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Using Pythagorean Fuzzy Sets (PFS) in Multiple Criteria Group Decision Making (MCGDM) Methods for Engineering Materials Selection Applications

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USING PYTHAGOREAN FUZZY SETS (PFS) IN MULTIPLE CRITERIA GROUP
DECISION MAKING (MCGDM) METHODS FOR ENGINEERING MATERIALS
SELECTION APPLICATIONS

by

Alaa Momena

A Dissertation Submitted in
Partial Fulfillment of the
Requirements for the Degree of

Doctor of Philosophy

in Engineering

at

The University of Wisconsin-Milwaukee

May 2019

ABSTRACT

USING PYTHAGOREAN FUZZY SETS (PFS) IN MULTIPLE CRITERIA GROUP DECISION MAKING (MCGDM) METHODS FOR ENGINEERING MATERIALS SELECTION APPLICATIONS

by

Alaa Momena

The University of Wisconsin, Milwaukee, 2019
Under the Supervision of Professor Nidal Abu-Zahra

The process of materials' selection is very critical during the initial stages of designing manufactured products. Inefficient decision-making outcomes in the material selection process could result in poor quality of products and unnecessary costs. In the last century, numerous materials have been developed for manufacturing mechanical components in different industries. Many of these new materials are similar in their properties and performances, thus creating great challenges for designers and engineers to make accurate selections. Our main objective in this work is to assist decision makers (DMs) within the manufacturing field to evaluate materials alternatives and to select the best alternative for specific manufacturing purposes.

In this research, new hybrid fuzzy Multiple Criteria Group Decision Making (MCGDM) methods are proposed for the material selection problem. The proposed methods tackle some challenges that are associated with the material selection decision making process, such as aggregating decision makers' (DMs) decisions appropriately and modeling uncertainty. In the proposed hybrid models, a novel aggregation approach is developed to convert DMs crisp decisions to Pythagorean fuzzy sets (PFS). This approach gives more flexibility to DMs to express their opinions than the traditional fuzzy and intuitionistic sets (IFS). Then, the proposed aggregation approach is integrated with a ranking method to solve the Pythagorean Fuzzy Multi

Criteria Decision Making (PFMCGDM) problem and rank the material alternatives. The ranking methods used in the hybrid models are the Pythagorean Fuzzy TOPSIS (The Technique for Order of Preference by Similarity to Ideal Solution) and Pythagorean Fuzzy COPRAS (COMplex PROportional Assessment). TOPSIS and COPRAS are selected based on their effectiveness and practicality in dealing with the nature of material selection problems.

In the aggregation approach, the Sugeno Fuzzy measure and the Shapley value are used to fairly distribute the DMs weight in the Pythagorean Fuzzy numbers. Additionally, new functions to calculate uncertainty from DMs recommendations are developed using the Takagai-Sugeno approach. The literature reveals some work on these methods, but to our knowledge, there are no published works that integrate the proposed aggregation approach with the selected MCDM ranking methods under the Pythagorean Fuzzy environment for the use in materials selection problems. Furthermore, the proposed methods might be applied, due to its novelty, to any MCDM problem in other areas.

A practical validation of the proposed hybrid PFMCGDM methods is investigated through conducting a case study of material selection for high pressure turbine blades in jet engines. The main objectives of the case study were: 1) to investigate the new developed aggregation approach in converting real DMs crisp decisions into Pythagorean fuzzy numbers; 2) to test the applicability of both the hybrid PFMCGDM TOPSIS and the hybrid PFMCGDM COPRAS methods in the field of material selection.

In this case study, a group of five DMs, faculty members and graduate students, from the Materials Science and Engineering Department at the University of Wisconsin-Milwaukee, were selected to participate as DMs. Their evaluations fulfilled the first objective of the case study. A computer application for material selection was developed to assist designers and engineers in

real life problems. A comparative analysis was performed to compare the results of both hybrid MCGDM methods. A sensitivity analysis was conducted to show the robustness and reliability of the outcomes obtained from both methods. It is concluded that using the proposed hybrid PFMCGDM TOPSIS method is more effective and practical in the material selection process than the proposed hybrid PFMCGDM COPRAS method. Additionally, recommendations for further research are suggested.

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**This is for my Mom, Dad
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CHAPTER ONE

Introduction

Materials selection is very important in design and manufacturing because it maximizes the performance and minimizes the cost (Thakker, Jarvis, Buggy, & Sahed, 2008). Materials selection decisions should be taken within the initial design stage of the product life cycle once the component is first designed or redesigned (Edwards, 2005; Deng & Edwards, 2007). Any false decision making in this initial stage could lead to early failure of the component and unwanted cost. New materials have been discovered and manufactured throughout the last few decades to provide many innovative opportunities to companies in different industries through using those materials to maximize performance and minimize cost. However, due to the rapid development and manufacturing of materials, many new alternatives in materials can be used for the same product or component. Thus, engineers and designers encounter some challenges in decision making on selecting the best material from a large set of options and alternatives, taking in consideration different criteria that affect the material performance (Jajimoggala & Karri, 2013). Also, it is essential to prioritize between these criteria in order to decide which property is more critical than the others (Zhao, Su, Chen, & Yu, 2016).

A main problem that decision makers face is the lack of information in the materials selection process. Using proper decision making procedures and appropriate material samples could be helpful in overcoming this lack of information. These decision making processes would involve picking the material that best suits the designated product, prioritizing the different alternatives and then ranking these alternatives. Therefore, applying a suitable decision making tool sets the basis for selecting and prioritizing the alternatives (Baghel, Jha, & Jindal, 2014).

Designers and engineers have to consider a large number of criteria when choosing materials. Karana, Hekkert, & Kandachar (2008) indicate that when engineers select a material they must consider the following quantitative requirements:

- Mechanical properties such as Young's modulus, ultimate strength, yield strength, elasticity, fatigue, hardness, toughness, etc.
- Fabrication requirements such as machining ability, welding ability, heat treatability, etc.
- Maintenance
- Thermal and radiation properties such as conductivity, transmissivity, reflectivity, specific heat, etc.
- Physical properties such as density, crystal structure, melting point, viscosity, vapor pressure, etc.
- Chemical properties
- Electrical properties
- Availability
- Life of component factors
- Business issues

Material properties play an important role in the design process because they define the performance of the product. For example, the product components might need to meet a specific performance requirement such as cost, stiffness, or strength. Consequently, each of the aforementioned needs should affect the material selection process. Edwards & Deng (2007) classified material selection problems into two groups:

1. Material selection based on the material properties such as mechanical, thermal and chemical properties.
2. Materials selection based on the design requirements, where the material properties are combined with those of the physical structure and the appropriate structural properties of the component.

In previous studies, material performances are usually assessed by considering multiple criteria and attributes (Jahan, Edwards, & Bahraminasab, 2016). Materials selection decisions may be affected by several aspects such as product performance, quality, cost, or customer satisfaction (Ashby, Shercliff, & Cebon, 2007; Van Kesteren, 2008). In addition, selecting the incorrect material for a specific product or component may result in such bad consequences such as profit loss, undesirable cost, and poor reputation of the company that manufactured this product (Edwards, 2005). Therefore, multiple criteria decision making (MCDM) problems have been used as an effective technique to select the most suitable material for numerous engineering designs and products. The application of MCDM usually consists of defining a finite set of feasible alternatives and related criteria, determining the importance of the criteria and the influence of the alternatives on these criteria, and defining the performance measures of the alternatives in order to complete the final step of ranking (Jahan, Ismail, Sapuan, & Mustapha, 2010; Chakraborty & Chatterjee, 2013). The MCDM problem can be mathematically represented in a decision making matrix that involve the assigned material alternatives and the listed weighted criteria (Jahan, Ismail, Sapuan, & Mustapha, 2010).

One of the permanent issues and challenges for materials engineers, designers and decision makers is to choose the suitable material to meet complex design problems. Therefore, implementing MCDM methods is going to help decision makers to create effective and efficient

designs through developing the decision making process in engineering materials selection applications. Progressively, MCDM methods have been used to materials selection, showing significant potential for tackling complex design problems in engineering materials applications through the last decades. These methods enhance existing quantitative approaches, such as selection charts, by allowing simultaneous consideration of design criteria, component configurations and types of material (Jahan, Edwards, & Bahraminasab, 2016).

MCDM is categorized into two groups; the first one is the multiple-objective decision-making (MODM) method and the second is the multiple-attribute decision-making (MADM) method (Jahan & Edwards, 2013). MODM can be defined as a technique that is used for optimization by prioritizing functional relationships such as minimizing cost or maximizing profit. On the other hand, MADM aims to compare and then rank the alternatives by weighting assigned criteria (Zhou, Yin, & Hu, 2009; Chauhan & Vaish, 2013).

Past researchers have focused on implementing diverse MCDM techniques and then defining the performance measures in order to rank the alternatives. MODM is totally disregarded by early researchers because of variable limitations. Instead, they tend to use MADM frequently for the process of considering relevant criteria to choose the proper material at the end (Yurdakul & İç, 2009).

Furthermore, different MADM techniques were implemented to improve the accuracy of the decision making process for materials selection such as TOPSIS (technique for order performance by similarity to ideal solution) (Shanian & Savadogo, 2006; Dağdeviren, Yavuz, & Kılınç, 2009), VIKOR (Vlse Kriterijumska Optimizacija Kompromisno Resenje means Multicriteria Optimization and Compromise Solution) (Jahan, Mustapha, Ismail, Sapuan, & Bahraminasab, 2011; Jeya Girubha, & Vinodh, 2012), ELECTRE (elimination and choice

expressing the reality) (Shanian, Savadogo, 2006) and AHP (analytical hierarchy process) (Jiang, Zhang, & Sutherland, 2011).

Usually there is one phase that supposed to be implemented in MADM problems before applying a decision making ranking method such as TOPSIS to rank the material alternatives. These methods are generally used for weighting and prioritizing criteria such as Analytical AHP, the entropy method, the digital logic method, the Simos method, etc. (Xie, 2014). Most of these weighting tools consider decision makers' (DMs) opinions and judgments to determine the weight for each criterion (Jajimoggala, & Karri, 2013).

1.1 Broad Application of MCDM

Çalışkan, Kurşuncu, Kurbanoğlu, & Güven (2013) use various MADM methods such as extended PROMETHEE II (EXPROM2) (preference ranking organization method for enrichment evaluation), TOPSIS and VIKOR to select the best material for the tool holder used under hard milling conditions as well as to rank the alternative materials. A compromised weighting technique such as AHP and entropy methods also have been performed to weight the material criteria and attributes. Moreover, Spearman's rank correlation coefficient was used to evaluate the correlation between the three ranking methods. It was found that MADM techniques are a practical and useful way to solve the complex material selection decision problems.

Chakraborty and Chatterjee (2013) test how the number of attributes and criteria can influence the ranking performance of some MCDM methods such as VIKOR, TOPSIS and PROMETHEE by addressing five material selection problems from different areas of applications. It was found that identifying the most significant criteria with the highest priority is very critical and can affect the selection of the best and the worst material. The authors suggested

that instead of focusing on building the comprehensive material selection decision models, designers may only concentrate on identifying the most important criterion that affect the entire decision making process. As a result, the application of this approach of selecting the best material based on the single highest priority criterion can decrease the complexity that is included in implementing MADM methods. Consequently, the weighting tool that determines criteria weights has a very important function in the material selection process. It also has been shown that VIKOR surpasses all of the other methods due to its advantages.

Zhao, Su, Chen, & Yu (2016) discuss an integrated multi-attribute decision-making (MADM) approach to assist selection of three different alternatives materials, poly (vinyl chloride) (PVC), polypropylene (PP), and polyethylene (PE). These alternatives are investigated to define their best economic, environmental, and social performances in designing sustainable plastic pipes. The authors implemented two popular MADM methods, specifically AHP and Gray rational Analysis (GRA). AHP was used to inspect the relative importance and appoint the weighting values for each indicator. In addition, GRA was employed to rank the materials based on the several attributes and to offer a better order of different inspected items in terms of a specific performance index.

Consequently, integrating these two previous methods can maximize their benefits as well as facilitate the multi-attributive decision-making of a complex system although it was found that limitations still exist in this MADM approach. First, the selected criteria need more examination to see if they can be properly used to help the selection of material alternatives in terms of sustainability. Second, engineers' and designers' opinions should be considered in determining the weight of each criterion (Zhao, Su, Chen, & Yu, 2016).

Jahan et al. (2011) present a mathematical aggregation technique in multi-criteria decision-making method to overcome the common issue of the similarity of the material alternatives to each other. This technique used ranking orders that were achieved through different MCDM methods such as the results of the ranking process obtained by various MCDM methods being used as the input of the proposed methodology. Thus, the application of this approach will aid designers and engineers to achieve a consensus on selecting the best material for a certain application. Researchers concluded that the recommended method is more practical than current methods as it takes into account each material being appointed to each ranking. Hence, an optimal aggregation ranking will be reached by applying this method.

Jajimoggala & Karri (2013) developed a multiple criteria decision making methodology based on two phases to select the best suitable material for an impellor. The first phase weighted and prioritized the criteria by implementing fuzzy AHP and the second phase ranked the material alternatives using fuzzy TOPSIS. In addition, the use of ranking scores in the TOPSIS methodology allowed decision makers to identify alternatives from best to worst. Moreover, employing fuzzy methods improved uncertainty and imprecise evaluation of material alternatives and criteria.

Fuzzy methods application focused on implementing triangular numbers into traditional AHP method. Therefore, this allows DMs to get a clear improvement in the overall criteria weighting and material alternatives ranking results. It was found that the proposed method is very reasonable to rank material alternatives considering several differing criteria and attributes. As well, it is capable of reinforcing the existing MCDM material selection techniques mainly when there are many alternatives (Jajimoggala & Karri, 2013).

Gupta (2011) discusses in his research how selecting the best suitable material for engineering design requires a strong knowledge of the functional condition for each material alternative as well as the need to respect different essential criteria and attributes. Despite the complication that may occur in material selection for the absorbent layer of thin-film solar cells because of the limitation in each proposed material, the author suggested a strategy that relies on the MADM approach using the TOPSIS method to enhance machine performance by picking the best material. The suggested materials that were analyzed for selecting the best alternative were Copper Indium Gallium Diselenide (CIGS), Amorphous Silicon (a-Si), Copper Indium Disulphide (CuInS₂), Cadmium Telluride (CdTe), and Perylene Diamine.

The main criteria that were considered for their weight evaluation are band gap, absorption coefficient, diffusion length, thermodynamic compatibility and recombination velocity. The evaluation of the weight priorities was performed by applying fuzzy linguistic variables. Fuzzy linguistic variables technique is used as a method to transform decision makers' (DMs) judgment and opinions, which are considered as linguistic values and description, into quantitative and numerical values to help build a mathematical model to make a flexible framework for solving decision making issues. In addition, this type of approach was used effectively in many areas such as marketing, clinical diagnosis, decision making, educational grading systems, scheduling, materials selection, personnel management, education, etc. Thus, a suitable linguistic expression or word is selected to define the inaccurate or unclear data based on the issue area (Gupta, 2011).

Xie (2014) suggests a new MADM method for the material selection problem. This method is implemented by using a G1 method for weighting attributes instead of the popular AHP. The reason for using G1 in this material selection problem is that it is easier than AHP for

calculation and it does not require any test for the consistency of the judgment matrix.

Subsequently, the TOPSIS method is applied to rank all the material alternatives. Finally, the new proposed method is tested and studied through a case study to determine its validation. This method can be used in other applications in many various domains such as robot selection and investment project selection.

Arul, Arumugam, & Parthiban (2014) also have applied multiple criteria decision making fuzzy TOPSIS to evaluate lean manufacturing in small and medium sized Indian industries. The TOPSIS method usually provides criteria and performance weights as crisp values. Crisp values are not always perfect in demonstrating real world situations because some of these cannot be projected with numerical values. Consequently, fuzzy set theory has been suggested to solve the vagueness and uncertainty of this issue; as a result, the model of fuzzy TOPSIS is founded.

Criteria have been identified based on its effect on the application of lean manufacturing in Indian industries through literature review and expert consultation. These criteria were strong management and leadership, resistance to change\organization culture, employee trust, skills and expertise, financial capabilities, communication of the transformation process and goals, performance measures, education and training, plan and strategy, thinking development, and customer focus. Finally, a comparison was performed between companies' alternatives in order to compare their relative closeness. The one with the highest relative closeness will ranked as the best industry.

Yang, Nasr, Ong, & Nee (2017) also concentrate on developing a multi-criteria decision-making framework by using the fuzzy TOPSIS method to facilitate materials selection from remanufacturing perspective and to determine the performance of alternative automotive materials. Also, this developed methodology can aid designers to improve their options about

materials and increase the opportunity for product remanufacturing. This method is illustrated by using two case studies in the automotive materials, an engine block and an intake manifold.

The material performance and criteria are weighed from the remanufacturing perspective and consider durability, cleanability, restorability, EHS, cost and density. The outcomes have shown the effect of using light duty materials on automotive materials from the remanufacturing evaluation. As well, the outcomes emphasized the significance of selecting the best material from a life-cycle prospective.

Mansor, Sapuan, Zainudin, Nuraini, & Hambali (2013) perform the multi-criteria decision making methodology by using AHP to select the most appropriate natural fiber material to be hybridized with glass fiber reinforced polymer composites for the manufacturing of an automotive brake lever component. Thirteen materials alternatives were chosen for the decision making process through three main criteria, which are performance, weight and cost. These criteria have been selected through a literature review based on the specifications of the automotive component product design.

Consistency analysis was completed to determine the degree of consistency for the achieved outcomes during the pair-wise comparison analysis. After using AHP and selecting the best suitable material, which appear to be the kenaf bast fiber, a sensitivity analysis was applied to ensure that the selected material alternative is the best choice over the other two components and to verify that AHP methodology outcomes are accurate. It was concluded that AHP method was shown to be very applicable to solve multi-criteria decision making problems throughout the theoretical product design phase as exhibited in this project that enabled specified rating values to be provided between the comparable set of criteria and alternatives (Mansor, Sapuan, Zainudin, Nuraini, & Hambali, 2013).

Jahan and Edwards (2013) explain in their research the development of a proper technique for better management of weighting criteria in the material selection procedure. In addition, a different method has been proposed to efficiently integrate three types of weighting criteria, correlation, objective, and subjective weightings, when there is uncertainty in the importance of these criteria. This concern is very critical for inexperienced engineers and designers.

This method has been implemented by biomedical applications where the first application shows the importance of correlation weighting in amalgam tooth filling material selection. The second one illustrates the implementation of target criteria through the application of a hip joint prosthesis material selection problem. The optimal decision-making in material selection in the entire process comprises integrating these three types of weightings, choosing the proper ranking method, and using the aggregation method for different cases where there are many similar comparable alternatives. Furthermore, a continuous improvement model in product development is defined. This model emphasized that material selection is a constant and stable mission for increasing sustainability and profitability.

Kumar and Ray (2015) discuss a problem on gear material selection problem. Some MADM significant methods have been applied such as MOOSRA, EXPROM-2, ORESTE, OCRA methods. The results of implementing those methods have been compared to determine the most suitable method for material selection. For the comparison, the Spearman rank correlation coefficient and the coefficient of variance have been considered.

The Spearman rank correlation coefficient can be very helpful for defining the degree of relationship between ranks achieved by various MCDM methods. As well, the coefficient of variance has been used to compare the amounts of variation from one data series to a different

one. In addition, the performance value of the MADM methods was determined in order to calculate the coefficient of variance. Results showed that the best rank correlation coefficient is achieved between EXPROM-2 and OCRA whereas the worst correlation is obtained between MOOSRA and EXPROM-2. Moreover, the OCRA coefficient of variance result is ranked the best with 74.59 % compared with other MADM methods. It can be deduced from this study that the Spearman rank correlation coefficient method is a useful tool to measure the correlation between various MADM techniques (Kumar and Ray, 2015).

Zhao, Neighbour, Deutz, & McGuire (2012) introduce in their study a suitable method that designers and engineers can use to support decision-making on selecting the most proper material while also considering environmental evaluation. A computational method has been created through a spreadsheet by integrating a binary- dominance matrix with grey relational analysis. The main focus of this research is selecting between comparable materials to help develop the environmental performance of the product and motivating engineers to include sustainability into manufacturing and production processes. The authors used a case study to illustrate how implementing this method could be effective in real life cases. This case study is about choosing variety of poly(vinyl chloride) (PVC) materials for handbag manufacturing.

However, after implementing this method the authors noted some room for improvement. First, the evaluation measures are broad in the setting of sustainable development. In order to improve this issue, other parameters can be considered during the material selection process to make it more specific and to make the evaluation outcomes more suitable for other cases. Next, the binary dominance matrix technique should be compared with other weighting factors techniques such as the analytic Hierarchy Process (AHP) and the entropy method in order to assure the effectiveness and efficiency of this method. Furthermore, the computational process

can be further developed as a decision-making aid tool for materials selection by embracing other essential features such as market demand and materials performance (Zhao et al., 2012).

Patel and Prajapati (2017) present a MCDM method to address material selection problems that weighs both qualitative and quantitative attributes. The authors study an approach to solve blanking die material selection manufactured by WEDM Process problems by one of MCDM methods, namely, the multi-objective optimization on the basis of ratio analysis (MOORA) method. This method works by ranking the most proper material among all other alternatives.

The MOORA method proved its ease, effectiveness and steadiness through previous applications in manufacturing, construction management and engineering (Brauers, Zavadskas, Peldschus, & Turskis, 2008; Chakraborty, 2011). MOORA's process optimizes more than two conflicting attributes simultaneously that are exposed to some constraints. The implementation of MOORA is very smooth because it requires minimum statistical calculations and estimating time. The application of this method can be executed through several phases. It starts with choosing the objective and defining the appropriate criteria, then preparing a decision matrix that displays information regarding all chosen criteria, and finally normalizing each alternative's performance with respect to criteria.

MOORA technique implementation is strongly suggested for decision making issues and problems especially in the manufacturing area. The reason behind this recommendation is because of the ease of applying this tool to select the best material option between many alternatives for a certain application. It also assists the designer to select any of the most proper alternatives to move a new product from the design phase to the manufacturing and production process phase. (Patel & Prajapati, 2017).

Mangera, Kienhöfer, Carlson, Conning, Brown, & Govender (2018) apply the ELECTRE III multiple attribute decision-making method in selecting the most appropriate alternative for a pediatric prosthetic knee. The purpose of this research is to select the best material for the polycentric component. Alternatives for material selection were from light metals. The evaluation of the materials was based on critical criteria such as patient comfort, cost and structural stability.

The measurement of patient comfort was done by evaluating material density since the weight of the prosthesis influences the comfort level. The application of the ISO 10328:2006 standard on assessing the lower limb prostheses was employed in measuring the structural strength suitability of the material alternatives. Furthermore, the cost of all material alternatives was evaluated (Mangera et al, 2018).

. Kumar & Ray (2015) explain an MADM integrated approach that consisted of entropy and multiple objectives on the basis of a simple ratio analysis (MOOSRA) method to handle the material selection problem. The suggested technique has been analyzed by applying it on a case study on exhaust manifolds.

The criteria selected for the alternatives are surface hardness, core hardness, surface fatigue limit, bending fatigue limit, ultimate tensile strength, and cost. Material alternatives in this case study were ductile iron, cast iron, cast alloy steel, hardened alloy steel, surface hardened alloy steel, carburized steels, and nitride steels. It can be concluded that carburized steel is the most suitable material whereas cast steel alloy is the worst one. Furthermore, from this case study, it can be deduced that the application of this method is easy and depends on the simple ratio of useful and useless criteria embodying output and input through a decision making process by engineers. It is also noted that implementing the MOOSRA method for material

selection is easier than using other MADM methods such as ELECTRE, VIKOR, PROMETHEE, and EVAMIX (Kumar & Ray, 2015).

Wang, Lei, Chang, Shi, Xiao, Li, He, Zhu, & Yang (2015) represent a mathematical model for an integrated evaluation that was developed based on grey relational analysis (GRA) and the analytic hierarchy process (AHP) theories. The developed model has been used to select the best biomass briquette fuel (BBF) system scheme. During the development of this model, several hierarchies were considered, such as economy, cleanliness and environmental protection, production capacity, product quality, and production stability. Moreover, criteria or indices that affect the previous mentioned hierarchies include the following: labor cost, capital investment, dust content, permitted moisture of material, lifetime of major parts, briquette rate, machine repair cycle, cooling capacity, noise, maintenance cost, and moisture content.

After developing the AHP model, GRA is employed to control and calculate data. Then, normalization is performed to the data in order to make the comparison possible. After that, implementing grey relational coefficient equations to the normalized data complete the calculations for the next step, quantifying the qualitative factors. The last step in developing this multi-objective optimization model is to determine the weight vector and perform a coherence test in order to confirm that the comparative results are not in contradiction and that the AHP results are reasonable. Sensitivity analysis was applied on the most important criteria that influenced each hierarchy (Wang et al., 2015).

The best scheme that was selected includes six sets of briquetting machines, three sets of chopping machines, six sets of drying machines, and one set of cooling machines. The selection of the evaluated indices and weight coefficients was made impartially. Hence, the overall performance of the certain BBF system scheme was enhanced. The major criteria that have an

impact on each hierarchy (economy, cleanliness and environmental protection, production capacity, product quality, and production stability) were analyzed using a sensitivity analysis. These major criteria included energy costs, dust density, drying capacity, density, and machine repair cycle.

It can be concluded from the outcomes that the major machines used in a BBF system scheme, which are drying, chopping, briquetting, and cooling, were integrated rationally and were improved. Consequently, the represented mathematical model gives a well-developed technological solution and creates a foundation for economical and steady system operation (Wang et al., 2015).

Berdie, Osaci, Muscalagiu, & Barz (2017) suggest an approach of a multi-criteria analysis model that combines AHP and TOPSIS methods implemented in the integrated software systems area. By implementing the mentioned methods, the study developed a hierarchy among three ABAP UI SAP (System Applications and Products in Data Processing) technologies that include Web Dynpro -WD, Floorplan Manager - FPM and CRM WebClient UI - CRM WCUI. Also, these technologies are assessed based on their performances in being implemented on similar web business applications.

The hybrid multi-criteria selection model was created through applying the SuperDecision software that define the weights for certain groups of criteria. After determining the seven chosen weights (response time, database request time, time consumed at the end of a transaction (CBU), the time obtained in front-end (HTTP Watch time), technology development, technology maintenance and technology enhancement by using SuperDecision software with AHP), decision makers assign importance and priorities to these determined weights. The TOPSIS method was employed to achieve the ranking of the technologies alternatives. It was

concluded that Web Dynpro ABAP was ranked as the most suitable technology that can be implemented in developing a SAP UI project (Berdie et al., 2017).

Mohagheghi, Mousavi, & Vahdanib (2017) introduce a new decision making methodology based on Pythagorean fuzzy sets (PFSs). PFSs have been extended from intuitionistic fuzzy sets and are characterized by increasing the influence of decision makers (DMs) to express their ideas and opinions. Also, PFSs gives the DMs more smoothness and flexibility in expressing uncertainty unlike many other methods, making it more credible in solving real-life decision making issues.

This new method handles DMs weights as well as the entropy of criteria and uses a new ranking index. The PFSs method uses most of the collected information and also is more specific in cases where the DMs have conflicts and vary in their opinions. As well, it offers a greater feasible region in values referring to the levels of agreement, disagreement and hesitancy. Moreover, PFSs' new ranking methodology is based on the model of ideal positive and negative solutions (Mohagheghi et al., 2017).

