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Equilibrium Modeling and Policy Analysis of a Biofuel Supply Chain with a Hydroelectric Reservoir

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EQUILIBRIUM MODELING AND POLICY ANALYSIS OF A BIOFUEL SUPPLY CHAIN WITH A HYDROELECTRIC RESERVOIR

by

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ABSTRACT

EQUILIBRIUM MODELING AND POLICY ANALYSIS OF A BIOFUEL SUPPLY CHAIN WITH A HYDROELECTRIC RESERVOIR

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This research proposed a game theoretic model of a biofuel supply chain (BSC) where a utility company supplies reservoir water to two farmers, located in downstream and upstream of a hydropower dam. The decision-making process of the model is formulated as a three-stage Stackelberg game. We analyze the equilibrium of the decentralized systems and the effect of the government subsidy on energy crop (switchgrass) production for cellulosic biofuel industries, with two forms of subsidy: (1) discriminated subsidies and (2) equalized subsidies.

The results show that both forms of subsidy improve social welfare in the BSC unless the amount of subsidy exceeds certain limits, in which case there are negative margins for the farmers, and disappearance or monopoly of the markets. Increasing the subsidy to the upstream farmer is more efficient in improving social welfare than equalizing the subsidies to the two farmers. Increasing the subsidy to the downstream farmer shows the least efficiency in improving social welfare.

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1. INTRODUCTION

Water, energy, and food are essential resources. Since demand for these resources for human activities, technologies, industries, and even survival. Since the demand for these resources has been rising steadily with increase in the world's population and industrial development, resource security is becoming a major issue for policymakers and government departments. Currently, the rapid population growth in many regions of the world and associated economic development are increasing demand for electricity and putting pressure on freshwater resources (IEA, 2012). The population increase could also threaten food security. It is estimated that energy consumption worldwide in particular will have increased by 50% upto 2030 (Hightower and Pierce, 2008). These factors will exacerbate the energy crises and water shortages in the world (Zhang and Vesselinov, 2016). Thus, we need to approach the water-energy-food nexus through an understanding of the interaction between the water, energy, and food sectors in order to improve their security.

Security of water, energy, and food is inextricably linked, and therefore they should not only be carefully managed as individual resources but also be understood in the perspective of the interaction between them. The proper management of their connection should be given priority, as it has significant potential to increase the efficiency of resource allocation and utilization and to reduce social costs. A comprehensive understanding of the resource use and flow would help induce and maintain equilibrium of supply and demand, as against an imbalance that may result in inefficient resource allocation and the consequent excessive social costs. Efficiency of use of the water, energy, and food system could be improved and managed effectively through an analysis of the interaction between the three resource sectors. Conventional policy and decision-making processes need to adopt a nexus approach which would reduce the trade-offs and build synergies across whole sectors through integration (Hoff, 2011). The water–energy–food (WEF) nexus is an approach to assessment, policy development, and implementation that focuses on water, energy, and food security simultaneously (Bizikova et al., 2014). The 2011 Bonn conference provides evidence that improved water, energy, and food security can be achieved through the nexus approach that integrates management and governance across the three sectors, supporting the transition to a green economy that has greater policy coherence and uses resources more efficiently (Hoff, 2011).

Amidst research on the water-energy-food nexus, research on the biofuel supply chain (BSC) is one of the most rapidly developing areas, since biofuel is a promising renewable energy source that can substitute for scarce fossil fuels. Until now, it has been produced mainly from food (first-generation) crops such as corn. In recent years, 40% of US corn was converted into ethanol (GRACE, 2014). However, biofuels are responsible for a 25–60% increase in corn prices (Sexton et al., 2009). In addition to price, land availability is an important factor affecting food security (Cobuloglu and Büyüktahtakın, 2015). Dependence on biofuel from food crops could undermine the security of food supplies such as corn (Cobuloglu and Büyüktahtakın, 2017).

Cellulosic ethanol, refined from energy (second-generation) crops, is one of the most promising alternatives to food-based bioethanol. Switchgrass, especially, one of the cellulosic feedstocks, is widely recognized as a leading crop for ethanol production in the U.S. according to social, economic, and environmental criteria (Bai et al., 2010). Based on life cycle assessment (LCA), the production of switchgrass ethanol has been shown to cause lower greenhouse gas (GHG) emissions than that of corn-based ethanol, because of higher yield, ability to store carbon in soil, and fewer fertilizer and energy inputs (Davis et al., 2012; Larson, 2006; Wright, 2010).

Security of water supply is vital for both first-generation and second-generation biofuel crops, since a huge amount of water is consumed by irrigation. Agriculture accounts for the largest consumptive water use which is not returned to a water source. If the return flow is polluted or heated, that may also be considered consumptive use because the changed water properties compromise further uses (Hoff, 2011). Agriculture consumes about 70% of fresh water in the world and accounts for 80–90% of consumptive water use in the United States (Pimentel et al., 2004; Schaible and Aillery, 2012).

As one of the largest consumers of water, agriculture competes directly with the energy sector for water resources. However, agriculture also contributes indirectly to the energy sector through biofuel production. Both connections will be strained by increasing concerns over water availability and quality (U.S. Department of Energy, 2014).

Hydropower is another major source of renewable energy. In 2012, global hydroelectricity generation reached 3,646 TWh, which accounted for about 77% of total renewable electricity generation and it supplied 18% of the total electricity consumed (Zhang et al., 2018). Since they are usually a domestic source of energy and water, hydropower plants with reservoirs could help manage energy security and water security if the hydropower systems could be developed as integrated systems for hydroelectricity generation and water supply (IEA, 2012). Water stored in a hydropower reservoir can be used for irrigation, industry and domestic supply. Since hydropower plants are usually located in upstream regions, the water released to generate hydroelectricity can be made available for irrigation in the downstream regions (Zhang et al., 2018). In other words, the water stored in reservoirs and the water released to the downstream areas can be used for irrigation in upstream regions and downstream regions, respectively. So, considering hydropower systems with water supply for irrigation could improve food security (Water Resources and Environment Administration, 2008).

Water systems supply water for human use such as drinking, irrigation, or industry. Although water is a public resource for everyone, excessive use and unequal water supply would cause scarcity of water and compromise the right to equal access to water. Water pricing could be a key factor in promoting efficient resource allocation and preventing anyone's exclusive possession of water. In this context, the price does not need to be the same for all units sold; non-linear pricing is shown to permit people to enjoy low prices for their essential uses of water but pay higher prices if they consume beyond a certain threshold quantity. Non-linear marginal cost pricing of water permits separation of the relatively more essential (low volume, low demand elasticity) uses of water from the more optional (high volume, higher demand elasticity) uses of water. On the supply side, many sources of water are shared in "common" and therefore unregulated markets tend to deplete and degrade sources of water at rates greater than the efficient rates (Holahan, 2010). In this research, it is assumed that water price is imposed in the form of a convex quadratic price function to reconcile the supply imbalance.

In this research, our model considers a utility company as a private water supply firm that tries to maximize its own profit rather than other values such as social welfare. In economics, a private firm maximizes its own profit while a public firm maximizes social welfare, in general. However, in the privatization neutrality theorem, social welfare is the same before and after privatization when the government gives optimal subsidies to both public and private firms. Fulton and Karp (1989) studied the performance of a public firm in a natural-resource industry and concluded that the public firm pursues objectives other than welfare maximization. As shown in previous research, private firms and public firms may make decisions for the same objective of profit maximization. Thus, while this research assumes that the utility company is a private firm, it could be regarded as a kind of public firm under certain conditions. However, this paper only focuses on the situation in which the utility company pursues profit maximization rather than social welfare maximization.

2. LITERATURE REVIEW

This section reviews research on the biofuel supply chain (BSC) and game theory in water-energy-food nexus.

2.1. Biofuel Supply Chain

The BSC, one of the most popular research areas in the water- energy-food nexus, had been developing even before the concept of the water-energy-food nexus appeared. Our model is also based on the BSC and incorporates the conventional BSC with water supply and hydropower generation.

Several studies on the BSC have been formulated in centralized optimization models, where a single decision maker makes a decision to maximize or minimize the objective function (Del-Mas et al., 2011; Awudu and Zhang, 2013; Marufuzzaman et al., 2014; Xie et al., 2014; Cobuloglu and Büyüktahtakın, 2014; Kim et al., 2011). For example, Xie et al. (2014) proposed a mixed-integer linear programming (MILP) model to minimize transportation costs of cellulosic feedstock through optimal location of biorefineries, hubs, and terminals. Cobuloglu and Büyüktahtakın (2014) developed a MILP model to find best decisions on seeding method, harvesting time, and land types, while considering the economic and environmental impacts of switchgrass biomass production. Kim et al. (2011) developed a MILP model to maximize profits of biofuel production through best transportation method, biomass locations, and biorefinery capacity and technologies. Cobuloglu and Büyüktahtakın, (2015) proposed a multi-objective mixed-integer optimization model to maximize economic and environmental benefits with optimal decisions on land allocation, seeding time, harvesting time and amount, and budget allocation. Cobuloglu and Büyüktahtakın (2017) extended their previous model to a two-stage stochastic mixed-integer programming model. Azadeh et al. (2014) proposed a stochastic linear programming model within a multi-period planning framework to maximize the expected profit. Papapostolou et al. (2011) proposed a mixed integer linear programming model to maximize performance of the BSC.

Compared to the centralized framework, the BSC model with a decentralized framework could better consider and analyze rational behaviors of each entity in the BSC (e.g. farmers and refineries). The entities, as decision makers in the model, make decisions independently to optimize their own objectives which can be in conflict with one another. Bai et al. (2012) proposed a bilevel Stackelberg leader-follower game theoretic model of an integrated BSC with farmers' decision on land uses and markets, and dynamic feedstock prices under market equilibrium. Under the decentralized framework, the government policy can be applied to the

BSC model as a form of regulation or subsidy. Bai et al (2016) proposed a Stackelberg game theoretic model to incorporate more options on land use and possibility of marginal land reclamation in a land market, with cap-and-trade regulatory mechanism for land-use constraints. Luo and Miller (2013) proposed a game theoretic model of Cournot competition between farmers and Stackelberg between switchgrass and corn ethanol producers, while considering the farmers and the ethanol producers. However, this research does not study the socio-economic impact of the subsidies on the BSC. Another game theoretical model, proposed by Bajgiran (2018), is modeled as a Cournot-Stackelberg game between a farmer and multiple biofuel refineries, and analyzes the effect of government subsidies on the BSC.

