August 2018

Online HVAC Temperature and Air Quality Control for Cost-efficient Commercial Buildings Based on Lyapunov Optimization Technique

Atiye MalekSadati
University of Wisconsin-Milwaukee

ONLINE HVAC TEMPERATURE AND AIR QUALITY CONTROL FOR COST-EFFICIENT COMMERCIAL BUILDINGS BASED ON LYAPUNOV OPTIMIZATION TECHNIQUE

by

Atiye MalekSadati

A Thesis Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Master of Science

in Engineering

at

The University of Wisconsin-Milwaukee

Augest 2018

ABSTRACT

ONLINE HVAC TEMPERATURE AND AIR QUALITY CONTROL FOR COST-EFFICIENT COMMERCIAL BUILDINGS BASED ON LYAPUNOV OPTIMIZATION TECHNIQUE
by

Atiye MalekSadati

The University of Wisconsin-Milwaukee, 2018
Under the Supervision of Professor Lingfeng Wang

Commercial buildings consume up to 35.5% of total electricity consumed in the United States. As a subsystem in the smart building management system, Heating, Ventilation, and Air Conditioning (HVAC) systems are responsible for 45% of electricity consumption in commercial buildings. Therefore, energy management of HVAC systems is of interest. The HVAC system brings thermal and air quality comfort to the occupants of the building, designing a controller that maximizes this comfort is the first objective. Inevitably, ideal comfort tracking means more energy consumption and energy cost. Hence, the more advanced objective is balancing the comfort-cost tradeoff.

Since HVAC systems have nonlinear, complex and MIMO characteristics, modeling the system and formulating an optimization problem for them is challenging. Moreover, there are physical and comfort constraints to be satisfied, and randomness of parameters such as thermal disturbances, number of occupants in the building that affects the air quality, thermal and air quality setpoints we want to track, electricity price and outside temperature to be considered. Adding real time analysis to this problem furthers the challenge.

In this thesis, utilizing Lyapunov optimization technique, we first transform the constraints to stability equations, and formulate a stochastic optimization problem, then we minimize the time average of the expected cost of the system while the cost is a weighted sum of the discomfort and energy cost. Results show that using the proposed algorithm and real data, the algorithm is feasible, and an optimal solution for the problem is achieved.

To my parents, Mehdi and Zohre

And my sister Sara.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

AHU	Air Handling Unit
BAS	Building Automation System
CMS	Communication Management System
CO_2	Carbon Dioxide
COP	Coefficient of Performance
EMS	Energy Management System
HVAC	Heating, Ventilation and Air Conditioning
IOT	Internet of Things
LOT	Lyapunov Optimization Technique
VAV	Variable Air Volume
VFD	Variable Frequency Drive
MPC	Model Predictive Control

ACKNOWLEDGEMENTS

I would like to thank my advisor, Dr. Lingfeng Wang, whose guidance has extended my insight in research and whose brilliance has always astonished me. I am grateful for the opportunity of working with him throughout my master's and will never forget this experience, I hope I can keep contributing to the Cyber-Physical Energy Systems Laboratory in the future.

I also want to thank Dr. Yi Hu and Dr. Zhi Zheng for their support and invaluable input in serving as committee members for this thesis.

Many thanks to Israel, who has helped me a lot in the past months, both personally and academically. Doing this research would have not been possible without the support and encouragement of my fellow lab members, my friends in UWM and my old dear friends in Iran.

At last, I want to thank my parents, whom I love the most and have always gone to incredible length so that my sister and I can thrive doing what makes us happy, they have raised us to be independent and genuine and caring and I cannot thank them enough for this gift. I thank my sister and her husband Rasool, for being so supportive during my ten years of academic studies, in Tehran and in here. I want to send my love to my beautiful niece and nephew, Hanna and Pendar, I hope I can see their success in the coming years.

Chapter 1. Introduction

Buildings are significant consumers of electricity; Residential buildings and commercial buildings accounted for 38.7% and 35.5% of the total electricity usage of U.S. in 2010 [1], respectively. Saving energy in buildings can reduce building cost as well as the adversary impacts on the environment [2]. In commercial buildings (e.g. warehouses, offices, stores, restaurants, other buildings used for commercial purposes), the Heating, Ventilation, and Air Conditioning (HVAC) systems consume up to 45% of the electrical energy [3]. Therefore, controlling the HVAC system in a building is effective in energy management of the building.

In this chapter, the background and motivation of this research is explained. First, we introduce the concepts of smart buildings as a major pillar of next-generation cities. Then, as a subsystem of the smart building, the HVAC systems are described and their role in the building management is discussed. After establishing why control of this subsystem is of importance, we move on to how a control objective can be defined. At the end of this chapter, the research objective and the thesis structure are given.

1.1 Smart buildings and Energy Management System

Smart buildings are the next-generation's buildings and a core component of the trending vision of smart cities. A smart building should be able to control and respond to the time-varying needs of the building while satisfying constraints such as financial thresholds, comfort thresholds and physical thresholds [4]. If appropriately designed, by use of state of the art technologies smart buildings can

- Eliminate human error in building management,
- Improve and facilitate the occupants' health, safety, comfort, security, and productivity,
- Maximize building performance and efficiency,
- Promote the building's sustainability and environment friendliness via minimizing energy consumption,
- Minimize the building's operation and management cost [5].

For these reasons, many efforts are made for developing and advancing innovative technologies related to smart buildings. These efforts include the development of Internet of Things (IoT) based sensors [6], building communication protocols [7], system integration on the internet [8], and building subsystem controls.

A smart building needs system integration. System integration is the process of integrating protocols and regulations, algorithms, devices and sensors in a standard architecture that shares data and performs control. Typically, system integration has three levels shown in Figure 1.1.

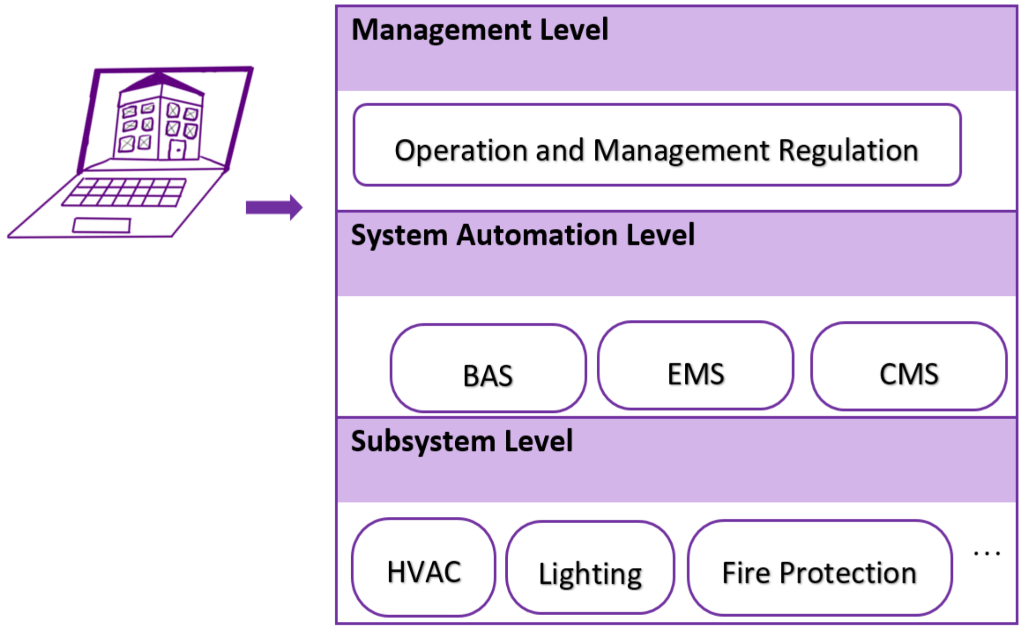

Figure 1.1 System integration for a smart building, adapted from [9].

The subsystems of a smart building are at the lowest level of system integration which can be controlled separately. These include the lighting system [10][11], fire protection system [10], security system [10] and HVAC system [8][12].

The middle level of system integration is the system automation level. Its role is to coordinate, supervise and control the subsystems mentioned above. Building Automation System (BAS) controls subsystems such as the HVAC system. The Energy Management System (EMS) supervises the energy consumption of the subsystems and tries to minimize the building's operation cost. The Communication Management System (CMS) [10] provides communication between subsystem controllers and the endpoint and helps the cooperation between automation systems [13].

The highest level of system integration is the management level; this level provides proper operation and management regulations for buildings with different functions [9]. A building can have Different functions such as commercial, residential, educational, and industrial. In this thesis, our focus is on a commercial building.

1.2 Heating Ventilation and Air Conditioning (HVAC) Systems

A typical HVAC system used in a commercial building with multiple thermal zones is shown in Figure 1.2. Different zones can have different characteristics and desired setpoints. The main components of HVAC systems are the Air Handling Unit (AHU), the supply fan, and the Variable Air Volume (VAV) boxes. Other components such as filters, dehumidifiers and humidifiers, and reheaters are usually added to a commercial building HVAC system to further the abilities of the system [14].

For a multi zone building shown in Figure 1.2, each zone has a VAV box. The mechanism works as follows: Outside fresh air enters the AHU, where the dampers control the ratio of outside to return air to satisfy the ventilation requirements. The mixed air is then passed through the cooling coil, which cools the air down and reduces its humidity if needed. The air is then delivered to the VAV boxes in each zone by a Variable Frequency Drive (VFD) supply fan. The VFD is a fast responding power electronic device that could be programmed to change the fan motor speed by changing motor input frequency. In the VAV box at each zone, there are a damper and a reheating coil; the damper manipulates the mass flow rate of air going into the zone so that the temperature of the zone tracks a prespecified desired temperature. This desired temperature is called the thermal setpoint. If the zone temperature is lower than the thermal setpoint, and flow rate cannot be reduced further due to ventilation requirements, the reheating coil reheats the supply air to maintain the

zone temperature at the desired level. Also, changing this damper and the outside air damper changes the amount of fresh air that is entering each zone. Fresh air dilutes the CO2 concentration of the zone because its CO2 concentration is always lower than the zone's CO2 concentration level. Therefore, manipulating the damper positions in the VAV and AHU can lead to tracking the CO2 concentration setpoint for a zone.