Furthermore, this research developed a new integrated approach that focuses on measuring the weight of each DM using two main concepts. First, an appropriate computation of the weight is done by gathering most information and judgments in order to evaluate the importance of each DM opinion and thought. Second, the importance of each DM in his/her field and experience is considered. The proposed approach manages subjective and objective ways in setting the weight of each DM through using PFSs.

All criteria weights have been processed by the entropy concept. Specifically, the significance of each criterion is addressed by collecting DMs opinions and by determining the

weights regarding the collected ideas and opinions. Moreover, the use of entropy as a concept in the given decision-making method is very important in determining and measuring uncertainty in various environments. Furthermore, a novel index is introduced to rank the alternatives. It should be noted that the index can be applied to estimate the utility degree of alternatives. Also, this index can display to what degree each alternative is superior or inferior to the others by comparing them with percentages.

Finally, the authors suggested examining the efficiency of the PFSs in further research by adjusting it for application in a decision support system (DSSs). This step could assist the DMs in making accurate decisions under uncertain environments. In addition, implementing a PFS-DSS in some real life decision making problems could effectively aid the DMs to reach agreement, disagreement or hesitance with high flexibility (Mohagheghi et al., 2017).

Qu, Meng, & You (2016) study the use of a multi-stage group decision making approach under the fuzzy TOPSIS method based on interval-valued trapezoidal fuzzy sets to determine the optimal emergency technology with respect to different threat level criteria for water source pollution. The multi-stage approach consists of the following steps, (a) defining feasible technologies using a CBR-based tool, (b) estimating threat criteria, (c) assessing the alternatives by using the proposed fuzzy TOPSIS method.

1.2 MCDM Applications in Material Selections

Various types of material selection methods are made based on an MCDM approach such as the Ashby approach (Prashant Reddy, Gupta, 2010; Rashedi, Sridhar, & Tseng, 2012), the TOPSIS (technique for order performance by similarity to ideal solution) (Shanian & Savadogo, 2006; Dağdeviren, Yavuz, & Kılınç, 2009), VIKOR (Vlse Kriterijumska Optimizacija

Kompromisno Resenje, means Multicriteria Optimization and Compromise Solution) (Jahan et al., 2011; Jeya Girubha & Vinodh, 2012), ELECTRE (elimination and choice expressing the reality) (Shania & Savadogo, 2006), PROMETHEE (preference ranking organization method for enrichment evaluation) (Chatterjee & Chakraborty, 2012), COPRAS (complex proportional assessment) (Chatterjee, Athawale, & Chakraborty, 2011; Maity, Chatterjee, & Chakraborty, 2012) and COPRAS-G (Maity et al., 2012). The main idea behind building those methods is to find a solution for the material selection problems of engineering components and products in order to increase the performance and efficiency of the manufacturing process.

Jahan et al. (2010) stated that the most popular MCDM approaches for materials selection in the last decade are TOPSIS, ELECTRE and AHP. However, the authors mentioned several disadvantages in the implementation of ELECTRE and AHP, which makes TOPSIS the best option to be used in materials selection. One of ELECTRE's drawbacks can be noticed in the absence of presenting clear numerical values to provide knowledge of the differences between materials alternatives. Another disadvantage of implementing the ELECTRE method is the rapid increase in calculations and computational complications when the number of alternatives increases. Moreover, ELECTRE analyzes the effect of the alternatives under each criterion individually, which is not applicable in a materials selection problem due to the importance of measuring the effect of all criteria with respect to each alternative. AHP as well has its own limitations and barriers in the application of materials selection (Mousavi-Nasab & Sotoudeh-Anvari, 2017). One such barrier is the limited number of material alternatives and criteria to be included in the process (not more than 15), which makes this approach impractical due to the huge set of alternatives and criteria usually used in material selection problems. In other words, the traditional AHP method is ineffective when the given alternatives are in

hundreds and thousands (Jahan et al., 2010). Furthermore, ELECTERE and AHP cannot deal with benefit and cost criteria in one process at the same time, which makes using these methods not valid in the materials selection process.

On the other hand, TOPSIS can be the best option to be implemented in MCDM problems for material selections due to the following reasons:

- It is applicable for qualitative and quantitative data.
- It is simple with a systematic process.
- It allows including an unlimited amount of material alternatives and criteria, which is very practical in materials selection MCDM problems.
- It allows benefit and cost factors to be included easily within one process.
- It is a compensatory tool and gives clear compromise among criteria, which means it allows trade-offs between various decision making criteria where a negative impact in one criteria may be rewarded with a positive one in other criteria.
- Results can be expressed in a preferential ranking of the alternatives with a numerical value that delivers a clear idea about the differences and similarities between alternatives (Jahan et al., 2010; Mousavi-Nasab & Sotoudeh-Anvari, 2017; Yang et al., 2017).

Based on the above advantages, TOPSIS can be considered as a crucial MCDM approach compared with other techniques in the materials selection field, such as AHP and ELECTRE. TOPSIS has been used widely and efficiently in many material selection problems (Jahan et al., 2010). The traditional TOPSIS, which was first proposed by Yoon and Hwang (1980), can be

defined as a practical method to solve the MCDM problems with crisp numbers by selecting the alternative with the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). Based on an overall literature review, Jahan et al. (2010) conclude that TOPSIS is a very effective and reliable MCDM tool to solve materials selection problems. Yousefpour & Rahimi (2014) state that TOPSIS is a decision-making approach that will be suitable in material selection problem. Kumar & Singal (2015) report that TOPSIS is very appropriate as an MCDM tool for penstock material selection. Rathod & Kanzaria (2011) infer that TOPSIS and fuzzy TOPSIS are applicable approaches in materials selection problems when the assessments are accurate and inaccurate. Lin, Wang, Chen, & Chang (2008) recommend a method that combines the AHP and TOPSIS to aid decision makers to find optimal design alternatives. Shanian & Savadogo (2006a) use the TOPSIS tool to select the most suitable materials for metallic bipolar plates for polymer electrolyte fuel cells. Çalışkan, Kursuncu, Kurbanoglu, & Güven (2013) implement the TOPSIS concept to choose the best material option for tool holder production. In addition, Anupam, Lal, Bist, Sharma, & Swaroop (2014) successfully prove the applicability of using the TOPSIS method as an MCDM approach in the raw material selection problem by considering chemical and morphological characteristics of Eucalyptus and Leucaena varieties to select the most reliable material alternative among them for pulping and papermaking.

Another recommended MCDM approach to be used in solving the material selection problem is the complex proportional assessment (COPRAS method), introduced by Zavadskas et al. (1994). This method determines the most preferred solution by comparing between the direct and proportional ratio of the best solution and the ideal-worst solution. Recent studies that extensively reviewed the applicability of MCDM approaches within the material selection field

suggested that COPRAS is one of the best MCDM techniques along with TOPSIS that can be used in this area. For example, according to Mousavi-Nasab and Sotoudeh-Anvari (2017), TOPSIS and COPRAS are selected as the best MCDM methods for material selection in general practice. They also introduced a comprehensive MCDM approach using TOPSIS, COPRAS and data envelopment analysis (DEA) to be implemented in general material selection problems. As well, Mousavi-Nasab and Sotoudeh-Anvari (2018) studied the rank reversal phenomena that occur in applying MCDM to material selection problems. They conclude that DMs should implement TOPSIS, COPRAS, and simple additive weight (SAW) in a comparison approach to verify the ranking results. Chatterjee et al. (2011) studied the performance of different MCDM approaches. He indicated that COPRAS can be considered as one of the best material selection techniques due to its easy procedure and low calculation time.

Moreover, researchers have investigated the implementation of COPRAS in different material selection applications. Zavadskas et al. (2014) reviewed various MCDM methods and pointed out that COPRAS is a rapid developed method to deal appropriately with real problems. Chatterjee and Chakraborty (2012) studied implementing COPRAS and additive ratio assessment (ARAS)-based techniques in the application of gear material selection by performing a comparative analysis. The outcome of this study showed that both methods can be effectively applied to any type of industrial material selection decision making processes, including problems with a large number of criteria and alternatives. Aghdaie, Zolfani, & Zavadskas (2013) evaluated and selected the most suitable machine tool using a hybrid model that integrates step-wise assessment ratio analysis (SWARA) and COPRAS-G. By studying the results of implementing the hybrid model in a machine tool selection problem, the implementation of COPRAS-G proves its validity by assisting the model in ranking the alternatives precisely from

best to worst. Petković, Madić, Radenković (2015) implemented COPRAS with weighted aggregated sum product assessment (WASPAS) in a novel MCDM approach to the machining of materials specifically in a non-conventional machining process for ceramics machining. Despite the challenge of requiring a large number of criteria and alternatives in most NCMP selection problems, this method has shown its efficiency in selecting the best NCMP for ceramics machining; therefore, it can be successfully employed for solving any type of decision-making problems in the manufacturing area. Yazdani (2015) tests the effect of implementing various normalization methods in COPRAS on decision making about selecting the most proper materials and designs. He mentioned that selecting COPRAS to be tested as MCDM method in this study is due to its recognition as a promising approach in the material selection field. It is constructed based on the attributes of alternatives to handle the complex real-world problems where the properties of attributes are conflicting. In summary, it can be inferred from many studies that the reasons for selecting TOPSIS and COPRAS as the most proper MCDM techniques to solve real material selection problems are briefly reviewed as follow:

- Both can deal with a large number of criteria and material alternatives.
- Both can treat beneficial (benefit) and non-beneficial (cost) criteria in one process.
- Both can rank material alternatives in a descending order (from best to worst).
- Both can manage quantitative as well as qualitative factors (Shanian & Savadogo, 2006; Jahan et al., 2010; Chatterjee & Chakraborty, 2011; Podvezko, 2011; Özcan, Çelebi & Esnaf, 2011; Mulliner, Smallbone & Maliene, 2013; Yazdani & Payam, 2015; Mousavi-Nasab & Sotoudeh-Anvari, 2017).

As mentioned above, TOPSIS and COPRAS have been implemented efficiently in various real manufacturing and materials selection applications. However, despite the applicability of both techniques in addressing MCDM models, they have a major weakness,

specially the inability to model the vagueness and uncertainty involved in the judgment process.

This problem can be treated by applying both methods within the fuzzy logic environment (Yang et al., 2017). The fuzzy logic concept will be explained in the subsequent section.

CHAPTER TWO

Theoretical Background

2.1 Fuzzy Logic Applications to MCDM Problem

Multicriteria decision making (MCDM) is used to achieve the optimal alternative that has the maximum level of satisfaction among a group of feasible alternatives that contain multiple criteria. Many MCDM tools such as the traditional TOPSIS express the problem in the form of a decision matrix filled with crisp data, with the assumption that the provided information is precisely defined. However, this might not be applicable in some real world MCDM problems. Due to the time barrier and the limited knowledge about the problem domain, decision makers tend to express their preferences with ranges and verbal descriptions, instead of crisp data or exact numbers. For this reason, fuzzy logic is suggested by many researchers to be used in MCDM problems to overcome this issue. The concept of fuzzy logic has been presented by Zadeh to form human logic from lacking and imprecise information by providing definitions to vague expressions (Zadeh, 1965, 1973). Fuzzy logic also can integrate human experiential data and give it an engineering perception to control and form such unclear systems with uncertainty (Siddique & Adeli, 2013). Therefore, implementing fuzzy logic in MCDM tools will include a higher degree of uncertainty in the decision making process. Hence, this will provide more realistic and accurate decision making outcomes and results (Yang et al., 2017). The combination of fuzzy logic and MCDM is often called the fuzzy-MCDM approach, which can be helpful in handling the imprecise information and data by converting them into linguistic variables (Siddique & Adeli, 2013).

In order to clearly demonstrate the concept of fuzzy logic, classical logic should be defined first. In classical logic a proposition is either true or false, ‘cold or hot,’ which is imprecise, unrealistic, and cannot define many real life problems. On the other hand, fuzzy logic is a change from absolute truth to partial truth. A clear example of using the fuzzy logic concept is by changing the obsolete truth in the variable x (Cold or Hot) to a partial truth in the linguistic variable ‘Very cold,’ ‘Slightly cold,’ ‘Warm,’ ‘Slightly hot,’ and ‘Very hot’ (Siddique & Adeli, 2013). These linguistic terms can be translated to crisp number such as ‘1’ for ‘very cold’ and ‘5’ for ‘warm.’ The process in which these crisp data are converted to fuzzy numbers is called the fuzzification process. The major issue in the fuzzification process in MCDM problems is choosing the suitable membership function that will be responsible for the conversion of fuzzy numbers. A membership function can be described as a curve that expresses how each point in the input space is represented by a membership value (degree of membership) between 0 and 1, which is usually given in the form of fuzzy numbers, that is, $u_{C_j}(x_i)$. The membership functions are usually developed from many basic functions such as piecewise linear function, Gaussian distribution function, sigmoid curve, and quadratic and cubic polynomial curves (Maity & Chakraborty, 2013).

Fuzzy sets theory, which was introduced by Bellman and Zadeh (1970), was the first implementation of fuzzy numbers in MCDM. This concept has been developed to assist decision makers in solving MCDM problems, considering the fuzziness in decision makers’ preferences and the uncertainty of the objects. They suggested that the decision makers could implement the $u_{C_j}(x_i)$ to show their preference about the membership grade of a specific alternative x_i with respect to a criterion C_j ; in other words, it means the grade to which the alternative x_i satisfies the criterion C_j . In the practical MCDM procedure, decision makers usually give their opinions

regarding the grade to which the alternative x_i not only satisfies but also dissatisfies the criterion C_j . For this reason, Atanassov (1986) developed the concept of intuitionistic fuzzy set (IFS), where decision makers can express their preferences about a membership grade and a nonmembership grade of an alternative x_i with respect to a criterion C_j meeting the condition that the sum of its membership grade and its nonmembership grade is equal to or less than 1. The IFS concept has been widely employed in real-world MCDM issues and problems. Moreover, much research has been conducted on the application of MCDM problems with IFSs.

Lately, Yager (2013) presented a Pythagorean fuzzy set (PFS) concept that can be defined as the alternative's membership grade and nonmembership grade meeting the condition that the square sum of both the membership grade and the nonmembership grade is equal to or less than 1, which is a generalization of IFS. The motive behind developing this concept is that in the real-life decision making practice, decision makers present their views about the support (membership) grade and the against (nonmembership) grade for an alternative meeting of a criterion and the sum of these membership grades and nonmembership grades may be bigger than 1 but their square sum is equal to or less than 1. For simplicity, Yager (2014) provided an example to clarify this case about a decision maker who states his preference taking in consideration the membership grade of an alternative x_i regarding a criterion C_j in which the membership grade of an alternative x_i that satisfies the criterion C_j is equal to $\frac{\sqrt{3}}{2}$ and a nonmembership grade of an alternative x_i that dissatisfies the criterion C_j is equal to $\frac{1}{2}$. It can be obviously noticed that $\frac{\sqrt{3}}{2} + \frac{1}{2} \geq 1$; hence, IFS cannot define this situation. On the other hand, $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 \leq 1$ can be defined by PFS. As a result, PFSs' ability in modeling uncertainty in real-life MCDM problems is stronger than that of IFS.

Furthermore, Yager (2014) developed a group of aggregation operations for PFSs in order to be implemented for future PFSs decision making applications. Yager also presented an effective decision making technique to deal with MCDM problems with Pythagorean fuzzy information based on the new aggregation operations and the PFSs concept. Additionally, Yager & Abbasov (2013) discussed the relation between Pythagorean membership degrees and complex numbers. They showed that the Pythagorean membership degrees are a subclass of complex numbers called Π -I numbers. As well, they recommended a decision-making method built on the Pythagorean fuzzy geometric mean and the order weighted geometric operator to address the MCDM issues with Pythagorean fuzzy information (i.e., Π -I numbers). As observed, the two decision making methods mainly focus on the extensions of the aggregation approaches that are used to solve the MCDM problems under Pythagorean fuzzy situations.

2.2 Fuzzy Topsis Applications to MCDM Problems

The TOPSIS method has been extended and studied by many researchers for solving MCDM problems in many fuzzy environments such as fuzzy number contexts, interval fuzzy set contexts, IFS contexts, hesitant fuzzy set contexts, and hesitant fuzzy linguistic term set contexts (Chen, 2000; Chen & Tsao, 2008; Boran, Genc, Kurt, & Akay, 2009; Xu & Zhang, 2013; Beg & Rashid, 2013; Chen, 2016). However, the fuzzy TOPSIS methods and its extensions have been ineffective to deal with Pythagorean fuzzy environments. For this reason, Zhang and Xu (2014) developed the TOPSIS approach to successfully address the MCDM problem with Pythagorean fuzzy information.

2.3 Applications of Fuzzy TOPSIS to MCDM Problems in Material Selections

In materials selection MCDM problems, weights are usually assigned by linguistic terms described by decision makers. Thus, the final decision results might encounter a risk due to this incomplete, vague and imprecise information. For this reason, using fuzzy logic in materials selection is to address the problem that might occur from assigning weight using linguistic terms described by decision makers. The application of fuzzy logic in this situation can be very helpful to deal with such materials selection problems because sometimes the evaluations completed by engineers, designers and decision makers throughout the initial design stage contain indefinite and uncertain requirements, parameters, and relationships. Thus, fuzzy logic is implemented as a supportive concept to express the action and approach of the decision makers while selecting the most suitable material alternative (Maity and Chakraborty, 2013).

Since TOPSIS has demonstrated its applicability in materials selection problems, many researchers combine the use of TOPSIS under fuzzy environments to address the uncertainty and vagueness issues that might occur in the decision making process and that affect the final decision results. Maity and Chakraborty (2013) adopted fuzzy logic effectively with the TOPSIS method in selecting the most suitable material for a list of eight grinding wheel abrasives whose efficiency is assessed based on multiple criteria. Maldonado-Macías, Alvarado, García, & Balderrama (2014) introduced the use of the TOPSIS method under an intuitionistic fuzzy environment to select the best alternative among three CNC milling machines for an advanced manufacturing technology training center considering human factors and ergonomics attributes. The intuitionistic fuzzy TOPSIS method was very beneficial in dealing with the vagueness or sometimes incorrect information provided by decision makers in the evaluation process. Yang et al. (2017) studied how applying fuzzy TOPSIS for material selection in the initial phase could

effectively improve the remanufacturability of automotive parts. Combining the TOPSIS method with fuzzy logic was very effective in reducing personal bias and uncertainty of the evaluation process. Liao (2015) developed a two-interval type 2 fuzzy TOPSIS method to handle the high nature of uncertainty and vagueness of material properties in the material selection process within the product design stage. Purohit and Ramachandran (2015) used a fuzzy Topsis approach in MCDM to select the best suitable alternative among five materials for making engine flywheels. They suggested that this approach can be implemented for any materials selection problem in a manufacturing field. However, to the best of our knowledge, no application of the MCDM method on material selection based on the use of the TOPSIS technique under the Pythagorean fuzzy environment is currently available in the open literature. To fill this gap, this research will suggest an effective Pythagorean fuzzy TOPSIS MCDM approach to be used in real life materials selection problems.

2.4 Applications of Fuzzy COPRAS to MCDM Problems in Material Selections

The COPRAS method has been commonly used as an effective MCDM technique to deal with real decision making problems (Zavadskas et al., 2014). Due to its simplicity, it has been successfully implemented for solving many problems in different fields and applications (Podvezko, 2011; Antucheviciene et al., 2012; Mulliner et al., 2013; Gadakh, 2014; Mousavi-Nasab and Sotoudeh-Anvari, 2017, Chatterjee et al, 2011). The idea of using the fuzzy COPRAS approach to address MCDM problems in material selection applications has been studied by several researchers. Dursun and Arslan (2018) introduced a fuzzy multi-criteria group decision-making framework (MCGDM) that incorporates customer and experts' requirements in the decision making process and used the fuzzy COPRAS technique as a ranking method for material selection purposes. They implemented the principles of quality function deployment

(QFD), 2-tuple fuzzy linguistic representation and linguistic hierarchies in the initial phase of the decision making framework, which is considered the assessment phase, and then applied fuzzy COPRAS to rank all material alternatives and choose the most suitable one. Nguyen et al. (2014) developed a hybrid approach of the fuzzy ANP (Analytic Network Process) and COPRAS-G (COMplex Proportional ASsessment of alternatives with Grey relations) for MCDM problems in the machine tool selection application. The proposed decision making model consists of three phases: setting the criteria and alternatives, determining the interaction and weights of the criteria using the fuzzy ANP, and selecting the most appropriate alternative by implementing COPRAS-G ranking method. They concluded that implementing this hybrid approach is effective in selecting the best machine tool and using the COPRAS-G method allows the interval values to consider the uncertainty about the criteria, which helps in ranking and identifying the most suitable option. Nguyen et al. (2015) presented a MCDM framework by integrating fuzzy AHP and fuzzy COPRAS for machine tool evaluation. The developed framework allows processing uncertain information within the data collection phase by integrating the fuzzy linguistic preference relation into AHP to determine the elements of MCDM matrix based on DMs thought and opinions and then applying fuzzy COPRAS in the ranking phase. The study inferred that the use of this integrated approach could be helpful in the machine tool selection process due to its applicability in measuring uncertainty and modeling DMs judgments to be used properly in the ranking procedure.

However, to the best of our knowledge, the contribution in the research area regarding the implementation of the fuzzy COPRAS method in material selection problems is still insufficient. For this reason, this research will introduce an effective Hybrid Pythagorean fuzzy MCGDM COPRAS model for the application of real life materials selection problems.

CHAPTER THREE

Research Significance, Novelty and Objectives

3.1 Research significance

The core significance of this research is in how the implementation of the proposed hybrid methods could affect selecting the best materials precisely in the design stage better than any other commonly used methods. Engineers, designers and decision makers may benefit from using these methods because it incorporates the Pythagorean fuzzy environment in modeling uncertainty in the given data, leading to better outcomes and decisions.

3.2 Research Novelty

The novelty in this research approach is based on the application of hybrid MCGDM methods with the Pythagorean fuzzy information in the decision making process of engineering materials selection. The Pythagorean fuzzy sets are superior to other properly used fuzzy sets because of their flexibility in allowing decision makers to express their preferences without any limitations or constraints. Furthermore, no publications have been cited with reference to the use of the Pythagorean fuzzy TOPSIS method in engineering materials selection applications. Another novelty in this research is the contribution of delivering a new set of aggregations to convert real life crisp decisions into Pythagorean fuzzy numbers.

3.3 Research Objectives

The objectives of this research can be summarized as follows:

1. To develop an approach to aggregate decision makers' and experts' knowledge from crisp decisions to Pythagorean Fuzzy Numbers (PFNs).
2. To build hybrid MCGDM methods through integrating the developed aggregation approach into ranking methods that deal with PFNs such as Pythagorean Fuzzy TOPSIS and Pythagorean Fuzzy COPRAS methods.
3. To implement the new hybrid Pythagorean Fuzzy MCGDM methods into real world materials selection problem.
4. To evaluate the accuracy and sensitivity of the proposed methods outcomes.

CHAPTER FOUR

Methodology

4.1 Fuzzy Method

Initially, the concept of the proposed fuzzy method, which is the Pythagorean fuzzy set, needs to be clarified for the purpose of this study. The Pythagorean fuzzy set (PFS) is a generalization of Intuitionistic Fuzzy Set (IFS). Therefore, IFS should be explained clearly first before introducing the concept of PFS.

4.1.1 Intuitionistic fuzzy set

DEFINITION 4.1. Let a set X be a universe of discourse. An IFS I is an object having the form,

$$I = \{ \langle x, I(\mu_I(x), v_I(x)) \rangle \mid x \in X \} \quad (4.1)$$

where the function $\mu_I: X \rightarrow [0,1]$ describes the degree of membership and $v_I: X \rightarrow [0,1]$ describes the degree of nonmembership of the element $x \in X$ to I , respectively, and for every $x \in X$, it holds that

$$0 \leq \mu_I(x) + v_I(x) \leq 1 \quad (4.2)$$

For any IFS I and $x \in X$, $\pi_I(x) = 1 - \mu_I(x) - v_I(x)$ is called the degree of indeterminacy of x to I . Moreover, Xu1 denoted $\alpha = I(\mu_\alpha, v_\alpha)$ as an intuitionistic fuzzy number (IFN), where μ_α and v_α is the degree of membership and the degree of nonmembership of the element $x \in X$ to I , respectively.

IFS is more practical to be used from decision makers in the assessment process of real life decision making issues than IFN. The basic operational laws of the IFNs $\alpha_j =$

$I(\mu_{\alpha_j}, v_{\alpha_j}) (j = 1, 2)$ and $= I(\mu_{\alpha}, v_{\alpha})$, developed by Xu and Yager (2006) and Xu (2007)

are presented as follows:

1. $\alpha_1 \oplus \alpha_2 = I(\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, v_{\alpha_1}v_{\alpha_2})$;
2. $\alpha_1 \otimes \alpha_2 = I(\mu_{\alpha_1}\mu_{\alpha_2}, v_{\alpha_1} + v_{\alpha_2} - v_{\alpha_1}v_{\alpha_2})$;
3. $\lambda\alpha = I(1 - (1 - \mu_{\alpha})^{\lambda}, v_{\alpha}^{\lambda}), \lambda > 0$;
4. $\alpha\lambda = I(\mu_{\alpha}^{\lambda} - (1 - v_{\alpha})^{\lambda}), \lambda > 0$;
5. $\alpha^c = I(v_{\alpha}, u_{\alpha})$

However, managers, designers and engineers in various real-life multiple criteria decision making issues may show their preferences about the degree of an alternative x_i with respect to a criterion C_j meeting the condition that the sum of the degree to which the alternative x_i meets the criterion C_j and the degree to which the alternative x_i does not meet the criterion C_j is more than

1. Clearly, this state cannot be expressed by using the IFS. Thus, Yager developed a novel model of PFS to cover this issue and to prevent decision makers from changing their preferences to fit within IFS's constraints (Zhang & Xu, 2014). PFS is introduced in the follow part.

4.1.2 Pythagorean Fuzzy Sets

Yager (2013, 2014) represented three key demonstrations for Pythagorean membership grades. The first one is (a, b) fulfilling the conditions that $a \in [0, 1], b \in [0, 1]$, and $a^2 + b^2 \leq 1$. The second one is the polar coordinates (r, θ) fulfilling the conditions that $r \in [0, 1], \theta \in [0, \pi/2]$. The third one is (r, d) close to the second one fulfilling the conditions that $r \in [0, 1], \theta \in [0, \pi/2]$, and $d = 1 - 2\theta/\pi$. Their relationship is that $a^2 + b^2 = R^2$, $a = r \cos(\theta)$, $b = r \sin(\theta)$. As well, he denoted a fuzzy subset having these Pythagorean membership grades as a PFS.

