retailer can decide whether to accept the contract and whether to produce SB. In addition, the retailer holds private cost information of the store brand, for which the NB supplier only has an estimate, i.e., high cost or low cost with certain probability. Due to the information asymmetry, a high-type retailer tends to mimic a low-type in order to pay lower procurement prices to the supplier. We applied revelation principle to design an optimal menu of contracts for the NB supplier to maximize her expected profit as well as induce the retailer to truthfully reveal his private information. Moreover, we quantify the value of information for each member as the difference between their optimal profits under symmetric information and asymmetric information, and illustrate how product characteristics (i.e., production cost and perceived quality) affect the value of information to both the supplier and the retailer. First, when the NB unit production cost increases, the private cost information becomes less valuable to both the supplier and the retailer. Second, when the gap between the high-type and low-type costs increases, the supplier considers the information to be more valuable, but the retailer's value of information may increase or decrease. Finally, we observed that when NB's perceived quality increases, the value of information to the retailer first decreases then increases, but the impact on the supplier's value of information can be positive or negative. In addition, our numerical results demonstrated that the respective value of information to the supplier and retailer may increase or decrease when SB's perceived quality increases. Such observations may be attributed to the complication caused by the two-dimensional relationship between the supplier and the retailer as both partners and competitors.

There are numerous ways to extend this study. First, we assume that the supplier's estimate for SB production cost follows a discrete probability with two possible values. One can generalize this assumption to study a realistic case where the possible cost values follow a continuous distribution. The model formulated in Corbett et al. (2004) may provide guidance in this direction. Second, we assume that the perceived quality of the SB is lower than that of the NB. However, it comes to our attention that there are rising evidences indicating that more consumers might perceive higher quality from some store brands than national brands in certain categories; see, e.g., Hale (2011), Tuttle (2012), Bronnenberg et al. (2015), Chung and Lee (2017). One can relax the assumption on quality values in a future study to investigate a more generalized case. Last but not the least, our model is formulated based on an assumption that the NB supplier is the game leader. One can consider a retailer to be the game leader and employ a signaling game framework to explore the interaction.

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2.A Appendix

Proof. Proposition 2.1 If $w \leq \frac{c_s q_n}{q_s}$, condition (1) is satisfied but condition (2) does not hold. Thus the local maximum exists at the endpoints of the interval $[p_n - q_n + q_s, \frac{p_n q_s}{q_n}]$. We have $\Pi_r^{II}|_{p_s=\frac{p_n q_s}{q_n}}= \frac{(q_n-w)^2}{4q_n} - T$ and $\Pi_r^{II}|_{p_s=p_n-q_n+q_s}= \frac{(q_s-c_s)^2}{4q_s} - T$. Given $w \leq \frac{c_s q_n}{q_s}$, we have $0 < \frac{w}{q_n} \leq \frac{c_s}{q_s} < 1$ following which $\Pi_r^{II}|_{p_s=\frac{p_n q_s}{q_n}} \geq \Pi_r^{II}|_{p_s=p_n-q_n+q_s}$. Indeed, when $w \leq \frac{c_s q_n}{q_s}$, Π_r^{II} is quasi-concave and monotonically increasing on p_s within the closed interval $[p_n - q_n + q_s, \frac{p_n q_s}{q_n}]$. Hence, Π_r^{II*} exists at the upper endpoint of the interval, such that

$$\Pi_r^{II*} = \Pi_r^{II}|_{p_s=\frac{p_n q_s}{q_n}}= \frac{(q_n - w)^2}{4q_n} - T = \Pi_r^{I*}$$

If w is in $(\frac{c_s q_n}{q_s}, c_s + q_n - q_s]$, both conditions (1) and (2) are satisfied. Then $p_n^{II*} = \frac{w+q_n}{2}$ and $p_s^{II*} = \frac{c_s+q_s}{2}$ are indeed the optimal solution. We can obtain the local optimum

$$\Pi_r^{II*} = \frac{q_n - w}{2}\left[\frac{1}{2} - \frac{w - c_s}{2(q_n - q_s)}\right] + \frac{q_s - c_s}{2}\left[\frac{w - c_s}{2(q_n - q_s)} - \frac{c_s}{2q_s}\right] - T. \qquad (2.A.1)$$

If $w > c_s + q_n - q_s$, we expect Π_r^{II*} to exist at the lower endpoint of the interval of p_s such that

$$\Pi_r^{II*} = \Pi_r^{II*}|_{p_s=p_n-q_n+q_s}= \frac{(q_s - c_s)^2}{4q_s} - T.$$

However, $\Pi_r^{II*}|_{p_s=p_n-q_n+q_s}< \Pi_r^{III*}$ for any $T > 0$. Then if $p_s = p_n - q_n + q_s$, it is no longer profitable for the retailer to carry both brands. Aware of such situation, we proceed to search for the global optimal profit, i.e., $\Pi_r^* = \max\{\Pi_r^{I*}, \Pi_r^{II*}, \Pi_r^{III*}\}$. The optimal profit is compared pair-wisely among three local optima. We will illustrate that the global optimum differentiates with respect to the contract parameters w and T offered by the national brand supplier.

We have shown that when $w \leq \frac{c_s q_n}{q_s}$, $\Pi_r^* = \Pi_r^{I*}$ and $\Pi_r^{I*}$is monotonically decreasing on w. For Π_r^{II}, if $w = \frac{c_s q_n}{q_s}$, $p_s^{II*} = \frac{p_n^{II*} q_s}{q_n}$, $\Pi_r^{II*} = \Pi_r^{II}|_{p_s^{II*}=\frac{p_n^{II*} q_s}{q_n}}= \Pi_r^{I*}|_{w=\frac{c_s q_n}{q_s}}$, that is $\Pi_r^{I*}$and Π_r^{II*} are continuous on w at $\frac{c_s q_n}{q_s}$.

Note the lump-sum transfer is not applicable in the case III. When comparing $\Pi_r^{I*}$and Π_r^{III*}, we need to discuss how the retailer's decision differentiates based on T. For any $w \leq \frac{c_s q_n}{q_s}$, there exist a threshold transfer $\bar{T}^I(w) \geq 0$, such that $\Pi_r^{I*}|_{T=\bar{T}^I(w)}= \Pi_r^{III*}$. We have It follows from (2.4.3) that $\bar{T}^I|_{w=0}= \frac{q_n}{4} - \frac{(q_s-c_s)^2}{4q_s}$ and $\bar{T}^I|_{w=\frac{c_s q_n}{q_s}}= \frac{(q_n-q_s)(q_s-c_s)^2}{4q_s^2}$. Directly differentiation on $\bar{T}^I(w)$w.r.t. w, we have $\frac{d\bar{T}^I(w)}{dw} = \frac{w-q_n}{2q_n} < 0$, i.e., $\bar{T}^I(w)$decreases in w. Hence, if the national brand supplier charges an wholesale price $w \in [0, \frac{c_s q_n}{q_s}]$ and a reasonable fixed payment that lower than the threshold value $\bar{T}^I(w)$, it is more profitable for the retailer to not introduce the SB and carry NB only; if the fixed payment is higher than the threshold, it would be better off to only produce SB.

Given any $w \in (\frac{c_s q_n}{q_s}, c_s + q_n - q_s)$, we firstly compare Π_r^{I*} with Π_r^{II*}. We have proved that Π_r^{I*} with Π_r^{II*} are continuous at $w = \frac{c_s q_n}{q_s}$. After some algebra, we notice that Π_r^{I*} with Π_r^{II*} are both monotonically decreasing on w, and $\frac{d\Pi_r^{II*}(w)}{dw} < \frac{d\Pi_r^{II*}(w)}{dw}$. Therefore, if $\frac{c_s q_n}{q_s} < w \leq c_s + q_n - q_s$, carrying both products is more profitable for the retailer. To compare Π_r^{II*} with Π_r^{III*}, we again use a threshold transfer $\bar{T}^{II}$ to determine whether to carry the national brand. For any $w \in (\frac{c_s q_n}{q_s}, c_s + q_n - q_s]$, there exists a $\bar{T}^{II}(w) > 0$ given by (2.4.4), such that $\Pi_r^{II*}|_{T=\bar{T}^{II}(w)}= \Pi_r^{III*}$. The value of $\bar{T}^{II}$ at the endpoints of the interval for w can be easily obtained that $\bar{T}^{II}|_{w=\frac{c_s q_n}{q_s}}= \frac{(q_n-q_s)(q_s-c_s)^2}{4q_s^2}$ and $\bar{T}^{II}|_{w=c_s+q_n-q_s}= 0$. Take direct derivative on $\bar{T}^{II}(w)$ w.r.t w, we have $\frac{d\bar{T}^{II}(w)}{dw} = -\frac{1}{2} + \frac{w-c_s}{2(q_n-q_s)} < 0$.

Thus, for any $w \in (\frac{c_s q_n}{q_s}, c_s+q_n-q_s)$, if $0 \leq T < \bar{T}^{II}(w)$, $\Pi_r^* = \Pi_r^{II*}$; otherwise, $\Pi_r^* = \Pi_r^{III*}$.