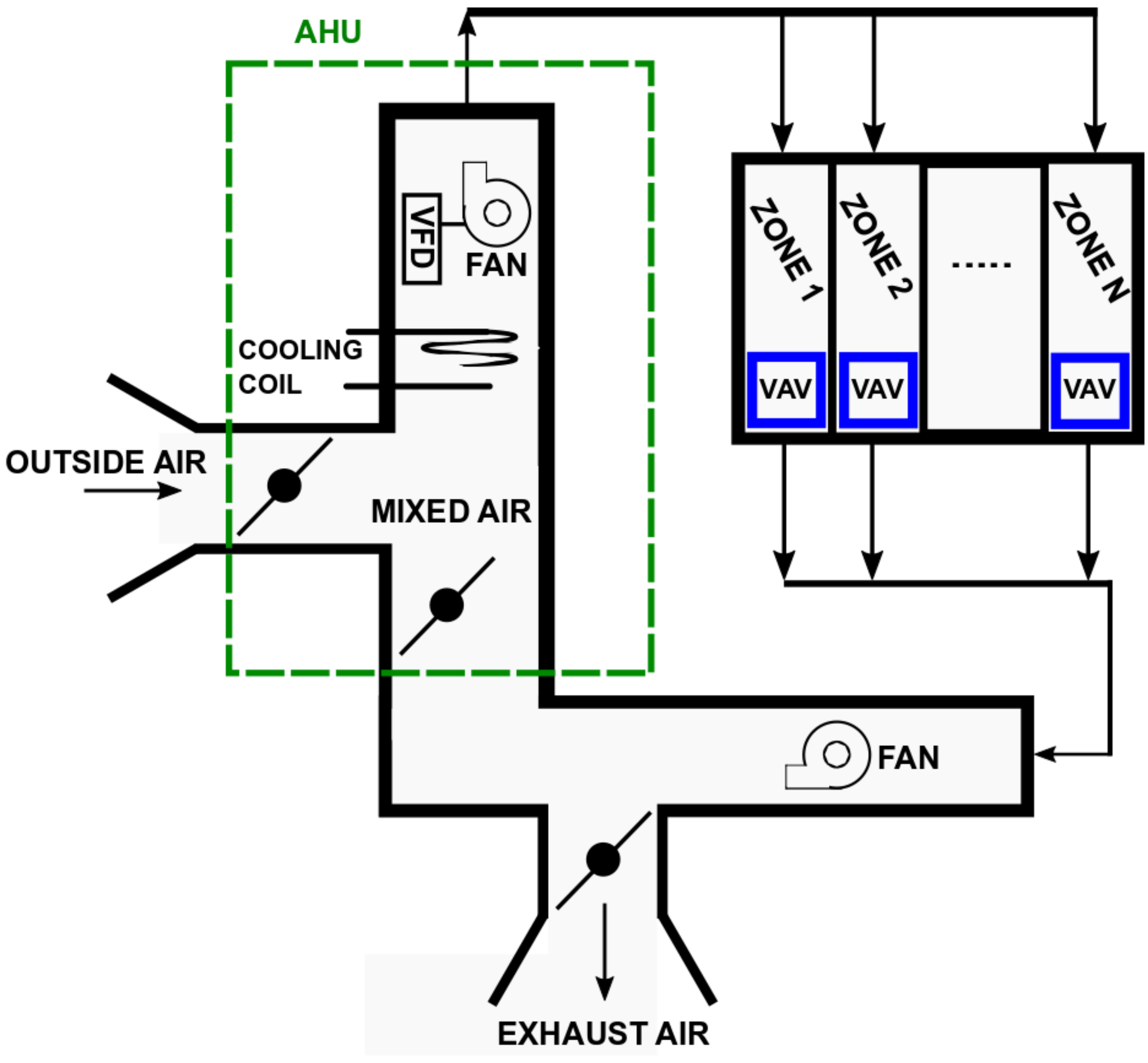

Figure 1.2. Commercial HVAC system [15]

For simplicity, throughout this thesis, we mainly focus on the case that all zones need cooling. Moreover, similar to [15],we assume the heat transfer between adjacent zones is negligible since the total heat gained from outside and inside of a zone is usually much more significant than the heat transfer between zones.

1.3 Control Objective for HVAC Systems

Although HVAC system control is a process control problem, there are certain features that render the problem unique and challenging [16], including:

- Nonlinear dynamics and nonlinear objective functions,
- Time varying setpoints and system dynamics,
- Time varying disturbances,
- Conflict and tradeoff between objectives.

The goal of HVAC system control is firstly meeting the occupants' comfort criteria; the system tries to keep a pleasant environment in each zone for all times. To be able to achieve comfort for the occupants, first, it should be formulated and quantized. What is used as comfort in this thesis is described in section 2.2.2, which is a function of the discrepancy between the actual and desired comfort indicators. Controlling the error of tracking is usually considered as *local control* in HVAC systems [17]. However, in smart buildings, *supervisory control* is desired. in the supervisory level for a building, the control objective is defined as one or a combination of the following:

- Maintain thermal comfort,

- Maintain indoor air quality (IAQ) at acceptable level,
- Minimize energy cost,
- Minimize energy consumption,
- Minimize thermal discomfort hours.

If control objective is only one of the above states, it is a single objective optimization and if the goal is achieving more than one objective simultaneously, it is a multi-objective optimization. Typically, in multi-objective problems, the objectives to be optimized conflict with each other. Therefore, there are more than one unique solution for this type of problems. Since we try to maintain thermal and CO_2 concentration comfort while minimizing the energy cost, our problem is a multi-objective optimization problem.

Commercial buildings consume electricity at a high level due to long hours of being highly occupied. Energy consumption and price for a commercial HVAC system is introduced in section 2.2.3. An ideal controller that tracks the comfort setpoints perfectly is consuming more energy and therefore is more expensive than a controller that tries to minimize the energy and disregards the comfort. Therefore, a reasonable tradeoff between these two is typically used as an objective for the HVAC system control.

In this thesis, we try to solve a multi-objective optimization problem for supervisory control of the system without the need to modify or predict the system setpoints, electricity price, external parameters and other uncertainties.

1.3.1 Different Control variables

There are four different control variables that can be chosen individually or together to achieve the objective:

- Controlling the return air damper position to minimize energy consumption while maintaining a desired level of comfort,
- Controlling the supply air temperature through heating and/or cooling coils to reduce energy consumption while maintaining comfort conditions,
- Controlling the outdoor and exhaust air dampers to control how much fresh air is introduced, which has an adversary effect on thermal energy consumption while keeping the constraints on the comfort,

  and
- Controlling the supply and return airflow rate to reduce the energy consumption while maintaining the comfort according to comfort setpoints [14].

In addition to the mentioned scenarios, the physical constraints of the system should be met. In this thesis, we use the mass air supply rate as the control variable for both temperature and CO_2 concentration. Problem formulation associated with this objective is described in detail in Chapter 2.

1.4 Research Objective and Thesis Structure

The objective of this research is to obtain an online HVAC controller for a commercial multi zone building. The controller needs to maintain the occupants' thermal and air quality comfort while minimizing the energy cost of the system. Also, it needs to satisfy the physical constraints of the system, without the need to modify or predict the system stochastic parameters.

The structure of this thesis is as follows: chapter 2 reviews the literature on different HVAC controllers, and formulates the stochastic optimization problem we want to solve. Chapter 3

describes the Lyapunov optimization method used to optimize the proposed objective function and introduces the proper algorithm that can be implemented in the building's energy management system, the feasibility of the algorithm and its assumptions are also discussed in this chapter. Chapter 4 goes over the simulated results of the algorithm for the time horizon of a month and discusses how the results are matching the expectations and how the constraints of the optimization problem are met. Chapter 5 concludes the thesis with the discussion and future work.

Chapter 2. Literature Review and Problem Formulation

In this thesis, a commercial building with multiple zones is considered. A zone is an area with its own thermal and air quality setpoints that we want to track, zones are typically rooms but a very large room can be considered as more than one zone. The goal is to adjust the temperature and Carbon Dioxide (CO_2) concentration level of each zone by controlling the HVAC system in a way that the comfort of the occupants in each zone is guaranteed and at the same time the energy cost of the HVAC system is minimized. Since keeping the occupant's demands met at all times could be costly, the designed controller in EMS of the smart building should resolve the tradeoff between the energy cost and the occupants comfort.

2.1 Review on Controls for HVAC Systems

Because of HVAC systems high, unknown power demand and their major part in building energy consumption, HVAC controllers have attracted a lot of attention and different controllers for them

are developed. In this section, we try to give a comprehensive review on methods for controlling these systems.

2.1.1 Classical Control

Controllers such as On/Off controllers, P (proportional), PI (proportional-integral), and PID (proportional-integral-derivative) controllers are considered as classical controllers. These controllers are simple and have low-initial cost [18].

The On/Off controllers are the most intuitive and the easiest to implement. They use an upper and lower threshold values to regulate a process within the range bounded by the thresholds. However, these controllers are unable to control processes that vary with delay. In an HVAC system that is controlled by an On/Off controller, the high thermal inertia of a building causes large discrepancies between the setpoints and the system performance.

The P, PI, and PID controllers use steady state error dynamics and modulate the controlled variable to obtain accurate tracking of the desired parameters in the process. In HVAC systems, classical controllers have been used for the dynamics of the cooling coil [19], room temperature control [20], supply air temperature control [21], and VAV temperature control [22].

A simple schematic of classical controllers is shown in Figure 2.1. While the PID controller produces more promising results than the other types, it is not fast in tuning the controller parameters and if the actual conditions are not around the assumed operating conditions, the performance decreases drastically [16]. Also, due to the linearity of PID controllers, using them in a nonlinear systems like HVAC systems causes inadequate performance [23]. To improve the performance of these controllers, they are usually combined with meta heuristic algorithms in hybrid controllers [24].

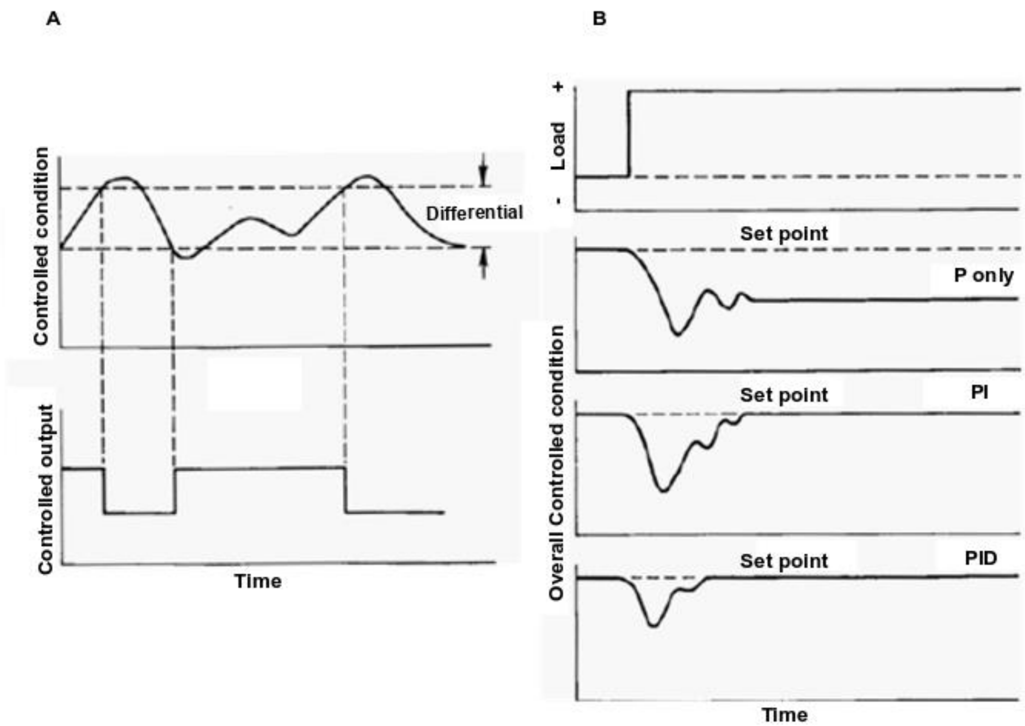

Figure 2.1. Simple schematic of A) On/Off controller and B) P, PI, and PID controller [25].

2.1.2 Hard Control

Hard control methods are nonlinear control, robust control, optimal control, and Model Predictive Control (MPC). These methods have better performance than classic controllers.