DEFINITION 4.2. Let a set X be a universe of discourse. A PFS P is an object having the form

$$P = \{ \langle x, P(\mu_P(x), \nu_P(x)) \rangle \mid x \in X \} \quad (4.3)$$

where the function $\mu_P: X \rightarrow [0, 1]$ describes the degree of membership and $\nu_P: X \rightarrow [0, 1]$ describes the degree of nonmembership of the element $x \in X$ to P , respectively, and for every $x \in X$, it holds that

$$(\mu_P(x))^2 + (\nu_P(x))^2 \leq 1 \quad (4.4)$$

For any PFS P and $x \in X$, $\pi_P(x) = \sqrt{1 - \mu_P^2(x) - \nu_P^2(x)}$ is called the degree of indeterminacy of x to P . Moreover, in order to simplify it, $P(\mu_P(x), \nu_P(x))$ is called a Pythagorean fuzzy number (PFN) denoted by $\beta = P(\mu_\beta, \nu_\beta)$, where $\mu_\beta, \nu_\beta \in [0, 1]$, $\pi_\beta = 1 - (\mu_\beta)^2 - (\nu_\beta)^2$, and $(\mu_\beta)^2 + (\nu_\beta)^2 \leq 1$.

Yager (2013, 2014), and Yager & Abbasov (2013) stated the major operations on three PFNs, which are $\beta_1 = P(\mu_{\beta_1}, \nu_{\beta_1})$, $\beta_2 = P(\mu_{\beta_2}, \nu_{\beta_2})$ and $\beta = P(\mu_\beta, \nu_\beta)$ as follows:

1. $\beta_1 \cup \beta_2 = P(\max\{\mu_{\beta_1}, \mu_{\beta_2}\}, \min\{\nu_{\beta_1}, \nu_{\beta_2}\})$
2. $\beta_1 \cap \beta_2 = P(\min\{\mu_{\beta_1}, \mu_{\beta_2}\}, \max\{\nu_{\beta_1}, \nu_{\beta_2}\})$
3. $\beta^c = P(\nu_\beta, \mu_\beta)$

Zhang and Xu (2014) developed some operations on the basis of a relationship between PFNs and IFNs, which can be expressed as follows:

4. $\beta_1 \oplus \beta_2 = P(\sqrt{\mu_{\beta_1}^2 + \mu_{\beta_2}^2 - \mu_{\beta_1}^2 \mu_{\beta_2}^2}, \nu_{\beta_1} \nu_{\beta_2})$

$$5. \beta_1 \otimes \beta_2 = P (u_{\beta_1} u_{\beta_2}, \sqrt{v_{\beta_1}^2 + v_{\beta_2}^2 - v_{\beta_1}^2 v_{\beta_2}^2})$$

$$6. \lambda \beta = P (\sqrt{1 - (1 - \mu_{\beta}^2)^{\lambda}}, (v_{\beta})^{\lambda}), \lambda > 0$$

$$7. \beta^{\lambda} = P ((\mu_{\beta})^{\lambda}, \sqrt{1 - (1 - v_{\beta}^2)^{\lambda}},) , \lambda > 0$$

THEOREM 4.1. For three PFNs $\beta_1 = P(u_{\beta_1}, v_{\beta_1})$, $\beta_2 = P(u_{\beta_2}, v_{\beta_2})$, and $\beta = P(\mu_{\beta}, v_{\beta})$, the following ones are valid:

$$1. \beta_1 \oplus \beta_2 = \beta_2 \oplus \beta_1$$

$$2. \beta_1 \otimes \beta_2 = \beta_2 \otimes \beta_1$$

$$3. \lambda(\beta_1 + \beta_2) = \lambda\beta_1 \oplus \lambda\beta_2, \lambda > 0 \quad \lambda(\beta_1 \oplus \beta_2) = \lambda\beta_1 \oplus \lambda\beta_2, \lambda > 0.$$

$$4. \lambda_1\beta \oplus \lambda_2\beta = (\lambda_1 + \lambda_2)\beta, \lambda_1, \lambda_2 > 0$$

$$5. (\beta_1 \otimes \beta_2)^{\lambda} = \beta_1^{\lambda} \otimes \beta_2^{\lambda}, \lambda > 0$$

$$6. \beta^{\lambda_1} \otimes \beta^{\lambda_2} = \beta^{(\lambda_1 + \lambda_2)}, \lambda_1, \lambda_2 > 0$$

DEFINITION 4.3. Let $\beta_j = P(u_{\beta_j}, v_{\beta_j})$ ($j = 1, 2$) be two PFNs; then the nature quasi-ordering on the PFNs can be defined as follows:

$$\beta_1 \geq \beta_2 \text{ if and only if } u_{\beta_1} \geq u_{\beta_2} \text{ and } v_{\beta_1} \leq v_{\beta_2}.$$

Also, a score function is developed and implemented in order to compare the magnitudes of two PFNs as defined in what follows.

DEFINITION 4.4. Let $\beta = P(u_\beta, v_\beta)$ be a PFN; then the score function of β can be defined as follows:

$$s(\beta) = (u_\beta)^2 - (v_\beta)^2 \quad (4.5)$$

It is observed that the score function $s(\beta)$ contains some desirable properties as below.

Rule 5.1. For any PFN $\beta = P(u_\beta, v_\beta)$, the proposed score function $s(\beta) \in [-1, 1]$.

Rule 5.2. For two PFNs $\beta_j = P(u_{\beta_j}, v_{\beta_j})$ ($j = 1, 2$), if $\beta_1 \leq \beta_2$, then $s(\beta_1) \leq s(\beta_2)$.

The subsequent laws are introduced on the basis of the score function of PFNs in order to make a comparison between two PFNs.

DEFINITION 4.5. Let $\beta_j = P(u_{\beta_j}, v_{\beta_j})$ ($j = 1, 2$) be two PFNs, $s(\beta_1)$ and $s(\beta_2)$ be the scores of β_1 and β_2 , respectively; then

1. If $s(\beta_1) < s(\beta_2)$, then $\beta_1 < \beta_2$;
2. If $s(\beta_1) > s(\beta_2)$, then $\beta_1 > \beta_2$;
3. If $s(\beta_1) = s(\beta_2)$, then $\beta_1 \sim \beta_2$;

Example 4.1. Let $\beta_1 = P(\frac{\sqrt{3}}{2}, \frac{1}{2})$ and $\beta_2 = P(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, according to Definitions 4.4-4.5, we have

$$s(\beta_1) = (\frac{\sqrt{3}}{2})^2 - (\frac{1}{2})^2 = \frac{1}{2}, s(\beta_2) = (\frac{\sqrt{2}}{2})^2 - (\frac{\sqrt{2}}{2})^2 = 0$$

Evidently, $s(\beta_1) > s(\beta_2)$, thus $\beta_1 > \beta_2$.

Furthermore, Yager (2014) presented the next weighted averaging aggregation operator in order to aggregate PFNs.

DEFINITION 4.6. Let $\beta_j = P(u_{\beta_j}, v_{\beta_j})$ ($j = 1, 2, \dots, n$) be a collection of PFNs and $w = (w_1, w_2, \dots, w_n)^T$ be the weight vector of β_j ($j = 1, 2, \dots, n$), where w_j specifies the magnitude degree of β_j , fulfilling $w_j \geq 0$ ($j = 1, 2, \dots, n$) $\sum_{j=1}^n w_j = 1$, and let Pythagorean fuzzy weighted averaging (PFWA): $\Theta^n \rightarrow \Theta$ if

$$PFWA(\beta_1, \beta_2, \dots, \beta_n) = P(\sum_{j=1}^n w_j u_{\beta_j}, \sum_{j=1}^n w_j v_{\beta_j}) \quad (4.6)$$

Then the function PFWA is called the PFWA operator.

4.2 MCDM Problem

This section demonstrates a brief explanation of the MCDM structure and how it can be effectively implemented in the proposed model.

4.2.1 General MCDM structure

The effective formation of an MCDM problem plays a significant role in having a successful decision making model with precise results. Typical MCDM problems are structured in a way to allow decision makers to rate each alternative with respect to each criterion. The MCDM matrix generally can be formed as follows:

$$D(a_{ij})_{m \times n} = \begin{matrix} & C_1 & C_2 & - & C_j \\ \begin{matrix} A_1 \\ A_2 \\ - \\ A_i \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & - & a_{1j} \\ a_{21} & a_{22} & - & a_{2j} \\ - & - & - & - \\ a_{i1} & a_{i2} & - & a_{ij} \end{bmatrix} & & & \end{matrix} \quad (\text{Model 4.1})$$

Where $D(a_{ij})$ called the decision matrix and a_{ij} is the performance value of the i th alternative ($A_i: i = 1, 2, \dots, m$) regarding the j th criterion ($C_j: j = 1, 2, \dots, n$). Weights should be assigned to each criterion ($w_j: j = 1, 2, \dots, n$) and satisfy $\sum_{j=1}^n w_j = 1$. Moreover, criteria are

divided into main categories: beneficial (profit) criteria and non-beneficial (cost) criteria. Specifically, the criterion that satisfies the condition that higher scores are desirable is a beneficial criterion, whereas the criterion that satisfies the condition that lower scores are desirable is a non-beneficial criterion.

4.2.2 MCDM for Group Decision Making (MCGDM)

The proposed method assumes that groups of decision makers are going to express their preferences in order to influence the decision making outcomes. Therefore, the designated MCDM problem should be developed to serve the purpose of aggregating the decision makers' group's knowledge and experiences to make the most suitable decision (Yue, 2014). In this case, the group decision matrix should be designed as shown below:

$$X_i = (r_{kj}^i)_{t \times n} = \begin{matrix} & C_1 & C_2 & - & C_n \\ \begin{matrix} d_1 \\ d_2 \\ - \\ d_t \end{matrix} & \begin{bmatrix} r_{11}^i & r_{12}^i & - & r_{1n}^i \\ r_{21}^i & r_{22}^i & - & r_{2n}^i \\ - & - & - & - \\ r_{t1}^i & r_{t2}^i & - & r_{tn}^i \end{bmatrix} & , & i = 1, 2, \dots, m \end{matrix} \quad (\text{Model 4.2})$$

$X_i = (r_{kj}^i)_{t \times n}$ refers to the group decision matrix for each i th alternative in which each decision maker $D = \{d_k: k = 1, 2, \dots, t\}$ evaluates the importance of each given criterion $C = \{C_j: j = 1, 2, \dots, n\}$ with respect to the alternative $x = \{x_i: i = 1, 2, \dots, m\}$.

4.1.1.1. Normalization

Normalizing decision makers' data regarding performance evaluation is a significant step in order to begin the fuzzification process. Appropriate normalization of the given numbers would facilitate converting them into Pythagorean fuzzy numbers efficiently, leading to an effective application of the MCGDM TOPSIS model. To start the normalization process, each

group decision matrix $X_i = (r_{kj}^i)_{t \times n}$ for each i th alternative needs to be normalized into $R_i = (s_{kj}^i)_{t \times n}$ using the following equations:

$$s_{kj}^i = \frac{r_{kj}^i - \min j}{\max j - \min j}, \quad \text{for beneficial criteria } C_j \quad (4.7)$$

$$s_{kj}^i = \frac{\max j - r_{kj}^i}{\max j - \min j}, \quad \text{for non - beneficial criteria } C_j \quad (4.8)$$

where the $\max j$ and $\min j$, respectively, are the maximum grade and the minimum grade used by a decision maker d_k in the evaluation system. In this case, for example, the hundred-mark system that consists of 100 as the maximum grade ($\max j$) and 0 as the minimum grade ($\min j$) is suggested to be used by the decision makers in the evaluation process. For this reason, equations (4.7) & (4.8) will be re-written as follow:

$$s_{kj}^i = \frac{r_{kj}^i}{100}, \quad \text{for beneficial criteria } C_j \quad (4.9)$$

$$s_{kj}^i = \frac{100 - r_{kj}^i}{100}, \quad \text{for non - beneficial criteria } C_j \quad (4.10)$$

After normalization, $R_i = (s_{kj}^i)_{t \times n}$ matrix will be presented as follows:

$$R_i = (s_{kj}^i)_{t \times n} = \begin{matrix} & C_1 & C_2 & - & C_n \\ \begin{matrix} d_1 \\ d_2 \\ - \\ d_t \end{matrix} & \begin{bmatrix} s_{11}^i & s_{12}^i & - & s_{1n}^i \\ s_{21}^i & s_{22}^i & - & s_{2n}^i \\ - & - & - & - \\ s_{t1}^i & s_{t2}^i & - & s_{tn}^i \end{bmatrix} & , & i = 1, 2, \dots, m \end{matrix} \quad (\text{Model 4.3})$$

However, normalizing column vectors in Model (4.3) should be done in which all values are within $[0,1]$ and satisfy that the maximum value is 1 and the minimum is 0. The satisfactory

and dissatisfactory values in $R_i = (s_{1j}^i, s_{2j}^i, \dots, s_{tj}^i)$, which is the attribute/column vector in R_i , still need to be defined to pursue the aggregation approach.

4.2.3 Aggregation of crisp decisions into PFNs for MCGDM

Before explaining this part, it is important to note that the following developed technique has been inspired by the You (2014) method in deriving intuitionistic fuzzy numbers from crisp data. However, You's method has been criticized by Lin & Zhang (2016) for having weaknesses that result in illogical outcomes, which makes it impractical for real applications. In addition, Lin & Zhang (2016) have suggested a new revised aggregation approach for intuitionistic fuzzy numbers to overcome the gaps in You's method. Accordingly, we have built our novel aggregation approach to convert crisp numbers into Pythagorean Fuzzy sets based on the aforementioned aggregation techniques.

In order to determine a PFN efficiently, all crisp decisions in R_i need to be aggregated within the process. The aggregation process of PFN should consider determining three main elements: membership degree μ_α (degree of satisfaction), non-membership degree ν_α (degree of dissatisfaction) and hesitation degree π_α (degree of uncertainty) in which the induced numbers satisfy the following:

$$0 \leq \mu_p(x), \nu_p(x), \pi_p(x) \leq 1 \quad (4.11)$$

$$\sqrt{\mu_p^2(x) + \nu_p^2(x) + \pi_p^2(x)} = 1 \quad (4.12)$$

Initially, some rules should be followed to define the satisfactory, dissatisfactory or uncertain values in R_{ij} . As seen in (Model 4.3) all values are within $[0,1]$; thus, "0.5" will be the bound for identifying the satisfactory and dissatisfactory value in R_i . These values can be represented as follows:

$$s_{kj}^u = \{s_{kj}^i > 0.5, s_{kj}^i \text{ are the elements of } R_{ij}\}$$

$$s_{kj}^c = \{s_{kj}^i < 0.5, s_{kj}^i \text{ are the elements of } R_{ij}\}$$

Then, a linear transformation should be made to each element to determine the satisfaction, dissatisfaction, and uncertainty degrees. The calculation of this process should take in consideration that (1) the closer the element value (s_{kj}^u) is to 1, the larger the satisfactory level is; (2) the closer the element value (s_{kj}^c) is to 0, the larger the dissatisfactory level is; and (3) the closer the element value (s_{kj}^u) or (s_{kj}^c) is to 0.5, the larger the uncertainty level is. The linear transformation formula can be represented as follows:

$$o_{kj} = \frac{s_{kj}^u - 0.5}{1 - 0.5} \quad (4.13)$$

$$\xi_{kj} = \frac{0.5 - s_{kj}^c}{0.5 - 0} \quad (4.14)$$

where o_{ij} represents each satisfactory element of i th alternative with respect to j th attribute and ξ_{ij} represents each dissatisfactory element of i th alternative with respect to j th attribute.

The next step is to aggregate the satisfaction and dissatisfaction elements of each j th criterion in i th alternative with respect to all decision makers' (DMs) importance (weights). We need to take in consideration that DM weights may not all be equal. In fact, the importance of a decision maker should be assigned based on several factors such as his/her educational background, experience level and authority (Jian & Quing, 2016). In order to assign the priority of each decision maker effectively, an evaluation process can be done by conducting surveys and interviews with the assigned committee members. Since each DM has its own weight, we need to assure that the effect of each DM power is properly aggregated to the calculation of the

satisfactory and dissatisfactory weighted average degrees. For this reason, Jian & Quing (2016) have suggested the use of the Shapely value method, which is the most popular method that is applied for solving cooperative games in this stage. The Shapley value will help us in covering the correlative and interaction phenomenon among DMs, and therefore, weights can be assigned more fairly and efficiently. DM's Shapley value reflects how much his opinion affects or contributes to a coalition, specifically, how much he adds to a coalition. For example, a DM whose opinion never adds much to a coalition has a small Shapley value, while a DM whose opinion has significant effect on the coalition has a high Shapley value. As a result, it is a useful way for a fair division of DMs weights based on their contributions to the system (Fatima, Wooldridge & Jennings, 2008). The following formula can clearly express the Shapely value mathematically:

$$\Phi_k = \frac{1}{t!} \sum_{F \subseteq D \setminus \{d_k\}} |F|! (t - |F| - 1)! [\mu(F \cup \{d_k\}) - \mu(F)], \quad (4.15)$$

$$\forall k \in \{1, 2, \dots, t\}$$

Φ_k is the Shapely value, which indicates the weighted averaging value of the marginal contribution with respect to a player (decision maker) d_k . μ is a fuzzy measure on a finite set that represents decision makers $D = \{d_1, d_2, \dots, d_t\}$ with: (1) $\mu(\emptyset) = 0, \mu(\{d_1, d_2, \dots, d_t\}) = 1$; (2) If $A, B \subseteq D$ and $A \subseteq B$, then $\mu(A) \leq \mu(B)$ (Monotonic). Since $\mu(D) = 1$, therefore, the Shapley value $\Phi_k = (\Phi_1, \Phi_2, \dots, \Phi_k)$ is a weighting vector and $\sum_{k=1}^t \Phi_k = 1$. $\mu(\{d_k\})$ is a singleton sets called the fuzzy density value that expresses the importance value of each decision maker $d_k, k = 1, 2, \dots, t$ and it can be assigned based on his/her influence and power that will be derived from surveys and interviews. In the group decision making, $\mu(F)$ can be

viewed as the importance of the subset (or coalition) F , $F \subseteq D$ and it can be determined using the λ -fuzzy measures of Sugeno as follows (Sugeno, 1974; Leszczyński, Penczek & Grochulski, 1985):

$$\lambda + 1 = \prod_{k=1}^t (1 + \lambda \mu(\{d_k\})), \quad (4.16)$$

where $\lambda > -1$

$$\mu(F) = \mu(\{d_A\} \cup \{d_B\}) = \mu(d_A) + \mu(\{d_B\}) + \lambda \mu(d_A) \mu(\{d_B\})$$

where $A \& B \in (1, 2, \dots, k)$, and $A \neq B$

As seen above, the key property of a fuzzy measure is monotonicity with respect to set inclusion. In addition, the Sugeno λ -fuzzy measures is a nondecreasing function with respect to the values of the measure of the constituent fuzzy density values $\mu(\{d_k\})$. Finding the unique value of the parameter λ requires obtaining a root of a high order polynomial in λ (Mohamed & Xiao, 2003).

After calculating the Shapely value of each DM, the weighted averaging satisfactory and dissatisfactory values can be measured, respectively, as follows:

$$\kappa_{ij} = \sum_{k=1}^t \Phi_k o_k \quad (4.17)$$

$$\varsigma_{ij} = \sum_{k=1}^t \Phi_k \xi_k \quad (4.18)$$

where κ_{ij} and ς_{ij} are the weighted average satisfactory degree and the weighted average dissatisfactory degree, respectively, and Φ is the weight of importance of each DM, with $\Phi_k \in [0,1]$.

After finding the weighted average satisfactory and dissatisfactory values, the only phase left to complete this aggregation is to measure the uncertainty of the induced numbers. The following uncertainty calculations are based on Pythagorean fuzzy sets concept in multi criteria decision making that is defined by Yager (2014). In order to pursue this step, r computation should be measured by determining the sum of the square root of satisfactory and dissatisfactory degrees. It can be shown as follows:

$$r = \sqrt{\kappa_{ij}^2 + \varsigma_{ij}^2} \quad (4.19)$$

Then, the determination of theta (θ) would play a very important role to identify the performance of the uncertainty function. Using r , (θ) can be calculated as follows:

$$\cos(\theta) = \frac{\kappa}{r} \quad (4.20)$$

The calculated $\cos(\theta)$ can be used to find (θ) value through trigonometric tables. It can be also measured by the following formula:

$$(\theta) = \cos^{-1}(\cos(\theta)) \quad (4.21)$$

For function modeling purposes, (θ) will be transformed as follows:

$$d = 1 - \frac{\theta}{\pi} \quad (4.22)$$

Hence, when (θ) = 0 we have $d = 1$ and when (θ) = $\frac{\pi}{2}$ we have $d = 0$. To define the uncertainty degree in an appropriate logical and mathematical approach, a fuzzy modeling method by using Takagi–Sugeno (T-K) approach will be applied to build the required functions (Takagi & Sugeno, 1985). The fuzzy modeling application relies on forming fuzzy base rules that aim to explain local input–output relations between the previous experimental data in the

function (Salgado, Viegas, Azevedo, Ferreira, Vieira & Sousa, 2017). The rule should be formed as follows:

$$R^i : \text{IF } x_1 \text{ is } A_1 \text{ and } \dots \text{ and } x_k \text{ is } A_i$$

$$\text{THEN } y_i = g_i(x),$$

$$\text{for } i = 1, 2, \dots, n, k = 1, 2, \dots, K$$

where R^i refers to i th rule of total number of rules n , x_k is the input vector of K inputs, and y_i is the output of rule R^i . A_k^i is the antecedent fuzzy subset with a linear membership function $E_{A_{ij}}(x_j)$. The model output is computed by aggregating n rules contributions as shown below:

$$f(x) = \frac{\sum_{i=1}^n \beta_i g_i(x)}{\sum_{i=1}^n \beta_i} \quad (4.23)$$

$$\text{Where } \beta_i = \prod_{j=1}^k E_{A_{ij}}(x_k)$$

In order to form reasonable fuzzy base rules, the required function $F(x)$ behavior should be demonstrated precisely. As mentioned before, 0.5 is the bound that has been used to distinguish between the satisfactory and dissatisfactory degrees in each element; therefore, it will be used again for identifying the amount of uncertainty. In other words, if $d = 0.5$ then the uncertainty will be at its highest degree $F = 1$ and the farthest the value from the bound whether it is $(d < 0.5)$ or $(d > 0.5)$, the lower the uncertainty degree will be. Consequently, the fuzzy base rules that are going to be implemented in the modeling technique can be defined as follows:

A) First function modeling rules (When $d > 0.5$, $\theta < \frac{\pi}{4}$):

$$R^1 : \text{IF } r \text{ is close to } 1 \text{ and } d \text{ is close to } 1$$

THEN $g_1(x) = 0$,

R^2 : IF r is close to 1 and d is close to 0.5

THEN $g_2(x) = 2$,

R^3 : IF r is close to 0

THEN $g_3(x) = 1$,

B) Second function modeling rules (When $d < 0.5$, $\theta > \frac{\pi}{4}$) :

R^1 : IF r is close to 1 and d is close to 0

THEN $g_1(r, d) = 0$,

R^2 : IF r is close to 1 and d is close to 0.5

THEN $g_2(r, d) = 2$,

R^3 : IF r is close to 0

THEN $g_3(r, d) = 1$,

For more clarification, r is close to 1 will be represented as a fuzzy subset A_1 on the unit interval with a membership function ($E_{A_1}(r) = r$). For r is close to 0, it will be represented as a fuzzy subset A_2 on the unit interval with a membership function ($E_{A_2}(r) = 1 - r$). In the first function rules, $d \in [0.5, 1]$; thus, in this situation, d is close to 1 will be represented as a fuzzy subset C_1 on the unit interval with a membership function ($E_{C_1}(d) = d$) and d is close to 0.5 will be represented as a fuzzy subset C_2 on the unit interval with a membership function ($E_{C_2}(d) = 1 - d$). In the second function rules, $d \in [0, 0.5]$, accordingly, d is close to 0 will be represented as a fuzzy subset N_1 on the unit interval with a membership

function $(E_{N_1}(d) = 1 - d)$ and d is close to 0.5 will be represented as a fuzzy subset N_2 on the unit interval with a membership function $(E_{N_2}(d) = d)$. $g_i(r, d)$ is the output of applying the i th rule.

Therefore, the uncertainty functions will be developed by applying a T-K approach to aggregate fuzzy rule bases as the model output represented in equation (4.23). As a result, the following functions: function (A) and function (B) are modeled, respectively, as follows:

$$f_A(r, d) = \frac{E_{A_1}(r)E_{C_1}(d)(2)+E_{A_1}(r)E_{C_2}(d)(0)+E_{A_2}(r)(1)}{E_{A_1}(r)E_{C_1}(d)+E_{A_1}(r)E_{C_2}(d)+E_{A_2}(r)} \quad (4.24)$$

$$f_B(r, d) = \frac{E_{A_1}(r)E_{N_1}(d)(0)+E_{A_1}(r)E_{N_2}(d)(2)+E_{A_2}(r)(1)}{E_{A_1}(r)E_{N_1}(d)+E_{A_1}(r)E_{N_2}(d)+E_{A_2}(r)} \quad (4.25)$$

It can be observed from the first function behavior that if r has been placed as a fixed arc of radius, then the uncertainty function will decrease from $f_A(r, d) = 1$ to $f_A(r, d) = 0$ as it goes from $d = 0.5$ to $d = 1$, which is similar to $\theta = \pi/4$ to $\theta = 0$. Also, if d is any fixed value from $(0.5 < d \leq 1)$ then the function will decrease from $f_A(r, d) = 1$ to $f_A(r, d) = 0$ as the radius increases from $r = 0$ to $r = 1$. Lastly, if $d = 0.5$ then $f_A(r, d) = 1$ and it stays the same at any r value. For this reason, the output of implementing the first rule R^1 has been assigned as $g_2(x) = 0$, where d is close to 1, and the output of implementing the second rule R^2 has been assigned as $g_2(x) = 2$, where d is close to 0.5. Therefore, if d goes from 0.5 to 1 the function will decrease from $f_A(r, d) = 1$ to $f_A(r, d) = 0$. For the second function performance, it will decrease from $f_B(r, d) = 1$ to $f_B(r, d) = 0$ as r is constant and d decreases from 0.5 to 1. As well, if d is any fixed value from $(0 \leq d < 0.5)$ then the function will decrease from $f_A(r, d) = 1$ to $f_A(r, d) = 0$ as the radius increases from $r = 0$ to $r = 1$ and similarly to the previous function if $d = 0.5$ then $f_A(r, d) = 1$ and it stays the same at any r value. For the same reason mentioned before in the first function behavior, the output of implementing the first rule R^1 has

been assigned as $g_2(x) = 0$, where d is close to 0, and the output of implementing the second rule R^2 has been assigned as $g_2(x) = 2$, where d is close to 0.5. Hence, if d goes from 0.5 to 0 the function will decrease from $f_A(r, d) = 1$ to $f_A(r, d) = 0$.

After modeling the functions (4.24) & (4.25), the uncertainty degree can be measured clearly as part of the aggregation process in this piecewise function:

$$\tau_{ij} = \begin{cases} f_A(r, d) & ; \quad \{d \mid 0.5 < d \leq 1\} \\ f_B(r, d) & ; \quad \{d \mid 0 \leq d < 0.5\} \\ 1 & ; \quad \{d \mid d = 0.5\} \end{cases} \quad (4.26)$$

where τ_{ij} is the uncertainty degree. The last step is to make κ_{ij} , ς_{ij} and τ_{ij} aggregated to satisfy the conditions of PFN in equation (4.4). This can be determined as follows:

$$\mu_{ij} = \sqrt{\frac{\kappa_{ij}}{\kappa_{ij} + \varsigma_{ij} + \tau_{ij}}}, \quad (4.27)$$

$$\nu_{ij} = \sqrt{\frac{\varsigma_{ij}}{\kappa_{ij} + \varsigma_{ij} + \tau_{ij}}}, \quad (4.28)$$

$$\pi_{ij} = \sqrt{\frac{\tau_{ij}}{\kappa_{ij} + \varsigma_{ij} + \tau_{ij}}}, \quad (4.29)$$

where μ_{ij} and ν_{ij} are PFNs that refer to the Pythagorean Fuzzy satisfactory degree and dissatisfactory degree of i th alternative with respect to j th attribute, respectively, and can be represented in a PFS: $P(\mu_{ij}, \nu_{ij})$.