When $w > c_s + q_n - q_s$, we can directly claim that for any $T > 0$, it is never profitable to carry NB, i.e., $\Pi_r^* = \Pi_r^{III*}$. □

Proof. Lemma 2.1. Direct differentiation on Π_r^I given by $2.A.1$ with respect to p_n yields $\frac{d\Pi_r^I}{dp_n^I} = \frac{w+q_n-2p_n}{q_n}$ and $\frac{d^2\Pi_r^I}{dp_n^{I\,2}} = -\frac{2}{q_n} < 0$. Solving $\frac{d\Pi_r^I}{dp_n^I} = 0$, we can get $p_n^{I*} = \frac{w+q_n}{2}$. Substituting p_n^{I*} into Π_r^I, we have $\Pi_r^{I*} = \frac{(q_n-w)^2}{4q_n} - T$. The derivations of p_s^{III*} and Π_r^{III*} are obtained in a similar manner and omitted here. □

Proof. Lemma 2.2. The occurrence of case I implies that $0 \le w \le \frac{c_s q_n}{q_s}$ and $0 < T \le \bar{T}^I(w)$ given which the retailer will price NB as p_n^{I*}. We can easily observe that Π_n^I is increasing in T. Thus in the optimal contract, $T^* = \bar{T}^I(w)$. The retailer consequently gets the reservation profit Π_r^{III*}.

Substitute in p_n^{I*} and T^* and taking direct differentiation on Π_n^I with respect to w, we have $\frac{d\Pi_n^I(w)}{dw} = \frac{c_n - w}{2q_n}$ and $\frac{d^2\Pi_n^I(w)}{dw^2} = -\frac{1}{2q_n} < 0$. We can obtain $w^* = c_n$ from $\frac{d\Pi_n^I(w)}{dw} = 0$. If $c_n \le \frac{c_s q_n}{q_s}$, then $0 \le w^* \le \frac{c_s q_n}{q_s}$ is indeed satisfied. As a result, the retailer will choose to carry the national brand only, i.e.,

$$\Pi_n^{I*} = \Pi_n^I|_{w=c_n} = \frac{(q_n - c_n)^2}{4q_n} - \frac{(q_s - c_s)^2}{4q_s}. \tag{2.A.2}$$

If $c_n > \frac{c_s q_n}{q_s}$, since the profit function is concave and decreasing on w for any $w \le \frac{c_s q_n}{q_s}$, the optimal solution exists at the endpoint of the interval, i.e., $w^* = \frac{c_s q_n}{q_s}$. We have

$$\Pi_n^{I*} = \Pi_n^I|_{w=\frac{c_s q_n}{q_s}} = \frac{2(c_s q_n - c_n q_s)(q_s - c_s) + (q_n - q_s)(q_s - c_s)^2}{4q_s^2}.$$

Case II: If the retailer decides to carry both products, then the national brand supplier's profit function is

$$\max_{(w,T)} \Pi_n^{II} = (w - c_n)(1 - \frac{p_n - p_s}{q_n - q_s}) + T \tag{2.A.3}$$

Following the Proposition 1, the occurrence of case II suggests that $\frac{c_s q_n}{q_s} < w < c_s + q_n - q_s$ and $0 < T \le \bar{T}^{II}(w)$ given which the retailer will set the retailer prices for NB and SB as $\{p_n^{II*}, p_s^{II*}\}$. Similar to case I, we can easily verify that $T^* = \bar{T}^{II}(w)$ and the retailer only earns the reservation profit Π_r^{III*}.

After some algebra, we have $w^* = c_n$ by solving $\frac{d\Pi_n^{II}(w)}{dw} = 0$, noting $\frac{d^2\Pi_n^{II}(w)}{dw^2} < 0$. If $c_n \le \frac{c_s q_n}{q_s}$, Π_n^{II} is quasi-concave and decreasing on w for any $w \in (\frac{c_s q_n}{q_s}, c_s + q_n - q_s]$. w^* is then forced to take the left ending-point value of the interval, i.e., $w^* = \frac{c_s q_n}{q_s}$. Thus

$$\Pi_n^{II*} = \Pi_n^{II}|_{w=\frac{c_s q_n}{q_s}} = \frac{2(c_s q_n - c_n q_s)(q_s - c_s) + (q_n - q_s)(q_s - c_s)^2}{4q_s^2} = \Pi_n^I|_{w=\frac{c_s q_n}{q_s}}$$

If $\frac{c_s q_n}{q_s} < c_n \leq c_s + q_n - q_s$, $w^* = c_n$ is indeed the optimal solution. Then we have

$$\Pi_n^{II*} = \Pi_n^{II}|_{w=c_n} = \frac{(q_n - c_n - q_s + c_s)^2}{4(q_n - q_s)}. \tag{2.A.4}$$

If $c_n > c_s + q_n - q_s$, w^* then takes the value $c_s + q_n - q_s$ due to the quasi-concavity and monotonicity on w in $(\frac{c_s q_n}{q_s}, c_s + q_n - q_s]$. Thus,

$$\Pi_n^{II*} = \Pi_n^{II}|_{w=c_s+q_n-q_s} = 0.$$

In such extreme case that the national brand supplier's production cost is too high, selling products to the retailer is no longer a profitable option.

We compare the supplier's profit among the three cases to seek her optimal profit, i.e $\Pi_n^* = \max\{\Pi_n^{I*},\ \Pi_n^{II*},\ \Pi_n^{III*}\}$, on various intervals of c_n.

(1) If $c_n \leq \frac{c_s q_n}{q_s}$, $\Pi_n^{I*} = \Pi_n^{I}|_{w^{I*}=c_n}$, $\Pi_n^{II*} = \Pi_n^{II}|_{w^{II*}=\frac{c_s q_n}{q_s}}$. When we compare the expected profit among the three case, we have that $\Pi_n^{I*} - \Pi_n^{II*} = \frac{(c_s q_n - c_n q_s)^2}{4 q_n q_s^2} \geq 0$, and $\Pi_n^{I*} = \frac{(q_n - c_n)^2}{4q_n} - \frac{(q_s - c_s)^2}{4q_s} \geq 0 = \Pi_n^{III}$.

(2) If $\frac{c_s q_n}{q_s} < c_n \leq c_s + q_n - q_s$, $\Pi_n^{I*} = \Pi_n^{I}|_{w^{I*}=\frac{c_s q_n}{q_s}}$, $\Pi_n^{II*} = \Pi_n^{II}|_{w^{II*}=c_n}$. Again, we can easily derive that $\Pi_n^{II*} - \Pi_n^{I*} = \frac{(c_s q_n - c_n q_s)^2}{4q_s^2(q_n - q_s)} > 0$ and $\Pi_n^{II*} = \frac{(q_n - c_n - q_s + c_s)^2}{4(q_n - q_s)} \geq 0 = \Pi_n^{III}$.

(3) If $c_n > c_s + q_n - q_s$, $\Pi_n^{I*} = \Pi_n^{I}|_{w^{I*}=\frac{c_s q_n}{q_s}}$, $\Pi_n^{II*} = \Pi_n^{II}|_{w^{II*}=c_s+q_n-q_s} = 0$. By transforming Π_n^{I*}, we get $\Pi_n^{I*} = \frac{q_s - c_s}{4{q_s}^2}[(c_s q_n - c_n q_s) + q_s(c_s + q_n - q_s - c_n)] < 0$.

Given any $c_n > c_s + q_n - q_s$, we compare the profit functions cross three cases, obtaining that $\Pi_n^{I*} < \Pi_n^{II*} = \Pi_n^{III*} = 0$. □

Proof. Lemma 2.3. Proposition 2.1 implies that for any (w, T) satisfies $\Pi_r(w, T, c_s^i) \geq \Pi_r^{III}(c_s^i)$ iff $w \leq c_s^i + q_n - q_s$ and $T \leq \bar{T}^i(w), \forall i = L, H$. Next, we want to show that for any $w \in [0, c_s^L + qn - q_s]$, we have $\Delta(w) \stackrel{def}{=} \bar{T}^H(w) - \bar{T}^L(w) > 0$. It follows from ?? that $\Delta(w)$ have different forms in different ranges of w . Therefore, we consider the following cases:

(1) $0 \le w \le \frac{c_s^L q_n}{q_s}$. In conjunction with $c_s^L < c_s^H$, we have $w < \frac{c_s^H q_n}{q_s}$. Therefore, ?? implies that $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IL}(w) = \frac{(q_s - c_s^L)^2 - (q_s - c_s^H)^2}{4q_s} > 0$.

(2) $\frac{c_s^L q_n}{q_s} <$ w $\le c_s^L + q_n - q_s$. We consider the following two sub-cases:

(2-i) If $c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s}$, then we have $\frac{c_s^L q_n}{q_s} < w \le c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s} < c_s^H + q_n - q_s$. Therefore, ?? implies that $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IIL}(w) = \frac{(q_n - w)^2}{4q_n} - \frac{(q_s - c_s^H)^2}{4q_s} - \frac{(c_s^L - w + q_n - q_s)^2}{4(q_n - q_s)}$. Since $\frac{\partial \Delta}{\partial w} = \frac{c_s^L q_n - q_s w}{2q_n^2 - 2q_n q_s} < 0$ for all $w > \frac{c_s^L q_n}{q_s}$, it follows from $\Delta|_{w = c_s^L + q_n - q_s} = \frac{(q_s - c_s^L)^2}{4q_n} - \frac{(q_s - c_s^H)^2}{4q_s} > 0$ that $\Delta(w) > 0, \forall w \in (\frac{c_s^L q_n}{q_s}, c_s^L + q_n - q_s]$.