Besides the abovementioned research, mathematical programming models of the BSC have been developed, in a variety of research papers. Sharma et al. (2013) reviewed 32 research papers to analyze mathematical programming models for the BSC with focus on facility location and capacity. De Meyer et al. (2014) reviewed 71 research papers on biomass-for-bioenergy supply chain with focus on optimization methods used in the BSC. Ghaderi et al. (2016) reviewed 146 research papers on biomass supply chain network design (BSCND) and classified them into three classes: facility-related, biomass-related, and final product-related. Of the 146 papers, none considers decisions regarding water supply and irrigation for biomass production with BSC. Most of the research studied determination of facility capacity and location, biomass type, land allocation, and final products type.

Although there is a variety of research on the mathematical modeling of the BSC, Ghaderi et al. (2016) addressed the lack of multi-objective problem research which accounts for only 22.6% of the BSC papers. Out of the 146 papers, only 14 papers (9.6%) proposed non-linear programming, which is more flexible and practical to deal with real-world problems than mixed-integer linear programming (MILP), which is applied in 109 papers (74.6%) (Ghaderi et al., 2016). Consequently, the BSC field needs more research on multi-objective problems and non-linear programming approaches.

Hydropower systems can be integrated with irrigation for the biomass crops. For example, Lacombe et al. (2014) studied the effect of hydropower development on irrigation in the Nam Ngum River Basin. The research found that full hydropower development could increase river flow during the dry season and improve water availability for irrigation. Since this research only considers the impacts of development within the Nam Ngum sub-basin, additional analysis of collective influences in the wider Mekong basin needs further research (Zhang et al., 2018). However, to our knowledge, only a few researchers have studied the BSC with hydropower systems and water supply systems for irrigation, although the integrated system has a high potential to simultaneously consider a water system and the BSC in the water-energy-food nexus.

In this research, we propose a mathematical model for decision-making on biomass type and land allocation. Our BSC model also deals with hydropower generation and water supply for irrigation, which has not been dealt with much in previous BSC research. The BSC model is formulated as a non-linear programming with a three-stage Stackelberg game, where three players maximize their own objectives (profits) in the game and the government also promotes its objective (social welfare) out of the game. The Stackelberg game theoretic approach is proposed to solve multi-objective problems. Moreover, our BSC model consists of two biomass suppliers (farmers), two kinds of biofuel producers (biomass refineries) as a corn market and a switchgrass market, and a water supplier (utility company) that operates a hydropower plant and supplies water to the farmers for irrigation.

2.2. Game Theory in Water-Energy-Food Nexus

More than one decision maker can be involved, in the water-energy-food nexus. The decision makers take decisions to achieve their own objectives, interacting with each other.

Game theory can provide a framework to study the strategic actions of individual decision makers to develop acceptable solutions when the decisions of multiple firms mutually affect the outcomes of other decision makers. Also, game theory could derive practical results under conditions of competition between firms, since this method reflects the interaction between the involved parties, which is often neglected by conventional optimization methods of solving multi-criteria multi-decision-maker problems (Madani, 2010).

Hence, game theoretic approaches can be used to analyze the Nash equilibrium in a multi-stakeholder model for the water-energy-food nexus. Especially if one entity is more influential than the others or is an external arbiter such as a regulating agent, and they want to manage water, energy, and food flows between other parties, a leader-follower type game could prove valuable (Garcia and You, 2016). The mathematical models could describe and explain the rationalization of the players' decisions and their results in the water-energy-food nexus.

There are three traditional competition models in game theory: a Cournot model, a Bertrand model, and a Stackelberg model. The Bertrand model addresses price competition between firms in a simultaneous game, while the Cournot model and the Stackelberg model compete on the quantity produced in a simultaneous game and a sequential game, respectively.

The Cournot model was first proposed by a French mathematician, Antoine Augustin Cournot in 1838 (Siriruk, 2009). The basic Cournot model is a static model where each firm rationally forecasts other firms' decisions. Given the forecast, firms simultaneously make the decision to maximize their own profit (Varian, 2006). The Cournot competition is a quantity competition where the firms make decisions on quantity rather than on price. In each firm's problem, the quantities supplied by other firms are assumed to be fixed and do not change depending on price change (Siriruk, 2009). The Cournot model derives a Nash equilibrium solution for the optimal quantities produced by each firm. The market price of the output is determined by the equilibrium solution with a given demand function of the market. When solving the single-level game, we can solve an optimization problem through putting together KKT conditions of each firm's problem.

The Bertrand competition was first studied by Joseph Bertrand who pointed out that firms compete primarily in prices (Prokop et al., 2015). In the static model of price competition in duopoly, two firms produce a homogeneous good at identical and constant marginal cost. This game theoretical model is assumed to have no capacity constraint, so that each firm can satisfy the entire demand of the market. The firms set prices of their products simultaneously and independently. With identical prices quoted by the firms, the demand is split equally between the two firms. On the other hand, with discriminated prices, a firm that quotes lower price takes all the demand in the market since consumers would purchase the good at the lowest price. In the case of the Bertrand game, the winner takes all demand while the other firm takes nothing.

The Stackelberg was first described by a German economist, Heinrich Freiherr von Stackelberg, who in 1934 studied competition between two firms selling a homogeneous good (Von Stackelberg et al., 2010). The concept of the Stackelberg game is extended to various research areas to study situations containing a leader–follower relationship (Chu and You, 2014; Chu et al., 2014). In a standard Stackelberg game, a leader takes actions first, and then a follower makes best responses to the leader's decisions rationally. Hence, the two players make their best decisions sequentially in the Stackelberg game. In the game, the follower observes the leader's decisions and the leader knows the follower's best responses to its decisions. The leader has the advantage of moving first and the advantage lets the leader gain a larger profit than the follower. On the other hand, if the leader does not guarantee a certain degree of incentives to the follower, the follower may refuse to participate in the supply chain and the leader's strategic plan may become infeasible or unprofitable (Yue and You, 2014).

For example, a single-leader-single-follower Stackelberg game can be formulated as a bilevel programming problem (Bard, 1998; Colson et al., 2007), as follows:

$$\max_{x \in X} F(x, y)$$

$$\text{s.t.}\, A_i(x, y) \leq 0 \quad i = 1, \cdots, m$$

$$\text{s.t.}\, B_j(x, y) = 0 \quad j = 1, \cdots, n$$

$$\text{where } y \text{ solves } \max_{y \in Y} G(x, y)$$

$$\text{s.t.}\, C_k(x, y) \leq 0 \quad k = 1, \cdots, r$$

$$\text{s.t.}\, D_l(x, y) = 0 \quad l = 1, \cdots, s$$

In this Stackelberg game, the leader's decision variables are denoted by $x \in X$ and the follower's decision variables are denoted by $y \in Y$. The leader's objective function, inequality constraints, and equality constraints are denoted by $F(x, y)$, $A_i(x, y)$, and $B_j(x, y)$, respectively. The function and the constraints depend on both the leader's decisions, x, and the follower's decisions, y. The follower's objective function, inequality constraints, and equality constraints are denoted by $G(x, y)$, $C_k(x, y)$, and $D_l(x, y)$, respectively.

The bilevel program is also called a "mathematical program that contains an optimization problem in the constraints" (Bracken and McGill, 1973), because the value of y in the leader's problem is obtained by solving the follower's optimization problem. The leader's decision variables x are treated as given parameters in the follower's optimization problem since the leader's decisions have already been made at the time when the follower takes actions.

When solving a bilevel game such as a leader-follower game, a lower-level optimization problem usually can be embedded as constraints in an upper-level optimization problem. In case the lower-level optimization problem is replaced with the form of equivalent variational inequalities or KKT conditions, we can transform the bilevel problem into a single level optimization problem that consists of equilibrium constraints (Bajgiran, 2018). Such single level problems are called mathematical programs with equilibrium constraints (MPEC) (Luo et al. 1996). MPECs have been extensively employed in various research areas and industries including energy, transportation, and production. For example, Koh (2012), Allevi et al. (2018), and Siddiqui and Christensen (2016) considered MPECs as non-linear programming (NPL), and special algorithms have been developed to solve them.

In our biofuel supply chain, the two farmers compete for water allocations of the utility, and each farmer solves a trilevel problem where the farmers (leaders) maximize their own profit at the upper level problems and the lower level problems, and the utility company (follower) maximizes its own profit at the middle level problem. Because of the convexity of the farmers' problems in the third stage, we can replace the problems with their KKT conditions and embed them in the utility company's problem in the second stage as new constraints and solve the resultant single level problem (MPEC). Likewise, because of the convexity of the utility company's problem in the second stage, we can replace the problem with its KKT conditions and embed the KKT conditions in the farmers' problems in the first stage as new constraints and solve the resultant single level problem (MPEC). Having derived the MPEC for each of the farmers, we need to jointly consider all MPECs to obtain the generalized Nash equilibrium, which is one of the main objectives of this research. For that, we obtain the KKT conditions of each single level problem and combine them into one single optimization problem. The new problem is called equilibrium problem with equilibrium constraints (EPEC), which has been previously addressed in other works and industries, especially the electricity market (e.g, Pozo and Contreras 2011; Ruiz et al., 2012; and Kazempour et al., 2013), but not much in research on biofuel supply chain.

The Cournot game, the Stackelberg game, and the Bertrand game can be combined to model complicated game theoretic problems. For example, Assila et al. (2017), Caldentey and Haugh (2017), and Ruiz-Hernández et al. (2017) studied the combined Cournot-Stackelberg game to deal with game theoretic models with more than one leader, follower or both. Ma and

Li (2014) and Zhang et al. (2015) studied the combined Stackelberg-Bertrand game to deal with pricing game models with more than one leader, follower or both.

In this research, we study a biofuel supply chain where two farmers and a utility company independently make their own decisions throughout three stages. At the first stage, the farmers quote water prices to the utility company. In the second stage, the utility company allocates water to the farmers, based on their price announcements. In the third stage, the farmers produce crops and sell them at a corn market and a switchgrass market, competing against each other. Our mathematical model consists of a Bertrand game and a Cournot game between the two farmers in the first stage and the third stage, respectively, and a Stackelberg game between the farmers and the utility company. Our model is formulated as a three-stage Cournot-Stackelberg-Bertrand game.