After fuzzyfying all crisp data into PFNs, the collective evaluation Pythagorean fuzzy decision matrix $R = (C_j(x_i))_{m \times n}$ can be represented as follows:

$$R = (C_j(x_i))_{m \times n} = \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \\ \left[\begin{array}{cccc} P(\mu_{11}, v_{11}) & P(\mu_{12}, v_{12}) & \dots & P(\mu_{1n}, v_{1n}) \\ P(\mu_{21}, v_{21}) & P(\mu_{22}, v_{22}) & \dots & P(\mu_{2n}, v_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ P(\mu_{m1}, v_{m1}) & P(\mu_{m2}, v_{m2}) & \dots & P(\mu_{mn}, v_{mn}) \end{array} \right] \end{array} \quad (\text{Model 4.4})$$

Where each of elements $C_j(x_i) = P(\mu_{ij}, v_{ij})$ is a PFS, which indicates that the degree to which the alternative $x = \{x_i: i = 1, 2, \dots, m\}$ meets the criterion $C = \{C_j: j = 1, 2, \dots, n\}$ is the value μ_{ij} and the degree to which the alternative $x = \{x_i: i = 1, 2, \dots, m\}$ doesn't meet the criterion $C = \{C_j: j = 1, 2, \dots, n\}$ is the value v_{ij} .

4.2.4 Assigning criteria weights

Prior to the implementation of the ranking method, we need to assign criteria weights first. The final criteria weights need to fulfill the condition $\sum_{j=1}^n w_j = 1$. For this reason, the Sugeno λ -fuzzy measures and Shapley values methods mentioned in equation (4.16) and (4.15), respectively, will be applied in this phase for the fair distribution of criteria weights. The setting of the initial weights (fuzzy density values) ($\chi(Q)$) used in the fuzzy measure technique to calculate the marginal contributions for each weight will be through conducting information about each criteria priority and importance in the material selection process. The following linguistic terms can be employed to facilitate the assignment of the initial fuzzy density values for criteria weights:

Table 4. 1 The assignment of initial criteria weights and their Shapley method results:

Importance level	Initial weight (fuzzy density value) $\chi(\{q_j\})$
Very Important (VI)	0.9
Important (I)	0.7
Fair (F)	0.5
Unimportant (U)	0.3
Very Unimportant (VU)	0.1

Where $\chi(\{q_j\})$ is the initial weight assigned for each j th criteria.

4.3 TOPSIS Implementation

This section will illustrate how the TOPSIS method works under the Pythagorean fuzzy environment in MCDM; however, the standard form of traditional TOPSIS needs to be clarified first.

4.3.1 The Traditional TOPSIS Approach in MCDM

The traditional TOPSIS method implementation in MCDM can be represented in the following steps:

1- Normalization of the decision matrix (In model 1) using the following equation :

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^m a_{ij}^2}}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

2- Calculation of the weighted normalized matrix by using the following equation:

$$V_{ij} = a_{ij} \times w_j, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

3- Identification of Positive Ideal (A^*) and Negative Ideal (A^-) Solutions, which can be represented, respectively, as follow :

$$A^* = \left\{ \left(\max_i V_{ij} \mid j \in B \right), \left(\min_i V_{ij} \mid j \in C \right) \right\} = \{V_j^* \mid j = 1, 2, \dots, n\}$$

$$A^- = \left\{ \left(\min_i V_{ij} \mid j \in B \right), \left(\max_i V_{ij} \mid j \in C \right) \right\} = \{V_j^- \mid j = 1, 2, \dots, n\}$$

where B refers to beneficial criteria and C refers to non-beneficial criteria.

4- Calculation of positive separation measures S_i^* and negative separation measures S_i^- of alternatives can be expressed by the Euclidean distance ,respectively, as follows:

$$S_i^* = \sqrt{\sum_{j=1}^n (V_{ij} - V_j^*)^2}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

$$S_i^- = \sqrt{\sum_{j=1}^n (V_{ij} - V_j^-)^2}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

The calculations of both separation measures indicate the deviation of each alternative from its positive and negative ideal solutions, respectively.

5- Calculation of the relative closeness to the ideal solution can be defined as:

$$C_i = \frac{S_i^-}{S_i^- + S_i^*}, \quad i = 1, 2, \dots, m \quad 0 \leq C_i \leq 1$$

This step can define how near are an alternative's attributes to the best hypothetical solution.

- 6- Ranking the alternatives based on relative closeness scores in descending order (from highest to lowest), where the highest score is the best alternative and the lowest score is the worst one.

4.3.2 The Pythagorean fuzzy TOPSIS Approach

After describing the traditional TOPSIS in the previous section, a suggested approach that combines the TOPSIS form and Pythagorean fuzzy sets will be illustrated. This approach has been developed by Zhang & Xu to effectively address MCDM problems with PFNs (Zhang & Xu, 2014). Also, the illustrated method is built on the principle that the optimal alternative should have the shortest distance from the PIS and the farthest distance from the NIS.

Thus, this approach begins by setting of the Pythagorean fuzzy PIS and the Pythagorean fuzzy NIS (Zhang & Xu, 2014). The score function in eq. (4.5) based comparison technique presented in Definition 4.5 is used to find the Pythagorean fuzzy PIS and the Pythagorean fuzzy NIS, taking in consideration that the decision data take the form of PFNs. The Pythagorean fuzzy PIS is denoted by x^+ , which can be expressed by the following formula:

$$x^+ = \left\{ \left(C_j, \max_i \langle s(C_j(x_i)) \rangle \mid j \in B \right), \left(C_j, \min_i \langle s(C_j(x_i)) \rangle \mid j \in B \right) \mid j = 1, 2, \dots, n \right\}$$

$$= \{ \langle C_1, P(u_1^+, v_1^+) \rangle, \langle C_2, P(u_2^+, v_2^+) \rangle, \dots, \langle C_n, P(u_n^+, v_n^+) \rangle \} \quad (4.30)$$

Where B refers to beneficial criteria and C refers to non-beneficial criteria.

The Pythagorean fuzzy PIS normally does not exist in the real-world MCDM procedure. In other words, the Pythagorean fuzzy PIS x^+ typical is not the feasible alternative; that is, $x^+ \notin X$. Moreover, the optimal alternative of the MCDM problem is the Pythagorean fuzzy PIS x^+ .

Next, the distance between each alternative and the Pythagorean fuzzy PIS needs to be measured.

For this reason, the concept of measuring the distance for PFNs should be defined.

DEFINITION 4.7. Let $\beta_j = P(\mu_{\beta_j}, \nu_{\beta_j})$ ($j = 1, 2$) be two PFNs, then the distance between β_1 and β_2 can be defined as follows:

$$d(\beta_1, \beta_2) = \frac{1}{2} (|(\mu_{\beta_1})^2 - (\mu_{\beta_2})^2| + |(v_{\beta_1})^2 - (v_{\beta_2})^2| + |(\pi_{\beta_1})^2 - (\pi_{\beta_2})^2|) \quad (4.31)$$

THEOREM 4.2. Let $\beta_j = P(\mu_{\beta_j}, \nu_{\beta_j})$ ($j = 1, 2$) be two PFNs, then $0 \leq d(\beta_1, \beta_2) \leq 1$.

THEOREM 4.3. Let $\beta_j = P(\mu_{\beta_j}, \nu_{\beta_j})$ ($j = 1, 2$) be two PFNs, then $d(\beta_1, \beta_2) = 0$, if and only if $\beta_1 = \beta_2$.

THEOREM 4.4. Let $\beta_j = P(\mu_{\beta_j}, \nu_{\beta_j})$ ($j = 1, 2$) be two PFNs, then $d(\beta_1, \beta_2) = d(\beta_2, \beta_1)$

THEOREM 4.5. Let $\beta_j = P(\mu_{\beta_j}, \nu_{\beta_j})$ ($j = 1, 2, 3$) be three PFNs, if $\beta_1 \leq \beta_2 \leq \beta_3$, then $d(\beta_1, \beta_2) \leq d(\beta_1, \beta_3)$ and $d(\beta_2, \beta_3) \leq d(\beta_1, \beta_3)$.

Thus, the distance between the alternative x_i and the Pythagorean fuzzy PIS x^+ can be calculated by using Equation (4.14) as follows:

$$\begin{aligned} D(x_i, x^+) &= \sum_{j=1}^n w_j d(C_j(x_i), C_j(x^+)) \\ &= \frac{1}{2} \sum_{j=1}^n w_j (|(\mu_{ij})^2 - (\mu_j^+)^2| + |(v_{ij})^2 - (v_j^+)^2| + |(\pi_{ij})^2 - (\pi_j^+)^2|), \\ & \quad i = 1, 2, \dots, n \end{aligned} \quad (4.32)$$

Usually, the smaller $D(x_i, x^+)$ the better the alternative x_i , which means that the optimal alternative should have the shortest distance from the PIS and let

$$D_{min}(x_i, x^+) = \min_{1 \leq i \leq m} D(x_i, x^+) \quad (4.33)$$

It is also important to mention that the alternative with the closest distance to the Pythagorean fuzzy PIS may not necessarily be the farthest from the Pythagorean fuzzy NIS. The Pythagorean fuzzy NIS is denoted by x^- , which can be defined by the following formula:

$$\begin{aligned} x^- &= \left\{ (C_j, \min_i \langle s(C_j(x_i)) \rangle \mid j \in B), (C_j, \max_i \langle s(C_j(x_i)) \rangle \mid j \in B) \mid j = 1, 2, \dots, n \right\} \\ &= \{ \langle C_1, P(u_1^-, v_1^-) \rangle, \langle C_2, P(u_2^-, v_2^-) \rangle, \dots, \langle C_n, P(u_n^-, v_n^-) \rangle \} \end{aligned} \quad (4.34)$$

where B refers to beneficial criteria and C refers to non-beneficial criteria.

It is definitely observed from Equation (4.17) that the achieved value of the Pythagorean fuzzy NIS under each criterion is less between all the alternatives. Normally, the Pythagorean fuzzy NIS does not exist in the realistic MCDM process. In other words, the Pythagorean fuzzy PIS x^- is typically an unfeasible alternative, that is, $x^- \notin X$. In addition, the Pythagorean fuzzy NIS x^- is the worst alternative of the decision making approach, which should be removed in the process. The distance between the alternative x_i and the Pythagorean fuzzy NIS x^- can be found through applying Equation (4.35), as follows:

$$\begin{aligned} D(x_i, x^-) &= \sum_{j=1}^n w_j d(C_j(x_i), C_j(x^-)) \\ &= \frac{1}{2} \sum_{j=1}^n w_j \left(\left| (\mu_{ij})^2 - (\mu_j^-)^2 \right| + \left| (v_{ij})^2 - (v_j^-)^2 \right| + \left| (\pi_{ij})^2 - (\pi_j^-)^2 \right| \right), \end{aligned}$$

$$i = 1, 2, \dots, n \quad (4.35)$$

Usually, the bigger $D(x_i, x^+)$ the better the alternative x_i , which means that the optimal alternative should have the farthest distance from the NIS and let

$$D_{max}(x_i, x^-) = \max_{1 \leq i \leq m} D(x_i, x^-) \quad (4.36)$$

In order to rank the alternatives, determining the extent to which the alternative x_i is close to the Pythagorean fuzzy PIS x^+ and is far from the Pythagorean fuzzy NIS x^- , concurrently, should be done first. The relative closeness has been used usually in the traditional TOPSIS to perform this step. However, the relative closeness in some cases has failed to fulfill the objective that the optimal solution should have the shortest distance from the PIS and the farthest distance from the NIS, simultaneously. For this reason, the revised closeness has been suggested by Hadi-Vencheh & Mirjaberi (2014) to overcome this issue and measure the optimal alternative; it can be represented as follows:

$$\zeta(x_i) = \frac{D(x_i, x^-)}{D_{max}(x_i, x^-)} - \frac{D(x_i, x^+)}{D_{min}(x_i, x^+)} \quad (4.37)$$

It can be simply noticed that $\zeta(x_i) \leq 0 (i = 1, 2, \dots, m)$ and the bigger $\zeta(x_i)$, the better the alternative x_i . An alternative x^* will be the best alternative if it meets the conditions that $D(x^*, x^-) = D_{max}(x_i, x^-)$ and $D(x^*, x^+) = D_{min}(x_i, x^+)$, simultaneously, then $\zeta(x^*) = 0$. As well, it will obviously be the closest alternative to the Pythagorean fuzzy PIS x^+ and farthest one from the Pythagorean fuzzy NIS x^- , concurrently.

4.4 The Algorithm of the Proposed Fuzzy Pythagorean MCGDM TOPSIS Method

In this section, a practical algorithm of the Pythagorean fuzzy TOPSIS approach is demonstrated based on the previous methodology explanation. The algorithm can be applied by following the next steps:

Step 1. Establish group decision matrix $X_i = (r_{kj}^i)_{t \times n}$ for each i th alternative in which each decision maker $D = \{d_k: k = 1, 2, \dots, t\}$ evaluates the importance of each given criterion $C = \{C_j: j = 1, 2, \dots, n\}$ with respect to each alternative $x = \{x_i: i = 1, 2, \dots, m\}$, as shown in Model (4.1).

Step 2. Normalize each group decision matrix $X_i = (r_{kj}^i)_{t \times n}$ into $R_i = (s_{kj}^i)_{t \times n}$ for each alternative by using equations (4.9) & (4.10).

Step 3. Do a linear transformation by applying equations (4.13) & (4.14), respectively, for each criterion vector in R_i .

Step 4. Determine the Shapley value (weight) of each DM by applying equations (4.16) & (4.15), respectively.

Step 5. Measure the weighted averaging satisfactory and dissatisfactory degrees for each j th attribute in i th alternative with respect to all DMs weights in R_i using equations (4.17) & (4.18), respectively.

Step 6. Calculate the uncertainty degree using equations (4.19), (4.20), (4.21), (4.22) & (4.26), respectively, for each criterion vector in R_i .

Step 7. Perform the calculations in equations (4.27), (4.28), & (4.29), respectively, to obtain the final PFNs.

Step 8. Form all the resulted PFNs from crisp data in $R_i = (s_{kj}^i)_{m \times n}$ to a collective evaluation Pythagorean fuzzy decision matrix $R = (C_j(x_i))_{m \times n} = (P(\mu_{ij}, \nu_{ij}))_{m \times n}$, in which the decision information is expressed in the aggregated PFSs $= P(\mu_{ij}, \nu_{ij})$, as shown in model (4.4).

Step 9. Determine the Shapley value of each criterion weight using equations (4.16) & (4.15), respectively.

Step 10. Use the score function in equation (4.5) to identify the Pythagorean fuzzy

PIS $x^+ = \{C_1(x^+), C_2(x^+), \dots, C_n(x^+)\}$ and the Pythagorean fuzzy NIS $x^- = \{C_1(x^-), C_2(x^-), \dots, C_n(x^-)\}$, respectively.

Step 11. By using the assigned weight for each criterion $w = (w_{C_1}, w_{C_2}, \dots, w_{C_j})$, $\sum_{j=1}^n w_{C_j} = 1$ where $j = (1, 2, \dots, n)$ $0 \leq w_{C_j} \leq 1$, implement Equations (4.32) and (4.35) to measure the distances between the alternative x_i and the Pythagorean fuzzy PIS x^+ as well as the Pythagorean fuzzy NIS x^- , respectively.

Step 12. Apply Equation (4.37) to determine the revised closeness $\zeta(x_i)$ of the alternative x_i ($i = 1, 2, \dots, m$)

Step 13. Define the optimal ranking order of the alternatives and find the optimal alternative. Based on the revised closeness $\zeta(x_i)$ achieved from Step 10, the alternatives will be ranked into order with respect to the declining values of $\zeta(x_i)$ ($i = 1, 2, \dots, m$) and the alternative with the highest revised closeness $\zeta(x_i)$ is the optimal one, namely,

$$x^* := \{x_i : \left(i : \zeta(x_i) = \max_{1 \leq i \leq m} \zeta(x_i) \right)\}$$

4.5 COPRAS Implementation

The following section will demonstrate the COPRAS method in the Pythagorean Fuzzy environment for MCDM problems. First, the standard form of the normal COPRAS is going to be clarified; then the Pythagorean Fuzzy COPRAS method will be explained.

4.5.1 The Traditional COPRAS Approach in MCDM

The traditional COPRAS method application for MCDM problems can be represented in the following steps:

- 1- Normalize the decision matrix (In model 1) using the following equation :

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^m a_{ij}} , \quad i = 1,2, \dots, m; j = 1,2, \dots, n$$

- 2- Calculate the weighted normalized matrix by using the following equation:

$$V_{ij} = a_{ij} \times w_j , \quad i = 1,2, \dots, m; j = 1,2, \dots, n$$

- 3- Measure the sums of weighted normalized elements for beneficial and non-beneficial criteria as follows:

$$S_{+i} = \sum_{j=1}^n V_{+ij} , \quad i = 1,2, \dots, m; j = 1,2, \dots, n$$

$$S_{-i} = \sum_{j=1}^n V_{-ij} , \quad i = 1,2, \dots, m; j = 1,2, \dots, n$$

where V_{+ij} is associated with maximizing criteria and V_{-ij} is associated with minimizing criteria, respectively. Thus, the bigger S_{+i} value, the higher is the alternative i as well as the

lower S_{+i} value, the higher is the alternative i . In fact, S_{+i} and S_{-i} show the level of objectives reached by alternative i .

- 4- Obtain the minimum value of S_{-i} which can be referred as S_{-min} .
- 5- Determine the relative weight of each alternative as follow :

$$Q_i = S_{+i} + \frac{S_{-min} \sum_{i=1}^m S_{-i}}{S_{-i} \sum_{i=1}^m \frac{S_{-min}}{S_{-i}}} = S_{+i} + \frac{\sum_{i=1}^m S_{-i}}{S_{-i} \sum_{i=1}^m \frac{1}{S_{-i}}}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

- 6- Determine the priority of alternatives by calculating the utility degree U_i of each alternative as follows:

$$U_i = \frac{Q_i}{Q_{max}} \times 100$$

- 7- Rank the alternatives based on their priorities. The larger the value of U_i , the more preference of the alternative i .

4.5.2 The Pythagorean fuzzy COPRAS Approach

After explaining the traditional COPRAS in the previous section, a suggested approach defining the COPRAS method within the Pythagorean fuzzy environment will be illustrated. This approach has been proposed by Peng & Selvachandran (2017) to effectively address MCDM problems with PFNs (Peng & Selvachandran, 2017). Also, the illustrated method is developed on the concept that the optimal alternative should have the highest utility level.

Initially, we need to set the Pythagorean fuzzy decision matrix $R = P(\mu_{ij}, \nu_{ij})_{m \times n}$ according to the following equation:

$$R' = P(\mu'_{ij}, \nu'_{ij})_{m \times n} = \begin{cases} P(\mu_{ij}, \nu_{ij}) & \text{is beneficial element} \\ P(\nu_{ij}, \mu_{ij}) & \text{is non - beneficial element} \end{cases} \quad (4.38)$$

Then, we need to measure the weighted Pythagorean fuzzy decision matrix through multiplying criteria C_j weights to the collective decision making matrix in Model (4.4). The new weighted Pythagorean fuzzy decision matrix $S = (s_{ij})_{m \times n}$ can be calculated using the following equation:

$$s_{ij} = w_j R' = \left(\sum_{i=1}^m w_j \mu'_{ij}, \sum_{i=1}^m w_j \nu'_{ij} \right) \quad (4.39)$$

After that, the summation of beneficial criteria values will be calculated for each alternative. Let BA refers to beneficial criteria, the higher values of which are better. As well, the summation of beneficial criteria values will be calculated for each alternative. Let CA refer to non-beneficial criteria, the lower values of which are better. The equations to calculate the sum of beneficial criteria and non-beneficial criteria for each alternative can be written, respectively, as follows:

$$P_{+i} = \bigoplus_{j \in BA} s_{ij} \quad (4.40)$$

$$P_{-i} = \bigoplus_{j \in CA} s_{ij} \quad (4.41)$$

where P_{+i} and P_{-i} represent the beneficial criteria values and non-beneficial criteria values, respectively. In fact, $CA \cup BA$ is all the criteria.

The next step is to measure the score of P_{+i} , P_{-i} and P_{-min} using the score function in equation (4.4). It is important to note that P_{-min} refers to alternative i with the lowest value that belongs to a non-beneficial criterion. The calculated score functions can be referred to as $s(P_{+i})$, $s(P_{-i})$ and $s(P_{-min})$. Also, $s(P_{-min})$ can be written as $\min_i s(P_{-i})$. Now, we can calculate the relative weight of each alternative. Thus, the relative priority $Q(x_i)$ for each alternative can be determined by the following formula:

$$Q(x_i) = s(P_{+i}) + \frac{s(P_{-min}) \sum_{i=1}^m e^{s(P_{-i})}}{e^{s(P_{-i})} \sum_{i=1}^m \frac{s(P_{-min})}{e^{s(P_{-i})}}} = s(P_{+i}) + \frac{\sum_{i=1}^m e^{s(P_{-i})}}{e^{s(P_{-i})} \sum_{i=1}^m \frac{1}{e^{s(P_{-i})}}}$$

, $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ (4.42)

After that, we obtain the maximum relative priority $Q_{max}(x_i)$ to calculate the level of utility $U(x_i)$ for each alternative i as follows:

$$U(x_i) = \frac{Q(x_i)}{Q_{max}(x_i)} \times 100$$

(4.43)

Finally, the alternatives can be ranked based on the level of utility $U(x_i)$ for each one: the larger the value of $U(x_i)$, the more preference of the alternative i . Accordingly, the best alternative is going to have the highest utility degree $U_{max}(x_i)$.

4.6 The Algorithm of the Proposed Fuzzy Pythagorean MCGDM COPRAS Method

In this section, a practical algorithm of the Pythagorean fuzzy COPRAS approach is demonstrated based on the previous methodology explanation. The algorithm can be applied by following the next steps:

Step 1. Follow step1-step 9 in the proposed MCGDM Fuzzy Pythagorean COPRAS algorithm in section (4.4).

Step 2. Set collective evaluation Pythagorean fuzzy decision matrix $R = P(\mu_{ij}, \nu_{ij})_{m \times n}$ into $R' = (\mu'_{ij}, \nu'_{ij})_{m \times n}$ as in equation (4.38).

Step 3. Determine the weighted Pythagorean fuzzy decision matrix $S = (s_{ij})_{m \times n}$ using equation (39).

Step 4. Calculate the sum of beneficial criteria P_{+i} and non-beneficial criteria P_{-i} for each alternative by implementing equations (4.40) and (4.41), receptively.

Step 5. Use the score function in equation (4.5) to measure the score of P_{+i} , P_{-i} and P_{-min} . The calculated score functions will be represented as $s(P_{+i})$, $s(P_{-i})$ and $s(P_{-min})$, respectively.

Step 6. Measure the relative priority $Q(x_i)$ for each alternative as well as determine $Q_{max}(x_i)$ using equation (4.42).

Step 7. Calculate the level of utility $U(x_i)$ for each alternative i by applying equation (4.43).

Step 8. Define the optimal ranking order of the alternatives and find the optimal alternative.

Based on the utility degree $U(x_i)$ achieved from Step 7, the alternatives will be ranked into order

with respect to the declining values of $U(x_i)$ ($i = 1, 2, \dots, m$) and the alternative with the highest utility degree $U(x_i)$ is the optimal one, namely,

$$x^* := \{x_i : (i: U(x_i) = \max_{1 \leq i \leq m} U(x_i))\}$$

CHAPTER FIVE

Validation of the Proposed MCGDM Methods in Material Selection Applications

In this section, a real life example in the material selection field will be examined and solved using the proposed hybrid PFMCGDM frameworks. Results of both methods will be discussed and compared. Furthermore, to test the validation of the model, a sensitivity analysis will be carried by adjusting some critical input parameters such as criteria and DMs weights.

5.1 Application of the methods in a Real Life Material Selection Case

The following scenario has been inspired by real problems in the aerospace industry. It was selected to achieve the purpose of this research in studying the suggested method through implementing it in a suitable real life example in the material selection field. The main objectives of the case study were: 1) investigating the new developed aggregation approach in converting real DMs crisp decisions into Pythagorean fuzzy numbers; 2) testing the applicability of both the hybrid PFMCGDM TOPSIS and the hybrid PFMCGDM COPRAS approaches in the field of material selection.

5.1.1 Case study: Material selection for high pressure turbine blades in jet engines

The design of an aircraft engine is composed of three major sections: the compressor, the combustion chamber and the gas turbine section. The efficiency of the turbine blades in turbojet engines can be harmfully affected in the long term by the high temperature gas that comes from the combustion chamber. Other problems that can damage the turbine blades performance are related to corrosion and fatigue cracking. In 2017, Trent 1000 high pressure turbine blades, which are manufactured by Rolls-Royce, have been reported to encounter some major issues regarding corrosion that affect turbofan engines in Boeing 787 jets (Johnson & Raina, 2018).

Due to these blade problems, many airlines had remove the Boeing 787 from service and Rolls-Royce had to pay approximately 35 M \$ for blade inspections and replacements. In 2018, the estimation of the total cost of fixing the blades was about 504 M \$ (Kollewe, 2018). To avoid such problem, materials should be designed and selected to properly suit the high pressure turbine blade environment. It is important to note that the high pressure turbine in modern jet engines operates at a temperature degree range of 1350–1450°C or higher. As well, lighter weight materials will be more desirable since the aerospace industry tends to favor materials that save weight (Boyer, Cotton, Mohaghegh, & Schafrik, 2015). Accordingly, in order to select an efficient material for the turbine blade, the material selection process requirements and criteria will be shown in (Table 4.1) as follows:

Table 5. 1. Material criteria descriptions and their level of importance

Criteria	Material requirements	Material indices	Importance
C1	It should be high-resistant to high temperature environment	Max service temperature	Very important (VI)
C2	It should be low in weight	Low density	Very Important (VI)
C3	It should be high-resistant to tensile stresses	High yield strength	Important (I)
C4	It should be high-resistant to bending stresses	High young's modulus	Important (I)
C5	It should be economic in its price	Low cost	Unimportant (U)

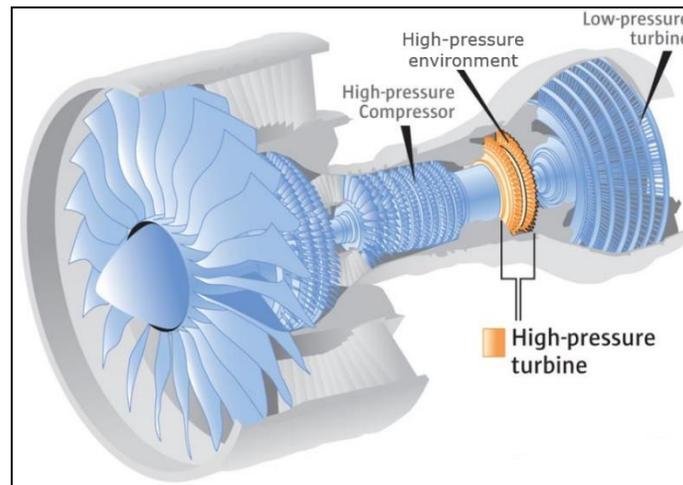


Figure 5. 1 High pressure turbine in the turbojet engine. From the seattletimes. Retrieved from <https://www.seattletimes.com/business/jet-engine-explosion-triggered-chaotic-2015-las-vegas-fire-ntsb-report-says/>. Copyright 2018 by General Electric & (Mark Nowlin/ theseattletimes)



Figure 5. 2 Rolls Royce jet engine on Boeing 787. From Rolls Royce for aircraft. Retrieved from <http://aircraft.sewaro.us/rolls-royce-engine-for-aircraft/>.