Since there is no general nonlinear control theory, different techniques such as Lyapunov stability theory, gain scheduling control, and adaptive control are used [26][27].in these methods, the control law tries to drive the system toward a stable state and simultaneously achieve the control objectives. In gain scheduling control, first the system is divided to piecewise linear regions, then a classic linear control is designed for each region with appropriate gains [28]. Understandably, the shortcomings of classical controllers are present in this method too. In addition, identifying the

linear regions and designing the switch logic between the regions is necessary and can be problematic or unrealistic. Generally, linearization around an operating point limits the operation range. Also, the uncertainty involved in HVAC system models make designing the controller challenging [29].

In robust control design, it is tried to make the performance resistant to time varying conditions of the system [18]. Optimal controls are supervisory controls [29]. These controls pursue the minimization or maximization of a function by systematically obtaining values for parameters or variables within their acceptable ranges. For HVAC systems, this function is a combination of comfort and energy consumption and therefore these methods are well exploited in controlling HVAC systems.

Robust control and optimal control designs can compensate for the disturbance presence and time-varying parameters of the system. However, optimal control needs an accurate modeling of the system parameters. Many of the HVAC system parameters such as outside temperature, electricity price, the comfort setpoints and the disturbances are uncertain and are subject to uncontrollable circumstances. Therefore, it is tried to model, predict, or estimate these parameters as well as possible beforehand. Specifications of these parameters and guaranteeing feasibility could be impractical and difficult. Another disadvantage of optimal control is that unless the problem has special configuration, such as linear, unconstrained models in classic Linear Quadratic Regulator (LQR), it is challenging to implement the complex method online [23].

MPC is a multivariable control technique which is based on a prediction model. In MPC, the memory of the system, i.e. the past information, and the future input of the system account for the prediction of the future output. At each time slot, the latest element in the control vector is used as the input and the remainder is discarded. Here, the cost function could be tracking setpoint error,

energy cost, power consumption, or other factors. MPC can be used in constrained cost functions too; the constraints of the HVAC system could be minimum and maximum inside temperature, minimum and maximum air flow rate, and other limitations of the system [30]. In theory, if the uncertainties and disturbances of the system such as occupant activities and outside whether parameters are modeled properly, their predicted influence on the system can be calculated in the control vector. This controller is robust to both time-varying system parameters and disturbances and it can regulate the process within the given bounds.

To sum up, among the hard control approaches, MPC is one of the most promising techniques because of its ability to integrate disturbance rejection, constraint handling, slow-moving dynamic control and energy conservation strategies into controller formulation. However, acceptable results depend on the disturbance and parameter predictions that are often different from actual realizations [31][32].

2.1.3 Soft control

As mentioned before, HVAC systems are nonlinear, MIMO and complex. Identifying an appropriate mathematical model for the system could be challenging because of disturbances, time delays, contingencies and vagueness about thermal comfort. In soft control methods, modeling of the system is not required and the controller design only depends on the occupants' thermal comfort feeling [32]. The soft control or intelligent control strategies include neural networks control method (NNs) [33][34], fuzzy logic control method (FL) [35],and genetic algorithms method (GAs) [36]. These methods are called black box methods since no mathematical and physical understanding is needed and only input and output of the system is required.

If trained properly, neural networks controls have a great capacity to predict nonlinear systems. However, the efficiency and performance of the control depends on huge amount of “good” data, training process needs a long time and each building needs its own data set to be trained. The long process of training means this method is not suitable for real time.

Fuzzy logic control methods consist of three steps: fuzzification, fuzzy inference and defuzzification. In the first step, the crisp values of the inputs are transferred to the fuzzy values using *membership functions*. Fuzzy inference means mapping fuzzy values to different fuzzy values by applying logical operations and if-then rules. Finally, the defuzzification process produces a single crisp value as an output. the FL can be implemented on both the local and supervisory levels of control.

One problem of fuzzy systems is that to get more accurate results, the fuzzy grades should increase exponentially. Also, the methods are rather slow and are not suitable for real time response. Since there is no learning strategy in the algorithm, it is hard to get feedback on the behavior of the system. Designing proper membership functions is also challenging.

The genetic algorithm is mostly used for automatic tuning or learning of the components in fuzzy systems. These systems are called Genetic Fuzzy Systems. Using the genetic algorithm, the optimum solution will be found among the other solutions.

Hybrid controllers or fusion controllers are a combination of soft and hard controllers. In these methods, it is tried to utilize advantages of one method to overcome the disadvantage of the other [37][38][17].

2.2 Problem Formulation

Based on what is discussed in the previous section, we want to use a hard controller that can act on real time, does not require modeling, predicting or estimating system uncertain parameters, and satisfies the constraints on comfort bounds and physical limits of the system.

2.2.1 Thermal and Ventilation Models

For zone i, the thermal dynamics is described by the following model [39]:

$$C_i \frac{dT_{i,t}}{dt} = \frac{T_{o,t} - T_{i,t}}{R_i} + C_a m_{i,t}\left(T_{supply} - T_{i,t}\right) + q_{i,t} \tag{1}$$

where R_i and C_i are the zone equivalent thermal resistance (°C/W) and capacitance (J/°C) respectively, we can find R_i and C_i by model identification. $T_{i,t}$ and $T_{o,t}$ are inside and outside air temperature of zone i at time t (°C), C_a is the specific heat of the air (J/g/°C), $m_{i,t}$ is the air supply rate of zone i at time t (g/s), T_{supply} is the air temperature of the supply fan (°C), and $q_{i,t}$ denotes the external thermal disturbance of zone i at time t (W). $q_{i,t}$ is associated with heat generated by lighting levels, occupants, and other unaccounted-for heat generations in the zone and can be measured using IOT sensors or smart devices.

For a well-ventilated space, the CO_2 concentration dynamics is described by the following model [40]:

$$V_i \frac{dC_{i,t}}{dt} = \frac{m_{i,t}}{\rho}\left(C_{supply} - C_{i,t}\right) + P_{i,t} G \tag{2}$$

where V_i is the volume of the zone (m^3), $C_{i,t}$ is the CO_2 concentration in the ventilated space (ppm), ρ is air density (g/m^3), $P_{i,t}$ is the number of people in the zone and G is the average CO_2 emission rate per person (g/s) and C_{supply} is the CO_2 concentration of supply air (ppm):

$$C_{supply} = \pi \frac{\sum_{i=1}^{N} m_{i,t}\, C_{i,t}}{\sum_{i=1}^{N} m_{i,t}} + (1-\pi)C_o \tag{3}$$

where $\pi \in [0,1]$ represents the damper position in the AHU, and C_o is the CO2 concentration of the outside air.

We can discretize (1) and (2) over time intervals τ using finite-difference methods as follows:

$$T_{i,t+1} = \left(1 - \frac{\tau}{R_i C_i}\right) T_{i,t} + \frac{\tau C_a}{C_i} m_{i,t}\left(T_{supply} - T_{i,t}\right) + \frac{\tau}{R_i C_i} T_{o,t} + \frac{\tau}{C_i} q_{i,t} \tag{4}$$

$$C_{i,t+1} = C_{i,t} + \frac{\tau}{V_i \rho} m_{i,t}\left(C_{supply} - C_{i,t}\right) + \frac{\tau}{V_i} P_{i,t} G \tag{5}$$

For simplicity, we define:

$$\alpha_i = \frac{\tau}{R_i C_i} \tag{6}$$

$$\beta_i = \frac{\tau C_a}{C_i} \tag{7}$$

$$\gamma_i = \frac{\tau}{C_i} \tag{8}$$

$$\varepsilon_i = \frac{\tau}{V_i \rho} \tag{9}$$

$$\xi_i = \frac{\tau G}{V_i} \tag{10}$$

Now (3) and (4) can be rewritten as:

$$T_{i,t+1} = (1-\alpha_i)T_{i,t} + \beta_i m_{i,t}\left(T_{supply} - T_{i,t}\right) + \alpha_i T_{o,t} + \gamma_i q_{i,t} \tag{11}$$

$$C_{i,t+1} = C_{i,t} + \varepsilon_i m_{i,t}\left(C_{supply} - C_{i,t}\right) + \xi_i P_{i,t} \tag{12}$$

Here, $t \in [1, M]$ denotes the time interval index and M is the total number of time intervals. For choosing τ, we consider a time interval during which all of time varying parameters could be regarded as constants.

The rate of the air supplied to each zone at each time $m_{i,t}$ is the only parameter in (10) and (11) that we can control by damper positions in the VAV box. Based on physical constraints of the dampers we have:

$$m_i^{min} \leq m_{i,t} \leq m_i^{max} \tag{13}$$

where m_i^{min} and m_i^{max} are minimum and maximum air supply rate of zone i. Our goal in this thesis is to find optimal $m_{i,t}$ at each time for each zone without exceeding the boundaries while keeping temperature and CO_2 concentration level at user setpoints and energy cost at the lowest level possible. In the next section, we define the occupants comfort and its associated cost which is part of the objective function we want to minimize.

2.2.2 Occupants' Comfort

Comfort for the occupants of a zone in a smart building is mainly determined by three factors: visual comfort, thermal comfort, and indoor air quality [9]. Visual comfort is achieved by appropriate illumination level and lighting systems, but the last two are controlled by the HVAC system. Therefore, the proper control of the HVAC system is important in occupants' comfort. Inside

temperature indicates the thermal comfort and inside CO_2 concentration is used as an index to measure the air quality in each zone. The heating and air conditioning system tries to keep the temperature in a comfortable range:

$$T_i^{min} \leq T_{i,t} \leq T_i^{max} \tag{14}$$

where T_i^{min} and T_i^{max} are minimum and maximum inside temperature for each zone, respectively. The ventilation system keeps the CO_2 concentration level in a comfortable range by changing the damper position in AHU unit to let more outside air come in and mix with return air from the zones:

$$C_i^{min} \leq C_{i,t} \leq C_i^{max} \tag{15}$$

where C_i^{min} and C_i^{max} are minimum and maximum inside CO_2 concentration for each zone, respectively.

In addition to these ranges, there are setpoints for the temperature and CO_2 concentration set by the occupants of a zone. The occupant can decide to change these setpoints at any time t and the HVAC system must be able to track the thermal setpoint $T_{i,t+1}^{set}$ and CO_2 concentration setpoint $C_{i,t+1}^{set}$ for next time interval in each zone. To measure how close the HVAC system can track the setpoints, we define the discomfort cost $\mathbf{\Phi}_{1,t}$ that should be minimized as part of our objective function:

$$\mathbf{\Phi}_{1,t} = \sum_{i=1}^{N} \phi_T \left(T_{i,t+1} - T_{i,t+1}^{set}\right)^2 + \sum_{i=1}^{N} \phi_C \left(C_{i,t+1} - C_{i,t+1}^{set}\right)^2 \tag{16}$$

Where N is the number of zones, ϕ_T is the cost coefficients associated with temperature and ϕ_C is the cost coefficients associated with CO_2 concentration.