3. MODEL DESIGN

3.1. Comprehensive Problem Description

This research models the equilibrium of a biofuel supply chain (BSC) with three stages of the decision-making process, which consists of three entities (players): a downstream farmer (F_d), an upstream farmer (F_u), and a utility company (U_0).

In this BSC, the two farmers are located in two discrete regions; the downstream side and the upstream side of a hydroelectric reservoir. Each of the abovementioned farmers could be regarded as a farmer union of small farmers in each region. Forming a union can bring them benefits such as having a more advantageous position in contract negotiation with refineries, avoiding unproductive competition with each other, and protecting themselves against large corporates.

Both farmers produce corn and switchgrass and sell their crops at a corn market consisting of corn-based refineries and at a switchgrass market consisting of switchgrass-based refineries. So, both markets are duopolies. The farmers compete against each other in the two markets. The downstream and upstream farmers both have identical technology, equipment, and capability to cultivate corn and switchgrass. The outputs of the farmers are homogenous. Both farmers make decisions to maximize their own profits, competing against each other at the corn market and the switchgrass market. The farmers quote the water prices at which they want to buy at the beginning of a season to secure the amount of water they will use during the season.

The utility company manages the reservoir, operates the hydropower dam, and sells water to both farmers. The amount of water the utility company sells to the farmers is determined based on the water prices quoted by the farmers. In addition to the revenue from the water sales to the farmers, the utility company also earns revenue from selling hydroelectricity at an electricity market. The utility company has a low market share and little power to influence the electricity price at the market; the utility company is a price-taker.

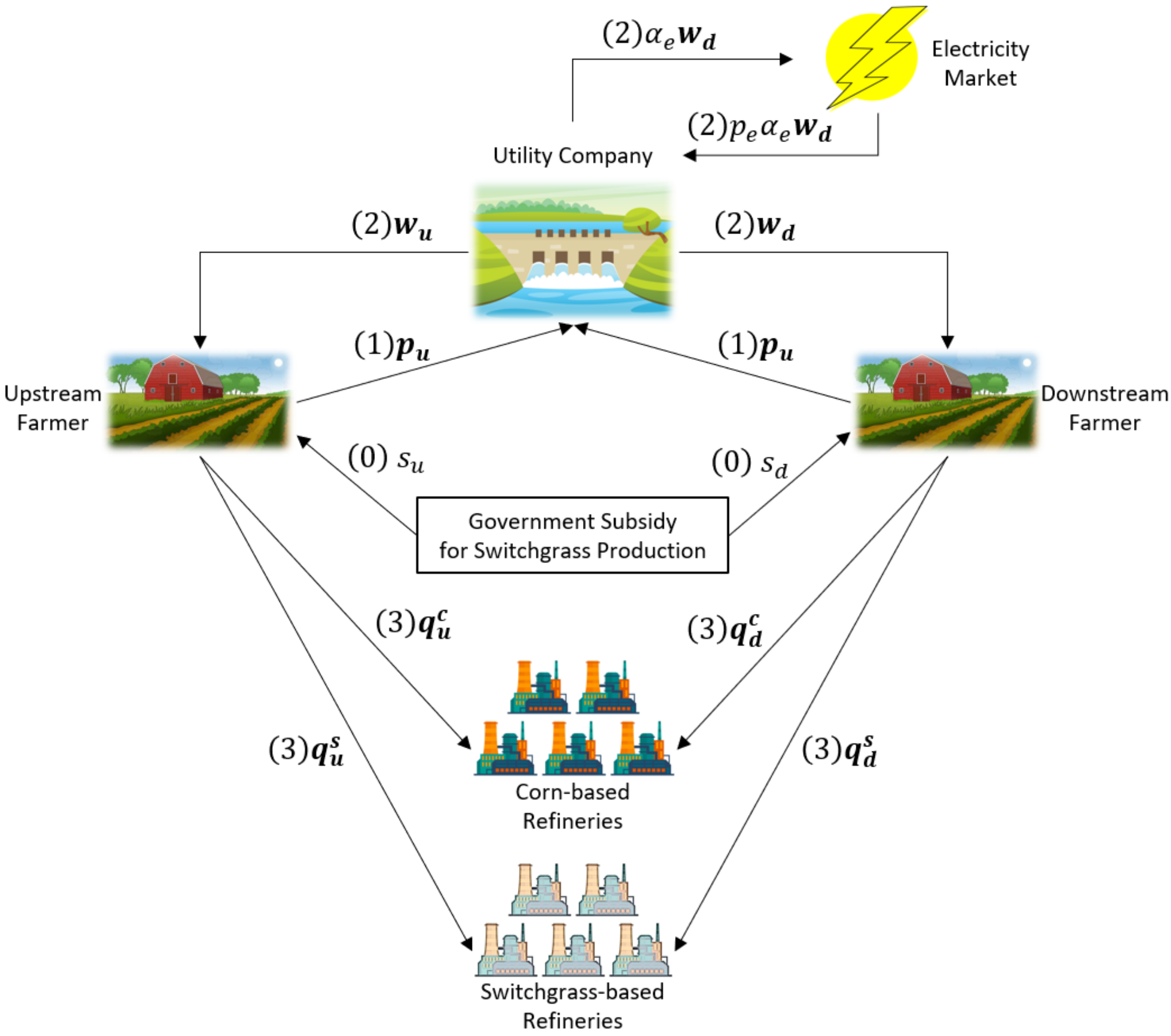

Figure 1. The entire framework of the interactive decision-making in the model (the numbers in parentheses are the order of decision-making in the BSC)

The structure of the BSC is depicted in Figure 1. The sequence of the decision process in the BSC is as follows:

(0) Out of the BSC and before the game, the government announces subsidies to the farmers (s_d, s_u).

(1) In the first stage, the farmers decide water prices (p_d, p_u) and announce them to the utility company for procurement. At this stage, they do not yet decide the land allocation to corn and switchgrass.

(2) In the second stage, after observing the price announcement, the utility company decides water quantities (w_d, w_u) to allocate to the farmers. The company knows that they can generate hydroelectricity using the water released to the downstream of the dam and sell the electricity to an electricity market.

(3) In the third stage, after observing the water allocation, the farmers decide land allocations $(q_d^c, q_d^s, q_u^c, q_u^s)$ for corn and switchgrass production in order to compete against each other in the corn market and the switchgrass market.

The utility company decides the water allocations to the two farmers depending on the water prices announced from the farmers. The water allocations are affected not only by the water prices but also by the sale of hydroelectricity generated by water released to the downstream. The amounts of water allocated affect the farmers' land allocation for the crops, since the two crops have different water requirements and different prices in the markets. At the two markets, the prices of the products are determined by the total amounts of commodities (corn and switchgrass) provided by the farmers and available at the markets.

This study analyzes the interdependency of the decisions and models the decision process as a three-stage Stackelberg game. At the first stage, the amount of water released to farmers depends not only on their own quote of the water price, but also on the water price quoted by the competitor. So, there is a Bertrand game between the downstream farmer and the upstream farmer for water allocation. At the third stage, since the revenues from the sales of corn and switchgrass by farmers are determined not only by their decisions on the crops' production amounts, but also by the market prices of the commodities, we have a Cournot game at both the corn market and the switchgrass market. Since we have a sequence of decision-making throughout the three stages, where decisions in a stage affect decisions in the following stages, we have a leader-follower Stackelberg game.

In this game, the decisions at each stage affect the decisions of other stages in a cyclic feedback structure. The farmers' decisions on the water prices (p_d, p_u) in the first stage affect the utility company's decisions on water allocation (w_d, w_u) in the second stage. The utility company's decisions affect the farmers' decisions on land allocation $(q_d^c, q_d^s, q_u^c, q_u^s)$ in the third stage. Then, the water allocation in the second stage and the land allocation in the third stage affect the farmers' decisions on water prices in the first stage. In this decision cycle, players' decisions at each stage are dependent on those in other stages.

In the model, we assume that the farmers' land sizes are big enough so that they can consume any amount of water allocated by the utility company based on their quoted water prices. If the farmers are small and the utility company has enough water to meet the demands of the farmers, there will be no game between the farmers for water; in such as case only the Cournot competition exists at the corn market and the switchgrass market.

Due to the societal benefits of cellulosic bioethanol, the government subsidizes the energy crop (switchgrass) production rather than the food crop (corn) production in the BSC. Because the government subsidies affect the decisions of the firms in the BSC, we consider the subsidies in the model. In the model, the government is not a player in the game, but an entity that sets the condition of the game environment before the game begins.

Farmers are willing to produce corn rather than switchgrass, since corn supply chains are well developed, contrary to the only recently currently emerging switchgrass supply chains. Since the government should consider not only energy security but also food security, the amounts of the subsidies are determined to prevent the corn market and the switchgrass market from disappearing or becoming monopolistic markets. We assume that the government decides the amounts of the subsidies to induce the farmers to produce both corn and switchgrass because consumer surplus and social welfare under duopoly would be higher than those under monopoly.

The decision variables and parameters are shown in Table 1 and Table 2, respectively.

Table 1. Decision variables

Decision Variables		Unit
p_d	Water price the downstream farmer offers to the utility company	\$/gal
p_u	Water price the upstream farmer offers to the utility company	\$/gal
w_d	Water quantity the utility company allocates to the downstream farmer	gal
w_u	Water quantity the utility company allocates to the upstream farmer	gal
q_d^c	Land area of the downstream farmer for corn production	ha
q_d^s	Land area of the downstream farmer for switchgrass production	ha
q_u^c	Land area of the upstream farmer for corn production	ha
q_u^s	Land area of the upstream farmer for switchgrass production	ha

Table 2. Parameters

Parameters		Unit
w	Water capacity of the utility company	gal
α_c	Amount of corn grown in unit land	t/ha

α_s	Amount of switchgrass grown in unit land	t/ha
α_e	Amount of hydroelectricity generated by release of unit water	kWh/gal
p_e	Price of the hydroelectricity at an electricity market	\$/kWh
c_c	Cost of corn production	\$/t
c_s	Cost of switchgrass production	\$/t
c_w	Cost of processing water supply	$\$/gal^2$
δ_c	Water requirement per unit land for corn production	gal/ha
δ_s	Water requirement per unit land for switchgrass production	gal/ha
a_c	Reservation price at a corn market	\$/t
b_c	Marginal price per unit quantity at a corn market	$\$/t^2$
a_s	Reservation price at a switchgrass market	\$/t
b_s	Marginal price per unit quantity at a switchgrass market	$\$/t^2$
$P_c(\cdot)$	Inverse demand function at a corn market	\$/t
$P_s(\cdot)$	Inverse demand function at a switchgrass market	\$/t
s_d	Subsidy to the downstream farmer for switchgrass production	\$/ha
s_u	Subsidy to the upstream farmer for switchgrass production	\$/ha

3.2. Mathematical Model

In this section, we introduce the objective functions, constraints, and decision variables of the players in the three stages of the decision process in the BSC. The farmers are leaders in the first stage and followers in the third stage: they decide water prices, p_d and p_u, in the first stage, and land allocation for crop production, $q_d^c, q_d^s, q_u^c,$ and q_u^s, in the third stage. The utility company is a follower and decides water allocation, w_d and w_u, in the second stage.