Figure 5. 3. Crews work on the engine of an Air New Zealand Boeing 787-9 Dreamliner, July 9, 2014 in Everett, Washington. From International Business Times. Retrieved from <https://www.ibtimes.co.uk/rolls-royce-engine-trouble-forces-air-new-zealand-ground-flights-1650546>. Photo by Stephen Brashear

Based on the requirements provided in the previous table, our material candidates will be between titanium- and nickel-based Super alloys and stainless steels since these materials are capable of withstanding the harsh high pressure and temperature conditions. The material candidates are listed in the following table 5.2:

Table 5. 2. Material candidates and their commercial names

Symbol	Material alternative	Commercial name
X ₁	Nickel-based Super alloy	Inconel 625
X ₂	Nickel-based Super alloy	Pyromet 680
X ₃	Nickel-based Super alloy	Haynes R-41
X ₄	Nickel-Chromium Super alloy	Inconel 706
X ₅	Nickel-Co-Cr Super alloy	Udimet 700
X ₆	Titanium alpha beta Super alloy	Ti-6Al-2Sn-2Zr-2Mo (Ti-6-6-6-6)
X ₇	Austenitic Stainless steel	AISI 202, Wrought
X ₈	Nickel-Cr-Co-Mo Super alloy	Rene 41
X ₉	Austenitic Stainless steel	AISI 302, Wrought
X ₁₀	Nickel based Super alloy	Rene 80

5.1.1.1 Implementing the proposed hybrid MCGDM methods

5.1.1.1.1 Aggregation phase

Initially, for the purpose of this study, we have assigned a total of five participants between graduate students and faculty members from the Material Science Department to participate as decision makers. Then, we provide each DM (participant) with the needed information about material indices and performances for each material candidate for the process of selecting the best material that suits the high pressure turbine environment and requirements along with an evaluation paper using the document in Appendix A. Based on his/her perspective, knowledge and experience level, every DM = $\{d_k : k = 1,2,3,4,5\}$ will rate each criterion C_j with respect to the material alternative X_i as explained in Step 1 through completing the evaluation document. The rating system will be performed based on grades from 0 to 100, with 0

being worst and 100 being best. After collecting the evaluation papers from the DMs, the data will be gathered and represented in the following tables:

Table 5. 3. Rating of DMs to all criteria with respect to material alternative Inconel 625 (X1)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	80	10	40	60	95
d ₂	80	12	35	60	90
d ₃	79	15	46	66	92
d ₄	82	5	32	70	80
d ₅	80	10	25	68	85

Table 5. 4. Rating of DMs to all criteria with respect to material alternative Pyromet 680 (X₂)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	15	18	20	62	75
d ₂	10	12	25	60	70
d ₃	5	14	10	70	72
d ₄	12	10	20	70	70
d ₅	8	5	18	68	68

Table 5. 5. Rating of DMs to all criteria with respect to material alternative Haynes R-41 (X₃)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	81	17	75	65	90
d ₂	80	12	70	67	82
d ₃	78	15	75	70	90
d ₄	80	10	68	75	80
d ₅	79	5	70	70	85

Table 5. 6. Rating of DMs to all criteria with respect to material alternative Inconel 706 (X₄)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	65	20	95	100	85
d ₂	55	15	95	95	75
d ₃	70	20	92	92	88
d ₄	65	18	90	90	73
d ₅	70	20	92	95	85

Table 5. 7. Rating of DMs to all criteria with respect to material alternative Udimet 700 (X₅)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	55	25	100	85	60
d ₂	50	20	100	80	70
d ₃	60	18	95	84	65
d ₄	40	20	97	79	35
d ₅	61	10	95	90	40

Table 5. 8. Rating of DMs to all criteria with respect to material alternative Ti-6-6-6-6 (X₆)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	70	100	95	10	40
d ₂	70	100	96	15	50
d ₃	75	90	100	20	45
d ₄	60	95	90	22	35
d ₅	81	100	95	8	25

Table 5. 9. Rating of DMs to all criteria with respect to material alternative AISI 202 (X₇)

Material indices (criteria)

Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	70	40	45	65	60
d ₂	73	30	35	72	70
d ₃	75	25	55	68	65
d ₄	65	20	33	75	30
d ₅	80	15	57	69	35

Table 5. 10. Rating of DMs to all criteria with respect to material alternative Rene 41 (X₈)

Material indices (criteria)

Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	93	20	83	100	10
d ₂	95	16	80	95	5
d ₃	90	12	82	92	13
d ₄	95	10	83	90	15
d ₅	90	10	75	95	18

Table 5. 11. Rating of DMs to all criteria with respect to material alternative AISI 302 (X₉)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	50	35	40	62	30
d ₂	45	25	30	60	15
d ₃	60	25	55	71	15
d ₄	45	18	30	70	18
d ₅	48	15	55	65	16

Table 5. 12. Rating of DMs to all criteria with respect to material alternative Rene 80 (X₁₀)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	100	40	90	95	10
d ₂	100	28	95	95	5
d ₃	95	25	90	88	15
d ₄	95	22	95	92	15
d ₅	95	15	85	90	12

We have three beneficial criteria (max service temperature, yield strength, and Young's modulus), which means the larger the points the better the criterion rate, and two non-beneficial criteria (density and cost), which means the lower the points the better the criterion rate.

Following step 2, all the elements in the previous evaluation tables will be normalized.

Therefore, normalization values can be displayed in the next tables as follows:

Table 5. 13. Normalized values of DMs evaluations for material alternative Inconel 625 (X₁)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.80	0.10	0.40	0.60	0.95
d ₂	0.80	0.12	0.35	0.60	0.90
d ₃	0.79	0.15	0.46	0.66	0.92
d ₄	0.82	0.5	0.32	0.70	0.80
d ₅	0.80	0.10	0.25	0.68	0.85

Table 5. 14. Normalized values of DMs evaluations for material alternative Pyromet 680 (X₂)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.15	0.18	0.20	0.62	0.75
d ₂	0.10	0.12	0.25	0.60	0.70
d ₃	0.05	0.14	0.10	0.70	0.72
d ₄	0.12	0.10	0.20	0.70	0.70
d ₅	0.08	0.05	0.18	0.68	0.68

Table 5. 15. Normalized values of DMs evaluations for material alternative Haynes R-41 (X₃)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.81	0.17	0.75	0.65	0.90
d ₂	0.80	0.12	0.70	0.67	0.82
d ₃	0.78	0.15	0.75	0.70	0.90
d ₄	0.80	0.10	0.68	0.75	0.80
d ₅	0.79	0.05	0.70	0.70	0.85

Table 5. 16. Normalized values of DMs evaluations for material alternative Inconel 706 (X₄)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.65	0.20	0.95	1.00	0.85
d ₂	0.55	0.15	0.95	0.95	0.75
d ₃	0.70	0.20	0.92	0.92	0.88
d ₄	0.65	0.18	0.90	0.90	0.73
d ₅	0.70	0.20	0.92	0.95	0.85

Table 5. 17. Normalized values of DMs evaluations for material alternative Udimet 700 (X₅)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.55	0.25	1.00	0.85	0.60
d ₂	0.50	0.20	1.00	0.80	0.70
d ₃	0.60	0.18	0.95	0.84	0.65
d ₄	0.40	0.20	0.97	0.79	0.35
d ₅	0.61	0.10	0.95	0.90	0.40

Table 5. 18. Normalized values of DMs evaluations for material alternative Ti-6-6-6-6 (X₆)

Material indices (criteria)					
Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.70	1.00	0.95	0.10	0.40
d ₂	0.70	1.00	0.96	0.15	0.50
d ₃	0.75	0.90	1.00	0.20	0.45
d ₄	0.60	0.95	0.90	0.22	0.35
d ₅	0.81	1.00	0.95	0.08	0.25

Table 5. 19. Normalized values of DMs evaluations for material alternative AISI 202 (X₇)

Material indices (criteria)

Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.70	0.40	0.45	0.65	0.60
d ₂	0.73	0.30	0.35	0.72	0.70
d ₃	0.75	0.25	0.55	0.68	0.65
d ₄	0.65	0.20	0.33	0.75	0.30
d ₅	0.80	0.15	0.57	0.69	0.35

Table 5. 20. Normalized values of DMs evaluations for material alternative Rene 41 (X₈)

Material indices (criteria)

Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.93	0.20	0.83	1.00	0.10
d ₂	0.95	0.16	0.80	0.95	0.05
d ₃	0.90	0.12	0.82	0.92	0.13
d ₄	0.95	0.10	0.83	0.90	0.15
d ₅	0.90	0.10	0.75	0.95	0.18

Table 5. 21. Normalized values of DMs evaluations for material alternative AISI 302 (X₉)

Material indices (criteria)

Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	0.50	0.35	0.40	0.62	0.30
d ₂	0.45	0.25	0.30	0.60	0.15
d ₃	0.60	0.25	0.55	0.71	0.15
d ₄	0.45	0.18	0.30	0.70	0.18
d ₅	0.48	0.15	0.55	0.65	0.16

Table 5. 22. Normalized values of DMs evaluations for material alternative Rene 80 (X₁₀)

Material indices (criteria)

Material alternative	Max service temperature (C ₁)	Density (C ₂)	Yield strength (C ₃)	Young's modulus (C ₄)	Cost (C ₅)
d ₁	1.00	0.40	0.90	0.95	0.10
d ₂	1.00	0.28	0.95	0.95	0.05
d ₃	0.95	0.25	0.90	0.88	0.15
d ₄	0.95	0.22	0.95	0.92	0.15
d ₅	0.95	0.15	0.85	0.90	0.12

Following step 3, we are going to linearly transform all the normalized values for each criterion vector in every material alternative expressed in each previous table in which $s_{kj}^i > 0.5$ and $s_{kj}^i < 0.5$ will be transformed to satisfactory o_{kj} and dissatisfactory ξ_{kj} elements, respectively. After that, we need to set the initial weight for each DM in order to proceed in measuring the marginal contributions using the Sugeno fuzzy measure. Next we need to determine the importance of each DM based on their contributions to the whole decision making process by implementing the Shapely value method as mentioned in step 4.

As aforementioned, the initial weight, which is the fuzzy density value $\mu(\{d_k\})$, can be assigned by conducting surveys or interviews with DMs. As well, many factors should be considered such as educational background and experience level. For the sake of this study, we are going to assign the weights based on our participants' educational background and research experience following the setting demonstrated in the next table:

Table 5. 23. The assignment of initial weight based on level of educational and research experience

Level of experience (years)	Initial weight (fuzzy density value) $\mu(\{d_k\})$
12+	0.8
10	0.6
8	0.4
6	0.2

The participants' (DMs) group is composed of one faculty member (12+ years), two PhD students (8+ years) and two master's students (6+ years). Thus, according to (Table 5.24) the DMs weights will be assigned as $\mu(\{d_1\}) = 0.8$, $\mu(\{d_2\}) = 0.4$, $\mu(\{d_3\}) = 0.4$, $\mu(\{d_4\}) = 0.2$, and $\mu(\{d_5\}) = 0.2$, respectively. Now we can use the fuzzy measure to find the value of the parameter λ and, then determine the marginal contribution of each DM to every coalition as shown in the following table:

Table 5. 24. λ -Sugeno Fuzzy measure coalition results of $\{d_1, d_2, d_3, d_4, d_5\}$

$\lambda = -0.93459 < -1$									
$\mu(\{\emptyset\}) = 0,$		$\mu(\{d_1\}) = 0.8,$		$\mu(\{d_2\}) = 0.4,$		$\mu(\{d_4\}) = 0.2,$		$\mu(\{d_5\}) = 0.2$	
$\mu(F \cup \{d_1\})$	$\mu(F \cup \{d_2\})$	$\mu(F \cup \{d_3\})$	$\mu(F \cup \{d_4\})$	$\mu(F \cup \{d_5\})$					
$\mu(\{\emptyset, \{d_1\}) = 0.8$	$\mu(\{\emptyset, \{d_2\}) = 0.4$	$\mu(\{\emptyset, \{d_3\}) = 0.4$	$\mu(\{\emptyset, \{d_4\}) = 0.2$	$\mu(\{\emptyset, \{d_5\}) = 0.2$					
$\mu(\{d_1, \{d_2\}) = 0.90092$	$\mu(\{d_1, \{d_2\}) = 0.90092$	$\mu(\{d_1, \{d_3\}) = 0.90092$	$\mu(\{d_1, \{d_4\}) = 0.85046$	$\mu(\{d_1, \{d_5\}) = 0.85046$					
$\mu(\{d_1, \{d_3\}) = 0.90092$	$\mu(\{d_2, \{d_3\}) = 0.65046$	$\mu(\{d_2, \{d_3\}) = 0.65046$	$\mu(\{d_2, \{d_4\}) = 0.52523$	$\mu(\{d_2, \{d_5\}) = 0.52523$					
$\mu(\{d_1, \{d_4\}) = 0.85046$	$\mu(\{d_2, \{d_4\}) = 0.52523$	$\mu(\{d_3, \{d_4\}) = 0.52523$	$\mu(\{d_3, \{d_4\}) = 0.52523$	$\mu(\{d_3, \{d_5\}) = 0.52523$					
$\mu(\{d_1, \{d_5\}) = 0.85046$	$\mu(\{d_2, \{d_5\}) = 0.52523$	$\mu(\{d_3, \{d_5\}) = 0.52523$	$\mu(\{d_4, \{d_5\}) = 0.36261$	$\mu(\{d_4, \{d_5\}) = 0.36261$					
$\mu(\{d_1, \{d_2, \{d_3\}) = 0.96412$	$\mu(\{d_1, \{d_2, \{d_3\}) = 0.96412$	$\mu(\{d_1, \{d_2, \{d_3\}) = 0.96412$	$\mu(\{d_1, \{d_2, \{d_4\}) = 0.93252$	$\mu(\{d_1, \{d_2, \{d_5\}) = 0.93252$					
$\mu(\{d_1, \{d_2, \{d_4\}) = 0.93252$	$\mu(\{d_1, \{d_2, \{d_4\}) = 0.93252$	$\mu(\{d_1, \{d_3, \{d_4\}) = 0.93252$	$\mu(\{d_1, \{d_3, \{d_4\}) = 0.93252$	$\mu(\{d_1, \{d_3, \{d_5\}) = 0.93252$					
$\mu(\{d_1, \{d_2, \{d_5\}) = 0.93252$	$\mu(\{d_1, \{d_2, \{d_5\}) = 0.93252$	$\mu(\{d_1, \{d_3, \{d_5\}) = 0.93252$	$\mu(\{d_1, \{d_4, \{d_5\}) = 0.89149$	$\mu(\{d_1, \{d_4, \{d_5\}) = 0.89149$					
$\mu(\{d_1, \{d_3, \{d_4\}) = 0.93252$	$\mu(\{d_2, \{d_3, \{d_4\}) = 0.72887$	$\mu(\{d_2, \{d_3, \{d_4\}) = 0.72887$	$\mu(\{d_2, \{d_3, \{d_4\}) = 0.72887$	$\mu(\{d_2, \{d_3, \{d_5\}) = 0.72887$					
$\mu(\{d_1, \{d_3, \{d_5\}) = 0.93252$	$\mu(\{d_2, \{d_3, \{d_5\}) = 0.72887$	$\mu(\{d_2, \{d_3, \{d_5\}) = 0.72887$	$\mu(\{d_2, \{d_4, \{d_5\}) = 0.62705$	$\mu(\{d_2, \{d_4, \{d_5\}) = 0.62705$					
$\mu(\{d_1, \{d_4, \{d_5\}) = 0.89149$	$\mu(\{d_2, \{d_4, \{d_5\}) = 0.62705$	$\mu(\{d_3, \{d_4, \{d_5\}) = 0.62705$	$\mu(\{d_3, \{d_4, \{d_5\}) = 0.62705$	$\mu(\{d_3, \{d_4, \{d_5\}) = 0.62705$					
$\mu(\{d_1, \{d_2, \{d_3, \{d_4\}) = 0.98391$	$\mu(\{d_1, \{d_2, \{d_3, \{d_4\}) = 0.98391$	$\mu(\{d_1, \{d_2, \{d_3, \{d_4\}) = 0.98391$	$\mu(\{d_1, \{d_2, \{d_3, \{d_4\}) = 0.98391$	$\mu(\{d_1, \{d_2, \{d_3, \{d_5\}) = 0.98391$					
$\mu(\{d_1, \{d_2, \{d_3, \{d_5\}) = 0.98391$	$\mu(\{d_1, \{d_2, \{d_3, \{d_5\}) = 0.98391$	$\mu(\{d_1, \{d_2, \{d_3, \{d_5\}) = 0.98391$	$\mu(\{d_1, \{d_2, \{d_4, \{d_5\}) = 0.95822$	$\mu(\{d_1, \{d_2, \{d_4, \{d_5\}) = 0.95822$					
$\mu(\{d_1, \{d_2, \{d_4, \{d_5\}) = 0.95822$	$\mu(\{d_1, \{d_2, \{d_4, \{d_5\}) = 0.95822$	$\mu(\{d_1, \{d_3, \{d_4, \{d_5\}) = 0.95822$	$\mu(\{d_1, \{d_3, \{d_4, \{d_5\}) = 0.95822$	$\mu(\{d_1, \{d_3, \{d_4, \{d_5\}) = 0.95822$					
$\mu(\{d_1, \{d_3, \{d_4, \{d_5\}) = 0.95822$	$\mu(\{d_2, \{d_3, \{d_4, \{d_5\}) = 0.79263$								
$\mu(\{d_1, \{d_2, \{d_3, \{d_4, \{d_5\}) = 1$	$\mu(\{d_1, \{d_2, \{d_3, \{d_4, \{d_5\}) = 1$	$\mu(\{d_1, \{d_2, \{d_3, \{d_4, \{d_5\}) = 1$	$\mu(\{d_1, \{d_2, \{d_3, \{d_4, \{d_5\}) = 1$	$\mu(\{d_1, \{d_2, \{d_3, \{d_4, \{d_5\}) = 1$					

By using the Shapley values method, the weight of each DM can be calculated as follow:

$$\Phi_1 = \frac{1}{5!} \sum_{F \subseteq D \setminus \{d_k\}} |F|! (5 - |F| - 1)! [\mu(F \cup \{d_1\}) - \mu(F)] = 0.45762$$

$$\Phi_2 = \frac{1}{5!} \sum_{F \subseteq D \setminus \{d_k\}} |F|! (5 - |F| - 1)! [\mu(F \cup \{d_2\}) - \mu(F)] = 0.18517$$

$$\Phi_3 = \frac{1}{5!} \sum_{F \subseteq D \setminus \{d_k\}} |F|! (5 - |F| - 1)! [\mu(F \cup \{d_3\}) - \mu(F)] = 0.18517$$

$$\Phi_4 = \frac{1}{5!} \sum_{F \subseteq D \setminus \{d_k\}} |F|! (5 - |F| - 1)! [\mu(F \cup \{d_4\}) - \mu(F)] = 0.08601$$

$$\Phi_5 = \frac{1}{5!} \sum_{F \subseteq D \setminus \{d_k\}} |F|! (5 - |F| - 1)! [\mu(F \cup \{d_5\}) - \mu(F)] = 0.08601$$

As a consequence, the weighted averaging satisfactory and dissatisfactory degrees of the material alternative X_i regarding the criteria C_j will be determined by following Step 5. Results can be expressed in the following table:

Table 5. 25. Results of the weighted averaging satisfactory and dissatisfactory degrees for Inconel 625 (X_1)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
<i>Inconel 625 (X_1)</i>	C_1	0.59973	0.00000
	C_2	0.00000	0.78627
	C_3	0.00000	0.23586
	C_4	0.25318	0.00000
	C_5	0.82736	0.00000

Table 5. 26. Results of the weighted averaging satisfactory and dissatisfactory degrees for Pyromet 680 (X₂)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
Pyromet 680 (X₂)	C₁	0.00000	0.77275
	C₂	0.00000	0.71316
	C₃	0.00000	0.62195
	C₄	0.28630	0.00000
	C₅	0.44972	0.00000

Table 5. 27. Results of the weighted averaging satisfactory and dissatisfactory degrees for Haynes R-41 (X₃)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
Haynes R-41 (X₃)	C₁	0.59259	0.00000
	C₂	0.00000	0.71861
	C₃	0.46083	0.00000
	C₄	0.35172	0.00000
	C₅	0.73716	0.00000

Table 5. 28. Results of the weighted averaging satisfactory and dissatisfactory degrees for Inconel 706 (X₄)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
<i>Inconel 706 (X₄)</i>	C₁	0.29008	0.00000
	C₂	0.00000	0.62195
	C₃	0.87512	0.00000
	C₄	0.92605	0.00000
	C₅	0.64576	0.00000

Table 5. 29. Results of the weighted averaging satisfactory and dissatisfactory degrees for Udimet 700 (X₅)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
<i>Udimet 700 (X₅)</i>	C₁	0.10000	0.01720
	C₂	0.00000	0.57884
	C₃	0.96772	0.00000
	C₄	0.67063	0.00000
	C₅	0.22114	0.04300

Table 5. 30. Results of the weighted averaging satisfactory and dissatisfactory degrees for Ti-6-6-6-6 (X_6)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
Ti-6-6-6-6 (X_6)	C₁	0.41851	0.00000
	C₂	0.95436	0.00000
	C₃	0.89139	0.00000
	C₄	0.00000	0.72724
	C₅	0.00000	0.17885

Table 5. 31. Results of the weighted averaging satisfactory and dissatisfactory degrees for AISI 202 (X_7)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
AISI 202 (X_7)	C₁	0.43822	0.00000
	C₂	0.00000	0.37000
	C₃	0.02711	0.12711
	C₄	0.35939	0.00000
	C₅	0.22114	0.06021

Table 5. 32. Results of the weighted averaging satisfactory and dissatisfactory degrees for Rene 41 (X_8)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
<i>Rene 41 (X_8)</i>	C_1	0.85457	0.00000
	C_2	0.00000	0.67884
	C_3	0.63142	0.00000
	C_4	0.92605	0.00000
	C_5	0.00000	0.78279

Table 5. 33. Results of the weighted averaging satisfactory and dissatisfactory degrees for AISI 302 (X_9)

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
<i>AISI 302 (X_9)</i>	C_1	0.03703	0.03055
	C_2	0.00000	0.43772
	C_3	0.02711	0.20000
	C_4	0.28484	0.00000
	C_5	0.00000	0.55411

Table 5. 34. Results of the weighted averaging satisfactory and dissatisfactory degrees for Rene 80 (X_{10})

<i>Material alternative</i>	<i>Material Criteria</i>	κ_{ij}	ζ_{ij}
<i>Rene 80 (X_{10})</i>	C_1	0.96427	0.00000
	C_2	0.00000	0.37396
	C_3	0.80000	0.00000
	C_4	0.86427	0.00000
	C_5	0.00000	0.78279

The computation of the uncertainty degree (τ_{ij}) will be completed by performing Step 6.