(2-ii) If $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, then we have $\frac{c_s^L q_n}{q_s} < \frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s < c_s^H + q_n - q_s$. For $\frac{c_s^L q_n}{q_s} < w \le \frac{c_s^H q_n}{q_s}$, (2-i) implies that $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IIL}(w)$ and $\frac{\partial \Delta}{\partial w} < 0, \forall w > \frac{c_s^L q_n}{q_s}$. Moreover, $\Delta|_{w = \frac{c_s^H q_n}{q_s}} = \frac{(q_s - c_s^H)^2 (q_n - q_s)^2 - (c_s^L q_s + q_n q_s - q_s^2 - c_s^H q_n)^2}{4(q_n - q_s) q_s^2} > 0$. Hence, we show $\Delta(w) > 0, \forall w \in (\frac{c_s^L q_n}{q_s}, \frac{c_s^H q_n}{q_s}]$. For $w \in (\frac{c_s^H q_n}{q_s}, c_s^L + q_n - q_s]$, we have

$\Delta = \bar{T}^{IIH}(w) - \bar{T}^{IIL}(w) = \frac{(c_s^H - c_s^L) t (c_s^H + c_s^L + 2q_n - 2q_s - 2w)}{4t(q_n - q_s)} > 0$ due to $c_s^L < c_s^H$.

In summary, we have shown that $\bar{T}^H(w) > \bar{T}^L(w), \forall w \in [0, c_s^L + q_n - q_s]$. Therefore, Proposition 1 implies that for any (w^L, T^L) that satisfies (I.R.L), we have $w^L \le c_s^L + q_n - q_s < c_s^H + q_n - q_s$ and $T^L \le \bar{T}^L(w^L) < \bar{T}^H(w^L)$, which implies such (w^L, T^L) satisfies $\Pi_r(w^L, T^L, c_s^H) \ge \Pi_r^{III}(c_s^H)$. Hence, we have $\Pi_r(w^H, T^H, c_s^H) \overset{(I.C.H)}{\ge} \Pi_r(w^L, T^L, c_s^H) \ge \Pi_r^{III}(c_s^H)$. □

Proof. Lemma 2.4

We have proved that following (I.C.H.), (I.C.L.) and (I.R.L.), (I.R.H.) is redundant. We now further show that under the (H^{II}, L^I) contract, (I.C.H.), (I.C.L.), (I.R.L.) and $T^H \ge 0$ cannot be simultaneously satisfied so that (H^{II}, L^I) contract is never feasible for the supplier.

By offering a (H^{II}, L^I) contract, the national brand supplier guides a retailer to carry both brands if she is high type, and to carry the national brand only if she is low type. It follows that w^H,T^H, w^L and T^L must be priced according to Proposition 1, i.e., $\frac{c_s^H q_n}{q_s} < w^H \le c_s^H + q_n - q_s, T^H \le \bar{T}^{IIH}(w^H), w^L \le \frac{c_s^L q_n}{q_s}, T^L \le \bar{T}^{IL}(w^L)$. In conjunction with Lemma

3, we can predict that if a high-type retailer lies about his type by choosing the (w^L, T^L), he will react as carrying the national brand only, i.e., $\Pi_r(w^L, T^L, c_s^H) \equiv \Pi_r^I(w^L, T^L, c_s^H)$. Hence, (I.C.H.) can be equivalently written as $\Pi_r^{II}(w^H,\ T^H,\ c_s^H) \geq \Pi_r^I(w^L, T^L, c_s^H)$, or $\frac{q_n-w^H}{2}(\frac{1}{2} - \frac{w^H-c_s^H}{2(q_n-q_s)}) + \frac{q_s-c_s^H}{2}(\frac{w^H-c_s^H}{2(q_n-q_s)} - \frac{c_s^H}{2q_s}) - T^H \geq \frac{(q_n-w^L)^2}{4q_n} - T^L$. Rearranging the inequality, we can obtain the relationship between T^H and T^L that $T^H - T^L \leq \frac{q_n-w^H}{2}(\frac{1}{2} - \frac{w^H-c_s^H}{2(q_n-q_s)}) + \frac{q_s-c_s^H}{2}(\frac{w^H-c_s^H}{2(q_n-q_s)} - \frac{c_s^H}{2q_s}) - \frac{(q_n-w^L)^2}{4q_n} \stackrel{def}{=} \Delta T^U$.

Together with $T^L \leq \bar{T}^{IL}(w^L)$, $T^H \leq \frac{q_n-w^H}{2}(\frac{1}{2} - \frac{w^H-c_s^H}{2(q_n-q_s)}) + \frac{q_s-c_s^H}{2}(\frac{w^H-c_s^H}{2(q_n-q_s)} - \frac{c_s^H}{2q_s}) - \frac{(q_s-c_s^L)^2}{4q_s} \stackrel{\text{def}}{=} \hat{T}^H$. Since $\frac{\partial\ \hat{T}^H}{\partial\ w^H} = \frac{w^H-c_s^H-q_n+q_s}{2(q_n-q_s)} \leq 0$, $\hat{T}^H \leq \hat{T}^H|_{w^H=\frac{c_s^H q_n}{q_s}} = \frac{q_n(q_s-c_s^H)^2-(q_s-c_s^L)^2 q_s}{4q_s^2}$. If $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$, then in conjunction with $\frac{1}{q_n} < \frac{1}{q_s}$, $\frac{q_n(q_s-c_s^H)^2-(q_s-c_s^L)^2 q_s}{4q_s^2} < 0$ so that T^H has to satisfy $T^H < 0$. In this case, (H^{II}, L^{I}) contract is not feasible. If $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, $\frac{q_n(q_s-c_s^H)^2-(q_s-c_s^L)^2 q_s}{4q_s^2} - \bar{T}^{IIL}(w^H) = -\frac{(c_s^H-c_s^L)(2w^H q_s - c_s^H q_n - c_s^L q_n)}{4q_s(q_n-q_s)} \leq 0$. If $w^H < c_s^L + q_n - q_s$, then according to Proposition 1 we can predict that if a low-type retailer chooses the high-type contract, he would carry both brands, i.e., $\Pi_r(w^H,\ T^H,\ c_s^L) \equiv \Pi_r^{II}(w^H,\ T^H,\ c_s^L)$. Further checking (I.C.L.), if follows from $\Pi_r^I(w^L,\ T^L,\ c_s^L) \geq \Pi_r^{II}(w^H,\ T^H,\ c_s^L)$ that $T^H - T^L \geq \frac{q_n-w^H}{2}(\frac{1}{2} - \frac{w^H-c_s^L}{2(q_n-q_s)}) + \frac{q_s-c_s^L}{2}(\frac{w^H-c_s^L}{2(q_n-q_s)} - \frac{c_s^L}{2q_s}) - \frac{(q_n-w^L)^2}{4q_n} \stackrel{\text{def}}{=} \Delta T^D$. However, $\Delta T^U - \Delta T^D = -\frac{(c_s^H-c_s^L)(2w^H q_s - c_s^H q_n - c_s^L q_n)}{4q_s(q_n-q_s)} < 0$, implying (I.C.H.) and (I.C.L.) cannot be simultaneously satisfied. If $w^H < c_s^L + q_n - q_s$, $T^H \leq \hat{T}^H \leq \hat{T}^H|_{w^H=c_s^L+q_n-q_s} = \frac{(c_s^H-c_s^L)(c_s^H q_n + c_s^L q_n - 2(c_s^L+q_n-q_s)q_s)}{4(q_n-q_s)q_s} < 0$. Therefore, under the parameter setting $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, ($H^{\text{II}}$, L^{I}) contract is not feasible regardless of the value of w^H.

Hence we have that (H^{II}, L^I) contract is not a feasible option for the supplier. □

Proof. Lemma 2.5. Proposition 2.1 implies that a (w, T) contract, from which a retailer will earn a profit no less than his reservation profit $\Pi_r^{III}(c_s^i)$, must satisfy $w \leq c_s^i + q_n - q_s$ and $T \leq \bar{T}^i(w), \forall i = L, H$. We now define $\Delta(w) \stackrel{def}{=} \bar{T}^H(w) - \bar{T}^L(w)$ for any $w \in [0, c_s^L + q_n - q_s]$. It follows from (2.5.3) that $\Delta(w)$ has different forms in different ranges of w. Therefore, we consider the following cases:

(1) If $0 \le w \le \frac{c_s^L q_n}{q_s}$, then it follows from $c_s^L < c_s^H$ that $w \le \frac{c_s^L q_n}{q_s}$. Therefore, equation (2.5.3) implies that $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IL}(w) = \frac{(q_s - c_s^L)^2 - (q_s - c_s^H)^2}{4q_s} > 0$.