We find the equilibrium of the farmers' decisions in the first stage and the third stage, and the utility company's decision in the second stage by the backward induction procedure using their best response (BR) functions. First, we derive the best responses of the farmers through Karush-Kuhn-Tucker (KKT) conditions of their problems for the decision variables of the third stage. Second, we derive the best response of the utility company through KKT conditions of its problems for the decision variables of the second stage. Third, we derive leaders' optimal decisions through KKT conditions of their problems for the decision variables of the first stage.

In this section, the backward induction solution process is followed in this order:

(1) We derive the best responses of the farmers in the third stage.

(2) We derive the best responses of the utility company in the second stage.

(3) We derive the optimal decision of the farmers in the first stage by using the utility company's best responses in the second stage and the farmers' best responses in the third stage.

(4) Then, we find the equilibrium of the farmers and the utility company by substituting the optimal decisions in the first stage into the best responses in the second and third stages.

3.2.1. Farmers' decision on land use in the third stage

In this section, we introduce the objective functions of the downstream and upstream farmers that maximize their own profits through their best decisions on land allocation, (q_d^c, q_d^s) and (q_u^c, q_u^s) respectively. The water capacity constraints are also introduced. In this third stage, the farmers make their best responses for land allocation based on the decision of the utility company on water allocation (w_d, w_u) in the second stage and each other's decisions on land allocation $(q_d^c, q_d^s, q_u^c, q_u^s)$.

All parameters in the model are strictly positive. However, water allocation (w_d, w_u) from the second stage can be zero. We have four cases to consider based on the positivity of the water allocation in the third stage: Case [1] $w_d = 0, w_u = 0$; Case [2] $w_d > 0, w_u > 0$; Case [3] $w_d = 0, w_u > 0$; and Case [4] $w_d > 0, w_u = 0$. The best responses for these cases of this third stage subgame are derived below and summarized in Table 3 and the derivation is presented in Appendix A. It is assumed that the parameter values meet the non-negativity conditions of the land allocation $(q_d^c, q_d^s, q_u^c, q_u^s)$ and two shadow prices (λ_1, λ_4) shown in the second column of Table 3.

3.2.1.1. Best response for Case [1] $(w_d = 0, w_u = 0)$

In this case neither the downstream farmer nor the upstream farmer produces any crop $(q_d^c = q_d^s = q_u^c = q_u^s = 0)$, since the farmers cannot utilize water $(w_d = w_u = 0)$. They do not make profit $(\pi_d = \pi_u = 0)$.

3.2.1.2. Best response for Case [2] $(w_d > 0, w_u > 0)$

(1) Formulation

For the downstream farmer, we have

$$\max_{q_d^c, q_d^s} \pi_d = [a_c - b_c\alpha_c(q_d^c + q_u^c)]\alpha_c q_d^c + [a_s - b_s\alpha_s(q_d^s + q_u^s)]\alpha_s q_d^s - c_c\alpha_c q_d^c$$

$$-c_s\alpha_s q_d^s + s_d q_d^s - w_d p_d \qquad (1)$$

$\delta_c q_d^c + \delta_s q_d^s \leq w_d$ $\quad(\lambda_1)\quad$ water capacity constraint (2)

$q_d^c, q_d^s \geq 0$ $\quad(\lambda_2, \lambda_3)\quad$ non-negativity constraints (3,4)

and, for the upstream farmer, we have

$$\max_{q_u^c, q_u^s} \pi_u = [a_c - b_c\alpha_c(q_d^c + q_u^c)]\alpha_c q_u^c + [a_s - b_s\alpha_s(q_d^s + q_u^s)]\alpha_s q_u^s - c_c\alpha_c q_u^c$$

$$-c_s\alpha_s q_u^s + s_u q_u^s - w_u p_u \qquad (5)$$

$\delta_c q_u^c + \delta_s q_u^s \leq w_u$ $\quad(\lambda_4)\quad$ water capacity constraint (6)

$q_d^c, q_d^s \geq 0$ $\quad(\lambda_5, \lambda_6)\quad$ non-negativity constraints (7,8)

In Equation (1) and Equation (5), the first two terms are the farmers' revenues from the corn market and the switchgrass market, respectively. The next two terms are the costs of crop production. The last two terms are the government subsidy for the switchgrass production and cost of water purchased from the utility company. In Equations (2) – (4) and (6) – (8), λs' are KKT multipliers, or the shadow prices, of the constraints.

(2) KKT condition

Now, we use KKT conditions to derive the best responses of the downstream farmer and the upstream farmer in the third stage. The KKT conditions of the problems are as follows:

For the downstream farmer,

$$\mathcal{L}_1 = \pi_d + \lambda_1(w_d - \delta_c q_d^c - \delta_s q_d^s) + \lambda_2 q_d^c + \lambda_3 q_d^s \tag{9}$$

$$\frac{\partial \mathcal{L}_1}{\partial q_d^c} = (a_c\alpha_c - c_c\alpha_c - b_c{\alpha_c}^2 q_u^c) - 2b_c{\alpha_c}^2 q_d^c - \delta_c\lambda_1 + \lambda_2 = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}_1}{\partial q_d^s} = (a_s\alpha_s - c_s\alpha_s - b_s{\alpha_s}^2 q_u^s + s_d) - 2b_s{\alpha_s}^2 q_d^s - \delta_s\lambda_1 + \lambda_3 = 0 \tag{11}$$

$$0 \le \lambda_1 \perp [w_d - \delta_c q_d^c - \delta_s q_d^s] \ge 0 \tag{12}$$

$$0 \le \lambda_2 \perp [q_d^c] \ge 0 \tag{13}$$

$$0 \le \lambda_3 \perp [q_d^s] \ge 0, \tag{14}$$

and, for the upstream farmer,

$$\mathcal{L}_2 = \pi_u + \lambda_4(w_u - \delta_c q_u^c - \delta_s q_u^s) + \lambda_5 q_u^c + \lambda_6 q_u^s \tag{15}$$

$$\frac{\partial \mathcal{L}_2}{\partial q_u^c} = (a_c\alpha_c - c_c\alpha_c - b_c{\alpha_c}^2 q_d^c) - 2b_c{\alpha_c}^2 q_u^c - \delta_c\lambda_4 + \lambda_5 = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}_2}{\partial q_u^s} = (a_s\alpha_s - c_s\alpha_s - b_s{\alpha_s}^2 q_d^s + s_u) - 2b_s{\alpha_s}^2 q_u^s - \delta_s\lambda_4 + \lambda_6 = 0 \tag{17}$$

$$0 \le \lambda_4 \perp [w_u - \delta_c q_u^c - \delta_s q_u^s] \ge 0 \tag{18}$$

$$0 \leq \lambda_5 \perp [q_u^c] \geq 0 \tag{19}$$

$$0 \leq \lambda_6 \perp [q_u^s] \geq 0 \tag{20}$$

(3) Best response

Because the lands of the farmers are large, the farmers consume all the water allocated to them by the utility company, and the water capacity constraints of the farmers are binding. So, from the complementary slack conditions of the KKT conditions of the water capacity constraints shown above, we have $[w_d - \delta_c q_d^c - \delta_s q_d^s] = 0, [w_u - \delta_c q_u^c - \delta_s q_u^s] = 0, \lambda_1 \geq 0,$ and $\lambda_4 \geq 0$.

The downstream and upstream farmers' land allocations for both corn and switchgrass are equal to or greater than zero ($q_d^c > 0, q_d^s > 0, q_u^c > 0, q_u^s > 0$). So, we assume $\lambda_2 = 0, \lambda_3 = 0, \lambda_5 = 0,$ and $\lambda_6 = 0$. In summary, we have these conditions in Case [2]: $\lambda_1 \geq 0, \lambda_2 = 0, \lambda_3 = 0, \lambda_4 \geq 0, \lambda_5 = 0,$ and $\lambda_6 = 0$.

These conditions are used to solve the KKT conditions, and the results are shown in Table 3.

3.2.1.3. Best response for Case [3] $w_d = 0, w_u > 0$

(1) Formulation

In this case, the downstream farmer does not produce any crop $(q_d^c = q_d^s = 0)$, since the downstream farmer cannot utilize water $(w_d = 0)$. Also, the downstream farmer does not make profit $(\pi_d = 0)$. In this case, the corn market and the switchgrass market are monopoly markets of the upstream farmer without competition. The objective function and the constraints of the upstream farmer are as follows:

$$\max_{q_u^c, q_u^s} \pi_u = (a_c - b_c\alpha_c q_u^c)\alpha_c q_u^c + (a_s - b_s\alpha_s q_u^s)\alpha_s q_u^s - c_c\alpha_c q_u^c - c_s\alpha_s q_u^s + s_u q_u^s - w_u p_u \qquad (5')$$

$\delta_c q_u^c + \delta_s q_u^s \leq w_u$ $\quad (\lambda_4)$ $\quad$ water capacity constraint $\quad$ (6')

$q_u^c, q_u^s \geq 0$ $\quad (\lambda_5, \lambda_6)$ $\quad$ non-negativity constraints $\quad$ (7',8')

(2) KKT condition

Now, we use KKT conditions to derive the best responses of the upstream farmer in the third stage. The KKT conditions of the problems are as follows:

$$\mathcal{L}_{2'} = \pi_u + \lambda_4(w_u - \delta_c q_u^c - \delta_s q_u^s) + \lambda_5 q_u^c + \lambda_6 q_u^s \qquad (21)$$

$$\frac{\partial \mathcal{L}_{2'}}{\partial q_u^c} = (a_c\alpha_c - c_c\alpha_c) - 2b_c{\alpha_c}^2 q_u^c - \delta_c\lambda_4 + \lambda_5 = 0 \quad (22)$$

$$\frac{\partial \mathcal{L}_{2'}}{\partial q_u^s} = (a_s\alpha_s - c_s\alpha_s + s_u) - 2b_s{\alpha_s}^2 q_u^s - \delta_s\lambda_4 + \lambda_6 = 0 \quad (23)$$

$$0 \le \lambda_4 \perp [w_u - \delta_c q_u^c - \delta_s q_u^s] \ge 0 \quad (24)$$

$$0 \le \lambda_5 \perp [q_u^c] \ge 0 \quad (25)$$

$$0 \le \lambda_6 \perp [q_u^s] \ge 0 \quad (26)$$

(3) Best response

Because the land of the upstream farmer is large, the upstream farmer consumes all the water allocated by the utility company, and the water capacity constraint of the farmer is binding. So, from the complementary slack conditions of the KKT conditions of the water capacity constraint shown above, we have $[w_u - \delta_c q_u^c - \delta_s q_u^s] = 0$ and $\lambda_4 \ge 0$.