The related parameters and uncertainty functions results are displayed in the following tables:

Table 5. 35. Results of the related parameters to calculate uncertainty for each criteria in Inconel 625 (X_1)

<i>Material alternative</i>	<i>Material criteria</i>	r	$\cos(\theta)$	θ	d	τ_{ij}
<i>Inconel 625 (X_1)</i>	C_1	0.59973	1.00000	00.00	1.00000	0.40026
	C_2	0.78267	0.00000	90.00	0.00000	0.21732
	C_3	0.23586	0.00000	90.00	0.00000	0.76413
	C_4	0.25318	1.00000	00.00	1.00000	0.74681
	C_5	0.82736	1.00000	00.00	1.00000	0.17263

Table 5. 36. Results of the related parameters to calculate uncertainty for each criteria in Pyromet 680 (X₂)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
Pyromet 680 (X₂)	C₁	0.77275	0.00000	90.00	0.00000	0.22724
	C₂	0.71316	0.00000	90.00	0.00000	0.28683
	C₃	0.62195	0.00000	90.00	0.00000	0.37804
	C₄	0.28630	1.00000	00.00	1.00000	0.71369
	C₅	0.44972	1.00000	00.00	1.00000	0.55027

Table 5. 37. Results of the related parameters to calculate uncertainty for each criteria in Haynes R-41 (X₃)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
Haynes R-41 (X₃)	C₁	0.59259	1.00000	00.00	1.00000	0.40740
	C₂	0.71861	0.00000	90.00	0.00000	0.28138
	C₃	0.46083	1.00000	00.00	1.00000	0.53916
	C₄	0.35172	1.00000	00.00	1.00000	0.64827
	C₅	0.73716	1.00000	00.00	1.00000	0.26283

Table 5. 38. Results of the related parameters to calculate uncertainty for each criteria in Inconel 706 (X₄)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
<i>Inconel 706 (X₄)</i>	<i>C₁</i>	0.29008	1.00000	00.00	1.00000	0.70991
	<i>C₂</i>	0.62195	0.00000	90.00	0.00000	0.37804
	<i>C₃</i>	0.87512	1.00000	00.00	1.00000	0.12487
	<i>C₄</i>	0.92605	1.00000	00.00	1.00000	0.07394
	<i>C₅</i>	0.64576	1.00000	00.00	1.00000	0.35423

Table 5. 39. Results of the related parameters to calculate uncertainty for each criteria in Udimet 700 (X₅)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
<i>Udimet 700 (X₅)</i>	<i>C₁</i>	0.10146	0.98552	9.761	0.89154	0.92054
	<i>C₂</i>	0.57884	0.00000	90.00	0.00000	0.42115
	<i>C₃</i>	0.96772	1.00000	00.00	1.00000	0.03227
	<i>C₄</i>	0.67063	1.00000	00.00	1.00000	0.32936
	<i>C₅</i>	0.22528	0.98160	11.01	0.87771	0.82981

Table 5. 40. Results of the related parameters to calculate uncertainty for each criteria in Ti-6-6-6-6 (X₆)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
Ti-6-6-6-6 (X₆)	C₁	0.41851	1.00000	00.00	1.00000	0.58148
	C₂	0.95436	1.00000	00.00	1.00000	0.04563
	C₃	0.89139	1.00000	00.00	1.00000	0.10860
	C₄	0.72724	0.00000	90.00	0.00000	0.27275
	C₅	0.17885	0.00000	90.00	0.00000	0.82114

Table 5. 41. Results of the related parameters to calculate uncertainty for each criteria in AISI 202 (X₇)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
AISI 202 (X₇)	C₁	0.43822	1.00000	00.00	1.00000	0.56177
	C₂	0.37000	0.00000	90.00	0.00000	0.62999
	C₃	0.12997	0.20863	77.96	0.13380	0.90480
	C₄	0.35939	1.00000	00.00	1.00000	0.64060
	C₅	0.22919	0.96487	15.23	0.83076	0.84838

Table 5. 42. Results of the related parameters to calculate uncertainty for each criteria in Rene 41 (X₈)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
<i>Rene 41 (X₈)</i>	<i>C₁</i>	0.85457	1.00000	00.00	1.00000	0.14542
	<i>C₂</i>	0.67884	0.00000	90.00	0.00000	0.32115
	<i>C₃</i>	0.63142	1.00000	00.00	1.00000	0.36857
	<i>C₄</i>	0.92605	1.00000	00.00	1.00000	0.07394
	<i>C₅</i>	0.78279	0.00000	90.00	0.00000	0.21720

Table 5. 43. Results of the related parameters to calculate uncertainty for each criteria in AISI 302 (X₉)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
<i>AISI 302 (X₉)</i>	<i>C₁</i>	0.04801	0.77130	39.53	0.56079	0.99416
	<i>C₂</i>	0.43772	0.00000	90.00	0.00000	0.56227
	<i>C₃</i>	0.20183	0.13436	82.28	0.08579	0.83280
	<i>C₄</i>	0.28484	1.00000	00.00	1.00000	0.71515
	<i>C₅</i>	0.55411	0.00000	90.00	0.00000	0.44588

Table 5. 44. Results of the related parameters to calculate uncertainty for each criteria in Rene 80 (X₁₀)

<i>Material alternative</i>	<i>Material criteria</i>	<i>r</i>	<i>cos (θ)</i>	<i>θ</i>	<i>d</i>	<i>τ_{ij}</i>
<i>Rene 80 (X10)</i>	<i>C₁</i>	0.96427	1.00000	00.00	1.00000	0.03572
	<i>C₂</i>	0.37396	0.00000	90.00	0.00000	0.62603
	<i>C₃</i>	0.80000	1.00000	00.00	1.00000	0.20000
	<i>C₄</i>	0.86427	1.00000	00.00	1.00000	0.13572
	<i>C₅</i>	0.78279	0.00000	90.00	0.00000	0.21720

By applying the Steps 7 and 8, the aggregated PFNs can be simply obtained and expressed in a collective evaluation PF decision making matrix as shown on the next page:

	C_1	C_2	C_3	C_4	C_5
X_1	$P(0.77442, 0.00000)$	$P(0.00000, 0.88469)$	$P(0.00000, 0.48565)$	$P(0.50317, 0.00000)$	$P(0.90959, 0.00000)$
X_2	$P(0.00000, 0.87906)$	$P(0.00000, 0.84448)$	$P(0.00000, 0.78864)$	$P(0.53507, 0.00000)$	$P(0.67061, 0.00000)$
X_3	$P(0.76980, 0.00000)$	$P(0.00000, 0.84770)$	$P(0.67885, 0.00000)$	$P(0.59306, 0.00000)$	$P(0.85850, 0.00000)$
X_4	$P(0.53859, 0.00000)$	$P(0.00000, 0.78864)$	$P(0.93548, 0.00000)$	$P(0.96231, 0.00000)$	$P(0.80359, 0.00000)$
X_5	$P(0.31042, 0.12875)$	$P(0.00000, 0.76082)$	$P(0.98372, 0.00000)$	$P(0.81892, 0.00000)$	$P(0.44961, 0.00000)$
X_6	$P(0.64692, 0.00000)$	$P(0.97691, 0.00000)$	$P(0.94413, 0.00000)$	$P(0.00000, 0.85278)$	$P(0.00000, 0.42291)$
X_7	$P(0.66198, 0.00000)$	$P(0.00000, 0.60827)$	$P(0.16002, 0.34645)$	$P(0.59949, 0.00000)$	$P(0.44243, 0.23086)$
X_8	$P(0.92443, 0.00000)$	$P(0.00000, 0.82392)$	$P(0.79462, 0.00000)$	$P(0.96231, 0.00000)$	$P(0.00000, 0.88475)$
X_9	$P(0.18676, 0.16965)$	$P(0.00000, 0.66161)$	$P(0.15995, 0.43438)$	$P(0.53371, 0.00000)$	$P(0.00000, 0.74438)$
X_{10}	$P(0.98197, 0.00000)$	$P(0.00000, 0.61153)$	$P(0.89442, 0.00000)$	$P(0.92966, 0.00000)$	$P(0.00000, 0.88475)$

The next phase is to assign criteria weights and to implement the MCDM approaches to solve the induced PF matrix and rank the alternatives.

5.1.1.1.2 Assigning criteria weights

The initial weight (fuzzy density value) $\chi(\{q_j\})$ to each criterion will be assigned based on the provided data in (Table 5.1). Then, the Sugeno λ -fuzzy measure and the Shapley value methods will be applied to distribute the weights of the criteria equally as mentioned in Step 9. The initial criteria weights and their Shapley values are displayed in the following table:

Table 5. 45. The assignment of initial criteria weights and their Shapley method results:

	service temperature (C1)	Density (C2)	Yield strength (C3)	Young's modulus (C4)	Cost (C5)	\sum Total weight
Initial weight $\chi(\{q_j\})$	0.9	0.9	0.7	0.7	0.3	3.5
Shapley value (w_j)	0.26863	0.26863	0.19403	0.19403	0.074662	1

As seen above, the weighting vector is $w = (0.26863, 0.26863, 0.19403, 0.19403, 0.074662)$ and satisfies $\sum_{j=1}^n w_j = 1$.

5.1.1.1.3 First method implementation: Pythagorean fuzzy MCGDM TOPSIS

To start implementing the PF TOPSIS approach, we will apply the score function as in Step 10 to obtain the PFPIS and PFNIS, respectively. The results can be presented as:

$$X^+$$

$$= \{P(0.98197,0.00000),P(0.97691,0.00000),P(0.98372,0.00000),P(0.96231,0.00000),P(0.90959,0.00000)\}$$

$$X^-$$

$$= \{P(0.00000,0.87906),P(0.00000,0.88469),P(0.00000,0.78864),P(0.00000,0.85278),P(0.00000,0.88475)\}$$

Then, we will calculate the distances and revised closeness, and rank the alternatives as shown in Steps 11, 12, and 13. The outcomes are displayed in the following table:

Table 5. 46. Results of the Pythagorean fuzzy TOPSIS method

Material alternative (X_i)	$D(X_i, X^+)$	$D(X_i, X^-)$	$\zeta(X_i)$	<i>Ranking</i>
Inconel 625 (X_1)	0.67263	0.48538	-1.15297	8
Pyromet 680 (X_2)	0.85551	0.21823	-2.04342	10
Haynes R-41(X_3)	0.57274	0.54503	-0.79133	6
Inconel 706 (X_4)	0.46900	0.65870	-0.34098	4
Udimet 700 (X_5)	0.58576	0.64228	-0.68664	5
Ti-6-6-6-6 (X_6)	0.40287	0.68201	-0.12494	2
AISI 202 (X_7)	0.73760	0.61141	-1.14992	7
Rene 41 (X_8)	0.41286	0.55966	-0.32935	3
AISI 302 (X_9)	0.87503	0.53477	-1.63966	9
Rene 80 (X_{10})	0.36267	0.69176	0.00000	1

For more illustration of the proposed method and aforementioned steps, processes can be clearly simplified in Fig (4.2) as follows:

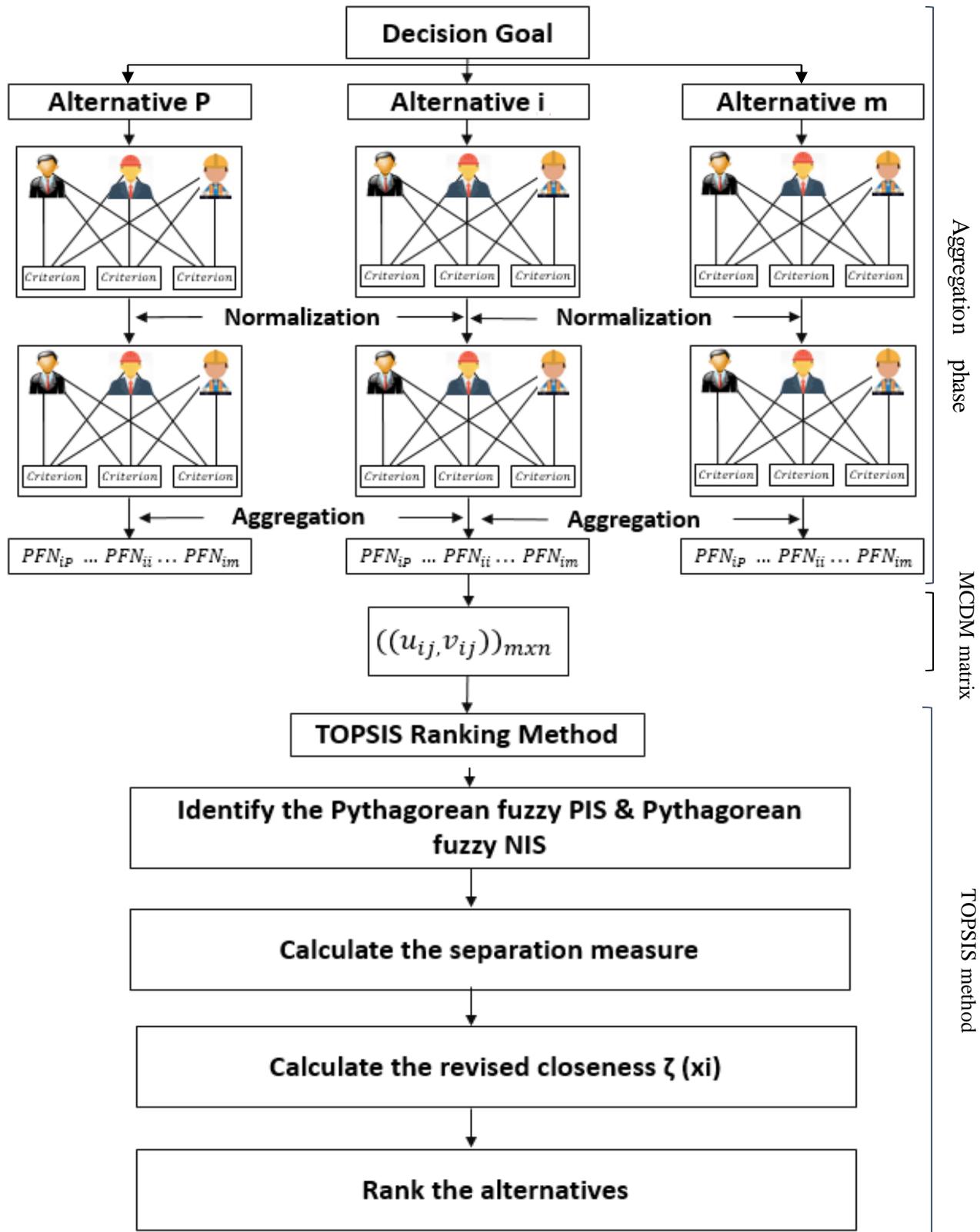


Figure 5. 4. Illustration of the Pythagorean fuzzy MCGDM TOPSIS method

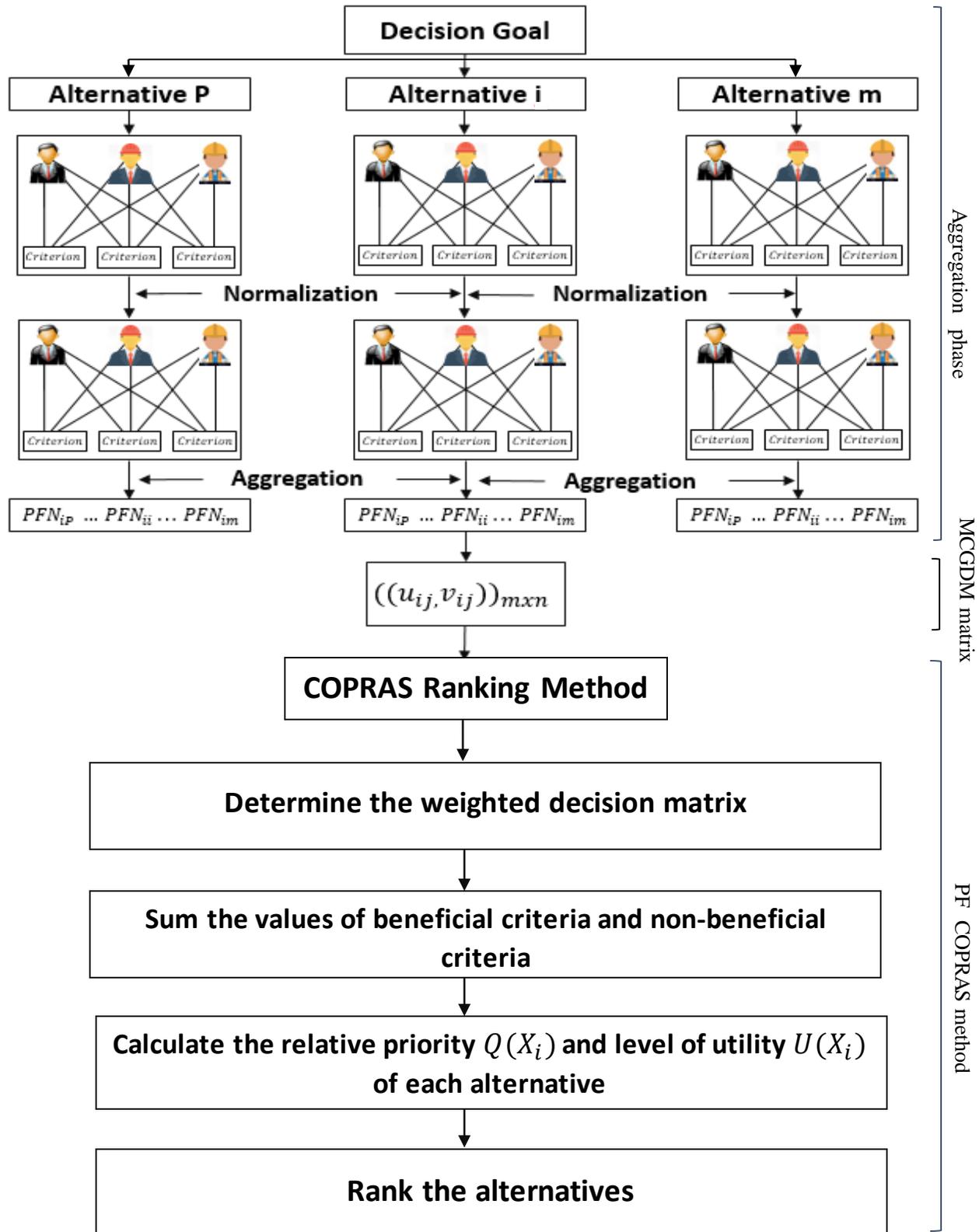
5.1.1.1.4 Second model implementation: Pythagorean fuzzy MCGDM COPRAS

First, we will find the weighted Pythagorean fuzzy decision matrix following step 2 and step 3. After that, to proceed with the ranking process, we will calculate the summations of the beneficial criteria P_{+i} and the non-beneficial criteria P_{-i} , and calculate their score functions $s(P_{+i})$, $s(P_{-i})$ and $s(P_{-min})$ as shown in step 4 and 5. Then, in order to rank the material alternatives, we should calculate the relative priority $Q(X_i)$ and level of utility $U(X_i)$ as explained in steps 6, 7 and 8. All the calculations are displayed in the next table:

Table 5. 47.Results of the Pythagorean fuzzy COPRAS method

Material alternative (X_i)	P_{+i}	P_{-i}	$s(P_{+i})$	$s(P_{-i})$	$s(P_{-min})$	$Q(x_i)$	$U(x_i)$	<i>Ranking</i>
Inconel 625 (X_1)	(0.30567,0.09424)	(0.23765,0.06791)	0.08455	0.05186	-0.06787	1.11109	78.97	8
Pyromet 680 (X_2)	(0.10382,0.38917)	(0.22685,0.05006)	-0.14067	0.04895		0.88886	63.18	10
Haynes R-41(X_3)	(0.45359,0.00000)	(0.22772,0.06410)	0.20574	0.04774		1.23652	87.89	5
Inconel 706 (X_4)	(0.51292,0.00000)	(0.21185,0.05999)	0.26309	0.04128		1.30056	92.44	3
Udimet 700 (X_5)	(0.43317,0.03458)	(0.21185,0.03356)	0.18644	0.04691		1.21808	86.58	6
Ti-6-6-6-6 (X_6)	(0.35698,0.16547)	(0.03157,0.26243)	0.10005	-0.06787		1.25718	89.36	4
AISI 202 (X_7)	(0.32520,0.06722)	(0.18063,0.03303)	0.10124	0.03153		1.14886	81.66	7
Rene 41 (X_8)	(0.58924,0.00000)	(0.28738,0.00000)	0.34720	0.08259		1.34269	95.43	2
AISI 302 (X_9)	(0.18476,0.12986)	(0.23330,0.00000)	0.01727	0.05443		1.04119	74.00	9
Rene 80 (X_{10})	(0.61773,0.00000)	(0.00000,0.23033)	0.38159	0.05305		1.40692	100	1

For more illustration of the proposed method and aforementioned steps, processes can be clearly simplified in (Figure 5.1) as follow:



Aggregation phase

MCGDM matrix

PF COPRAS method

Figure 5. 5. Illustration of the Pythagorean Fuzzy MCGDM COPRAS method

5.1.2 Computational application

For the implementation of the proposed models in real life material selection situations, an application has been developed using Microsoft Excel and Python to achieve this purpose. Microsoft Excel was used to develop and model the proposed algorithms mathematical approaches including Sugeno λ fuzzy measure, model solver to find the root of high order polynomial equations, Shapley value, uncertainty function, the Pythagorean Fuzzy TOPSIS method and the Pythagorean Fuzzy COPRAS method. Then, Python was employed to design the Graphical User Interface (GUI) for this application. The developed application characterized by its simplicity and applicability to be used by designers and engineers in real life material selection problems.

We used the aforementioned case study data to illustrate the processes and steps of using the developed application in a practical material selection problem. Initially, we need to select the number of materials and their commercial names as displayed in (Figure 5.6) below:

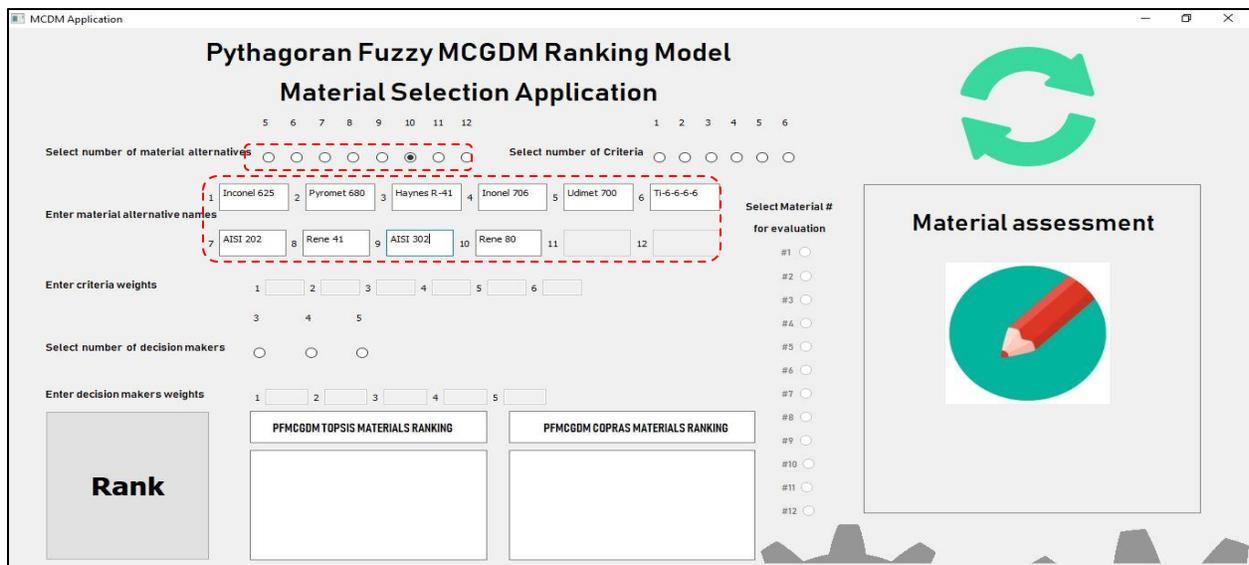


Figure 5. 6. Illustration of using the material selection application (select the number of materials & enter their commercial names)

Next, we will select the number of criteria and enter their initial weights as shown in (Figure 5.7) below:

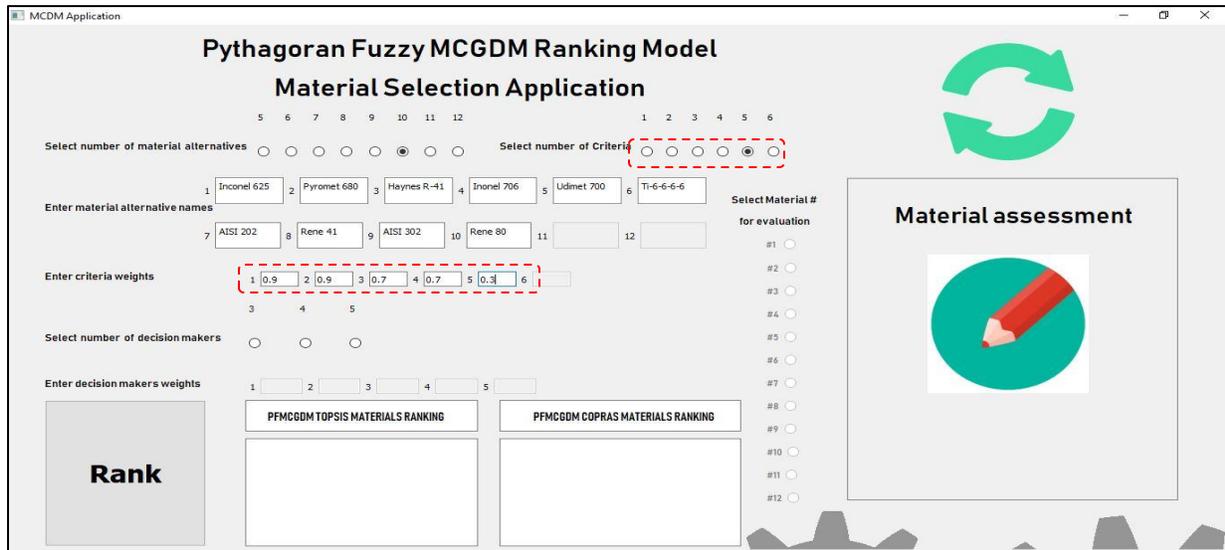


Figure 5. 7. Illustration of using the material selection application (select the number of criteria & enter their initial weights)

After that, we are going to choose the number of decision makers and enter their initial weights as shown in (Figure 5.8) below:

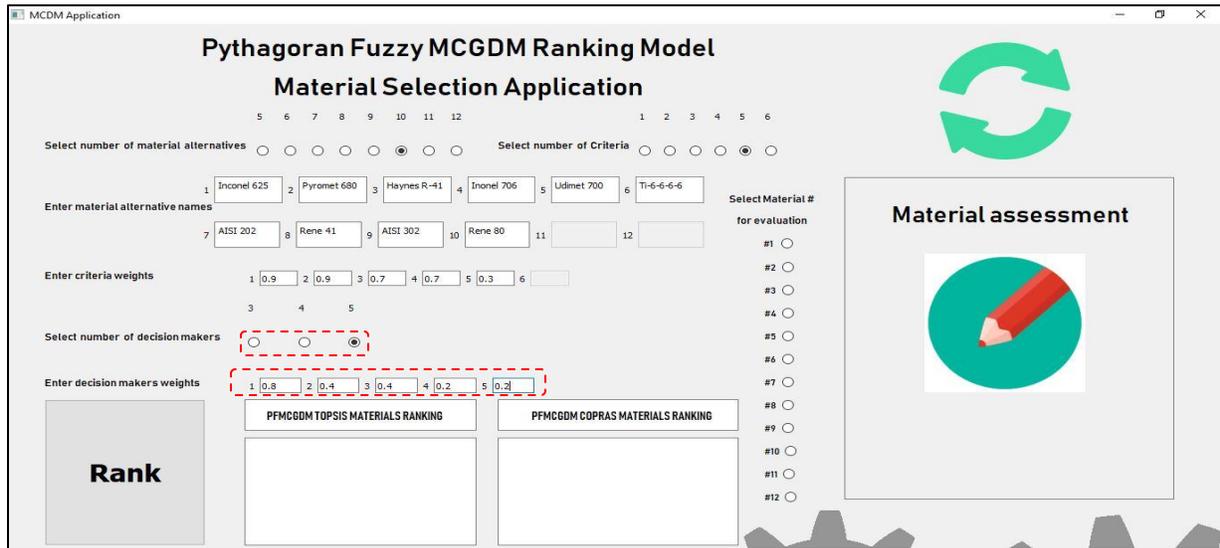


Figure 5. 8. Illustration of using the material selection application (select the number of decision makers & enter their initial weights)

Then, we will select each material and fill the decision makers' assessments as explained in (Figure 5.9) below:

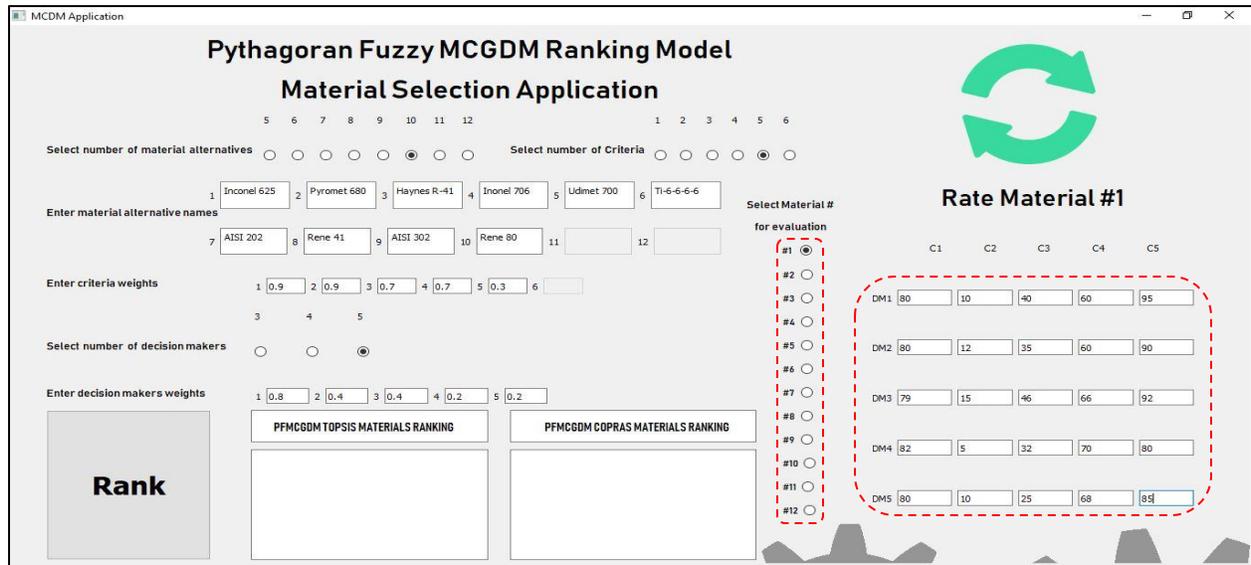


Figure 5. 9. Illustration of using the material selection application (select each material & fill the decision makers' assessments)