(2) If $\frac{c_s^L q_n}{q_s} < w \le c_s^L + q_n - q_s$, then the following two parameter settings should be discussed separately.

(2-i) Under the parameter setting $c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s}$, we have $\frac{c_s^L q_n}{q_s} < w \le c_s^L + q_n - q_s \le \frac{c_s^H q_n}{q_s} < c_s^H + q_n - q_s$. Therefore, equation (2.5.3) implies that $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IIL}(w) = \frac{(q_n - w)^2}{4q_n} - \frac{(q_s - c_s^H)^2}{4q_s} - \frac{(c_s^L - w + q_n - q_s)^2}{4(q_n - q_s)}$. Since $\frac{\partial \Delta(w)}{\partial w} = \frac{c_s^L q_n - q_s w}{2q_n^2 - 2q_n q_s} < 0$ for all $w > \frac{c_s^L q_n}{q_s}$, it follows from $\Delta(w)|_{w = c_s^L + q_n - q_s} = \frac{(q_s - c_s^L)^2}{4q_n} - \frac{(q_s - c_s^H)^2}{4q_s} > 0$ that $\Delta(w) > 0, \forall w \in (\frac{c_s^L q_n}{q_s}, c_s^L + q_n - q_s]$.

(2-ii) Under the parameter setting $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, we naturally have $\frac{c_s^L q_n}{q_s} < \frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s < c_s^H + q_n - q_s$. For $\frac{c_s^L q_n}{q_s} < w \le \frac{c_s^H q_n}{q_s}$, (2-i) has shown that $\Delta(w) = \bar{T}^{IH}(w) - \bar{T}^{IIL}(w)$ and $\frac{\partial \Delta}{\partial w} < 0, \forall w > \frac{c_s^L q_n}{q_s}$. Moreover, $\Delta(w)|_{w = \frac{c_s^H q_n}{q_s}} = \frac{(q_s - c_s^H)^2 (q_n - q_s)^2 - (c_s^L q_s + q_n q_s - q_s^2 - c_s^H q_n)^2}{4(q_n - q_s) q_s^2} > 0$. Hence, we obtain $\Delta(w) > 0, \forall w \in (\frac{c_s^L q_n}{q_s}, \frac{c_s^H q_n}{q_s}]$. For $\frac{c_s^H q_n}{q_s} < w \le c_s^L + q_n - q_s$, we have $\Delta(w) = \bar{T}^{IIH}(w) - \bar{T}^{IIL}(w) = \frac{(c_s^H - c_s^L) t (c_s^H + c_s^L + 2q_n - 2q_s - 2w)}{4t(q_n - q_s)} > 0$ due to $c_s^L < c_s^H$.

Hence, we have that $\bar{T}^H(w) > \bar{T}^L(w), \forall w \in [0, c_s^L + q_n - q_s]$. □

Proof. Proposition 2.2

Under each parameter setting, we compare the local optima from each subcontract option throughout all the intervals of c_n. (I.R.L.) constraint was set to be binding at the optimality, implying that a low-type retailer will only obtain his reservation profit, i.e., $\Pi_r(w^L,\ T^L,\ c_s^L) = \Pi_r^{III}(c_s^L)$. Consequently, (I.C.L.) can be rewritten as $\Pi_r^{III}(c_s^L) \ge \Pi_r(w^H,\ T^H,\ c_s^L)$. In order to check the feasibility of the optimal solution in the (I.C.L.) constraint, it follows the Proposition 1 that if $w^{H*} \le \frac{c_s^L q_n}{q_s}$ and $T^{H*} \ge \bar{T}^{IL}(w^{H*})$, or $\frac{c_s^L q_n}{q_s} < w^{H*} \le c_s^L + q_n - q_s$ and $T^{H*} \ge \bar{T}^{IIL}(w^{H*})$, or $w^{H*} > c_s^L + q_n - q_s$ and $T^{H*} > 0$, then (I.C.L) is satisfied. We will show that with substituting the resulting optimal parameters (w^{H*}, T^{H*}), (w^{L*}, T^{L*}) into (I.C.L), (I.C.L) is indeed satisfied.

1. $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$

(1-i) If $c_n \leq \frac{c_s^L q_n}{q_s}$, $\Pi_n^*(H^I, L^I) - \Pi_n^*(H^I, L^{II}) = \frac{(1-v)(c_s^L q_n - c_n q_s)^2}{4q_n q_s^2} \geq 0$; $\Pi_n^*(H^I, L^{II}) - \Pi_n^*(H^{II}, L^{II}) = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_n q_s^2} > 0$. Hence, (H^I, L^I) contract is more profitable, and $w^{H*} = c_n, T^{H*} = \frac{(q_n-c_n)^2}{4q_n} - \frac{(q_s-c_s^L)^2}{4q_s}, w^{L*} = c_n, T^{L*} = \frac{(q_n-c_n)^2}{4q_n} - \frac{(q_s-c_s^L)^2}{4q_s}$. Since $w^{H*} = c_n \leq \frac{c_s^L q_n}{q_s}$, $T^{H*} - \bar{T}^{IL}(w^{H*}) = 0$, (I.C.L.) is satisfied and binding at optimality.

(1-ii) If $\frac{c_s^L q_n}{q_s} < c_n < \frac{(c_s^L+q_n-q_s)[(1-v)q_n - vq_s] + vc_s^L q_n}{(1-v)q_n}$, $\Pi_n^*(H^I, L^{II}) - \Pi_n^*(H^I, L^I) = \frac{(1-v)^2 q_n (c_s^L q_n - c_n q_s)^2}{4(q_n-q_s)q_s^2((1-v)q_n - vq_s)} > 0$; $\Pi_n^*(H^I, L^{II}) - \Pi_n^*(H^{II}, L^{II}) = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_n q_s^2} > 0$. Therefore, (H^I, L^{II}) contract is more profitable, and $w^{L*} = \frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}, T^{L*} = \frac{\left(c_s^L + q_n - q_s - \frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}\right)^2}{4(q_n-q_s)}, w^{H*} = c_n, T^{H*} = \frac{(q_n-c_n)^2}{4q_n} - \frac{\left(q_n - \frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}\right)^2}{4q_n} + \frac{\left(c_s^L + q_n - q_s - \frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}\right)^2}{4(q_n-q_s)}$. Since $w^{H*} = c_n$ and $\frac{c_s^L q_n}{q_s} < c_n < c_s^L + q_n - q_s$, we compare T^{H*} with $\bar{T}^{IIL}(w^{H*})$ and obtain $T^{H*} - \bar{T}^{IIL}(w^{H*}) = \frac{v(2(1-v)q_n - vq_s)(c_s^L q_n - c_n q_s)^2}{4q_n(q_n-q_s)((1-v)q_n - vq_s)^2} > 0$. (I.C.L.) is satisfied.

(1-iii) If $\frac{(c_s^L+q_n-q_s)((1-v)q_n - vq_s) + vc_s^L q_n}{(1-v)q_n} < c_n \leq c_s^L + q_n - q_s$, $\Pi_n^*(H^I, L^{II}) - \Pi_n^*(H^I, L^I) = \frac{v(q_s-c_s^L)^2(q_n-q_s)}{4q_n q_s} - \frac{(1-v)(q_s-c_s^L)((q_n-q_s-2c_n)q_s + c_s^L(q_n+q_s))}{4q_s^2} \geq \frac{(q_s-c_s^L)^2(q_n-q_s)((1-v)q_n - vq_s)}{4q_n q_s^2} > 0$, $\Pi_n^*(H^I, L^{II}) - \Pi_n^*(H^{II}, L^{II}) = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_n q_s^2} > 0$. Hence, (H^I, L^{II}) contract is more profitable,

$$w^{H*} = c_n, T^{H*} = \frac{(q_n - c_n)^2}{4q_n} - \frac{(q_s - c_s^L)^2}{4q_n},$$

$$w^{L*} = c_s^L + q_n - q_s,\ T^{L*} = 0.$$

Since $w^{H*} = c_n$ and $\frac{c_s^L q_n}{q_s} < c_n \leq c_s^L + q_n - q_s$, we compare T^{H*} with $\bar{T}^{IIL}(w^{H*})$, resulting in that $T^{H*} - \bar{T}^{IIL}(w^{H*}) = \frac{(c_s^L + q_n - q_s - c_n)(c_n q_s - c_s^L q_n + (q_n-q_s)(q_s-c_s^L))}{4q_n(q_n-q_s)} \geq 0$. Therefore, (I.C.L.) is satisfied.