The upstream farmer's land allocations for both corn and switchgrass are equal to or greater than zero $(q_u^c > 0, q_u^s > 0)$. So, we assume $\lambda_5 = 0$, and $\lambda_6 = 0$. In summary, we have these conditions in Case [3]: $\lambda_4 \ge 0, \lambda_5 = 0,$ and $\lambda_6 = 0$.

These conditions are used to solve the KKT conditions, and the results are shown in Table 3.

3.2.1.4. Best response for Case [4] $w_d > 0, w_u = 0$

(1) Formulation

In this case, the upstream farmer does not produce any crop $(q_u^c = q_u^c = 0)$, since the farmer cannot utilize water $(w_u = 0)$. Also, the upstream farmer does not make profit $(\pi_u = 0)$. In this case, the corn market and the switchgrass market are monopoly markets of the downstream farmer without competition. The objective function and the constraints of the downstream farmer are as follows:

$$\max_{q_d^c, q_d^s} \pi_d = [a_c - b_c\alpha_c q_d^c]\alpha_c q_d^c + [a_s - b_s\alpha_s q_d^s]\alpha_s q_d^s - c_c\alpha_c q_d^c - c_s\alpha_s q_d^s + s_d q_d^s - w_d p_d \quad (1')$$

$\delta_c q_d^c + \delta_s q_d^s \leq w_d$ (λ_1) water capacity constraint (2')

$q_d^c, q_d^s \geq 0$ (λ_2, λ_3) non-negativity constraints (3',4')

(2) KKT condition

Now, we use KKT conditions to derive the best responses of the downstream farmer in the third stage. The KKT conditions of the problems are as follows:

$$\mathcal{L}_{1'} = \pi_d + \lambda_1(w_d - \delta_c q_d^c - \delta_s q_d^s) + \lambda_2 q_d^c + \lambda_3 q_d^s \tag{27}$$

$$\frac{\partial \mathcal{L}_{1'}}{\partial q_d^c} = (a_c\alpha_c - c_c\alpha_c) - 2b_c{\alpha_c}^2 q_d^c - \delta_c\lambda_1 + \lambda_2 = 0 \tag{28}$$

$$\frac{\partial \mathcal{L}_{1'}}{\partial q_d^s} = (a_s\alpha_s - c_s\alpha_s + s_d) - 2b_s{\alpha_s}^2 q_d^s - \delta_s\lambda_1 + \lambda_3 = 0 \tag{29}$$

$$0 \le \lambda_1 \perp [w_d - \delta_c q_d^c - \delta_s q_d^s] \ge 0 \tag{30}$$

$$0 \le \lambda_2 \perp [q_d^c] \ge 0 \tag{31}$$

$$0 \le \lambda_3 \perp [q_d^s] \ge 0 \tag{32}$$

(3) Best response

Because the land of the downstream farmer is large, the downstream farmer consumes all the water allocated by the utility company, and the water capacity constraint of the farmer is binding. So, from the complementary slack conditions of the KKT conditions of the water capacity constraint shown above, we have $[w_d - \delta_c q_d^c - \delta_s q_d^s] = 0$ and $\lambda_1 \ge 0$.

The downstream farmer's land allocations for both corn and switchgrass are equal to or greater than zero $(q_d^c > 0, q_d^s > 0)$. So, we assume $\lambda_2 = 0,$ and $\lambda_3 = 0$. In summary, we have these conditions in case [4]: $\lambda_1 \ge 0, \lambda_2 = 0,$ and $\lambda_3 = 0$. These conditions are used to solve the KKT conditions, and the results are shown in Table 3.

3.2.1.3. Solution

From the subgame, we obtain the farmers' best response in the third stage towards the utility company's decision in the second stage. Cases with the conditions of water allocation, and the farmers' best responses and profits are shown in Table 3. The calculation and the proof of Table 3 are shown in Appendix A.

Table 3. Cases with conditions of water allocation, and the farmers' best responses and profits in the third stage

Case	Condition	Best Response and Profits of the Farmers in the Third Stage $(q_d^c)^{BR}, (q_d^s)^{BR}, (q_u^c)^{BR}, (q_u^s)^{BR}, \pi_d, \pi_u$
[1]	$w_d = 0$ $w_u = 0$	$(q_d^c)^{BR} = 0$ $(q_d^s)^{BR} = 0$ $(q_u^c)^{BR} = 0$ $(q_u^s)^{BR} = 0$
		$\pi_d = 0$ $\pi_u = 0$
[2]	$w_d > 0$ $w_u > 0$ $\lambda_1 > 0$ $\lambda_2 = 0$ $\lambda_3 = 0$ $\lambda_4 > 0$	$(q_d^c)^{BR} = \frac{\delta_s{}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 3b_s\alpha_s{}^2\delta_c w_d - \delta_c\delta_s(2s_d - s_u)}{3(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} \geq 0$ $(q_d^s)^{BR} = \frac{-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + \delta_c{}^2(a_s\alpha_s - c_s\alpha_s) + 3b_c\alpha_c{}^2\delta_s w_d + \delta_c{}^2(2s_d - s_u)}{3(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} \geq 0$ $(q_u^c)^{BR} = \frac{\delta_s{}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 3b_s\alpha_s{}^2\delta_c w_u - \delta_c\delta_s(-s_d + 2s_u)}{3(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} \geq 0$

	$\lambda_5 = 0$ $\lambda_6 = 0$	$(q_u^s)^{BR} = \frac{-\delta_c\delta_s(a_c\alpha_c-c_c\alpha_c)+\delta_c{}^2(a_s\alpha_s-c_s\alpha_s)+3b_c\alpha_c{}^2\delta_s w_u+\delta_c{}^2(-s_d+2s_u)}{3(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} \geq 0$ $\lambda_1 = \frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c-c_c\alpha_c)+b_c\alpha_c{}^2\delta_s(a_s\alpha_s-c_s\alpha_s)-b_cb_s\alpha_c{}^2\alpha_s{}^2(2w_d+w_u)+b_c\alpha_c{}^2\delta_s s_d}{(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$ $\lambda_4 = \frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c-c_c\alpha_c)+b_c\alpha_c{}^2\delta_s(a_s\alpha_s-c_s\alpha_s)-b_cb_s\alpha_c{}^2\alpha_s{}^2(w_d+2w_u)+b_c\alpha_c{}^2\delta_s s_u}{(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$
		$\pi_d = [a_c\alpha_c - c_c\alpha_c - b_c\alpha_c{}^2\{(q_d^c)^{BR} + (q_u^c)^{BR}\}](q_d^c)^{BR}$ $+[a_s\alpha_s - c_s\alpha_s - b_s\alpha_s{}^2\{(q_d^s)^{BR} + (q_u^s)^{BR}\}](q_u^s)^{BR} + s_d(q_u^s)^{BR} - w_d p_d$ $\pi_u = [a_c\alpha_c - c_c\alpha_c - b_c\alpha_c{}^2\{(q_d^c)^{BR} + (q_u^c)^{BR}\}](q_u^c)^{BR}$ $+[a_s\alpha_s - c_s\alpha_s - b_s\alpha_s{}^2\{(q_d^s)^{BR} + (q_u^s)^{BR}\}](q_u^s)^{BR} + s_u(q_u^s)^{BR} - w_u p_u$
[3]	$w_d = 0$ $w_u > 0$ $\lambda_4 > 0$ $\lambda_5 = 0$ $\lambda_6 = 0$	$(q_d^c)^{BR} = 0$ $(q_d^s)^{BR} = 0$ $(q_u^c)^{BR} = \frac{\delta_s{}^2(a_c\alpha_c-c_c\alpha_c)-\delta_c\delta_s(a_s\alpha_s-c_s\alpha_s)+2b_s\alpha_s{}^2\delta_c w_u-\delta_c\delta_s s_u}{2(b_c\alpha_c{}^2\delta_s{}^2+b_s\alpha_s{}^2\delta_c{}^2)} \geq 0$ $(q_u^s)^{BR} = \frac{-\delta_c\delta_s(a_c\alpha_c-c_c\alpha_c)+\delta_c{}^2(a_s\alpha_s-c_s\alpha_s)+2b_c\alpha_c{}^2\delta_s w_u+\delta_c{}^2 s_u}{2(b_c\alpha_c{}^2\delta_s{}^2+b_s\alpha_s{}^2\delta_c{}^2)} \geq 0$ $\lambda_4 = \frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c-c_c\alpha_c)+b_c\alpha_c{}^2\delta_s(a_s\alpha_s-c_s\alpha_s)-2b_c\alpha_c{}^2b_s\alpha_s{}^2 w_u+b_c\alpha_c{}^2\delta_s s_u}{(b_c\alpha_c{}^2\delta_s{}^2+b_s\alpha_s{}^2\delta_c{}^2)} > 0$
		$\pi_d = 0$ $\pi_u = [a_c\alpha_c - c_c\alpha_c - b_c\alpha_c{}^2(q_u^c)^{BR}](q_u^c)^{BR} + [a_s\alpha_s - c_s\alpha_s - b_s\alpha_s{}^2(q_u^s)^{BR}](q_u^s)^{BR}$ $+s_u(q_u^s)^{BR} - w_u p_u$
[4]	$w_d > 0$ $w_u = 0$ $\lambda_1 > 0$ $\lambda_2 = 0$ $\lambda_3 = 0$	$(q_d^c)^{BR} = \frac{\delta_s{}^2(a_c\alpha_c-c_c\alpha_c)-\delta_c\delta_s(a_s\alpha_s-c_s\alpha_s)+2b_s\alpha_s{}^2\delta_c w_d-\delta_c\delta_s s_d}{2(b_c\alpha_c{}^2\delta_s{}^2+b_s\alpha_s{}^2\delta_c{}^2)} \geq 0$ $(q_d^s)^{BR} = \frac{-\delta_c\delta_s(a_c\alpha_c-c_c\alpha_c)+\delta_c{}^2(a_s\alpha_s-c_s\alpha_s)+2b_c\alpha_c{}^2\delta_s w_d+\delta_c{}^2 s_d}{2(b_c\alpha_c{}^2\delta_s{}^2+b_s\alpha_s{}^2\delta_c{}^2)} \geq 0$ $(q_u^c)^{BR} = 0$ $(q_u^s)^{BR} = 0$