2.2.3 Energy Cost

We also want to minimize the energy cost of the system; energy consumption of HVAC systems consists of two main parts: energy consumption associated with the supply fan and energy consumption associated with the cooling coil. For each part, the energy cost can be obtained by multiplying the consumed energy by the electricity price S_t at time t. Consumed energy for the supply fan can be approximated by [41]:

$$E_{fan,t} = \tau\mu\left(\sum_{i=1}^{N} m_{i,t}\right)^3 \qquad (17)$$

where μ is a given constant coefficient associated with the supply fan power consumption. The energy consumption of the cooling coil can be represented by the following model:

$$E_{coil,t} = \tau\frac{C_a}{\eta\ \mathrm{COP}}\sum_{i=1}^{N} m_{i,t}(T_m - T_{supply}) \qquad (18)$$

where η is the efficiency factor of the cooling coil, COP (Coefficient of Performance) is a given constant which is the ratio of the produced cold/heat to the consumed energy, and T_m is the temperature of the mixed air:

$$T_m = \pi\frac{\sum_{i=1}^{N} m_{i,t}\,T_{i,t}}{\sum_{i=1}^{N} m_{i,t}} + (1-\pi)T_{o,t} \qquad (19)$$

where $\pi \in [0,1]$ represents the damper position in the AHU. Substituting (18) into (17), $E_{coil,t}$ can be rewritten as:

$$E_{coil,t} = \tau\frac{C_a}{\eta\ \mathrm{COP}}\sum_{i=1}^{N} m_{i,t}(\pi T_{i,t} + (1-\pi)T_{o,t} - T_{supply}) \qquad (20)$$

With the above-mentioned models for energy consumption, energy cost $\mathbf{\Phi}_{2,t}$ can be modelled as:

$$\mathbf{\Phi}_{2,t} = S_t(E_{fan,t} + E_{coil,t}) \qquad (21)$$

2.2.4 Optimization Problem

With the above-mentioned models, we can have a cost function that consists of the discomfort cost and the energy cost to be minimized:

$$\boldsymbol{\Phi}_t = \boldsymbol{\Phi}_{1,t} + \boldsymbol{\Phi}_{2,t} \tag{22}$$

$$subject\ to\ (11) - (15)$$

The control variable here is the air mass flow rate $m_{i,t}$ and our goal is to find the optimal $m_{i,t}^*$ at each time t for each zone i that satisfies the constraints and minimizes the cost function. But there are uncertainties about other parameters in $\boldsymbol{\Phi}_t$**,** namely the next time slot temperature setpoint $T_{i,t+1}^{set}$, the next time slot CO_2 concentration setpoint $C_{i,t+1}^{set}$, outside temperature $T_{o,t}$, external thermal disturbances $q_{i,t}$, number of people in the zone $P_{i,t}$ and electricity price S_t. To consider these uncertainties, we change the cost to the expected cost and formulate the cost function as:

$$\mathbb{E}_{T_{i,t+1}^{set}, C_{i,t+1}^{set}, T_{o,t}, q_{i,t}, S_t, P_{i,t}}\{\boldsymbol{\Phi}_t\} \tag{23}$$

$$subject\ to\ (11) - (15)$$

where $\mathbb{E}$ denotes the expectation operator, acting on all its subscripts. Since we are dealing with stochastic processes it makes sense to take the long-term time average of expected cost. Therefore, the last step in problem formulation is:

$$(\mathbf{OP1})\ \min_{m_{i,t}}\ \lim_{M\to\infty} \frac{1}{M-1} \sum_{t=1}^{M} \mathbb{E}_{T_{i,t+1}^{set}, C_{i,t+1}^{set}, T_{o,t}, q_{i,t}, S_t, P_{i,t}}\{\boldsymbol{\Phi}_t\} \tag{24}$$

$$subject\ to\ (11) - (15)$$

This is a constrained stochastic optimization problem and finding the optimal value of the decision variable $m_{i,t}$ is very complicated. To solve this problem, we use Lyapunov Optimization Technique that is described in the next chapter.

Chapter 3. Online Cost and Comfort Efficient Controller

This chapter describes the Lyapunov optimization technique that is used in this thesis to solve the optimization problem $(\mathbf{OP1})$ described in previous chapters. That leads to a controller that minimizes the discomfort and the energy cost of the HVAC system at each time in each zone of a commercial multizone building. Lyapunov optimization uses *Lyapunov function* to optimally control a dynamical system. One significant feature of LOT is that it does not need any model or prediction of system parameters and only observes the current state to stabilize the queues and minimize some performance objective.

This framework has been widely used in energy management of data centers [42], microgrids [43] and smart buildings [44], as well as online temperature control of HVAC systems [15]. In [44], online energy management of a smart building with HVAC system and other baseline and controllable loads is investigated. They assume that same as electric vehicle loads, the HVAC system has a specific power demand and that is not accurate. HVAC systems have unknown power demand that is related to many factors, such as temperature setpoints by the user, upper and lower abounds of inside temperature, outside temperature, number of people in the building and external

thermal disturbances. Therefore, even generating random power demand cannot truly reflect the true demand of the system. This issue is addressed in [15], in which unknown power demand is considered for the HVAC system and an objective function related to the temperature tracking and expected energy cost is formulated. However, Neither of [15] and [44] investigate the dynamics of ventilation in the zones of the building which is investigated in this thesis.

3.1 Lyapunov Optimization Technique (LOT)

LOT tends to transform the evolving dynamics of a system to queue stability problems. If $\mathbf{Q}(t)$ is the state of the system which evolves over time, a Lyapunov function is a nonnegative scalar measure of $\mathbf{Q}(t)$. This function is typically defined in a way to grow large when the system moves towards undesirable states. By obtaining control actions that make the Lyapunov function move in the negative direction towards zero, system's state stability is achieved. In our case system stability means the constraints (11) – (15) are met.

Using LOT, A typical goal is to stabilize all system's evolving dynamics while optimizing some performance objective, in our case that performance objective is the time average expected cost defined in (23). Minimizing the weighted sum of performance objective and *Lyapunov drift* of the evolving system dynamics is referred to as minimizing *drift-plus-penalty* expression [45]. Optimizing this expression leads to joint minimization of time average expected cost and system stability; meaning a solution for $(\mathbf{OP1})$ is achieved.

The process can be summarized as follows:

- Constructing *virtual queues* associated with the states of the system: CO_2 concentration $C_{i,t}$ and inside temperature $T_{i,t}$ of all zones,

- Defining a suitable Lyapunov function,
- Obtaining the Lyapunov drift,
- Obtaining the drift plus penalty expression, and its upper bound,
- Minimize the upper bound of the drift plus penalty expression according to LOT framework.

A remarkable property of LOT is that it does not need to know the probability distribution of the uncertainties, it observes the parameters at the start of each time slot and then decides the optimum value for the control action. It does not model or predict any parameter beforehand.

3.1.1 Constructing virtual queues

First step is to construct virtual queues to satisfy the constraints of the optimization problem. For each zone we defined a queue associated with the temperature $Q_{i,t}$ and a queue associated with the CO_2 concentration $Q'_{i,t}$:

$$Q_{i,t} = T_{i,t} + \delta_i^t \tag{25}$$

$$Q'_{i,t} = C_{i,t} + \delta_i^c \tag{26}$$

where δ_i^t and δ_i^c are constants that act as slack variables to change the inequalities (14) and (15) into equalities. With the above definition, for the next time slot we have:

$$Q_{i,t+1} = T_{i,t+1} + \delta_i^t \tag{27}$$

$$Q'_{i,t+1} = C_{i,t+1} + \delta_i^c \tag{28}$$

3.1.2 Lyapunov Function and Lyapunov Drift

This function is a scalar measure of the problem congestion. In our system we have N zones and $2N$ queues, let $\mathbf{Q}(t) = (Q_{1,t}, \dots, Q_{N,t}, Q'_{1,t}, \dots, Q'_{N,t})$ be the queue vector and assume it evolves over time slots $t \in \{1, \dots, M\}$ according to (27) and (28). Now we can define a quadratic Lyapunov function:

$$L(\mathbf{Q}(t)) \triangleq \frac{1}{2} \sum_{i=1}^{N} (w_i Q_{i,t}^2 + w_i' Q'^2_{i,t}) \tag{29}$$

where $\{w_i\}_{i=1}^{N}$ and $\{w'_i\}_{i=1}^{N}$ are positive integer weights. Typically, the weights are defined to be equal to one.

Based on its definition, a small value of $L(\mathbf{Q}(t))$ means every queue is small, and a large value means at least one of the queues is large. Now we define one slot conditional Lyapunov drift (or simply, Lyapunov drift) $\mathbf{\Delta}(t)$ as follows:

$$\mathbf{\Delta}(t) = \mathbb{E}\{L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) | \mathbf{Q}(t)\} \tag{30}$$

$\mathbf{\Delta}(t)$ is the conditional expected change in the Lyapunov function over one slot, given that the current state in slot t is $\mathbf{Q}(t)$. minimizing this drift pushes Lyapunov functions towards a lower congestion level (i.e. stability of queues).

It can be proven by Lyapunov drift theorem [45] that if there exists a $B > 0, \epsilon_i > 0$ such that

$$\mathbf{\Delta}(t) < B + \sum_{i=1}^{N} \epsilon_i \, Q_i(t), \tag{31}$$

then all queues are strongly stable (i.e. the constraints are satisfied).

Now, we incorporate this into our problem using (27), (28) and (29), we have:

$$
\begin{aligned}
L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) &= \frac{1}{2}\sum_{i=1}^{N}\left(Q_{i,t+1}^2 + {Q'}_{i,t+1}^2\right) - \frac{1}{2}\sum_{i=1}^{N}\left(Q_{i,t}^2 + {Q'}_{i,t}^2\right) \\
&= \frac{1}{2}\sum_{i=1}^{N}\left(\left[(1-\alpha_i)Q_{i,t} + \beta_i m_{i,t}\left(T_{supply} - T_{i,t}\right) + \alpha_i\left(T_{o,t} + \delta_i^t\right) + \gamma_i q_{i,t}\right]^2 - Q_{i,t}^2\right) \\
&+ \frac{1}{2}\sum_{i=1}^{N}\left(\left[{Q'}_{i,t} + \varepsilon_i m_{i,t}\left(C_{supply} - C_{i,t}\right) + \xi_i P_{i,t}\right]^2 - {Q'}_{i,t}^2\right) \\
&\leq \frac{1}{2}\sum_{i=1}^{N}\left(\left[(1-\alpha_i)Q_{i,t} + \beta_i m_{i,t}\left(T_{supply} - T_{i,t}\right) + \alpha_i\left(T_{o,t} + \delta_i^t\right) + \gamma_i q_{i,t}\right]^2\right. \\
&\left.- (1-\alpha_i)Q_{i,t}^2\right) + \frac{1}{2}\sum_{i=1}^{N}\left(\left[{Q'}_{i,t} + \varepsilon_i m_{i,t}\left(C_{supply} - C_{i,t}\right) + \xi_i P_{i,t}\right]^2 - {Q'}_{i,t}^2\right) \\
&\leq \frac{1}{2}\sum_{i=1}^{N}(B_i + B'_i) + \sum_{i=1}^{N}\left((1-\alpha_i)Q_{i,t}\beta_i m_{i,t}\left(T_{supply} - T_{i,t}\right)\right) \\
&+ \sum_{i=1}^{N}\left({Q'}_{i,t}\varepsilon_i m_{i,t}\left(C_{supply} - C_{i,t}\right)\right) \qquad (32)
\end{aligned}
$$

where B_i and B'_i are positive constants that satisfy the following:

$$
\begin{aligned}
B_i \geq &\left(\beta_i m_i^{max}\left(T_{supply} - T_i^{min}\right) + \alpha_i(T_o^{max} + |\delta_i^t|) + \gamma_i q_i^{max}\right)^2 \\
&+ 2(1-\alpha_i)(T_i^{max} + |\delta_i^t|)(\alpha_i(T_o^{max} + |\delta_i^t|) + \gamma_i q_i^{max})
\end{aligned} \qquad (33)
$$

and

$$
{B'}_i \geq \left(\varepsilon_i m_i^{max}\left(C_{supply} - C_i^{min}\right) + \xi_i P_i^{max}\right)^2 + 2(C_i^{max} + |\delta_i^c|)\xi_i P_i^{max} \qquad (34)
$$