2. $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$

(2-i) If $c_n \leq \frac{c_s^L q_n}{q_s}$, $\Pi_n^*(H^I,\ L^I) - \Pi_n^*(H^I,\ L^{II}:\ \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) = \frac{(1-v)(c_s^L q_n - c_n q_s)^2}{4q_n q_s^2} > 0$; $\Pi_n^*(H^I,\ L^{II}:\ \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^I,\ L^{II}:\ \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) = \frac{(c_s^H - c_s^L)q_n(c_s^L((1-v)q_n + vq_s) + c_s^H((1-v)q_n - vq_s) - 2(1-v)c_n q_s)}{4q_s^2(q_n-q_s)} \geq \frac{(c_s^H - c_s^L)^2 q_n((1-v)q_n - vq_s)}{4(q_n-q_s)q_s^2} > 0$; $\Pi_n^*(H^I,\ L^{II}:\ \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^{II},\ L^{II}:\ \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_n q_s^2} > 0$; $\Pi_n^*(H^I,\ L^{II}:$

$\frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) - \Pi_n^*(H^{II},\ L^{II}: \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) = \frac{v(c_s^H q_n - c_n q_s)^2}{4 q_n q_s^2} > 0.$ Hence, $(H^I,\ L^I)$ contract is the most profitable contract among six options, and

$$w^{H*} = c_n, \quad T^{H*} = \frac{(q_n - c_n)^2}{4q_n} - \frac{(q_s - c_s^L)^2}{4q_s},$$

$$w^{L*} = c_n, \quad T^{L*} = \frac{(q_n - c_n)^2}{4q_n} - \frac{(q_s - c_s^L)^2}{4q_s}.$$

Since $w^{H*} = c_n \leq \frac{c_s^L q_n}{q_s}$, $T^{H*} - \bar{T}^{IL}(w^{H*}) = 0$, (I.C.L.) is satisfied and binding at optimality.

(2-ii) If $\frac{c_s^L q_n}{q_s} < c_n \leq \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$, $\Pi_n^*(H^I, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^I, L^I) = \frac{(1-v)^2 q_n (c_n q_s - c_s^L q_n)^2}{4(q_n - q_s) q_s^2 ((1-v)q_n - v q_s)} > 0$; $\Pi_n^*(H^I, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^I, L^{II}: \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) = \frac{q_n(((1-v)c_n - v c_s^L) q_s + c_s^H((1-v)q_n - v q_s))^2}{4(q_n - q_s) q_s^2 ((1-v)q_n - v q_s)} > 0$; $\Pi_n^*(H^I, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^{II}, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) = \frac{v(c_s^H q_n - c_n q_s)^2}{4 q_n q_s^2} > 0$; $\Pi_n^*(H^{II}, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^{II}, L^{II}: \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) = \frac{q_n(((1-v)c_n - v c_s^L) q_s + c_s^H((1-v)q_n - v q_s))^2}{4(q_n - q_s) q_s^2 ((1-v)q_n - v q_s)} > 0$. Therefore, in this case, $(H^I, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s})$ is the most profitable option for the national brand supplier, and

$$w^{H*} = c_n, T^{H*} = \frac{(q_n - c_n)^2}{4q_n} - \frac{\left(q_n - \frac{(c_n - v c_n - v c_s^L) q_n}{q_n - v q_n - v q_s}\right)^2}{4q_n} + \frac{\left(c_s^L + q_n - q_s - \frac{(c_n - v c_n - v c_s^L) q_n}{q_n - v q_n - v q_s}\right)^2}{4(q_n - q_s)},$$

$$w^{L*} = \frac{(c_n - v c_n - v c_s^L) q_n}{q_n - v q_n - v q_s}, T^{L*} = \frac{\left(c_s^L + q_n - q_s - \frac{(c_n - v c_n - v c_s^L) q_n}{q_n - v q_n - v q_s}\right)^2}{4(q_n - q_s)}.$$

Since $w^{H*} = c_n$ and $\frac{c_s^L q_n}{q_s} < c_n < c_s^L + q_n - q_s$, we compare T^{H*} with $\bar{T}^{IIL}(w^{H*})$ and obtain $T^{H*} - \bar{T}^{IIL}(w^{H*}) = \frac{v(2(1-v)q_n - v q_s)(c_s^L q_n - c_n q_s)^2}{4 q_n (q_n - q_s)((1-v)q_n - v q_s)^2} > 0$, implying that (I.C.L.) is satisfied.

(2-iii) If $\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v} < c_n \leq \frac{c_s^H q_n}{q_s}$, $\Pi_n^*(H^I, L^{II}: \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) - \Pi_n^*(H^I, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) = \frac{(((1-v)c_n - v c_s^L) q_s - c_s^H((1-v)q_n - v q_s))^2}{4 q_s^2 (1-v)(q_n - q_s)} > 0$; $\Pi_n^*(H^I, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^I, L^I) = \frac{(c_s^H - c_s^L) q_n (2(1-v) c_n q_s - c_s^L((1-v)q_n + v q_s) - c_s^H((1-v)q_n - v q_s))}{4(q_n - q_s) q_s^2} \geq \frac{(c_s^H - c_s^L)^2 q_n ((1-v)q_n - v q_s)}{4(q_n - q_s) q_s^2} > 0$; $\Pi_n^*(H^I, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^{II}, L^{II}: \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) = \frac{v(c_s^H q_n - c_n q_s)^2}{4 q_n q_s^2} > 0$; $\Pi_n^*(H^I, L^{II}: \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) - \Pi_n^*(H^{II}, L^{II}: \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) = \frac{v(c_s^H q_n - c_n q_s)^2}{4 q_n q_s^2} > 0$. Therefore, $(H^I, L^{II}: \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s)$ is the most profitable option for the national brand supplier, and

$$w^{H*} = c_n,$$

$$\begin{aligned} T^{H*} =& \frac{(q_n - c_n)^2}{4q_n} + \frac{\left(c_s^L + q_n - q_s - (c_n + \frac{v(c_s^H - c_s^L)}{1-v})\right)^2}{4(q_n - q_s)} \\ & - [\frac{q_n - (c_n + \frac{v(c_s^H - c_s^L)}{1-v})}{2}(\frac{1}{2} - \frac{c_n + \frac{v(c_s^H - c_s^L)}{1-v} - c_s^H}{2(q_n - q_s)}) + \frac{q_s - c_s^H}{2}(\frac{c_n + \frac{v(c_s^H - c_s^L)}{1-v} - c_s^H}{2(q_n - q_s)} - \frac{c_s^H}{2q_s})], \end{aligned}$$

$$w^{L*} = c_n + \frac{v(c_s^H - c_s^L)}{1-v},\ T^{L*} = \frac{\left(c_s^L + q_n - q_s - (c_n + \frac{v(c_s^H - c_s^L)}{1-v})\right)^2}{4(q_n - q_s)}.$$

Since $w^{H*} = c_n$ and $\frac{c_s^L q_n}{q_s} < c_n < c_s^L + q_n - q_s$, we compare T^{H*} with $\bar{T}^{IIL}(w^{H*})$, having $T^{H*} - \bar{T}^{IIL}(w^{H*}) = \frac{2((1-v)c_n c_s^H + v(c_s^H - c_s^L)^2)q_n q_s - (1-v)({c_s^H}^2 q_n^2 + c_n^2 q_s^2)}{4(1-v)(q_n - q_s)q_n q_s}$, which increases on c_n. As a result, $T^{H*} - \bar{T}^{IIL}(w^{H*}) \geq (T^{H*} - \bar{T}^{IIL}(w^{H*}))\big|_{c_n = \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}} = \frac{v(c_s^H - c_s^L)^2(2(1-v)q_n - vq_s)}{4(1-v)^2 q_n (q_n - q_s)} > 0$. (I.C.L.) constraint is satisfied.

In a summary for (2-ii) and (2-iii), for any $c_n \in (\frac{c_s^L q_n}{q_s}, \frac{c_s^H q_n}{q_s})$, (H^I, L^{II}) is the most profitable contract for the supplier.

(2-iv) If $\frac{c_s^H q_n}{q_s} < c_n < c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$, $\Pi_n^*(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) - \Pi_n^*(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_s^2(q_n - q_s)} > 0$; $\Pi_n^*(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) - \Pi_n^*(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) = \frac{(((1-v)c_n - vc_s^L)q_s - c_s^H((1-v)q_n - vq_s))^2}{4(1-v)(q_n - q_s)q_s^2} > 0$; $\Pi_n^*(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^I, L^{II} : \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_s^2(q_n - q_s)} > 0$; $\Pi_n^*(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^I, L^I) \geq \frac{v(c_s^H q_n - c_n q_s)^2}{4(q_n - q_s)q_s^2} + \frac{(c_s^H - c_s^L)^2 q_n((1-v)q_n + vq_s)}{4(q_n - q_s)q_s^2} > 0$. Therefore, $(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s)$ is most profitable contract for the supplier, and

$$w^{H*} = c_n, T^{H*} = \frac{{c_s^L}^2 + (c_n - q_n + q_s)^2 + 2c_s^L(q_n - q_s - (c_n + \frac{v(c_s^H - c_s^L)}{1-v})) + \frac{2vc_s^H(c_s^H - c_s^L)}{1-v}}{4(q_n - q_s)}$$

$$w_L^* = c_n + \frac{v(c_s^H - c_s^L)}{1-v}, T^{L*} = \frac{\left(c_s^L + q_n - q_s - (c_n + \frac{v(c_s^H - c_s^L)}{1-v})\right)^2}{4(q_n - q_s)}.$$

Since $w^{H*} = c_n$ and $\frac{c_s^L q_n}{q_s} < c_n < c_s^L + q_n - q_s$, we compare T^{H*} with $\bar{T}^{IIL}(w^{H*})$ and obtain $T^{H*} - \bar{T}^{IIL}(w^{H*}) = \frac{v(c_s^H - c_s^L)^2}{2(1-v)(q_n - q_s)} > 0$, which implies that (I.C.L.) is satisfied.