		$\lambda_1 = \frac{b_s{\alpha_s}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c{\alpha_c}^2\delta_s(a_s\alpha_s - c_s\alpha_s) - 2b_c{\alpha_c}^2 b_s{\alpha_s}^2 w_d + b_c{\alpha_c}^2\delta_s s_d}{(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2)} > 0$
		$\pi_d = [a_c\alpha_c - c_c\alpha_c - b_c{\alpha_c}^2(q_d^c)^{BR}](q_d^c)^{BR} + [a_s\alpha_s - c_s\alpha_s - b_s{\alpha_s}^2(q_d^s)^{BR}](q_u^s)^{BR}$ $+s_d(q_u^s)^{BR} - w_d p_d$ $\pi_u = 0$

In Case [2], the best responses of the farmers' land use are feasible only when the government subsidies meet following conditions.

$$2s_d - s_u \geq \frac{\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) - {\delta_c}^2(a_s\alpha_s - c_s\alpha_s) - 3b_c{\alpha_c}^2\delta_s w_d}{{\delta_c}^2}$$

$$2s_d - s_u \leq \frac{{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 3b_s{\alpha_s}^2\delta_c w_d}{\delta_c\delta_s}$$

$$-s_d + 2s_u \geq \frac{\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) - {\delta_c}^2(a_s\alpha_s - c_s\alpha_s) - 3b_c{\alpha_c}^2\delta_s w_u}{{\delta_c}^2}$$

$$-s_d + 2s_u \leq \frac{{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 3b_s{\alpha_s}^2\delta_c w_u}{\delta_c\delta_s}$$

$$s_d > \frac{b_c b_s{\alpha_c}^2{\alpha_s}^2(2w_d + w_u) - b_s{\alpha_s}^2\delta_c(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2\delta_s(a_s\alpha_s - c_s\alpha_s)}{b_c{\alpha_c}^2\delta_s}$$

$$s_u > \frac{b_c b_s{\alpha_c}^2{\alpha_s}^2(w_d + 2w_u) - b_s{\alpha_s}^2\delta_c(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2\delta_s(a_s\alpha_s - c_s\alpha_s)}{b_c{\alpha_c}^2\delta_s}$$

In Case [3], the best responses of the farmers' land use are feasible only when the government subsidies meet following conditions.

$$s_u \leq \frac{{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 2b_s{\alpha_s}^2\delta_c w_u}{\delta_c\delta_s}$$

$$s_u \geq \frac{\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) - {\delta_c}^2(a_s\alpha_s - c_s\alpha_s) - 2b_c{\alpha_c}^2\delta_s w_u}{{\delta_c}^2}$$

$$s_u > \frac{2b_c{\alpha_c}^2 b_s{\alpha_s}^2 w_u - b_s{\alpha_s}^2\delta_c(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2\delta_s(a_s\alpha_s - c_s\alpha_s)}{b_c{\alpha_c}^2\delta_s}$$

In Case [4], the best responses of the farmers' land use are feasible only when the government subsidies meet following conditions.

$$s_d \leq \frac{{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 2b_s{\alpha_s}^2\delta_c w_d}{\delta_c\delta_s}$$

$$s_d \geq \frac{\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) - {\delta_c}^2(a_s\alpha_s - c_s\alpha_s) - 2b_c{\alpha_c}^2\delta_s w_d}{{\delta_c}^2}$$

$$s_d > \frac{2b_c{\alpha_c}^2 b_s{\alpha_s}^2 w_d - b_s{\alpha_s}^2\delta_c(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2\delta_s(a_s\alpha_s - c_s\alpha_s)}{b_c{\alpha_c}^2\delta_s}$$

3.2.1.4. Analysis

We analyze the effect of the utility company's water allocation (w_d, w_u) in the second stage on the farmers' land allocation $(q_d^c, q_d^s, q_u^c, q_u^s)$ in the third stage. The best response of the farmers could be derived from the KKT conditions of their profit functions. In Case [1], Case [3], and Case [4], the land allocation is determined as a given value and not as a function of water allocation (w_d, w_u). So, the land allocation is not affected by the water allocation in case [1], [3], and [4], where the company either allocates all the water to a single farmer or does not allocate water to any farmer. Hence, we could analyze Case [2] with the effect of the water allocation.

Corollary 1. If the farmers' land capacity is enough to utilize all the water they purchase and the utility company allocates water to both farmers in Case [2] $(w_d \geq 0, w_u \geq 0)$, the following holds for the effect of the utility company's decision in the second stage on the farmers' decisions in the third stage:

$$(1)\ \frac{\partial(q_d^c)^{BR}}{\partial(w_d)^{BR}} = \frac{\frac{b_s{\alpha_s}^2}{\delta_s}}{\frac{b_c{\alpha_c}^2}{\delta_c}} \cdot \frac{\partial(q_d^s)^{BR}}{\partial(w_d)^{BR}} > 0 \quad (2)\ \frac{\partial(q_u^c)^{BR}}{\partial(w_d)^{BR}} = \frac{\partial(q_u^s)^{BR}}{\partial(w_d)^{BR}} = 0$$

$$(3)\ \frac{\partial(q_d^c)^{BR}}{\partial(w_u)^{BR}} = \frac{\partial(q_d^s)^{BR}}{\partial(w_u)^{BR}} = 0 \quad (4)\ \frac{\partial(q_u^c)^{BR}}{\partial(w_u)^{BR}} = \frac{\frac{b_s{\alpha_s}^2}{\delta_s}}{\frac{b_c{\alpha_c}^2}{\delta_c}} \cdot \frac{\partial(q_u^s)^{BR}}{\partial(w_u)^{BR}} > 0$$

Proof is shown in Appendix B.

Corollary 1 reveals the reaction of the farmers to the utility company's decision, as $(w_d)^{BR}$ and $(w_u)^{BR}$. (2,3) The land allocation of the downstream farmer is not directly affected by the water allocation to the upstream farmer, and vice versa. (1,4) The farmers would use more land to produce both corn and switchgrass in case of higher water allocation to them. The ratio of marginal land allocation between corn and switchgrass is determined by

$\frac{b_s {\alpha_s}^2}{\delta_s}$ and $\frac{b_c {\alpha_c}^2}{\delta_c}$ for both farmers. The meaning of the coefficients is division of the marginal crop market price per unit land use for the corresponding crop $(b_s {\alpha_s}^2$ and $b_c {\alpha_c}^2)$ by the water requirement of the crop per unit land (δ_s and δ_c), of the switchgrass and the corn respectively.

3.2.2. Utility company's decision on water allocation in the second stage

Here, we introduce the objective function of the utility company that maximizes its profit through its best decisions on water allocation (w_d, w_u) in the second stage. The constraints of water capacity and non-negativity of the decision variables used in the second stage are also introduced. In the second stage, the company makes its best response for water allocation (w_d, w_u), based on the decision of the farmers in the first stage.

3.2.2.1. Formulation

The objective function and the constraints of the utility company in the second stage are shown as follows:

$$\max_{\boldsymbol{w_d}, \boldsymbol{w_u}} \pi_h = p_d \boldsymbol{w_d} + p_u \boldsymbol{w_u} - c_w[(\boldsymbol{w_d})^2 + (\boldsymbol{w_u})^2] + p_e \alpha_e \boldsymbol{w_d} \qquad (33)$$

$\boldsymbol{w_d} + \boldsymbol{w_u} \leq w$ $\qquad (\lambda_7) \qquad$ water capacity constraint $\qquad$ (34)

$\boldsymbol{w_d}, \boldsymbol{w_u} \geq 0 \qquad (\lambda_8, \lambda_9) \qquad$ non-negativity constraints (35,36)

In Equation (33), the first two terms are revenue from sale of water to the downstream farmer and the upstream farmer, respectively. The third term is cost of crop production, and the last term is revenue from the sale of hydroelectricity generated by water release from the dam, at an electricity market. In equations (34) – (36), λs are KKT multipliers or the shadow price of the constraints.

3.2.2.2. KKT Conditions

Best responses of the utility company in the second stage could be derived by using the KKT conditions of the utility company's problem as follows:

$$\mathcal{L}_3 = \pi_h + \lambda_7(w - \boldsymbol{w_d} - \boldsymbol{w_u}) + \lambda_8 \boldsymbol{w_d} + \lambda_9 \boldsymbol{w_u} \tag{37}$$

$$\frac{\partial \mathcal{L}_3}{\partial \boldsymbol{w_d}} = p_d + p_e \alpha_e - 2c_w \boldsymbol{w_d} - \lambda_7 + \lambda_8 = 0 \tag{38}$$

$$\frac{\partial \mathcal{L}_3}{\partial \boldsymbol{w_u}} = p_u - 2c_w \boldsymbol{w_u} - \lambda_7 + \lambda_9 = 0 \tag{39}$$

$$0 \leq \lambda_7 \perp [w - \boldsymbol{w_d} - \boldsymbol{w_u}] \geq 0 \tag{40}$$

$$0 \leq \lambda_8 \perp [\boldsymbol{w_d}] \geq 0 \tag{41}$$

$$0 \leq \lambda_9 \perp [\boldsymbol{w_u}] \geq 0 \tag{42}$$

3.2.2.3. Solution

From the subgame, we obtain four domains, as shown in Figure 2 and Table 4, where the best response (BR) function of the company exists with five borders including x-axis and y-axis. The derivation of the best responses and the border lines are shown in Appendix B.