Now we can find the upper bound of $\mathbf{\Delta}(t)$ using (32):

$$\mathbf{\Delta}(t) = \mathbb{E}\{L(\mathbf{Q}(t+1)) - L(\mathbf{Q}(t)) | \mathbf{Q}(t)\} \tag{35}$$

$$\leq \frac{1}{2}\sum_{i=1}^{N}(B_i + B_i')$$

$$+ \mathbb{E}\left\{\sum_{i=1}^{N}\left((1-\alpha_i)Q_{i,t}\beta_i m_{i,t}(T_{supply} - T_{i,t})\right) | \mathbf{Q}(t)\right\}$$

$$+ \mathbb{E}\left\{\sum_{i=1}^{N}\left(Q'_{i,t}\varepsilon_i m_{i,t}(C_{supply} - C_{i,t})\right) | \mathbf{Q}(t)\right\}$$

where the expectation is taken over the randomness of next time slot temperature setpoint $T_{i,t+1}^{set}$, the next time slot CO_2 concentration setpoint $C_{i,t+1}^{set}$, outside temperature $T_{o,t}$, external thermal disturbances $q_{i,t}$, number of people in the zone $P_{i,t}$ and electricity price S_t. We found the upper bound of $\mathbf{\Delta}(t)$ in the form of (31) for our proposed problem. So far, the constrained optimization problem is half-solved and the constraints are transformed into queue stability problems.

3.1.3 Drift Plus Penalty Term

In addition to the queues that we want to stabilize, we have an associated stochastic "penalty" process $\mathbf{\Phi}_t$ whose time average of the expected value we want to minimize. The process $\mathbf{\Phi}_t$ represents the weighted sum of energy cost and discomfort cost and is introduced in (23). To incorporate this penalty and the constraints into one expression, we need to introduce drift plus penalty term $\mathbf{\Delta Y}(t)$:

$$\mathbf{\Delta Y}(t) = \mathbf{\Delta}(t) + V\mathbb{E}\{\mathbf{\Phi}_t\} \tag{36}$$

where V is a positive parameter and its role is to allow a smooth tradeoff between satisfying constraints and penalty minimization. Adding $V\mathbb{E}\{\mathbf{\Phi}_t\}$ to both sides of the inequality (35), we have:

$$\mathbf{\Delta Y}(t) = \mathbf{\Delta}(t) + V\mathbb{E}\{\mathbf{\Phi}_t|\mathbf{Q}(t)\} \tag{37}$$

$$< \frac{1}{2}\sum_{i=1}^{N}(B_i + B_i') + V\mathbb{E}\{\mathbf{\Phi}_t|\mathbf{Q}(t)\}$$

$$+ \ \mathbb{E}\left\{\sum_{i=1}^{N}\Big((1-\alpha_i)Q_{i,t}\beta_i m_{i,t}\big(T_{supply} - T_{i,t}\big)\Big)\,|\mathbf{Q}(t)\right\}$$

$$+ \ \mathbb{E}\left\{\sum_{i=1}^{N}\Big({Q'}_{i,t}\varepsilon_i m_{i,t}\big(C_{supply} - C_{i,t}\big)\Big)\,|\mathbf{Q}(t)\right\}$$

Using Lyapunov optimization theorem, it can be proven that minimizing drift plus penalty term $\mathbf{\Delta Y}(t)$ would minimize the time average of $\mathbb{E}\{\mathbf{\Phi}_t\}$, while the constraints are met [45] and a solution to $(\mathbf{OP1})$ is achieved.

In the next section, we explain the algorithm used to minimize $\mathbf{\Delta Y}(t)$.

3.2 Algorithm Design

The key idea of Lyapunov based algorithm is to minimize the upper bound of the drift plus penalty term $\mathbf{\Delta Y}(t)$ given in (37) for every slot t. It is useful for designing the algorithm to opportunistically minimize the expectation $\mathbb{E}\{\mathbf{\Phi}_t|\mathbf{Q}(t)\}$. If at each time slot t, we minimize $\mathbf{\Phi}_t$ given the current $\mathbf{Q}(t)$, the expectation is bound to be minimized. This helps us to simplify the problem further at each time slot. The proposed algorithm observes the queue vector $\mathbf{Q}(t)$ and other random parameters of the system, namely next time slot temperature setpoint $T_{i,t+1}^{set}$, the next time slot CO2 concentration setpoint $C_{i,t+1}^{set}$, outside temperature $T_{o,t}$, external thermal disturbances $q_{i,t}$, number of people in the zone $P_{i,t}$ and electricity price S_t. Then it seeks to minimize the upper bound of drift plus penalty term and find the optimal control action at time t. Next, using the found optimal control at time t, it updates the evolving dynamics of the system for

the next time slot represented in $\mathbf{Q}(t+1)$. This process is repeated for every time slot. The proposed algorithm is as follows:

Algorithm: Online HVAC Control
1. **For** each slot t do
2. At the beginning of slot t, observe $\mathbf{Q}(t), T_{i,t+1}^{set}, C_{i,t+1}^{set}, T_{o,t}, q_{i,t}, P_{i,t}, S_t$
3. Choose $m_{i,t}$ as the solution to optimization problem **OP2**:
4. (**OP2**) $\min_{m_{i,t}} V\mathbf{\Phi}_t + \sum_{i=1}^{N}\Big((1-\alpha_i)Q_{i,t}\beta_i m_{i,t}\big(T_{supply}-T_{i,t}\big)\Big) + \sum_{i=1}^{N}\Big({Q'}_{i,t}\varepsilon_i m_{i,t}\big(C_{supply}-C_{i,t}\big)\Big)$
5. $subject\ to$ (13)
6. Update $Q_{i,t}$ and ${Q'}_{i,t}$ according to (27) and (28);
7. **End**

A significant and useful property of this algorithm is that it does not need to know the probabilities of system parameters $T_{i,t+1}^{set}, C_{i,t+1}^{set}, T_{o,t}, q_{i,t}, P_{i,t}$ and S_t. There is no need for predicting or modeling these parameters beforehand. It makes this algorithm fast and suitable for real time control.

3.3 Solution to OP2

Because of the formulation of $E_{fan,t}$, **OP2** cannot be solved easily. To make the problem tractable, similar to [15], we approximate $E_{fan,t} = \mu\big(\sum_{i=1}^{N} m_{i,t}\big)^3\tau$ with one of its upper bounds. If we have $\bar{m} = \sum_{i=1}^{N} m_i^{max}$, then using Cauchy-Schwarz inequality, we have $\big(\sum_{i=1}^{N} m_{i,t}\big)^3 \leq$

$\overline{m}\left(\sum_{i=1}^{N} m_{i,t}\right)^2 \leq N\overline{m}\sum_{i=1}^{N} m_{i,t}^2$. Therefore, $(\mathbf{OP2})$ can be transformed into optimization problem $(\mathbf{OP3})$ as follows:

$$
\begin{aligned}
(\mathbf{OP3}) \qquad \min_{m_{i,t}} \sum_{i=1}^{N} \Bigg(& (1-\alpha_i)Q_{i,t}\beta_i\left(T_{supply} - T_{i,t}\right)m_{i,t} \\
& + Q'_{i,t}\varepsilon_i\left(C_{supply} - C_{i,t}\right)m_{i,t} \\
& + V\phi_T\left(T_{i,t+1} - T_{i,t+1}^{set}\right)^2 + V\phi_C\left(C_{i,t+1} - C_{i,t+1}^{set}\right)^2 \\
& + V\frac{C_a\tau}{\eta\,\mathrm{COP}}S_t\left(\pi T_{i,t} + (1-\pi)T_{o,t} - T_{supply}\right)m_{i,t} \\
& + V\tau S_t\mu N\overline{m}m_{i,t}^2\Bigg) \\
& \qquad \textit{subject to } (13)
\end{aligned} \tag{38}
$$

To solve $(\mathbf{OP3})$, we let the first derivative of the objective function be zero. We have:

$$
\begin{aligned}
2\left(V\tau S_t\mu N\overline{m} + V\phi_T\beta_i^2\left(T_{supply} - T_{i,t}\right)^2 + V\phi_C\varepsilon_i^2\left(C_{supply} - C_{i,t}\right)^2\right)m_{i,t} & \\
+ (1-\alpha_i)Q_{i,t}\beta_i\left(T_{supply} - T_{i,t}\right) + Q'_{i,t}\varepsilon_i\left(C_{supply} - C_{i,t}\right) & \\
+ 2V\phi_T\beta_i\left(T_{supply} - T_{i,t}\right)\left[(1-\alpha_i)T_{i,t} + \alpha_i T_{o,t} + \gamma_i q_{i,t} - T_{i,t+1}^{set}\right] & \\
+ 2V\phi_C\varepsilon_i\left(C_{supply} - C_{i,t}\right)\left[C_{i,t} + \xi_i P_{i,t} - C_{i,t+1}^{set}\right] & \\
+ V\frac{C_a\tau}{\eta\,\mathrm{COP}}S_t\left(\pi T_{i,t} + (1-\pi)T_{o,t} - T_{supply}\right) = 0 &
\end{aligned} \tag{39}
$$

For brevity, we will set

$$
\begin{aligned}
h_{i,t} = {} & 2V\phi_T\beta_i\left(T_{supply} - T_{i,t}\right)\left[(1-\alpha_i)T_{i,t} + \alpha_i T_{o,t} + \gamma_i q_{i,t} - T_{i,t+1}^{set}\right] \\
& + 2V\phi_C\varepsilon_i\left(C_{supply} - C_{i,t}\right)\left[C_{i,t} + \xi_i P_{i,t} - C_{i,t+1}^{set}\right] \\
& + V\frac{C_a\tau}{\eta\,\mathrm{COP}}S_t\left(\pi T_{i,t} + (1-\pi)T_{o,t} - T_{supply}\right)
\end{aligned} \tag{40}
$$

Now we can find optimal value $m_{i,t}^*$ as:

$$m_{i,t}^* = \frac{-(1-\alpha_i)Q_{i,t}\beta_i\left(T_{supply}-T_{i,t}\right) - Q'_{i,t}\varepsilon_i\left(C_{supply}-C_{i,t}\right) - h_{i,t}}{2\left(VS_t\mu N\bar{m} + V\phi_T\beta_i^2\left(T_{supply}-T_{i,t}\right)^2 + V\phi_C\varepsilon_i^2\left(C_{supply}-C_{i,t}\right)^2\right)} \tag{41}$$

To enforce $m_i^{min} \le m_{i,t} \le m_i^{max}$, we define the $m_{i,t}^{opt}$ as:

$$m_{i,t}^{opt} = \max(m_i^{min}, \min(m_i^{max}, m_{i,t}^*)) \tag{42}$$

This concludes the solution for $(\mathbf{OP3})$**,** and we can update the queues using $m_{i,t}^{opt}$ in the algorithm.