(2-v) If $c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v} < c_n < c_s^L + q_n - q_s$, $\Pi_n^*(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) - \Pi_n^*(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) \geq \frac{(1-vt)(q_s(c_s^L + q_n - q_s) - c_s^H q_n)^2}{4(q_n - q_s) q_s^2} > 0$; $\Pi_n^*(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^I, L^I) \geq \frac{v(c_s^H q_n - c_n q_s)^2}{4q_s^2(q_n - q_s)} + \frac{(c_s^H - c_s^L)^2 q_n((1-v)q_n + vq_s)}{4(q_n - q_s) q_s^2} > 0$; $\Pi_n^*(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) - \Pi_n^*(H^I, L^{II} : \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s) = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_s^2(q_n - q_s)} > 0$; $\Pi_n^*(H^{II}, L^{II} : \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) - \Pi_n^*(H^I, L^{II} : \frac{c_s^L q_n}{q_s} < w^L < \frac{c_s^H q_n}{q_s}) = \frac{v(c_s^H q_n - c_n q_s)^2}{4q_s^2(q_n - q_s)} > 0$. Hence, in this case, $(H^{II}, L^{II} : \frac{c_s^H q_n}{q_s} < w^L < c_s^L + q_n - q_s)$ is most profitable contract for the supplier;

$$w^{H*} = c_n, T^{H*} = \frac{v(c_s^L + q_n - q_s - c_n)(2c_s^H - c_s^L + q_n - q_s - c_n)}{4(q_n - q_s)},$$

$$w^{L*} = c_s^L + q_n - q_s, T^{L*} = 0.$$

Since $w^{H*} = c_n$ and $\frac{c_s^L q_n}{q_s} < c_n < c_s^L + q_n - q_s$, we compare T^{H*} with $\bar{T}^{IIL}(w^{H*})$ and obtain $T^{H*} - \bar{T}^{IIL}(w^{H*}) = \frac{(c_s^H - c_s^L)(c_s^L + q_n - q_s - c_n)}{2(q_n - q_s)} > 0$, which verifies that (I.C.L.) is satisfied.

From (ii-4) and (ii-5), we obtain that for any $c_n \in (\frac{c_s^H q_n}{q_s}, c_s^L + q_n - q_s]$, (H^{II}, L^{II}) is the most profitable contract for the national brand supplier.

In summary, throughout all the possible parameter settings of c_s^H, c_s^L and c_n, (I.C.L.) is automatically satisfied by the optimal solution. □

Proof. Proposition 2.3.

We present and compare the values of information to each entity throughout all possible parameter settings as follows.

Under the parameter setting $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$

1 If $c_n \leq \frac{c_s^L q_n}{q_s}$, $V_r = v(\frac{(q_s - c_s^H)^2}{4q_s} - \frac{(q_s - c_s^L)^2}{4q_s})$ and $V_n = v(\frac{(q_s - c_s^L)^2}{4q_s} - \frac{(q_s - c_s^H)^2}{4q_s})$. It is easy to obtain $V_n - |V_r| = 0$.

2 If $\frac{c_s^L q_n}{q_s} < c_n \leq \frac{(c_s^L + q_n - q_s)((1-v)q_n - vq_s) + vc_s^L q_n}{(1-v)q_n}$, $V_r = v(\frac{(q_s - c_s^H)^2}{4q_s} - \frac{\left(q_n - \frac{(c_n - vc_n - vc_s^L)q_n}{q_n - vq_n - vq_s}\right)^2}{4q_n} +$

$\frac{(c_s^L+q_n-q_s-\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s})^2}{4(q_n-q_s)}$) and $V_n = v[\frac{(q_n-\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s})^2}{4q_n} - \frac{(q_s-c_s^H)^2}{4q_s}]+(1-v)[\frac{(q_n-c_n-q_s+c_s^L)^2}{4(q_n-q_s)} - (\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s} - c_n)(\frac{1}{2} - \frac{\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s}-c_s^L}{2(q_n-q_s)})] - \frac{(c_s^L-\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s}+q_n-q_s)^2}{4(q_n-q_s)}$. It follows that $V_n - |V_r|= \frac{(1-v)v^2(c_nq_s-c_s^Lq_n)^2}{4(q_n-q_s)(q_n-vq_n-vq_s)^2} > 0$.

3 If $\frac{(c_s^L+q_n-q_s)((1-v)q_n-vq_s)+vc_s^Lq_n}{(1-v)q_n} < c_n \leq c_s^L + q_n - q_s$, then $V_r = v(\frac{(q_s-c_s^H)^2}{4q_s} - \frac{(q_s-c_s^L)^2}{4q_n})$ and $V_n = v(\frac{(q_s-c_s^L)^2}{4q_n} - \frac{(q_s-c_s^H)^2}{4q_s})+(1-v)\frac{(q_n-c_n-q_s+c_s^L)^2}{4(q_n-q_s)}$. Thus, $V_n-|V_r|= (1-v)\frac{(c_s^L+q_n-q_s-c_n)^2}{4(q_n-q_s)} \geq 0$.

Under the parameter setting $\frac{c_s^Hq_n}{q_s} < c_s^L + q_n - q_s$

1 If $c_n \leq \frac{c_s^Lq_n}{q_s}$, $V_r = v(\frac{(q_s-c_s^H)^2}{4q_s} - \frac{(q_s-c_s^L)^2}{4q_s})$ and $V_n = v(\frac{(q_s-c_s^L)^2}{4q_s} - \frac{(q_s-c_s^H)^2}{4q_s})$. Then we have $V_n - |V_r|= 0$.

2 If $\frac{c_s^Lq_n}{q_s} < c_n \leq \frac{c_s^Hq_n}{q_s} - \frac{v(c_s^H-c_s^L)}{1-v}$, $V_r = v(\frac{(q_s-c_s^H)^2}{4q_s} - \frac{(q_n-\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s})^2}{4q_n} + \frac{(c_s^L+q_n-q_s-\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s})^2}{4(q_n-q_s)})$ and $V_n = v[\frac{(q_n-c_n)^2}{4q_n} - \frac{(q_s-c_s^H)^2}{4q_s} + \frac{(q_n-\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s})^2}{4q_n}]+(1-v)[\frac{(q_n-c_n-q_s+c_s^L)^2}{4(q_n-q_s)} - (\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s} - c_n)(\frac{1}{2} - \frac{\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s}-c_s^L}{2(q_n-q_s)})] - \frac{(c_s^L+q_n-q_s-\frac{(c_n-vc_n-vc_s^L)q_n}{q_n-vq_n-vq_s})^2}{4(q_n-q_s)}$. After some algebra, we can show that $V_n - |V_r|= \frac{(1-v)v^2(c_nq_s-c_s^Lq_n)^2}{4(q_n-q_s)(q_n-vq_n-vq_s)^2} > 0$.

3 If $\frac{c_s^Hq_n}{q_s} - \frac{v(c_s^H-c_s^L)}{1-v} < c_n \leq c_s^L+q_n-q_s-\frac{v(c_s^H-c_s^L)}{1-v}$, $V_r = \frac{v(c_s^H-c_s^L)(2(1-v)(c_n-q_n+q_s)-(1-3v)c_s^H-c_s^L-vc_s^L)}{4(1-v)(q_n-q_s)}$ and $V_n = -\frac{v(c_s^H-c_s^L)(2(1-v)(c_n-q_n+q_s)-(1-2v)c_s^H-c_s^L)}{4(1-v)(q_n-q_s)}$. It follows that $V_n-|V_r|= \frac{v^2(c_s^H-c_s^L)^2}{4(1-v)(q_n-q_s)} > 0$.

4 If $c_s^L+q_n-q_s-\frac{v(c_s^H-c_s^L)}{1-v} < c_n \leq c_s^L+q_n-q_s$, $V_r = -\frac{v(c_s^H-c_s^L)^2}{4(q_n-q_s)}$ and $V_n = v\frac{(c_s^H+q_n-q_s-c_n)^2}{4(q_n-q_s)} + (1-v)\frac{(c_s^L+q_n-q_s-c_n)^2}{4(q_n-q_s)} - \frac{v(c_n-c_s^L-q_n+q_s)(c_n-2c_s^H+c_s^L-q_n+q_s)}{4(q_n-q_s)}$. Thus, $V_n-|V_r|= (1-v)\frac{(c_s^L+q_n-q_s-c_n)^2}{4(q_n-q_s)} \geq 0$.

Hence, we have that $V_n \geq |V_r|$ holds throughout all the parameter settings. □

Proof. Proposition 2.4.

As the value of information has different forms depending on the value of c_n, the impact of c_n on $|V_r|$ and V_n is discussed for each parameter setting and interval of c_n.