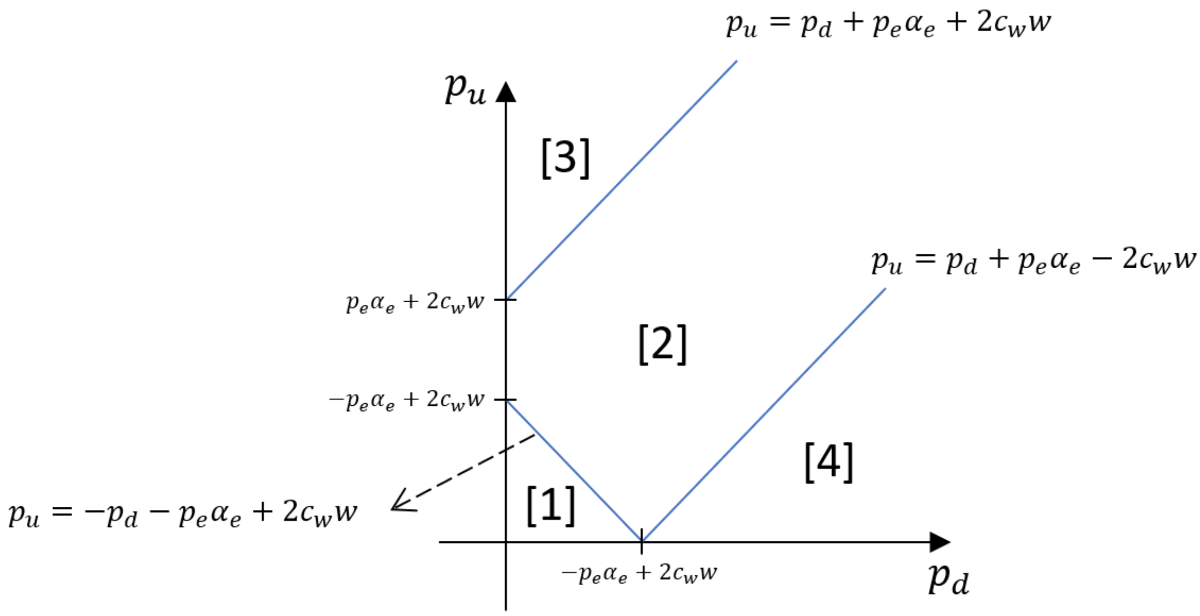

Figure 2. Domains of the Nash equilibria for the utility company's best response

Table 4. Best responses of the utility company and their conditions in the second stage

Domain	$\lambda_7, \lambda_8, \lambda_9$	Conditions	Best Responses of the Utility Company (w_d^{BR}, w_u^{BR})	KKT Multiplier $(\lambda_7, \lambda_8, \lambda_9)$	Profit of the Utility Company (π_h)
[1]	$\lambda_7 = 0$ $\lambda_8 = 0$ $\lambda_9 = 0$	$p_d + p_e\alpha_e + p_u \le 2c_w w$ $p_d + p_e\alpha_e \ge 0$ $p_u \ge 0$	$w_d^{BR} = \frac{p_d + p_e\alpha_e}{2c_w} > 0$ $w_u^{BR} = \frac{p_u}{2c_w} > 0$	–	$\frac{{p_d}^2 + {p_u}^2 + 2p_e\alpha_e p_d + (p_e\alpha_e)^2}{4c_w}$
[2]	$\lambda_7 > 0$ $\lambda_8 = 0$ $\lambda_9 = 0$	$p_d + p_e\alpha_e + p_u > 2c_w w$ $p_d + p_e\alpha_e - p_u \ge -2c_w w$ $p_d + p_e\alpha_e - p_u \le 2c_w w$	$w_d^{BR} = \frac{p_d + p_e\alpha_e - p_u + 2c_w w}{4c_w}$ $w_u^{BR} = \frac{-p_d - p_e\alpha_e + p_u + 2c_w w}{4c_w}$	$\lambda_7 = \frac{p_d + p_e\alpha_e + p_u - 2c_w w}{2}$	$\frac{{p_d}^2 + {p_u}^2 + 2p_e\alpha_e(p_d - p_u) - 2p_d p_u + 4c_w w(p_d + p_e\alpha_e + p_u + c_w w) + (p_e\alpha_e)^2}{8c_w}$
[3]	$\lambda_7 > 0$ $\lambda_8 > 0$ $\lambda_9 = 0$	$p_u > 2c_w w$ $p_d + p_e\alpha_e - p_u < -2c_w w$	$w_d^{BR} = 0$ $w_u^{BR} = w > 0$	$\lambda_7 = p_u - 2c_w w$ $\lambda_8 = -p_d - p_e\alpha_e + p_u - 2c_w w$	$wp_u - c_w w^2$
[4]	$\lambda_7 > 0$ $\lambda_8 = 0$ $\lambda_9 > 0$	$p_d + p_e\alpha_e > 2c_w w$ $p_d + p_e\alpha_e - p_u > 2c_w w$	$w_d^{BR} = w > 0$ $w_u^{BR} = 0$	$\lambda_7 = p_d + p_e\alpha_e - 2c_w w$ $\lambda_9 = p_d + p_e\alpha_e - p_u - 2c_w w$	$wp_d - c_w w^2$

- Domain [1] and Domain [2] result in Case [2] $(w_d > 0, w_u > 0)$ in the third stage.
- Domain [3] results in Case [3] $(w_d = 0, w_u > 0)$ in the third stage.
- Domain [4] results in Case [4] $(w_d > 0, w_u = 0)$ in the third stage.
- All the parameters used in Table 4 are strong positive.
- The calculation and the proof of Table 4 are shown in Appendix D.

Domain [1] result in Case [2] of the best responses of the farmers in the third stage under the conditions of $p_d + p_e\alpha_e + p_u \leq 2c_w w$. In this domain, the utility company utilizes its entire water capacity only if the sum of the prices per unit water from the farmers (p_d, p_u) and the revenue from sale of hydroelectricity per unit water $(p_e\alpha_e)$ is equal to the marginal cost of supplying all the water to a single farmer $(2c_w w)$. In other words, the utility company completely utilizes its entire water to allocate to the farmers under the condition $p_d + p_e\alpha_e + p_u = 2c_w w$. Otherwise, when the sum of the prices and the revenue is less than the marginal cost under a condition of $p_d + p_e\alpha_e + p_u < 2c_w w$, and the utility company does not completely allocate all of its water to the farmers. The farmers receive as much water as they want without competition against each other in the second stage. This case implies that the farmers do not compete on water price under a game in the first stage.

Domain [2] can results in a case among Case [2], Case [3], and Case [4] of the best responses of the farmers in the third stage under the conditions of $p_d + p_u > 2c_w w - p_e\alpha_e$, $p_d - p_u \geq -2c_w w - p_e\alpha_e$, and $p_d - p_u \leq 2c_w w - p_e\alpha_e$. In Domain [2], the downstream farmer and the upstream farmer would produce both crops $(q_d^c + q_d^s \geq 0, q_u^c + q_u^s \geq 0)$ in the third stage, since the utility company allocates water to both farmers $(w_d^{BR} > 0, w_u^{BR} > 0)$ in the second stage.

Domains [3] and [4] result in cases [3] and [4] respectively of the best responses of the farmers in the third stage. In Domain [3], the downstream farmer cannot cultivate any crop $(q_d^c = q_d^s = 0)$ to make profit $(\pi_d = 0)$ in the third stage, since the utility company does not allocate water to the downstream farmer $(w_d^{BR} = 0, w_u^{BR} = w)$ under the condition of $p_d - p_u < -2c_w w - p_e \alpha_e$, in the second stage. On the other hand, in Domain [4], the upstream farmer cannot cultivate any crop $(q_u^c = q_u^s = 0)$ to make profit $(\pi_u = 0)$ in the third stage, since the utility company does not allocate water to the upstream farmer $(w_d^{BR} = w, w_u^{BR} = 0)$ under the condition of $p_d - p_u > 2c_w w - p_e \alpha_e$ in the second stage.

Among all possible solutions of the above problem, we are interested only in the cases where additional water capacity of the dam increases the profit of the utility company, i.e., when $\lambda_7 > 0$. In this case, the water capacity constraint of the utility company becomes binding. Domains [2], [3], and [4] belong to this case whereas Domain [1] does not belong to this case. If the water capacity constraint is not binding, with there being enough water to meet the demand, the price competition between the farmers does not occur.

In domains [2], [3], and [4], we could analyze the result of a Bertrand game between the downstream farmers and the upstream farmers toward price-bidding for water allocation. In Domain [1], the downstream farmer and the upstream farmer share the water from the utility company after the competitive bidding. On the other hand, in domains [3] and [4], one of the farmers takes all the water from the utility company after winning the bidding against the other farmer. The upstream farmer and the downstream farmer are the winner in Domain [3] and Domain [4], respectively.

3.2.2.4. Analysis

We analyze the effect of the farmers' decision regarding water prices (p_d, p_u) in the first stage on the utility company's water allocation (w_d, w_u) in the second stage. The best response of the company could be derived from the KKT conditions. In domains [3] and [4], the water allocation to each farmer is w or 0, and not a function of the water prices from the farmers. The water allocation is not affected by the water prices in domains [3] and [4], where the utility company allocates all the water to only one of the farmers. Hence, we analyze Domain [2] with the effect of the water price from the farmers.

Corollary 2. If the farmers' land capacity is enough to utilize the water that they purchase from the utility company in Domain [2], the following holds for the effect of the farmers' decisions in the first stage on the utility company's decision in the second stage:

(1) $\frac{\partial w_d^{BR}}{\partial p_d} = -\frac{\partial w_u^{BR}}{\partial p_d} > 0$ (2) $\frac{\partial w_d^{BR}}{\partial p_u} = -\frac{\partial w_u^{BR}}{\partial p_u} < 0$

Proof is shown in Appendix D.

Corollary 2 reveals the reaction of the utility company, w_d^{BR} and w_u^{BR}, in the second stage to the farmers' decision, p_d and p_u, in the first stage. The company would allocate more water to a farmer when that farmer is offering a higher price. Consequently, announcement of a higher price offer by one farmer decreases the water allocation to each other.