Finally, we will press **'Rank'** and the results of the hybrid PFMCGDM TOPSIS model and hybrid PFMCGDM COPRAS model will show up as shown in (Figure 5.10) below:

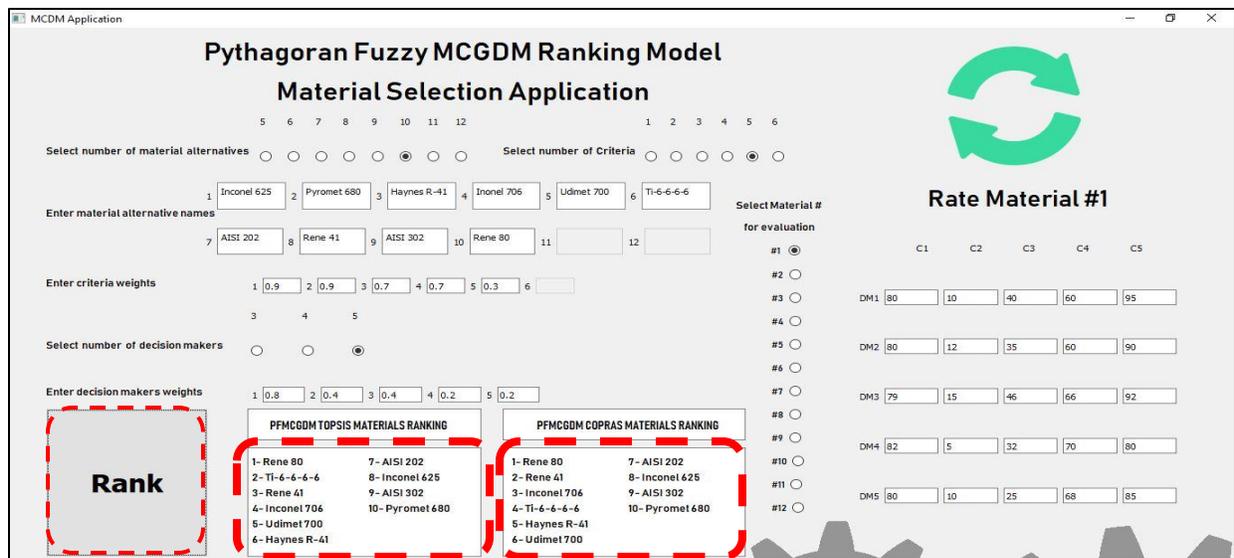


Figure 5. 10. Illustration of using the material selection application (Press 'Rank' to get the final ranking outcomes of both methods)

5.1.3 Discussion and comparative analysis of both MCGDM methods

In the PFMCGDM TOPSIS method, the material alternatives were ranked based on the revised closeness coefficient results in a descending order. As mentioned before, the revised closeness coefficient was used instead of the relative closeness coefficient due to its superiority in satisfying the purpose of the optimal solution having the shortest distance from the PIS and the farthest distance from the NIS, simultaneously (Hadi-Vencheh & Mirjaberli. 2014; Zhang & Xu, 2014). The ranking order of the MCGDM TOPSIS method is $X_{10} > X_6 > X_8 > X_4 > X_5 > X_3 > X_7 > X_1 > X_9 > X_2$ with X_{10} , which is Nickel-based Super alloy Rene 80, being the optimal material to be used in high pressure turbine blades in the turbojet engine and X_2 , which is Nickel-based Super alloy Pyromet 680, being the worst one. As seen in (Figure 5.11), the optimal alternative X_{10} has a revised closeness coefficient equal to zero, indicating that this alternative has satisfied the condition of having the shortest distance from the PIS and the farthest distance from the NIS at the same time. The second most preferred alternative is Titanium alpha beta Super alloy (X_6) with a revised closeness coefficient equal to -0.12. It can be seen that the difference between the coefficient of the best material alternative X_{10} and the second one X_6 is considered small (-0.12), which makes material X_6 performance close to the optimal one X_{10} . Furthermore, it is important to mention that there are no significant differences in coefficients between the top four material alternatives $X_{10} > X_6 > X_8 > X_4$; in particular, the difference among X_8 and X_4 is barely noticed (0.01), which implies that both materials have similar performances.

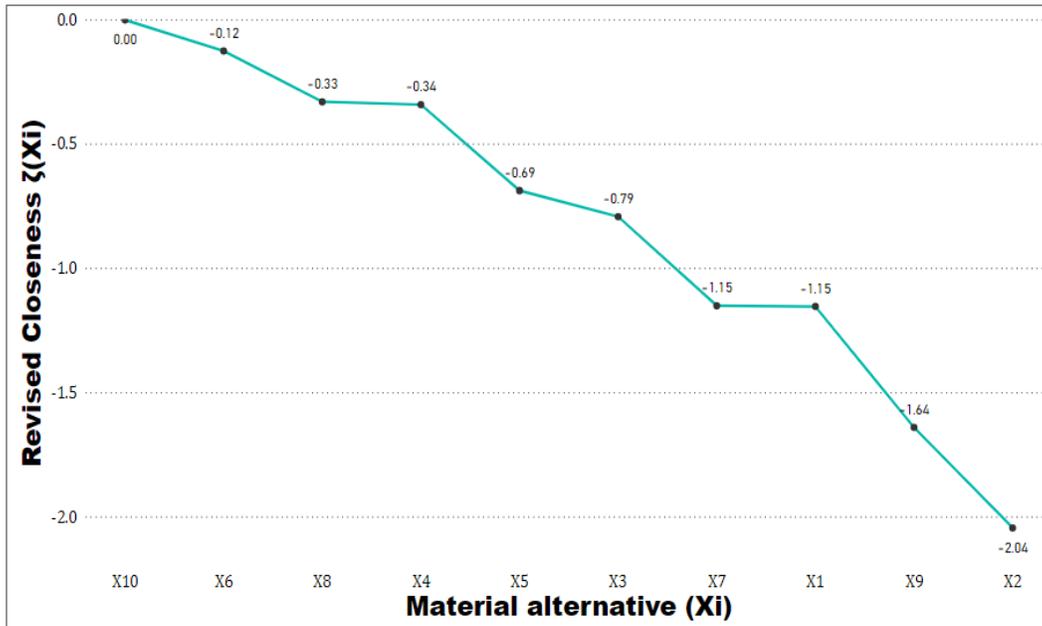


Figure 5. 11. Revised Closeness Coefficient $\zeta(X_i)$ for all material alternatives

On the other hand, the final ranking of material alternatives in the PFMCGDM COPRAS method was based on the degree of utility outcomes. The utility degree can be obtained by calculating the relative weight, which is the result of summing the maximization and minimization indices for each material alternative. The final ranking order of the PFMCGDM COPRAS method is $X_{10} \succ X_8 \succ X_4 \succ X_6 \succ X_3 \succ X_5 \succ X_7 \succ X_1 \succ X_9 \succ X_2$ with X_{10} (Nickel-based Super alloy Rene 80) being the optimal material alternative and X_2 (Nickel-based Super alloy Pyromet 680) being the worst option to be used to fit the high pressure turbine requirements. The difference between the level of utility between the optimal alternative X_{10} (100%) and the second one X_8 (95%) is slightly low, showing that X_8 performance is close to the best material X_{10} . Additionally, it can be observed from (Figure 5.12) that the differences between the materials X_3 and X_6 as well as X_3 and X_5 are very insignificant.

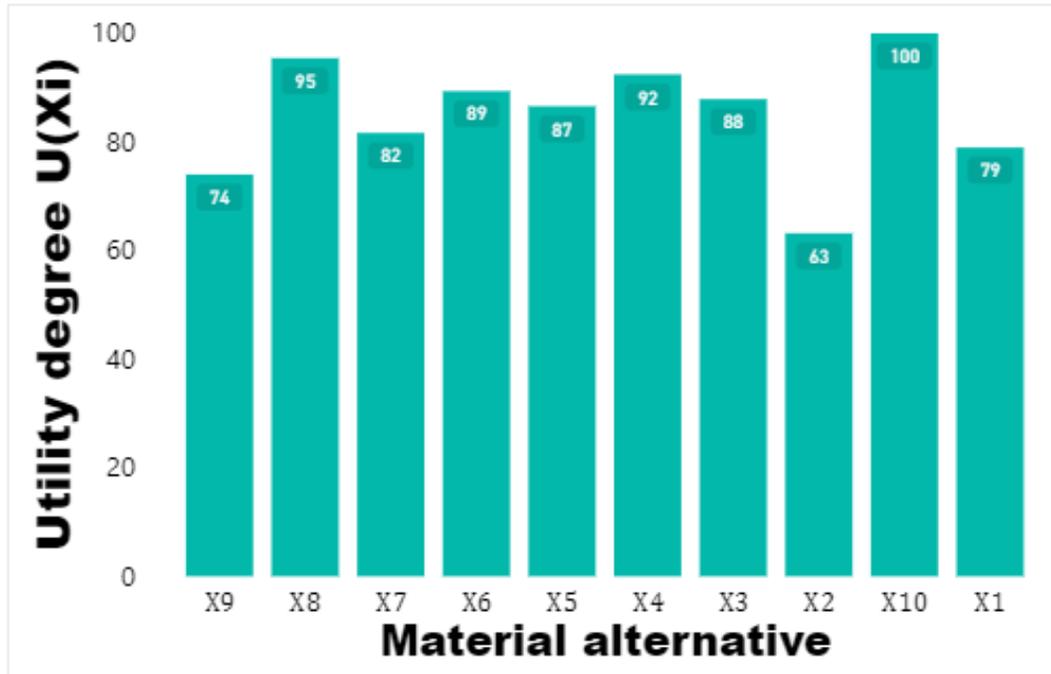


Figure 5. 12. Utility degree $U(X_i)$ for all material alternatives

5.1.4 Comparative analysis

It can be clearly noticed from the results displayed in final ranking processes that both methods recommended Nickel-based Super alloy Rene 80 (X_{10}) as the optimal material alternative in the application of high pressure turbine in jet engines. Note that, regardless of their ranking order, the top three proposed materials that follow the optimal alternative (X_8, X_4 & X_6) are the same materials given by both methods with a ranking order of $X_{10} > X_8 > X_4 > X_6$ in PFMCGDM TOPSIS and $X_{10} > X_6 > X_8 > X_4$ in PFMCGDM COPRSAS as shown in (Figure 5.13). From final ranking results of both approaches, it can be seen that Titanium alpha beta Super alloy, Ti-6-6-6-6 (X_6) has been placed as the fourth recommended material in PFMCGDM TOPSIS whereas in PFMCGDM COPRSAS has been ranked the second for most preferred materials. This indicates how both methods differ in terms of their mathematical approaches for

identifying the optimal alternative. To be more specific, the alternative X_6 has the positive ideal solution and closest number to an ideal solution in the most and second dominant criteria (density and tensile strength), respectively, within the PFMCGDM TOPSIS that results in the alternative being ranked second in the final ranking process, whereas in the PFMCGDM COPRAS method, the same alternative has the minimum index value for the summation of the non-beneficial criteria elements but it failed to have the maximum index value for the summation of the beneficial criteria elements. This outcome leads to decrease its level of utility, thus lowering it to fourth place in the COPRAS ranking process. Moreover, it is significant to note that the last four materials are ranked in the same order as $X_7 > X_1 > X_9 > X_2$ with Nickel-based Super alloy Pyromet 680 (X_2) being the worst option within the two methods.

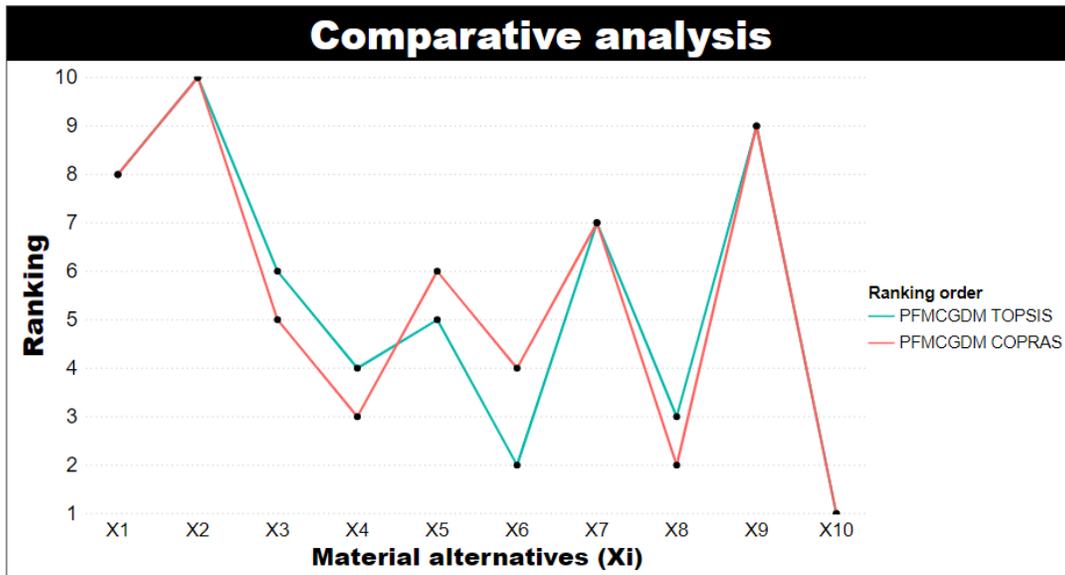


Figure 5. 13. Comparison of final ranking outcomes between Hybrid PFMCGDM TOPSIS and hybrid PFMCGDM COPRAS model

5.2 Sensitivity analysis

The implementation of MCDM usually faces some problems that affect the decision-making process in which inaccurate and instable results may occur in the final ranking procedure. Therefore, performing the proper sensitivity analysis can be helpful in investigating the initial input factors or parameters that can affect the accuracy of the final outcome performance of the tested model. Furthermore, this analysis can examine the main purpose of the decision making model by studying its applicability in the desired field, which is material selection in our case. Many approaches and methods have been proposed in research on this matter. One popular approach that has been used by researchers is to study the final results ranking behavior under various conditions. This can be done by defining the most important input and slightly manipulating its values to observe how sensitive the model responds to these small changes. However, it is important to mention that there is no consensus on such a tool or a general way to perform the sensitivity analysis and check the reliability of the MCDM method outcomes. Additionally, the measurement of the robustness and stability of the model outputs really depend on the purpose of designing and applying the tested decision making method. For this reason, to analyze the proposed MCGDM methods, the important parameters that might influence the outcome performance need to be defined first.

In the material selection field, DMs' priorities should be considered in selecting the best material that suits the design or product. As a result, our selected approach to test the reliability of the proposed method results will be based on changing the DMs weights and study the performance of the resultant ranking outputs.

To proceed with this approach, the fuzzy measure and Shapley value methods that are used to fairly distribute the weights of DMs and criteria in our MCGDM frameworks will be

implemented again to modify weights in this sensitivity analysis. These aforementioned techniques will allow other weights to be adjusted proportionally and hold the condition $\sum_{j=1}^n w_j = 1$ if we make any changes to a certain weight.

In order to provide accurate analysis in testing the sensitivity of the final outcomes to the changes of input parameters, specifically DMs' weights, we decided to slightly modify the original initial weight (fuzzy density value) used in the previous case study $\mu(\{d_1\}) = 0.8$, $\mu(\{d_2\}) = 0.4$, $\mu(\{d_3\}) = 0.4$, $\mu(\{d_4\}) = 0.2$, and $\mu(\{d_5\}) = 0.2$ through applying the following scenarios to each method :

- 1) Decreasing the weight of the first decision maker (the most influential one) (DM1) by 25%, 50% and 75%.
- 2) Increasing the weight of the second decision maker (DM2) by 50% and 100%.
- 3) Increasing the weight of the third decision maker (DM3) by 50% and 100%.
- 4) Increasing the weight of the fourth decision maker (DM4) by 50%, 100% and 200%.
- 5) Increasing the weight of the fifth decision maker (DM5) by 50%, 100% and 200%.

Comparing the previous scenarios ranking outcomes with the original ranking results of implementing each method in the last case study will allow us to observe carefully to what extent changing DMs weights within the fuzzy number aggregation process would affect the final ranking results of the proposed MCGDM methods.

5.2.1 Changing decision makers (DMs) weights in Hybrid PFMCGDM TOPSIS

There was no change noticed in the ranking order when the weight of DM1 increased by 25%. However, as seen in (Figure 5.14), the final ranking of material alternatives has changed after decreasing the weight of DM1 by 50%. Specifically, the material alternative X7 has shifted from the seventh to the eighth rank with a slight decrease of (- 0.00608) in the revised closeness coefficient value. As well, X1 alternative has placed seventh instead of eighth in the ranking order with a barely noticeable increase of (+ 0.00044) in the coefficient value. Note that we still get the same ranking results when the weight of DM1 increased from 50% up to 75% as seen in figure (Figure 5.15).

No difference has been observed in the ranking outcomes when DM2 weight increased by 50% and 100%. On the other hand, when the weight of DM3 increased to 50%, the ranking order varied, as shown in figure (Figure 5.16). The difference this time is significant since the change affected the order of the top four materials in the final ranking. To be more specific, the alternative X4 has exchanged places with X8 and transferred to the third rank, whereas X8 moved to the fourth place. Thus, the effect of changing DM3's weight on X4 alternative was noticeable since it caused a significant increase in the material's revised closeness coefficient from (-0.34098) to (-0.32323) as exhibited in (Figure 5.24), resulting in X4 surpassing X8 in the ranking process. Furthermore, when DM3 weight increased up to 100% as shown in (Figure 5.17), the revised coefficient of X4 increased to reach (-0.31649). The ranking outcomes remain the same as when DM3 weight increased 50%.

(Figure 5.18) shows that after increasing DM4 weight by 50%, the material alternative ranking changed from the original one. Alternative X7's revised closeness coefficient has notably decreased compared with the other materials, causing the material to shift from the

seventh to the eighth in final ranking outcomes. The decreasing in the coefficient occurred because of the poor rating of DM4 to the material alternative X7. Consequently, increasing DM4's weight has influenced alternative X7's Pythagorean fuzzy satisfactory numbers to decrease and dissatisfactory numbers to increase in their values. For this reason, we obtained the same pattern in behavior from material X7 when DM4 weight increased up to 100% and 200% as displayed in (Figure 5.19) and (Figure 5.20) as well as materials alternatives were ranked the same as in the 50% weight increasing .

It can be seen from (Figure 5.21), (Figure 5.22) and (Figure 5.23) that after increasing DM5 by 50%, 100%, and 200%, the coefficient closeness of alternative X4 has clearly increased similar to the results of increasing DM3's weight. The reason for this noticeable increase in the X4 coefficient is the high assessment of DM5 to the alternative X4 compared to other DMs. Subsequently, any change in DM5 weight will affect the results of X4.

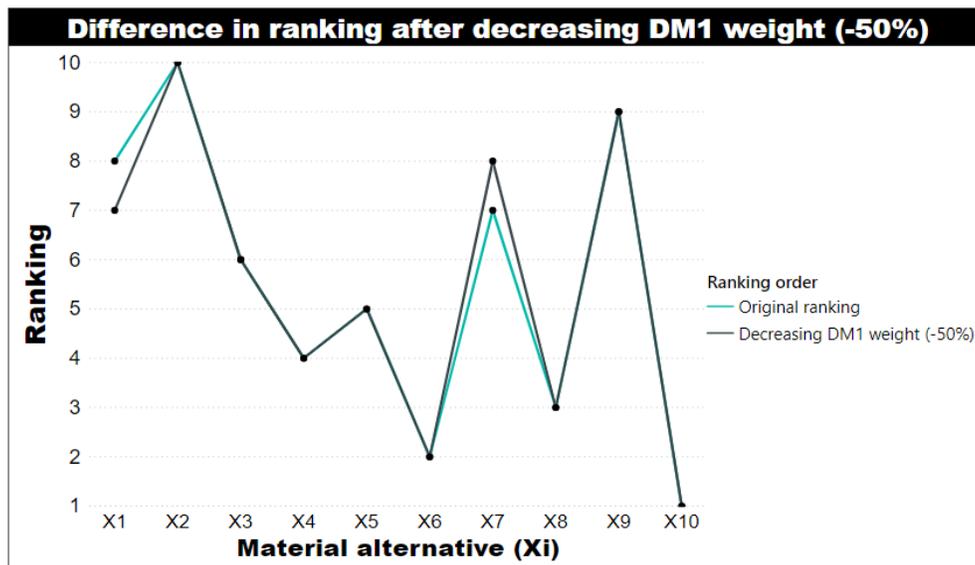


Figure 5. 14. Difference in the final ranking outcomes after decreasing 50% of DM1 weight in the hybrid PFMCGDM TOPSIS method

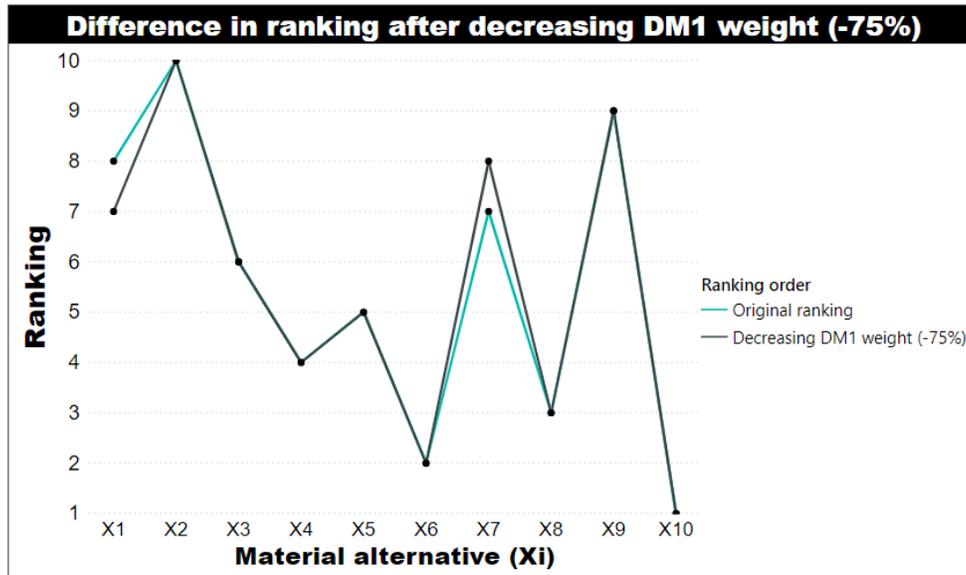


Figure 5. 15. Difference in the final ranking outcomes after decreasing 75% of DM1 weight in the hybrid PFMCGDM TOPSIS method

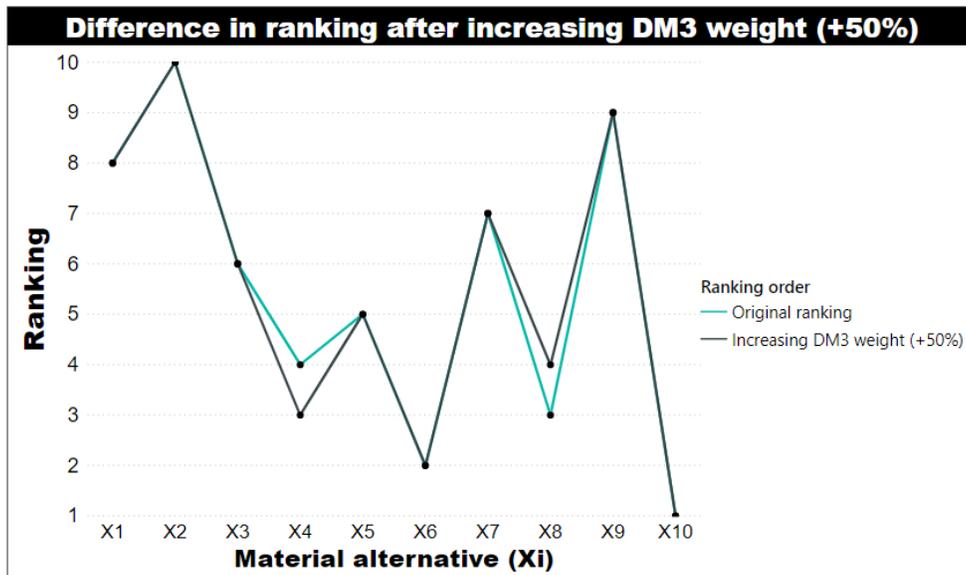


Figure 5. 16. Difference in the final ranking outcomes after increasing 50% of DM3 weight in the hybrid PFMCGDM TOPSIS method

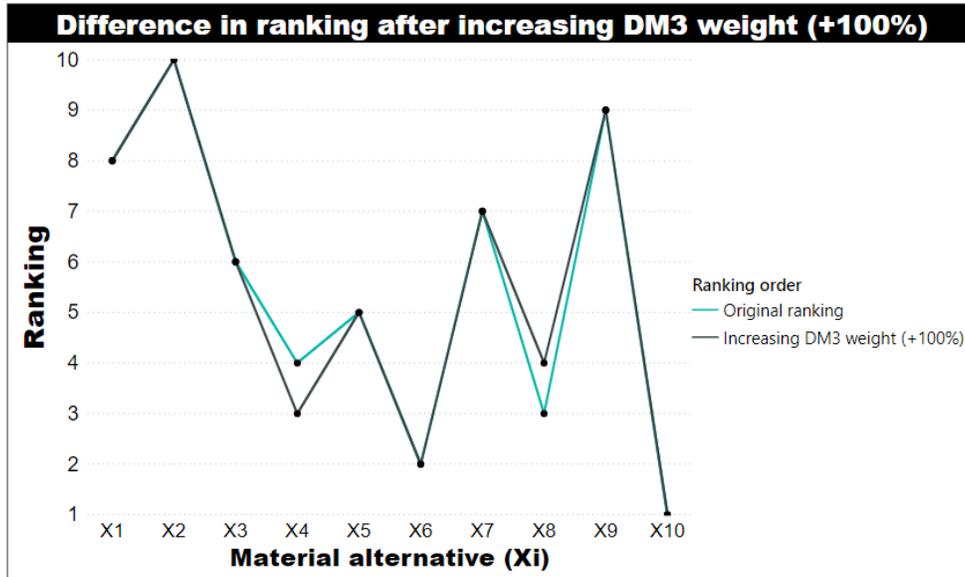


Figure 5. 17. Difference in the final ranking outcomes after increasing 100% of DM3 weight in the hybrid PFMCGDM TOPSIS method