(1) Parameter Setting 1: $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$

For the retailer, if $c_n \le \frac{c_s^L q_n}{q_s}$, $\frac{\partial |V_r|}{\partial c_n} = 0$; if $\frac{c_s^L q_n}{q_s} < c_n < \frac{(c_s^L+q_n-q_s)((1-v)q_n-vq_s)+vc_s^L q_n}{(1-v)q_n}$, $\frac{\partial |V_r|}{\partial c_n} = -\frac{(1-v)^2 vq_n(c_n q_s - c_s^L q_n)}{2(q_n-q_s)((1-v)q_n-vq_s)^2} < 0$; if $\frac{(c_s^L+q_n-q_s)((1-v)q_n-vq_s)+vc_s^L q_n}{(1-v)q_n} < c_n < c_s^L+q_n-q_s$, $\frac{\partial |V_r|}{\partial c_n} = 0$.

For the supplier, if $c_n \le \frac{c_s^L q_n}{q_s}$, $\frac{\partial V_n}{\partial c_n} = 0$; if $\frac{c_s^L q_n}{q_s} < c_n < \frac{(c_s^L+q_n-q_s)((1-v)q_n-vq_s)+vc_s^L q_n}{(1-v)q_n}$, $\frac{\partial V_n}{\partial c_n} = \frac{(1-v)v(c_s^L q_n - c_n q_s)}{2(q_n-q_s)((1-v)q_n-vq_s)} < 0$; if $\frac{(c_s^L+q_n-q_s)((1-v)q_n-vq_s)+vc_s^L q_n}{(1-v)q_n} < c_n < c_s^L + q_n - q_s$, $\frac{\partial V_n}{\partial c_n} = \frac{(1-v)(c_n - c_s^L - q_n + q_s)}{2(q_n-q_s)} < 0$.

(2) Parameter Setting 2: $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$

For the retailer, if $c_n \le \frac{c_s^L q_n}{q_s}$, $\frac{\partial |V_r|}{\partial c_n} = 0$; if $\frac{c_s^L q_n}{q_s} < c_n < \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial |V_r|}{\partial c_n} = \frac{(1-v)^2 vq_n(c_s^L q_n - c_n q_s)}{2(q_n-q_s)((1-v)q_n-vq_s)^2} < 0$; if $\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v} < c_n < c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial |V_r|}{\partial c_n} = \frac{v(c_s^L - c_s^H)}{2(q_n-q_s)} < 0$; if $c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v} < c_n < c_s^L + q_n - q_s$, $\frac{\partial |V_r|}{\partial c_n} = 0$.

For the supplier, if $c_n \le \frac{c_s^L q_n}{q_s}$, $\frac{\partial V_n}{\partial c_n} = 0$; if $\frac{c_s^L q_n}{q_s} < c_n < \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial V_n}{\partial c_n} = \frac{v(1-v)(c_s^L q_n - c_n q_s)}{2(q_n-q_s)((1-v)q_n-vq_s)} < 0$; if $\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v} < c_n < c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial V_n}{\partial c_n} = \frac{v(c_s^L - c_s^H)}{2(q_n-q_s)} < 0$; if $c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v} < c_n < c_s^L + q_n - q_s$, $\frac{\partial V_n}{\partial c_n} = \frac{(1-v)(c_n - c_s^L - q_n + q_s)}{2(q_n-q_s)} < 0$. □

Proof. Proposition 2.5

(1) Parameter Setting 1: $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$

If $c_n \le \frac{c_s^L q_n}{q_s}$, $\frac{\partial V_n}{\partial c_s^H} = \frac{v(q_s - c_s^H)}{2q_s} > 0$, $\frac{\partial V_n}{\partial c_s^L} = \frac{v(c_s^L - q_s)}{2q_s} < 0$; if $\frac{c_s^L q_n}{q_s} < c_n < \frac{(c_s^L+q_n-q_s)((1-v)q_n-vq_s)+vc_s^L q_n}{(1-v)q_n}$, $\frac{\partial V_n}{\partial c_s^H} = \frac{v(q_s - c_s^H)}{2q_s} > 0$, $\frac{\partial V_n}{\partial c_s^L} = \frac{v((1-v)c_n q_n - c_s^L(q_n - vq_s) - (q_n-q_s)((1-v)q_n - vq_s))}{2(q_n-q_s)((1-v)q_n-vq_s)} < 0$; if $\frac{(c_s^L+q_n-q_s)((1-v)q_n-vq_s)+vc_s^L q_n}{(1-v)q_n} < c_n < c_s^L + q_n - q_s$, $\frac{\partial V_n}{\partial c_s^H} = \frac{v(q_s - c_s^H)}{2q_s} > 0$, $\frac{\partial V_n}{\partial c_s^L} = \frac{(1-v)(c_s^L + q_n - q_s - c_n)}{2(q_n-q_s)} - \frac{v(q_s - c_s^L)}{2q_n} < 0$.

(2) Parameter Setting 2: $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$

If $c_n \le \frac{c_s^L q_n}{q_s}$, $\frac{\partial V_n}{\partial c_s^H} = \frac{v(q_s - c_s^H)}{2q_s} > 0$, $\frac{\partial V_n}{\partial c_s^L} = \frac{v(c_s^L - q_s)}{2q_s} < 0$; if $\frac{c_s^L q_n}{q_s} < c_n < \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial V_n}{\partial c_s^H} = \frac{v(q_s - c_s^H)}{2q_s} > 0$, $\frac{\partial V_n}{\partial c_s^L} = \frac{v((1-v)c_n q_n - c_s^L(q_n - vq_s) - (q_n-q_s)((1-v)q_n - vq_s))}{2(q_n-q_s)((1-v)q_n-vq_s)} < 0$; if $\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v} \le c_n < c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial V_n}{\partial c_s^H} = \frac{v((1-2v)c_s^H - (1-v)c_n + vc_s^L + q_n - vq_n - q_s + vq_s)}{2(1-v)(q_n-q_s)} > 0$, $\frac{\partial V_n}{\partial c_s^L} = -\frac{v(q_n - vq_n - q_s + vq_s - (1-v)c_n - vc_s^H + c_s^L)}{2(1-v)(q_n-q_s)} < 0$; if $c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v} \le c_n < c_s^L + q_n - q_s$, $\frac{\partial V_n}{\partial\ c_s^H} = \frac{v(c_s^H - c_s^L)}{2(q_n-q_s)} > 0$, $\frac{\partial V_n}{\partial\ c_s^L} = \frac{q_n - vq_n - q_s + vq_s - (1-v)c_n - vc_s^H + c_s^L}{2(q_n-q_s)} < 0$. □

EXAMPLE 1: Under parameter setting $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, if $\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v} < c_n \le$

$c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial |V_r|}{\partial c_s^H} = -\frac{v((1-v)(c_n - c_s^H - q_n + q_s) + 2v(c_s^H - c_s^L))}{2(1-v)(q_n - q_s)}$, directly following that $\frac{\partial |V_r|}{\partial c_s^H}$ decreases on c_n. Then for any $c_n \in (\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}, c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}]$, $\frac{\partial |V_r|}{\partial c_s^H} \leq \frac{\partial |V_r|}{\partial c_s^H}|_{c_n = \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}} = \frac{1}{2}v(1 - \frac{c_s^H}{q_s} - \frac{v(c_s^H - c_s^L)}{(1-v)(q_n - q_s)})$. Directly following $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$, we have $\frac{v(c_s^H - c_s^L)}{1-v} < c_s^L + q_n - q_s - \frac{c_s^H q_n}{q_s}$. With some algebra, we have that $\frac{1}{2}v(1 - \frac{c_s^H}{q_s} - \frac{v(c_s^H - c_s^L)}{(1-v)(q_n - q_s)}) > \frac{1}{2}v(\frac{(c_s^L + q_n - q_s)q_s - c_s^H q_n + (q_s - c_s^H)(q_n - q_s)}{q_s(q_n - q_s)}) > 0$, resulting in $\frac{\partial |V_r|}{\partial c_s^H}|_{c_n = \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}} > 0$. However, $\frac{\partial |V_r|}{\partial c_s^H}|_{c_n = c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}} = \frac{v(1-2v)(c_s^H - c_s^L)}{2(1-v)(q_n - q_s)}$ in which the sign depends on the value of v, the probability of high type. It leads to that $|V_r|$ consistently increases on c_s^H $\forall c_n \in (\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}, c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}]$ iff $v \leq \frac{1}{2}$; otherwise, if $v > \frac{1}{2}$, then there is a unique $\tilde{c}_n$ such that if $\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v} < c_n \leq \tilde{c}_n$, then $\frac{\partial |V_r|}{\partial c_s^H} \geq 0$; if $\tilde{c}_n < c_n \leq c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$, then $\frac{\partial |V_r|}{\partial c_s^H} < 0$. Hence, $|V_r|$ could either increase or decrease on c_s^H depending on v and c_n.