In the next chapter, the simulated results of the proposed algorithm are shown.

Chapter 4. Results

In this chapter, we show the simulated results of the online HVAC control algorithm for temperature and CO_2 concentration. The controller design and the temperature and CO_2 concentration for a typical zone are shown and discussed. Also, to see that the energy cost is getting optimized, different allowed temperature regions are considered and the corresponding energy cost is calculated. The results show that the algorithm is able to satisfy the constraints of the problem and optimize the objectives without knowing, modeling or estimating any of the stochastic parameters of the HVAC system in real time.

4.1 Simulation Setup

We considered a smart building with $N = 4$. For the time horizon, we consider a month with 31 days. We need a time step τ during which we can treat system parameters as constants. In practice, outside temperature and thermal disturbance vary at a time scale of minutes [41], while electricity price, Temperature setpoint, CO_2 concentration setpoint and number of people vary at a time scale

of hours [15]. We chose the length of the time step to be five minutes. That means the total number of our time slots is $M = 12 \times 24 \times 31 = 8928$ intervals. Values used for constant zone thermal parameters and HVAC system parameters are shown in Table 4-1. The values are taken from the published literature, specifically [15], [39], [41], [46] and [47].

For the electricity price information S_t, we used hourly retail price for the city Milwaukee in August 2017[1]. Figure 4.1 Shows the trend of hourly electricity retail price during the month (cents per kWhr). For the outdoor hourly temperature, we adopted Milwaukee's outdoor temperature for August 2017[2], the trend is shown in Figure 4.2. Similar to [15], it is assumed that external thermal disturbance $q_{i,t}$ follows a uniform distribution with parameters 0.1 and 0.2 (i.e. $q_{i,t} \sim U(0.1,0.2)$). Another stochastic parameter is number of people in each zone at each time slot. We used a random integer number that updates every hour.

---

[1] http://www.nationalgridus.com
[2] http://www.frontierweather.com

Table 4-1. Constant zone thermal parameters and HVAC system parameters used in the online controller algorithm.

$\pi = 0.95$	$R_i = [5.3, 6.0, 6.3, 6.7] \times 10^{-3}\,\frac{℃}{W}$
$T_{supply} = 12.8\,℃$	$C_i = [5.5, 5.7, 5.9, 6.2] \times 10^{5}\,\frac{J}{℃}$
$\mathrm{COP} = 5.9153$	$m_i^{min} = 0\,\frac{g}{s}$
$\eta = 0.8897$	$m_i^{max} = 450\,\frac{g}{s}$
$C_a = 1.012\,\frac{J}{g℃}$	$T_i^{min} = 17℃$
$\mu = 2 \times 10^{-6}\,\frac{W}{(g/s)^3}$	$T_i^{max} = 23℃$
$C_o = 400\,ppm$	$C_i^{min} = 400\,ppm$
$G = 4.9 \times 10^{-3}\,L/s$	$C_i^{max} = 700\,ppm$

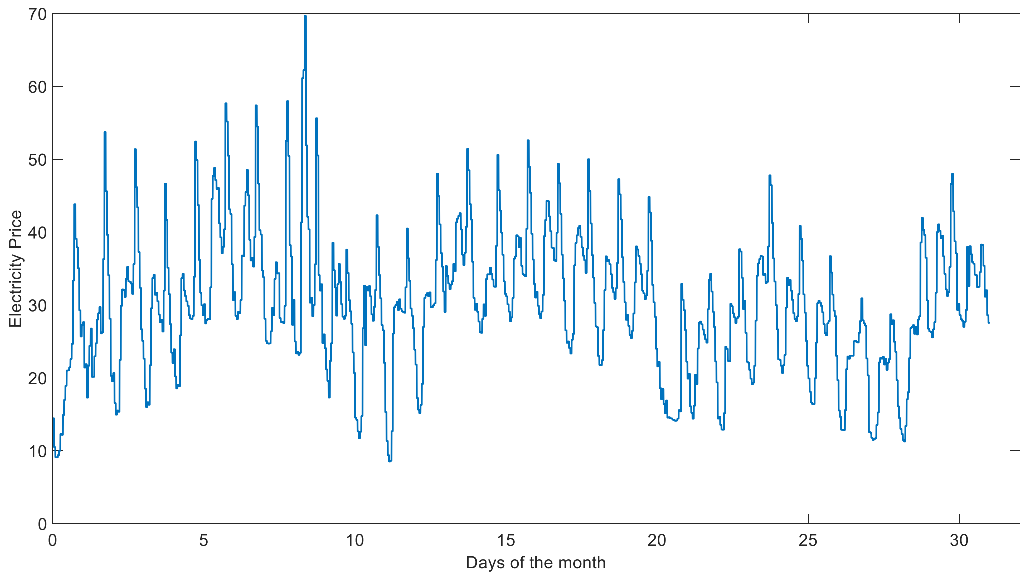

Figure 4.1. Hourly electricity price (cents/kWhr) for Milwaukee in August 2017.

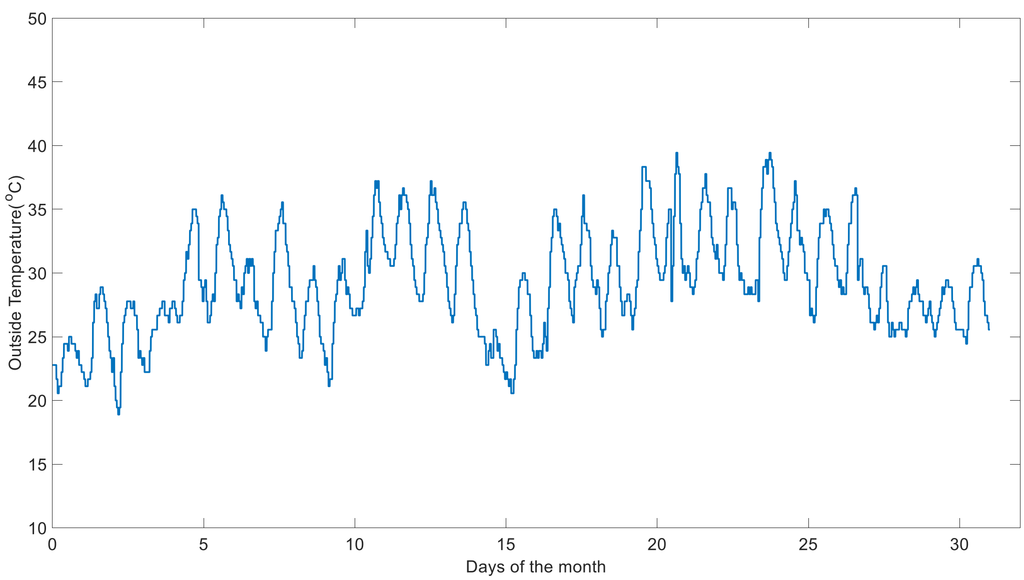

Figure 4.2. Outside temperature in degrees of centigrade for Milwaukee, in August 2017.

4.2 Simulated Results

After running the algorithm for the specified time horizon and with the mentioned parameters, we get optimal values for the air mass flow(g/s) at each time slot for each zone, indicated by $m_{i,t}^{opt}$. Figure 4.3 shows the values for $m_{1,t}^{opt}$ versus the timeslots for zone 1. As expected the constraints are staisfied and we have $m_{1,t}^{min} < m_{1,t}^{opt} < m_{1,t}^{max}$.

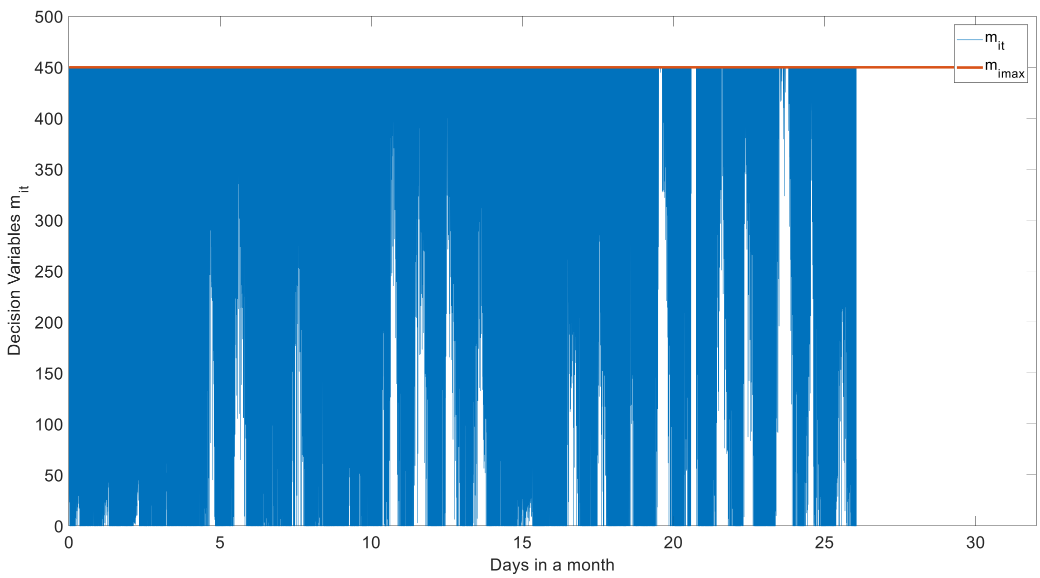

Figure 4.3. Obtained online controller $m_{1,t}^{opt}$ for zone 1.

Now that the physical constraint of the system is satisfied, we need to evaluate the comfort indicators, namely the temperature and the CO_2 concentration level. Optimally, these values should always fluctuate in their specified range and track the comfort setpoints well.

Results for the controlled temperature in zone 1 are shown in Figure 4.4. To avoid visual complexity, the setpoint for the temperature is assumed to be constant for the whole month $T_{i,t}^{set} =$

20°C, and the algorithm can track it well. As expected, the constraints on the temperature range are satisfied and we have $T_{1,t}^{min} < T_{1,t} < T_{1,t}^{max}$.

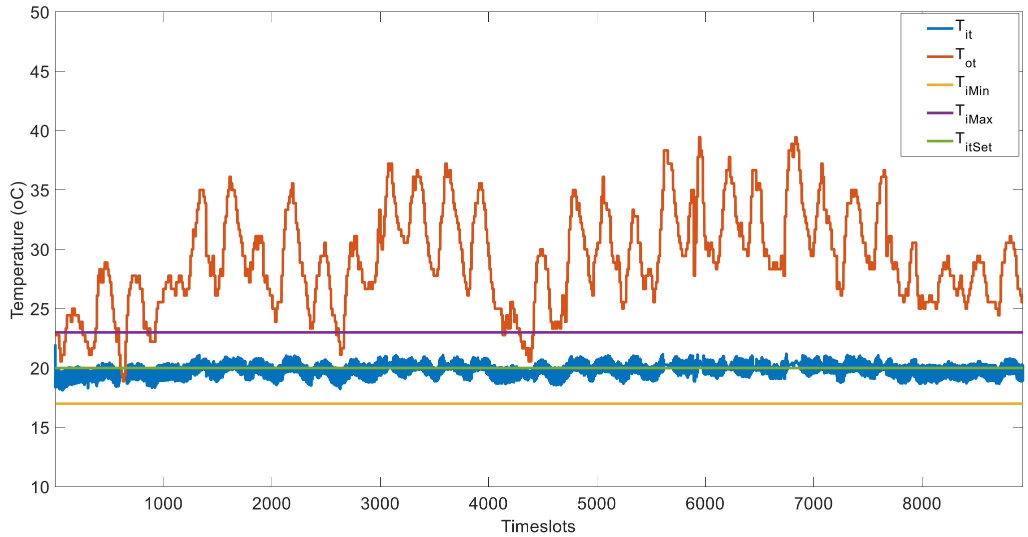

Figure 4.4. Results for the inside temperature in Centigrade vs. timeslots for zone 1 in the explained setup.