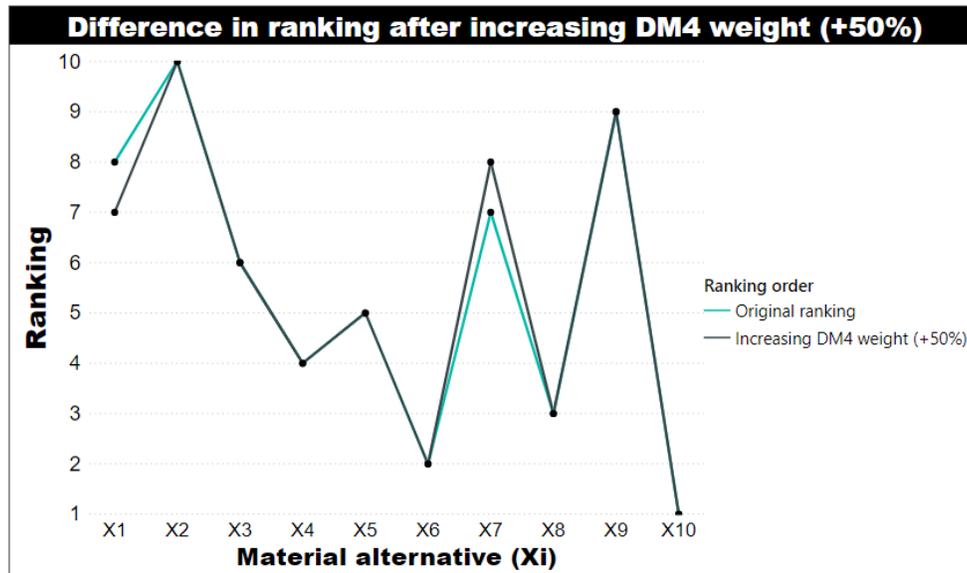


Figure 5. 18. Difference in the final ranking outcomes after increasing 50% of DM4 weight in the hybrid PFMCGDM TOPSIS method

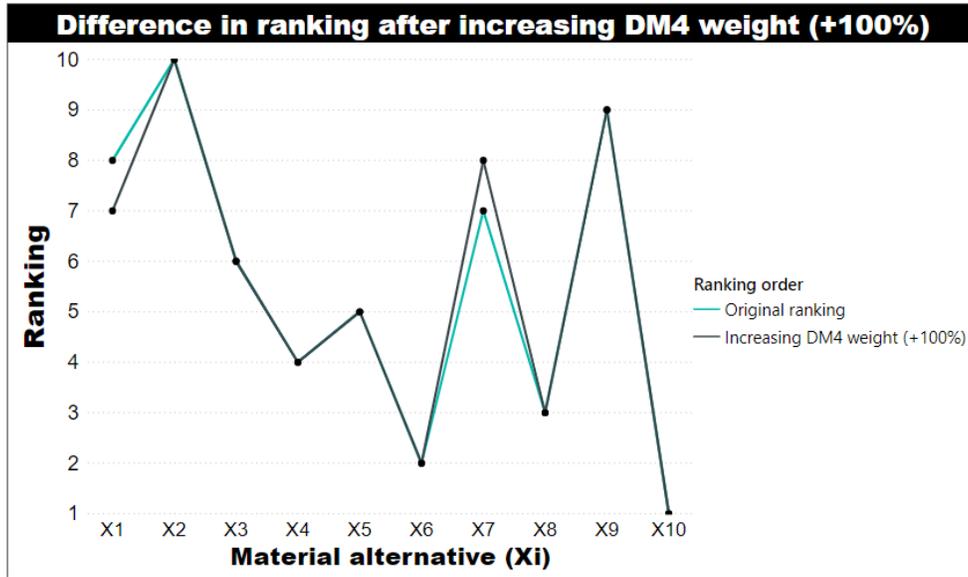


Figure 5.19. Difference in the final ranking outcomes after increasing 100% of DM4 weight in the hybrid PFMCGDM TOPSIS method

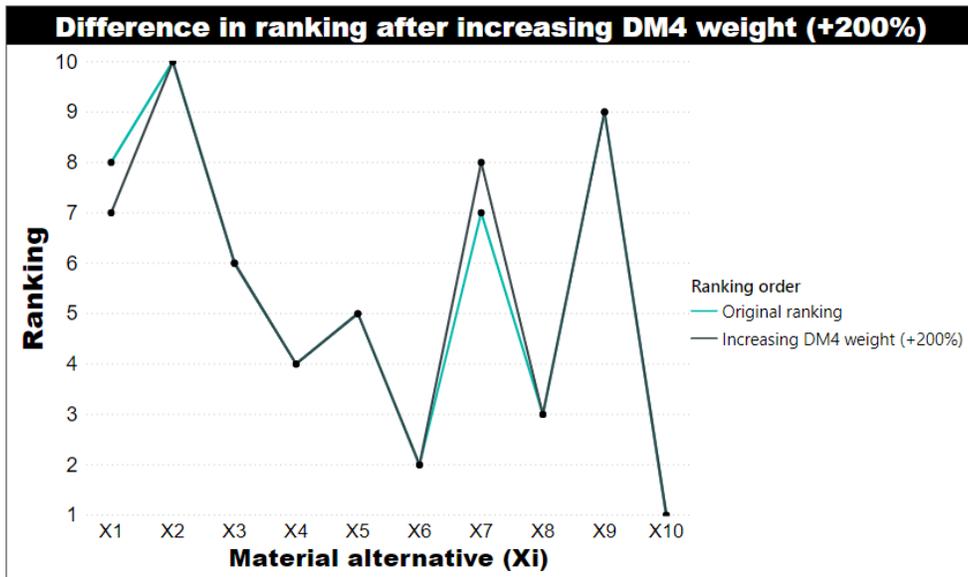


Figure 5.20. Difference in the final ranking outcomes after increasing 200% of DM4 weight in the hybrid PFMCGDM TOPSIS method

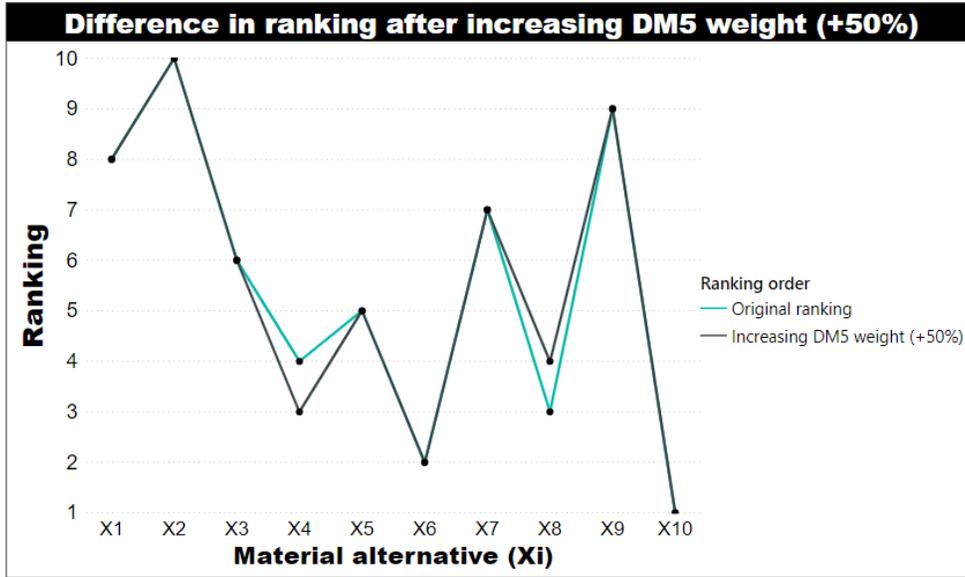


Figure 5. 21. Difference in the final ranking outcomes after increasing 50% of DM5 weight in the hybrid PFMCGDM TOPSIS method

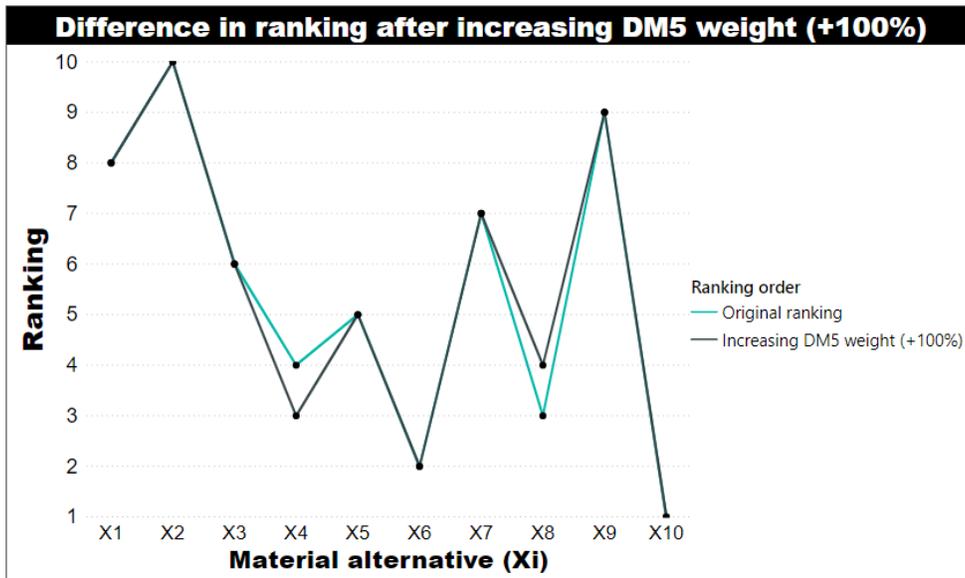


Figure 5. 22. Difference in the final ranking outcomes after increasing 100% of DM5 weight in the hybrid PFMCGDM TOPSIS method

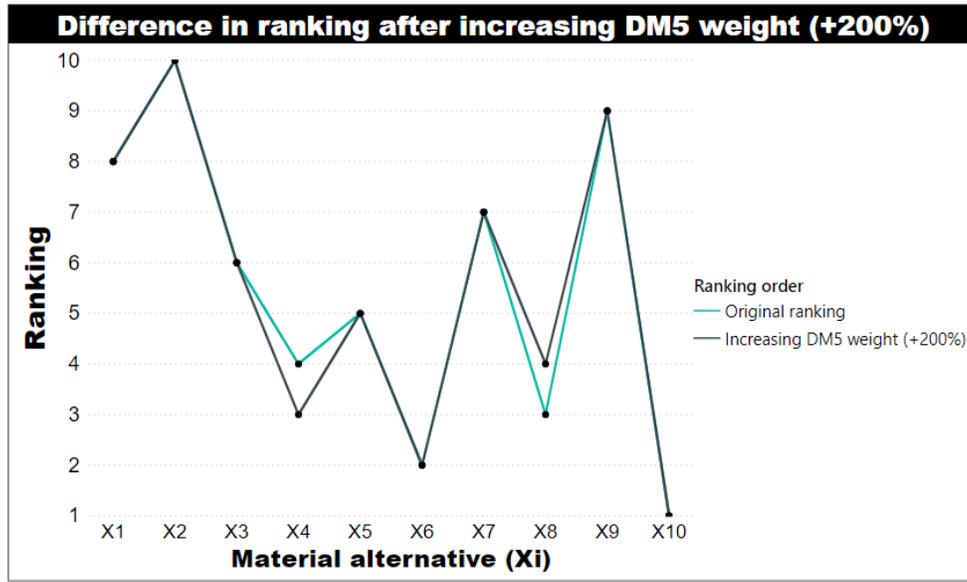


Figure 5. 23. Difference in the final ranking outcomes after increasing 200% of DM5 weight in the hybrid PFMCGDM TOPSIS method

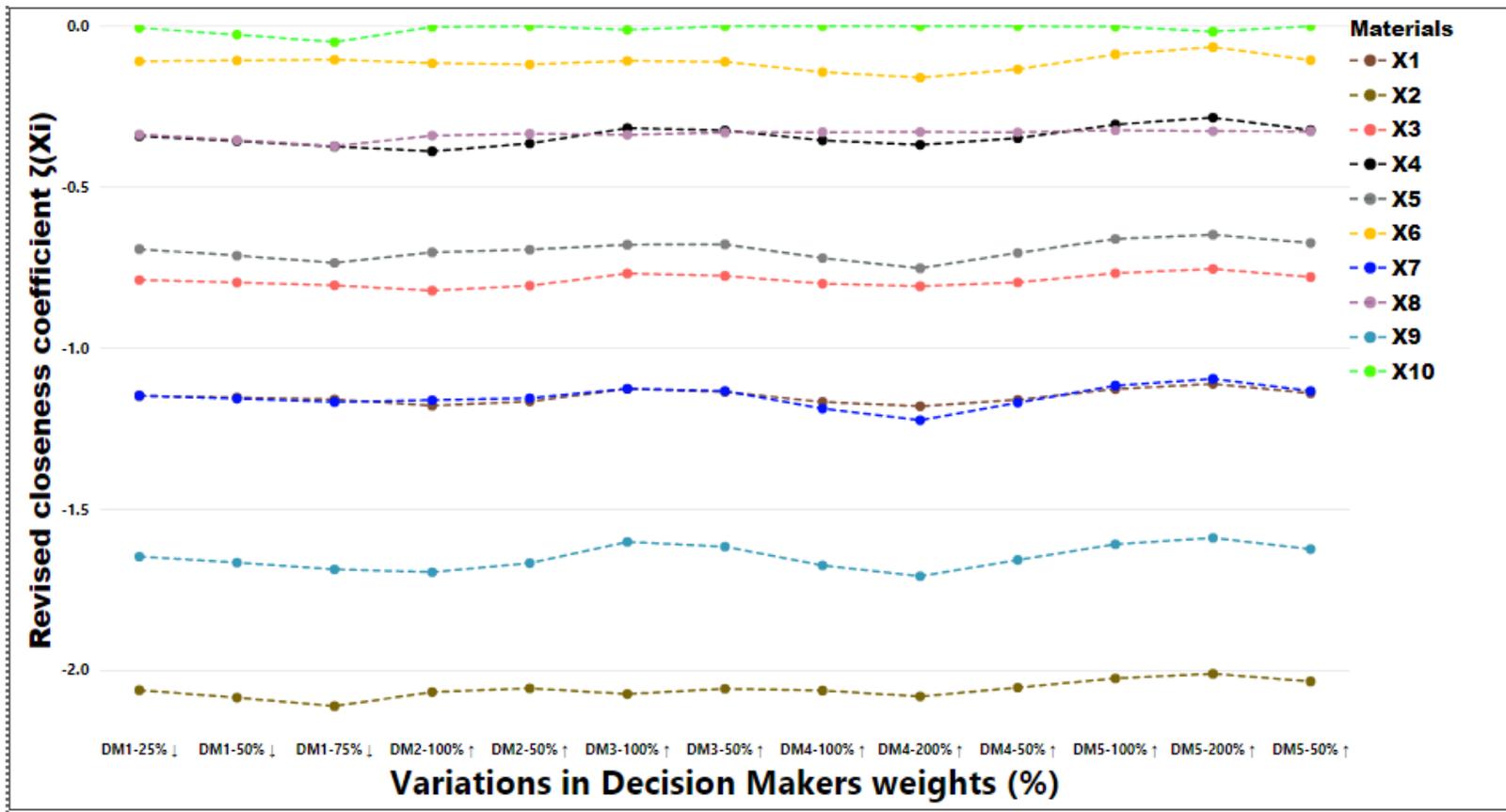


Figure 5. 24. Revised Closeness Coefficient $\zeta(X_i)$ behavior caused by changing DMs weights in the hybrid PFMCGDM TOPSIS method

5.2.2 Changing decision makers (DMs) weights in Hybrid PFMCGDM COPRAS

There were no changes in ranking order for all alternatives in the Hybrid PFMCGDM COPRAS method under any scenario. One can conclude that the PFMCGDM COPRAS method ranking results is less sensitive to be affected by changing DMs weights than the ranking outcomes in the PFMCGDM TOPSIS method. However, as shown in (Figure 5.25), the utility degree for all material alternatives has been changing with varying DMs' weight, indicating that the ranking order of the PFMCGDM COPRAS may be altered by the effect of DMs' weight under different scenarios and assessments.

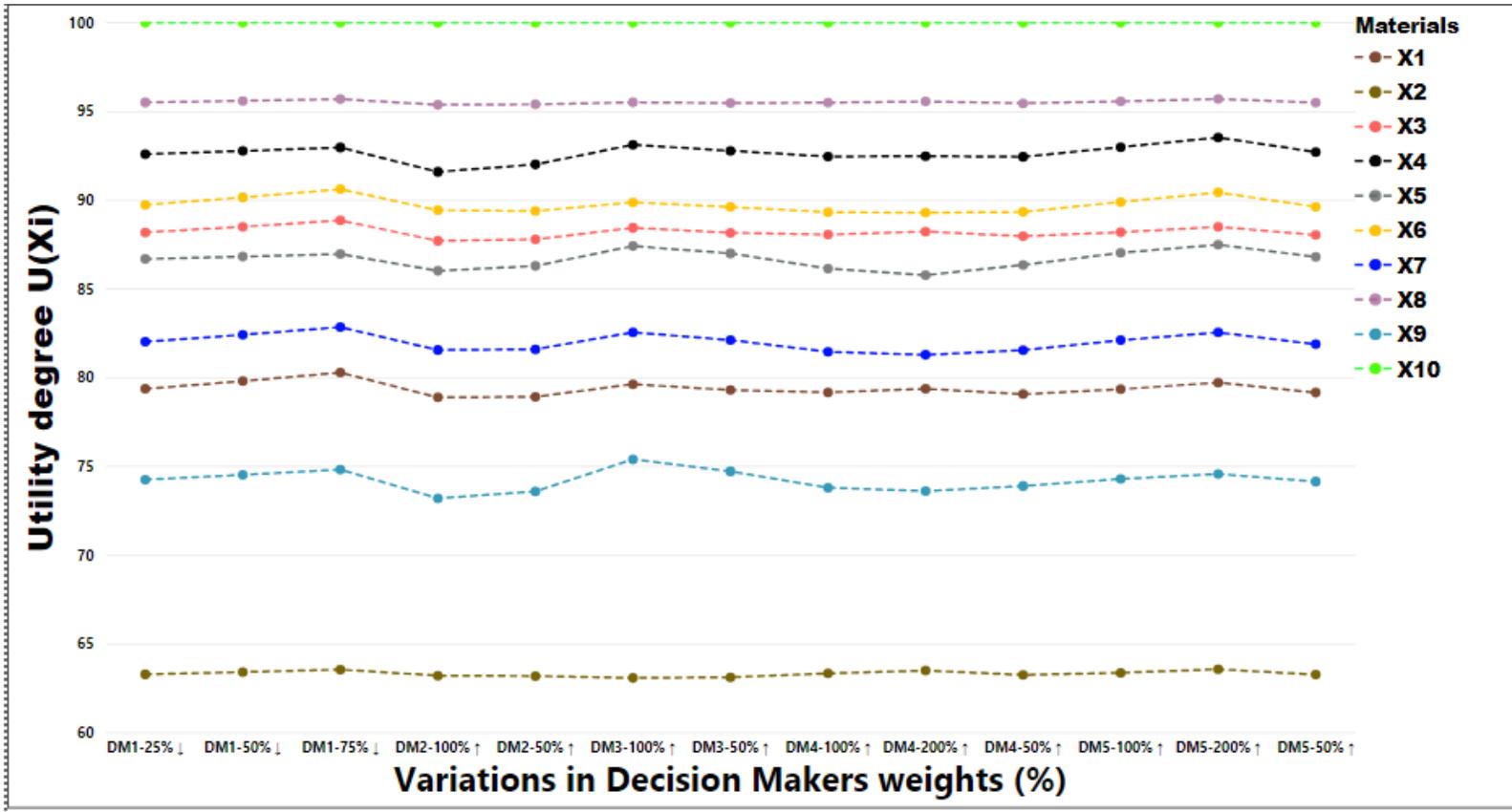


Figure 5. 25. Utility degree $U(X_i)$ behavior caused by changing DMs weights in the hybrid PFMCGDM COPRAS method

CHAPTER SIX

Summary and Conclusions

Research in the implementation of multi-criteria decision making (MCDM) tools in the material selection field has developed and grown over the last few decades. Many methods and approaches have been studied and examined for their use in selecting the optimal alternatives for material and mechanical designs. The TOPSIS and COPRAS MCDM approaches have been commonly used as ranking tools in the material selection applications due to their practicality and lack of complications. For this reason, many studies have extended the use of these methods, combining them with fuzzy sets and intuitionistic fuzzy sets (IFS) to overcome the lack of uncertainty modeling in these decision-making approaches. Therefore, this research has developed two hybrid MCDM approaches through integrating a new extension of fuzzy sets called the Pythagorean Fuzzy sets (PFS), which is more effective in modeling uncertainty than fuzzy and intuitionistic fuzzy sets, with TOPSIS and COPRAS MCDM methods.

This study has focused on developing practical hybrid multi criteria group decision making (MCGDM) methods by converting a group of human (decision makers) judgments and opinions (crisp data) to Pythagorean fuzzy numbers (PFN) and integrating them with a ranking method that is built to solve the induced Pythagorean fuzzy multi criteria group decision making (PFMCGDM) matrix and rank the alternatives. A novel aggregation approach has been suggested to derive crisp data into PFNs considering decision makers' (DMs) weights and uncertainty degrees. The Sugeno fuzzy measure and the Shapley value methods have been implemented to fairly distribute DMs powers within the aggregation process. Additionally, the Takagi-Sugeno approach has been applied to build the uncertainty degree functions based on fuzzy base rules to be aggregated within the PFNs.

The applicability of using PFMCGDM TOPSIS and COPRAS methods has been investigated by applying it to solve a real life materials selection problem, specifically in selecting the best material for high pressure turbine blades in jet engines. Both methods have expressed different results in the final ranking process, sharing in common the best and worst material alternatives that are Rene 80 (X10) and Inconel 625 (X2), respectively. Moreover, a computer application has been developed based on the proposed methods to assist designers and engineers in the real world material selection problems.

Sensitivity analysis has been applied to test the effect of the input parameters, specifically within the developed aggregation approach, on the ranking output results in both MCGDM methods. Different scenarios have been suggested to manipulate DMs weights. In the PFMCGDM TOPSIS method, the final rankings have changed and the revised closeness coefficient for some material alternatives have noticeably differed under certain scenarios. On the other hand, the PFMCGDM COPRAS method has shown no changes in the final ranking outcomes; however, small changes in the utility degrees for all alternatives have been observed in response of varying DMs weights.

It can be inferred from these results that the PFMCGDM COPRAS method is less sensitive than the PFMCGDM TOPSIS method when interacting with changes within the input parameter in the aggregation phase. The reason for that result is the nature of the mathematical approach of the Pythagorean fuzzy TOPSIS ranking method in measuring the Pythagorean fuzzy positive and negative ideal solutions for each criterion with respect to all alternatives. This is accomplished based on the score function taking in consideration the aggregated PFNs; therefore, the alternative with the farthest distance from the negative ideal solution and the closest distance from the positive ideal solution, simultaneously, will be named as the optimal

alternative. Contrarily, the Pythagorean fuzzy COPRAS method's mathematical approach to determine the optimal alternative is based on calculating the summation of values for all beneficial criteria and non-beneficial criteria in each alternative index and then obtaining their score functions, resulting in more resistance to the changes in DMs' weight, since all beneficial and non-beneficial criteria are being considered.

In summary, this research determined that the Pythagorean fuzzy TOPSIS interacts effectively with the novel aggregation approach for the aforementioned reasons. Therefore, using the developed hybrid PFMCGDM TOPSIS method in materials selection decision making problems is more recommended than the hybrid PFMCGDM COPRAS method.

CHAPTER SEVEN

Recommendations for Future Work

This study can be improved if numerical data are more involved in the hybrid MCGDM methods. For example, data measurements can be normalized and assigned weights as a decision maker to contribute to the aggregation of the Pythagorean Fuzzy satisfactory and dissatisfactory numbers. Another way to incorporate measured numerical data in the ranking method is through using it in calculating the criteria weights.

Moreover, the suggested MCGDM methods can be applied to many different fields such as logistics, finance, healthcare and facility locations. Furthermore, it is recommended to use the aforementioned methods in decision making processes that require involving DMs with their opinion and judgments.

Further investigations are needed to report the behavior of the PFMCGDM COPRAS method in response to varying the input parameters. In other words, different scenarios should be investigated such as changing the weight of multiple DMs at the same time for more accuracy in analysis.

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Material alternative	Criteria				
	Cost	Yield strength (MPa)	Tensile strength (MPa)	Young's modulus (GPa)	Service temperature (°C)
Inconel 625	20	460	880	160	982
Pyromet 680	25	241	421	165	540
Haynes R-41	25	869	1040	169	982
INCONEL 706	25	993	1300	210	872
UDIMET 700	30	1057	1361	190	780
Ti-6Al-2Sn-2Zr-2Mo	35	1015	1237	122	900
AISI 202, Wrought	30	580	980	170	910
Rene 41	45	938	1048	210	1000
AISI 302, Wrought	30	550	920	165	710
Rene 80	45	980	1123	200	1050

You are now asked based on your knowledge to rate each criterion with respect to nominated material to achieve the requirements for high pressure turbine blades from (0 – 100) with 0 being the worst grade and 100 being the best grade

Material alternative	Criteria				
	Cost	Yield strength (MPa)	Tensile strength (MPa)	Young's modulus (GPa)	Service temperature (°C)
Inconel 625					
Pyromet 680					
Haynes R-41					
INCONEL 706					
UDIMET 700					
Ti-6Al-2Sn-2Zr-2Mo					
AISI 202, Wrought					
Rene 41					
AISI 302, Wrought					
Rene 80					

CURRICULUM VITAE

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EDUCATION

- University of Wisconsin - Milwaukee (UWM), WI - USA*
Ph.D. in Industrial and Manufacturing Engineering 2019
- Milwaukee School of Engineering, Milwaukee WI - USA*
M.Sc. in Engineering Management 2013
Area of Concentration: Project and Operations Management
- Milwaukee School of Engineering, Milwaukee WI - USA*
Master's Certificate in Project Management 2012
- University of New Orleans, New Orleans LA - USA*
Management Courses 2010
- King Abdul Aziz University, Jeddah - Saudi Arabia*
B.Sc. in Industrial Engineering 2009

EXPERIENE

- Grader**, *University of Wisconsin, Milwaukee (UWM), Milwaukee, WI, USA* Spring, 2014
- Grade exams and quizzes for industrial and manufacturing engineering courses.
 - Teach the students and answer their questions regarding the material.
- Projects Control Engineer**, *National Water Company (NWC), Jeddah, Saudi Arabia* Oct, 2013- Aug, 2014
- Monitor project progress toward milestones and identify risks and roadblocks.
 - Review contractor's submittals regarding time and cost, and give recommendation to the PM.
 - Develop monthly project status reports that describe the overall performance of projects.
 - Check the delayed projects with PM and provide effective recommendations.
 - Prepare the monthly narrative, cost, schedule and financial reports, and monitor submission of same.
- Quality and Development Engineer**, *Jeddah Municipality, Jeddah, Saudi Arabia* Jul, 2009 - Feb, 2010
- Oversee and track the performance of the cleaning vehicles and improve its efficiency.
 - Evaluate and monitor transfer stations performance in north Jeddah and give detailed status reports along with suggested solutions for occurred issues.
 - One of the group members who were assigned to reengineer and build a new guidebook for all services and processes of each department in the Municipality.
- Trainee**, *Saline Water Conversion Corporation (SWCC), Jeddah, Saudi Arabia* Summer, 2008
- Trainee in Operation and maintenance Department.
 - Visiting all engineering departments, control room and plants to understand its processes and operations.

COMPUTER SKILLS

Software: C#, SAP system, Microsoft Office, Microsoft Project, Microsoft VISIO, Minitab, Primavera P6, Python.

Databases: Oracle database system, Microsoft Access.