EXAMPLE 2: Under parameter setting $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, if $\frac{c_s^L q_n}{q_s} < c_n < \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial |V_r|}{\partial\ c_s^L} = \frac{v((1-v)^2 c_n q_n^2 - (q_n - q_s)((1-v)q_n - v q_s)^2 - c_s^L((1-v^2)q_n^2 - (2-v)v q_n q_s + v^2 q_s^2))}{2(q_n - q_s)((1-v)q_n - v q_s)^2}$. It is easy to observe that $\frac{\partial |V_r|}{\partial c_s^L}$ monotonically increases on c_n. Thus $\frac{\partial |V_r|}{\partial c_s^L} > \frac{\partial |V_r|}{\partial c_s^L}|_{c_n = \frac{c_s^L q_n}{q_s}} = \frac{v(c_s^L - q_s)}{2q_s}$ so that $\frac{\partial |V_r|}{\partial c_s^L}|_{c_n = \frac{c_s^L q_n}{q_s}} = \frac{v(c_s^L - q_s)}{2q_s} < 0$. Also, $\frac{\partial |V_r|}{\partial c_s^L} \leq \frac{\partial |V_r|}{\partial c_s^L}|_{c_n = \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}} = \frac{v(q_n(c_s^H q_n - q_s(c_s^L + q_n - q_s)) + v(q_s(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2))}{2(q_n - q_s)q_s((1-v)q_n - v q_s)}$. In conjunction with $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, we have that $q_n(c_s^H q_n - q_s(c_s^L + q_n - q_s)) < 0$ and $q_s(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2 > q_s(q_n^2 - q_s(q_s - c_s^L)) - q_n q_s(c_s^L + q_n - q_s) = q_s(q_s - c_s^L)(q_n - q_s) > 0$. After some algebra, we have $\frac{v(q_n(c_s^H q_n - q_s(c_s^L + q_n - q_s)) + v(q_s(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2))}{2(q_n - q_s)q_s((1-v)q_n - v q_s)} < 0$ iff $v < \frac{q_n(q_s(c_s^L + q_n - q_s) - c_s^H q_n)}{q_s(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2}$. Our assumption 2 is equivalent to $v < \frac{q_n}{q_n + q_s}$. It directly follows from $\frac{q_n(q_s(c_s^L + q_n - q_s) - c_s^H q_n)}{q_s(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2} - \frac{q_n}{q_n + q_s} = -\frac{(c_s^H - c_s^L)q_n^2 q_s}{(q_n + q_s)(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2)} < 0$ that $0 < \frac{q_n(q_s(c_s^L + q_n - q_s) - c_s^H q_n)}{q_s(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2} < \frac{q_n}{q_n + q_s}$. Therefore, we have that $|V_r|$ consistently decreases on $c_s^L \forall c_n \in (\frac{c_s^L q_n}{q_s}, \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}]$ iff $v < \frac{q_n(q_s(c_s^L + q_n - q_s) - c_s^H q_n)}{q_s(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2}$; otherwise if $v > \frac{q_n(q_s(c_s^L + q_n - q_s) - c_s^H q_n)}{q_s(q_n^2 - q_s(q_s - c_s^L)) - c_s^H q_n^2}$, there exists a unique $\bar{c}_n$ such that if $\frac{c_s^L q_n}{q_s} < c_n \leq \bar{c}_n$, then $\frac{\partial |V_r|}{\partial c_s^L} \leq 0$; if $\bar{c}_n < c_n \leq \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$, then $\frac{\partial |V_r|}{\partial c_s^L} > 0$.

Proof. Proposition 2.6

We discuss the value of information to the retailer throughout all possible parameter settings and show that $|V_r|$ is firstly insensitive to q_n if c_n is very low. As c_n increases, $|V_r|$ starts to increase on q_n and then decreases on q_n. The trend with respect to q_n varies depending on the parameter range.

Under the parameter Setting $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$, if $c_n \leq \frac{c_s^L q_n}{q_s}$, $\frac{\partial |V_r|}{\partial q_n} = 0$. If $\frac{c_s^L q_n}{q_s} < c_n \leq \frac{(c_s^L + q_n - q_s)((1-v)q_n - vq_s) + vc_s^L q_n}{(1-v)q_n}$, $\frac{\partial |V_r|}{\partial q_n} = \frac{(1-v)^2 v(c_s^L q_n - c_n q_s)(c_s^L q_n((1+v)q_n - 3vq_s) - c_n(2(1-v)q_n^2 - (1-v)q_n q_s - vq_s^2))}{4(q_n - q_s)^2((1-v)q_n - vq_s)^3}$. If follows from $c_n \geq \frac{c_s^L q_n}{q_s}$ and $2(1-v)q_n^2 - (1-v)q_n q_s - vq_s^2 = (1-v)(q_n - q_s) + (1-v)q_n^2 - vq_s^2 > 0$ that $\frac{\partial |V_r|}{\partial q_n} > 0$ iff $c_n > \frac{c_s^L q_n(q_n + vq_n - 3vq_s)}{2(1-v)q_n^2 - (1-v)q_n q_s - vq_s^2}$. With some algebra, we can show that $\frac{c_s^L q_n(q_n + vq_n - 3vq_s)}{2(1-v)q_n^2 - (1-v)q_n q_s - vq_s^2} - \frac{c_s^L q_n}{q_s} = -\frac{2c_s^L q_n((1-v)q_n^2 - vq_s^2 + v(q_n - q_s))}{q_s(2(1-v)q_n^2 - (1-v)q_n q_s - vq_s^2)} < 0$. Thus we have $\frac{\partial |V_r|}{\partial q_n} > 0$ for $c_n \epsilon (\frac{c_s^L q_n}{q_s}, \frac{(c_s^L + q_n - q_s)((1-v)q_n - vq_s) + vc_s^L q_n}{(1-v)q_n}]$. If $\frac{(c_s^L + q_n - q_s)((1-v)q_n - vq_s) + vc_s^L q_n}{(1-v)q_n} < c_n \leq c_s^L + q_n - q_s$, $\frac{\partial |V_r|}{\partial q_n} = -\frac{v(q_s - c_s^L)^2}{4q_n^2} < 0$.

Under parameter setting $\frac{c_s^H q_n}{q_s} < c_s^L + q_n - q_s$, if $c_n \leq \frac{c_s^L q_n}{q_s}$, $\frac{\partial |V_r|}{\partial q_n} = 0$. If $\frac{c_s^L q_n}{q_s} < c_n \leq \frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}$, similar to the proof in parameter Setting $c_s^L + q_n - q_s < \frac{c_s^H q_n}{q_s}$ with $c_n \epsilon (\frac{c_s^L q_n}{q_s}, \frac{(c_s^L + q_n - q_s)((1-v)q_n - vq_s) + vc_s^L q_n}{(1-v)q_n})$, we can show that $\frac{\partial |V_r|}{\partial q_n} > 0$. If $\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v} < c_n \leq c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}$, $\frac{\partial |V_r|}{\partial q_n} = \frac{v(c_s^H - c_s^L)(2(1-v)c_n - (1-3v)c_s^H - (1+v)c_s^L)}{4(1-v)(q_n - q_s)^2}$ so that $\frac{\partial |V_r|}{\partial q_n} > 0$ iff $c_n > \frac{(1-3v)c_s^H + (1+v)c_s^L}{2(1-v)}$. After some algebra, we show that $\frac{(1-3v)c_s^H + (1+v)c_s^L}{2-2v} - (\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}) = \frac{c_s^H q_s + c_s^L q_s - 2c_s^H q_n}{2q_s} < 0$. Thus it directly follows that $\frac{\partial \ |V_r|}{\partial q_n} > 0$ for $c_n \epsilon (\frac{c_s^H q_n}{q_s} - \frac{v(c_s^H - c_s^L)}{1-v}, c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v}]$. If $c_s^L + q_n - q_s - \frac{v(c_s^H - c_s^L)}{1-v} < c_n \leq c_s^L + q_n - q_s$, $\frac{\partial |V_r|}{\partial q_n} = -\frac{v(c_s^H - c_s^L)^2}{4(q_n - q_s)^2} < 0$. $\square$

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RESEARCH PAPERS

Papers Under Review

"Component Procurement for an Assembly Supply Chain with Random Capacities and Random Demand," with Fang, X.

Work in Progress

"Optimal Procurement Design for a National Brand Supplier in the Presence of Store Brand," with Fang, X.

"Strategic R&D Policy with Intra-industry Spillover," with Fang, X.

ACADEMIC PRESENTATIONS

"Component Procurement for an Assembly Supply Chain with Random Capacities and Random Demand," with Fang, X.

- INFORMS Annual Meeting, Houston, TX, Oct. 2017
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"Optimal Procurement Design for a National Brand Supplier in the Presence of Store Brand," with Fang, X.

- DSI Annual Meeting, Austin, TX, Nov. 2016
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TEACHING EXPERIENCE

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BUSADM 370 – Intro. to Supply Chain Management Sept. 2017 – May 2018

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BUSMGMT 735 – Advanced Spreadsheet Tools (Online) Sept 2015 – Jun. 2016

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BUSADM 210 – Introduction to Management Statistics May – July 2015

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HONORS AND AWARDS

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Chancellor's Graduate Student Fellowship, University of Wisconsin-Milwaukee, 2013-2017

Graduate Student Scholarship, Marquette University, 2010-2011

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