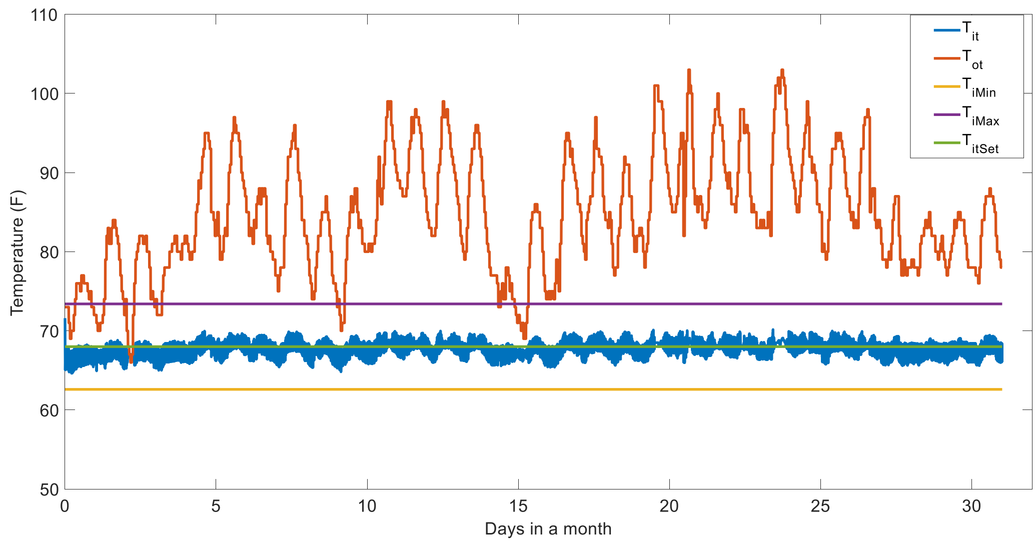

Figure 4.5. Results for the inside temperature in Fahrenheit vs. days of the month for zone 1 in the explained setup.

In Figure 4.4, the same results are shown with a slight change in the axis, it shows the temperatures in fahrenheit vs the days instead of timeslots.

Now we can move on to the other comfort indicator. Figure 4.6 shows the CO_2 concentration level (ppm) in the corresponding timeslots (shown in days) with the same setup for zone 1. The values obtained are in the desired range and the controller tries to track the CO_2 setpoint shown as $C_{1,t}^{set} = 450\ ppm$. By manipulating the weight on the discomfort cost for the CO_2 level, we can improve how the algorithm tracks the desired setpoint.

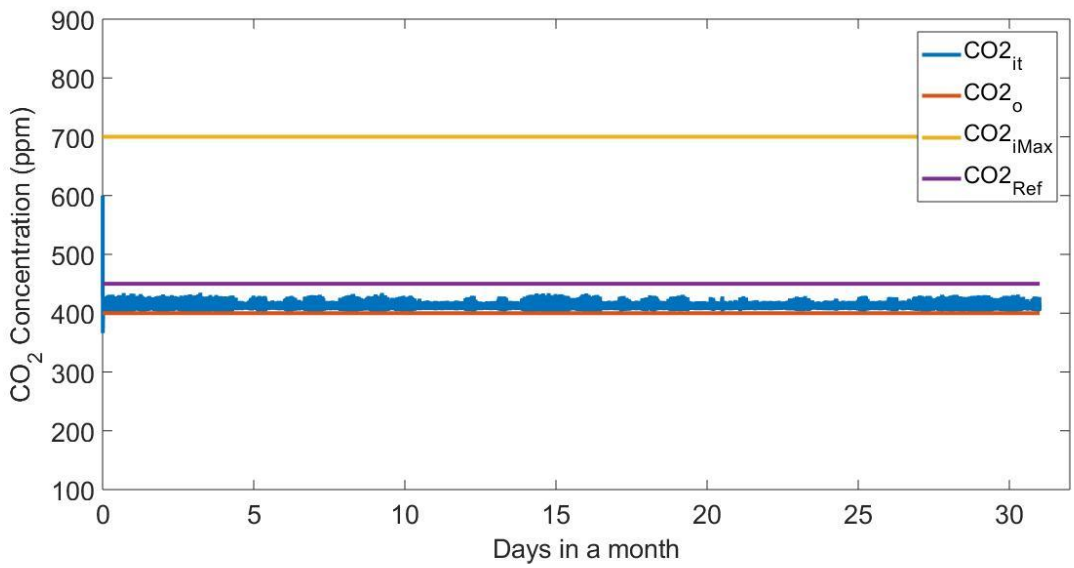

Figure 4.6. CO2 concentration level in the same zone vs. the days of month.

4.3 Impact of T_i^{max}

In this section, we evaluate the impact of T_i^{max} on the energy cost and we keep other parameters unchanged. Larger T_i^{max} contributes to a wider range of toleraence for inside temperature for every zone. Therefore with sacrificing average inside temperature, we will have lower energy cost. In fact, larger T_i^{max} means more opportunities for the cost reduction. Figure 4.7 shows how the

energy cost is erduced when we varied T_i^{max} from 23 to 33°C . however, this cost reduction is not free, we need to consider that the average inside temperature will grow larger because by widening the range of allowed inside temperatures, the actual inside temperature will be closer to T_i^{max}. This is shown in Figure 4.8.

4.4 Conclusion

The simulated results show the feasibility of the proposed algorithm in chapter 3. We were able to design an online controller that adjusts the air mass flow in a way that it does not exceed the physical limits of the system and can keep the comfort indicators of the occupants in the desired range. At the same time, the energy cost of the system is minimized and the algorithm can move the system to one optimal state. One way of reducing the energy even further is to allow a wider range for temperature, so that less electricity is consumed in the cooling coil.

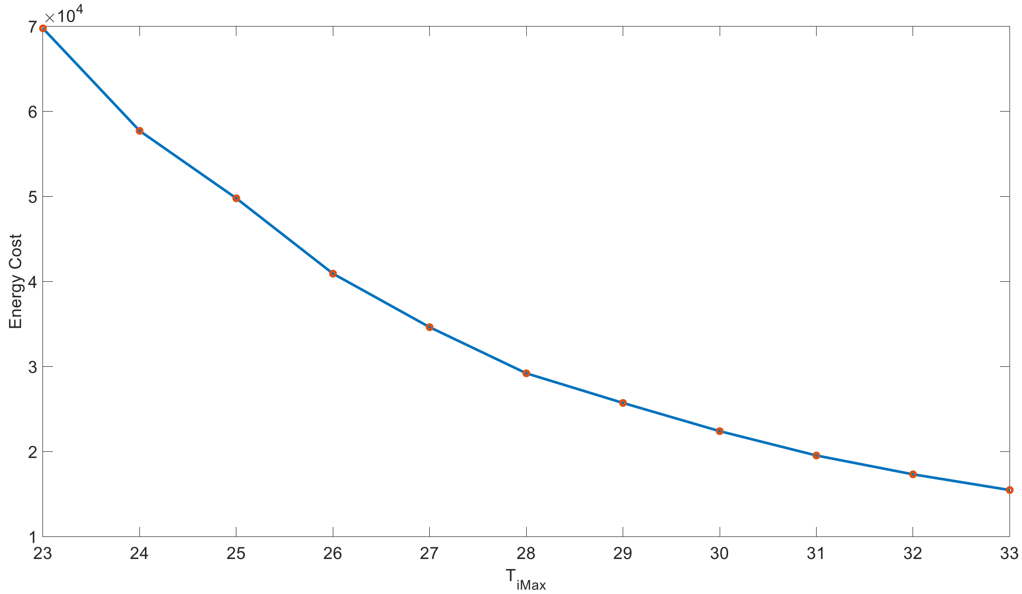

Figure 4.7. Energy cost reduction when varying T_i^{max}.

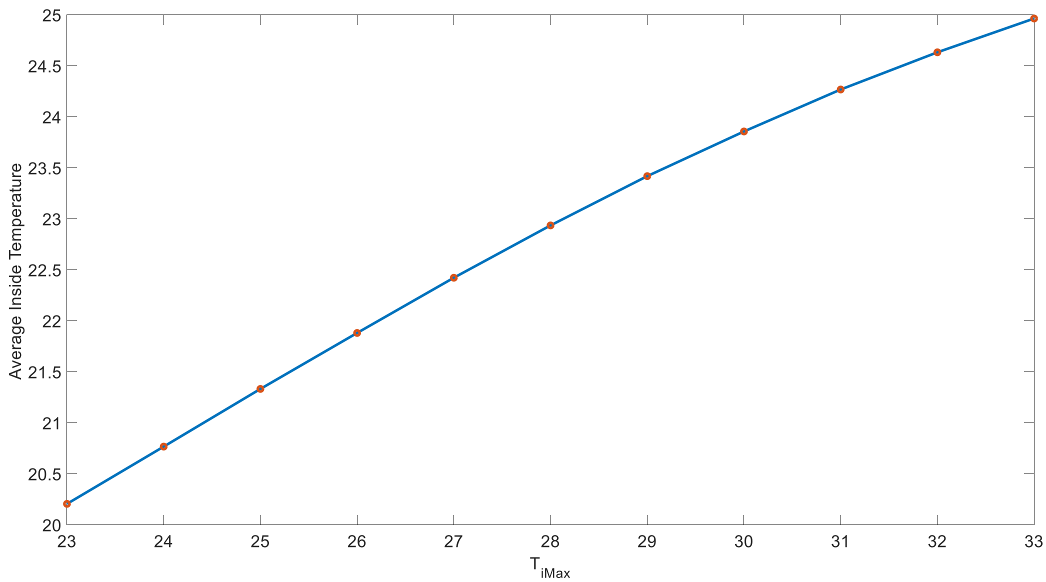

Figure 4.8. Average inside temperature variation while varying T_i^{max}.

Chapter 5. Conclusion and Future Work

In this chapter, the conclusion of this thesis is discussed and some ideas for the future direction are proposed.

We introduced an online algorithm for designing a control for HVAC systems in a multi zone building that can satisfy the physical limits and the comfort range for the occupants, while it minimizes the energy cost needed to operate the system. This algorithm is a solution to the optimization problem defined in chapter 2. We proposed a stochastic optimization problem for minimizing long term time average of the expected cost of the system. This optimization problem was constrained, nonlinear and complex. By using Lyapunov optimization technique, we transformed the original problem to a drift plus penalty term, where dynamics of the system were translated to stability of the processes. By minimizing the upper bound of the drift plus penalty

term, we obtained the optimal air mass flow rate at each time slot for each zone. Features of the proposed algorithm can be summarized as follows:

- It is worth noting that the HVAC system is implemented in a multi zone commercial building, and both air quality and thermal comfort of the occupants are considered.
- A simple model is used for the system dynamics.
- Although obtaining the control is mathematically intensive, the optimal control formula is fixed across time slots and implementing the algorithm is straightforward.
- There is no need to store the system information before the last timeslot, in the beginning of each timeslot, the parameters are observed from the sensors and the queue values are updated only from time slot $t-1$. From this point of view, the control algorithm has the same benefits of MPC.
- There is no need to model, predict or estimate the uncertainties in the system, the algorithm can perform without knowing the pdf of the random processes of the system. This is the most significant feature of the proposed method.
- Since there is a tradeoff between comfort and energy cost, by sacrificing one we can improve the other. The weights of each are accessible through the algorithm and changing them will not affect the performance of the algorithm.

In the future, the algorithm could be extended as a distributed algorithm to preserve occupants' privacy. That is possible if the information about the zones is not reported to the EMS, each zone will only send the updated queue values to EMS, and EMS calculates the optimal control. Adding a constraint on the sum of air mass flow rate in a way that not all zones can have m_i^{max}

could be considered. Furthermore, as a new control variable, the damper position in AHU could be taken into account.

References

[1] 2011 Buildings energy data book, 2017 [Online]. Available: *https://catalog.data.gov/dataset/buildings-energy-data-booktle.*

[2] Z. Wu, Q. S. Jia, and X. Guan, “Optimal Control of Multiroom HVAC System: An Event-Based Approach,” *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 2, pp. 662–669, 2016.

[3] P. B. Luh, “Building Energy Management: Integrated Control of Active and Passive Heating, Cooling, Lighting, Shading, and Ventilation Systems,” *IEEE Trans. Autom. Sci. Eng.*, vol. 10, no. 3, pp. 588–602, 2013.

[4] A. I. Dounis and C. Caraiscos, “Advanced control systems engineering for energy and comfort management in a building environment-A review,” *Renewable and Sustainable Energy Reviews*. 2009.

[5] R. Yang and L. Wang, “Optimal control strategy for HVAC system in building energy management,” *Pes T&D 2012*, pp. 1–8, 2012.

[6] and B. O. D. Minoli, K. Sohraby, “IoT consid erations, requirements, and architectures for smart buildings-ener gy optimization and next-generation building management systems,” *IEEE Internet things J.* , 2017.

[7] T.Bushby, “BACnetTM: a standard communication infrastructure for intelligent buildings Author links open overlay panelSteven.”

[8] L. Lu, W. Cai, L. Xie, S. Li, and Y. C. Soh, “HVAC system optimization - In-building section,” *Energy Build.*, vol. 37, no. 1, pp. 11–22, 2005.

[9] “Doctor of Philosophy,” *Wikipedia*, no. December 2013.

[10] AGuilleminNMorel, "An innovative lighting controller integrated in a self-adaptive building control system Author links open overlay panel."

[11] A. Barghi, A. R. Kosari, M. Shokri, and S. Sheikhaei, "Intelligent lighting control with LEDS for smart home," *Smart Grid Conf. 2014, SGC 2014*, pp. 1–5, 2014.

[12] V. Congradac and F. Kulic, "HVAC system optimization with CO2concentration control using genetic algorithms," *Energy Build.*, vol. 41, no. 5, pp. 571–577, 2009.

[13] W. Kastner, G. Neugschwandtner, S. Soucek, and H. M. Newman, "Communication systems for building automation and control," *Proc. IEEE*, vol. 93, no. 6, pp. 1178–1203, 2005.

[14] M. W. Ahmad, M. Mourshed, B. Yuce, and Y. Rezgui, "Computational intelligence techniques for HVAC systems: A review," *Build. Simul.*, vol. 9, no. 4, pp. 359–398, 2016.

[15] L. Yu, D. Xie, T. Jiang, Y. Zou, and K. Wang, "Distributed Real-Time HVAC Control for Cost-Efficient Commercial Buildings under Smart Grid Environment," *IEEE Internet Things J.*, vol. 5, no. 1, pp. 44–55, 2018.

[16] A. Afram and F. Janabi-Sharifi, "Theory and applications of HVAC control systems - A review of model predictive control (MPC)," *Build. Environ.*, vol. 72, pp. 343–355, 2014.

[17] A. Afram, F. Janabi-Sharifi, A. S. Fung, and K. Raahemifar, "Artificial neural network (ANN) based model predictive control (MPC) and optimization of HVAC systems: A state of the art review and case study of a residential HVAC system," *Energy Build.*, vol. 141, pp. 96–113, 2017.

[18] M. Anderson *et al.*, "MIMO robust control for HVAC systems," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 3, pp. 475–483, 2008.

[19] G. Y. Jin, P. Y. Tan, X. D. Ding, and T. M. Koh, "Cooling Coil Unit dynamic control of in

HVAC system," *Proc. 2011 6th IEEE Conf. Ind. Electron. Appl. ICIEA 2011*, pp. 942–947, 2011.

[20] J. Bai, S. Wang, and X. Zhang, "Development of an adaptive Smith predictor-based self-tuning PI controller for an HVAC system in a test room," *Energy Build.*, vol. 40, no. 12, pp. 2244–2252, 2008.

[21] M. Xu, S. Li, and W. Cai, "Practical receding-horizon optimization control of the air handling unit in HVAC systems," *Ind. Eng. Chem. Res.*, vol. 44, no. 8, pp. 2848–2855, 2005.

[22] Z. Xiaocheng, W. Ronghao, and X. Jianchun, "VAV air-conditioning system control of underground defensive project based on fuzzy predictive PID algorithm," *Proc. 28th Chinese Control Decis. Conf. CCDC 2016*, pp. 6286–6291, 2016.

[23] F. Behrooz, N. Mariun, M. Marhaban, M. Mohd Radzi, and A. Ramli, "Review of Control Techniques for HVAC Systems—Nonlinearity Approaches Based on Fuzzy Cognitive Maps," *Energies*, vol. 11, no. 3, p. 495, 2018.

[24] S. Soyguder, M. Karakose, and H. Alli, "Design and simulation of self-tuning PID-type fuzzy adaptive control for an expert HVAC system," *Expert Syst. Appl.*, vol. 36, no. 3 PART 1, pp. 4566–4573, 2009.

[25] H. Mirinejad, S. H. Sadati, M. Ghasemian, and H. Torab, "Control Techniques in heating, ventilating and air conditioning systems," *J. Comput. Sci.*, vol. 4, no. 9, pp. 777–783, 2008.

[26] H. Moradi, M. Saffar-Avval, and F. Bakhtiari-Nejad, "Nonlinear multivariable control and performance analysis of an air-handling unit," *Energy Build.*, vol. 43, no. 4, pp. 805–813, 2011.

[27] Z. Huaguang and L. Cai, "Decentralized nonlinear adaptive control of an HVAC system,"

IEEE Trans. Syst. Man Cybern. Part C Appl. Rev., vol. 32, no. 4, pp. 493–498, 2002.

[28] F. Tahersima, J. Stoustrup, H. Rasmussen, and P. G. Nielsen, "Thermal analysis of an HVAC system with TRV controlled hydronic radiator," *2010 IEEE Int. Conf. Autom. Sci. Eng. CASE 2010*, pp. 756–761, 2010.

[29] S. Wang and Z. Ma, "Supervisory and optimal control of building HVAC systems: A review," *HVAC R Res.*, vol. 14, no. 1, pp. 3–32, 2008.

[30] Y. Ma, J. Matusko, and F. Borrelli, "Stochastic Model Predictive Control for Building HVAC Systems: Complexity and Conservatism," *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 1, pp. 1–1, 2014.

[31] S. Goyal, H. A. Ingley, and P. Barooah, "Effect of various uncertainties on the performance of occupancy-based optimal control of HVAC zones," *Proc. IEEE Conf. Decis. Control*, pp. 7565–7570, 2012.

[32] Y. Ma, A. Kelman, A. Daly, and F. Borrelli, "Predictive Control for Energy Efficient Buildings with Thermal Storage: Modeling, Stimulation, and Experiments," *IEEE Control Syst.*, vol. 32, no. 1, pp. 44–64, 2012.

[33] H.-N. Wu and H.-X. Li, "A {Galerkin}/neural-network-based design of guaranteed cost control for nonlinear distributed parameter systems," *IEEE Trans. Neural Networks*, vol. 19, no. 5, pp. 795–807, 2008.

[34] X. Wei, A. Kusiak, M. Li, F. Tang, and Y. Zeng, "Multi-objective optimization of the HVAC (heating, ventilation, and air conditioning) system performance," *Energy*, vol. 83, pp. 294–306, 2015.

[35] Ž. Kristl, M. Košir, M. Trobec Lah, and A. Krainer, "Fuzzy control system for thermal and visual comfort in building," *Renew. Energy*, vol. 33, no. 4, pp. 694–702, 2008.

[36] H. Mirinejad, K. C. Welch, and L. Spicer, "A Review of Intelligent Control Techniques in HVAC Systems," *IEEE Int. Netw. Infrastruct. Digit. Content,* no. November, pp. 1–5, 2010.

[37] F. Behrooz, N. Mariun, M. H. Marhaban, M. A. M. Radzi, and A. R. Ramli, "A design of a hybrid non-linear control algorithm," *Energies,* vol. 10, no. 11, 2017.

[38] M. M. Gouda, S. Danaher, and C. P. Underwood, "Quasi-adaptive fuzzy heating control of solar buildings," *Build. Environ.*, vol. 41, no. 12, pp. 1881–1891, 2006.

[39] H. Hao, C. D. Corbin, K. Kalsi, and R. G. Pratt, "Transactive Control of Commercial Buildings for Demand Response," *IEEE Trans. Power Syst.*, vol. 32, no. 1, pp. 774–783, 2017.

[40] O. Access, "We are IntechOpen , the first native scientific publisher of Open Access books TOP 1 %."

[41] X. Zhang, W. Shi, B. Yan, A. Malkawi, and N. Li, "Decentralized and Distributed Temperature Control via HVAC Systems in Energy Efficient Buildings," no. Section II, pp. 1–10, 2017.

[42] L. Yu, T. Jiang, and Y. Zou, "Distributed Real-Time Energy Management in Data Center Microgrids," *IEEE Trans. Smart Grid,* pp. 1–1, 2016.

[43] Y. Huang, S. Mao, and R. M. Nelms, "Adaptive electricity scheduling in microgrids," *IEEE Trans. Smart Grid,* vol. 5, no. 1, pp. 270–281, 2014.

[44] W. Fan, N. Liu, and J. Zhang, "An Event-Triggered Online Energy Management Algorithm of Smart Home: Lyapunov Optimization Approach," *Energies*, vol. 9, no. 5, p. 381, 2016.

[45] M. Neely, *Stochastic Network Optimization with Application to Communication and Queueing Systems*. 2010.

[46] H. Hao, T. Middelkoop, P. Barooah, and S. Meyn, "How demand response from commercial

buildings will provide the regulation needs of the grid," *2012 50th Annu. Allert. Conf. Commun. Control. Comput. Allert. 2012*, pp. 1908–1913, 2012.

[47] N. Nassif, "A robust CO2-based demand-controlled ventilation control strategy for multi-zone HVAC systems," *Energy Build.*, vol. 45, pp. 72–81, 2012.