3.2.3. Farmers' decision on bidding water price in the first stage

In this subsection, we introduce the objective functions of the downstream farmer and the upstream farmer that maximize their own profits through their best decisions on the water price (p_d) and (p_u) respectively, in the first stage. Non-negativity constraints of the decision variables used in the first stage are also introduced.

We also find equilibrium of the supply chain's decision through the best decision of the farmers in the first stage, the best response of the utility company in the second stage, and the best responses of the farmers in the third stage.

In the first stage, the downstream farmer and the upstream farmer compete in water price under a Bertrand game. After the bidding competition, the farmers share the water from the utility company or one of the farmers takes all the water after winning the bidding. Both farmers make their decisions on water price in the belief that the corn market and the switchgrass market are duopolistic rather than monopolistic, since each farmer believes that the other may want to sell both corn and switchgrass at each market. After the farmers announce their water prices to the utility company in the first stage, the company decides the water allocation. In the price-bidding competition, the result shows that the farmers share water (Domain [1] and Domain [2] of the second stage), the downstream farmer is a winner of the bidding and takes all the water (Domain [3] of the second stage) or the upstream farmer is a winner of the bidding and takes all the water (Domain [4] of the second stage).

3.2.3.1. Formulation

In the first stage, the farmers make their best decisions for water prices (p_d, p_u) by using the best responses of the utility company in the second stage and of the farmers in the third stage. The objective function and the constraint of the farmers in the first stage are formulated as follows:

For the downstream farmer,

$$\max_{\boldsymbol{p_d}} \pi_d = [a_c - b_c\alpha_c\{(q_d^c)^{BR} + (q_u^c)^{BR}\}]\alpha_c(q_d^c)^{BR} + [a_s - b_s\alpha_s\{(q_d^s)^{BR} + (q_u^s)^{BR}\}]\alpha_s(q_d^s)^{BR} - c_c\alpha_c(q_d^c)^{BR} - c_s\alpha_s(q_d^s)^{BR} + s_d(q_d^s)^{BR} - {w_d}^{BR}\boldsymbol{p_d} \quad (43)$$

$$\boldsymbol{p_d} \geq 0 \qquad (\lambda_{10}) \qquad \text{non-negativity constraint} \quad (44)$$

and, for the upstream farmer,

$$\max_{\boldsymbol{p_u}} \pi_u = [a_c - b_c\alpha_c\{(q_d^c)^{BR} + (q_u^c)^{BR}\}]\alpha_c(q_u^c)^{BR} + [a_s - b_s\alpha_s\{(q_d^s)^{BR} + (q_u^s)^{BR}\}]\alpha_s(q_u^s)^{BR} - c_c\alpha_c(q_u^c)^{BR} - c_s\alpha_s(q_u^s)^{BR} + s_u(q_u^s)^{BR} - {w_u}^{BR}\boldsymbol{p_u} \quad (45)$$

$$\boldsymbol{p_u} \geq 0 \qquad (\lambda_{11}) \qquad \text{non-negativity constraint} \quad (46)$$

Then, we reformulate the above problems in each scenario into their KKT conditions to derive the best responses of the players in the problems of the downstream farmer and the upstream farmer in the third stage.

3.2.3.2. KKT Conditions

Best responses of the downstream farmer and the upstream farmer in the third stage could be derived by using the KKT conditions of its problems as follows:

For the downstream farmer's problem,

$$\mathcal{L}_4 = \pi_d + \lambda_{10} p_d \tag{47}$$

$$\frac{\partial \mathcal{L}_4}{\partial p_d} = [a_c\alpha_c - c_c\alpha_c - b_c{\alpha_c}^2(q_d^c + q_u^c)]\frac{\partial q_d^c}{\partial p_d} - b_c{\alpha_c}^2\left(\frac{\partial q_d^c}{\partial p_d} + \frac{\partial q_u^c}{\partial p_d}\right)q_d^c + \Big[a_s\alpha_s - c_s\alpha_s - b_s{\alpha_s}^2(q_d^s + q_u^s)\Big]\frac{\partial q_d^s}{\partial p_d} - b_s{\alpha_s}^2\left(\frac{\partial q_d^s}{\partial p_d} + \frac{\partial q_u^s}{\partial p_d}\right)q_d^s + s_d\frac{\partial q_d^s}{\partial p_d} - w_d - \frac{\partial w_d}{\partial p_d}p_d + \lambda_{10} = 0 \tag{48}$$

$$0 \le \lambda_{10} \perp [p_d] \ge 0 \tag{49}$$

and, for the upstream farmer's problem,

$$\mathcal{L}_5 = \pi_u + \lambda_{11} p_u \tag{50}$$

$$\frac{\partial \mathcal{L}_5}{\partial p_u} = [a_c\alpha_c - c_c\alpha_c - b_c{\alpha_c}^2(q_d^c + q_u^c)]\frac{\partial q_u^c}{\partial p_u} - b_c{\alpha_c}^2\left(\frac{\partial q_d^c}{\partial p_u} + \frac{\partial q_u^c}{\partial p_u}\right)q_u^c + \Big[a_s\alpha_s - c_s\alpha_s - b_s{\alpha_s}^2(q_d^s + q_u^s)\Big]\frac{\partial q_u^s}{\partial p_u} - b_s{\alpha_s}^2\left(\frac{\partial q_d^s}{\partial p_u} + \frac{\partial q_u^s}{\partial p_u}\right)q_u^s + s_u\frac{\partial q_u^s}{\partial p_u} - w_u - \frac{\partial w_u}{\partial p_u}p_u + \lambda_{11} = 0 \tag{51}$$

$$0 \le \lambda_{11} \perp [p_u] \ge 0 \tag{52}$$

From the above problems converted to their KKT conditions, we could find Nash equilibrium of the Cournot competition game between the downstream farmer and the upstream farmer by putting their KKT conditions, equations (48), (49), (51), and (52), together.

$$[(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2(q_d^c + q_u^c)]\frac{\partial q_d^c}{\partial p_d} + [(a_s\alpha_s - c_s\alpha_s) + s_d - b_s{\alpha_s}^2(q_d^s + q_u^s)]\frac{\partial q_d^s}{\partial p_d} - w_d - \frac{\partial {w_d}^{BR}}{\partial p_d}p_d + \lambda_{10} = 0 \quad (53)$$

$$\lambda_{10}[p_d] = 0 \quad (54)$$

$$p_d \geq 0 \quad (55)$$

$$[(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2(q_d^c + q_u^c)]\frac{\partial q_u^c}{\partial p_u} + [(a_s\alpha_s - c_s\alpha_s) + s_u - b_s{\alpha_s}^2(q_d^s + q_u^s)]\frac{\partial q_u^s}{\partial p_u} - w_u - \frac{\partial w_u}{\partial p_u}p_u + \lambda_{11} = 0 \quad (56)$$

$$\lambda_{11}[p_{\boldsymbol{u}}] = 0 \quad (57)$$

$$p_u \geq 0 \quad (58)$$

The downstream and upstream farmers' water price bids to the utility company are greater than zero $(p_d > 0, p_u > 0)$, since the utility company does not sell its water at a non-positive price. So, we assume that Equation (55) and Equation (58) are not binding, and $\lambda_{10} = 0$ and $\lambda_{11} = 0$.

3.2.3.3. Solution

The Nash equilibrium of the farmers' decisions in the first stage could be derived from the KKT conditions.

Proposition 1. Through the farmers' best responses in the third stage and the utility company's best response in the second stage, Nash equilibrium of the decisions of the farmers in the first stage is as follows:

(1) $p_d{}^{NE} = A - 2c_w w + \frac{2B_d + B_u - p_e\alpha_e}{3}$

(2) $p_u{}^{NE} = A - 2c_w w + \frac{B_d + 2B_u + p_e\alpha_e}{3}$

, where

$A = \frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c\alpha_c{}^2\delta_s(a_s\alpha_s - c_s\alpha_s) - b_c b_s\alpha_c{}^2\alpha_s{}^2 w}{\left(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2\right)}$,

$B_d = \frac{b_c\alpha_c{}^2\delta_s s_d}{(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)}$, and $B_u = \frac{b_c {\alpha_c}^2 \delta_s s_u}{(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2)}$.

The calculation and the proof of deriving the Nash equilibrium for Proposition 1 are shown in Appendix E.

3.2.4. Nash equilibrium of the farmers' and the utility company's decisions in the BSC

We derive the equilibria of the farmers' and the utility company's decisions by substituting the best responses.

After the downstream farmer and the upstream farmer decide their water prices, the utility company makes a decision on water allocation based on the prices. The farmers' best decisions in the first stage result in one of the four domains in the second stage: Domain [1], Domain [2], Domain [3], and Domain [4]. Among the four domains, we do not consider Domain [1], since this domain does not generate a competitive game in water prices between the farmers.

Domain [2] happens in the second stage only if the farmers' decisions on water prices meet the following Condition [2] which consists of three sub-conditions derived in the second stage:

Condition [2]:

$$p_d{}^{NE} + p_u{}^{NE} + p_e\alpha_e > 2c_w w \tag{2-1}$$

$$p_d{}^{NE} - p_u{}^{NE} + p_e\alpha_e \geq -2c_w w \tag{2-2}$$

$$p_d{}^{NE} - p_u{}^{NE} + p_e\alpha_e \leq 2c_w w \tag{2-3}$$

In Condition [2], sub-condition (2-1) presents that the utility company utilizes its entire water capacity and does not leave over its water since the revenue per unit water is greater than the marginal cost of supplying all the water. Moreover, sub-condition (2-2) and sub-condition (2-3) present that the utility company allocates its water to both downstream farmer and upstream farmer since the water prices from the farmers do not have significant difference in the second stage.

Domain [3] happens in the second stage only if the farmers' decisions on water prices meet the following Condition [3] derived in the second stage:

Condition [3] $p_d{}^{NE} - p_u{}^{NE} + p_e\alpha_e < -2c_w w$

Condition [2] presents that the utility company allocates all the water to the upstream farmer since the water price from the upstream farmer is significantly higher than that from the downstream farmer. So, the upstream farmer is a winner who takes all the water from the utility company in the price-bidding for water allocation in the second stage.

Domain [4] happens in the second stage only if the farmers' decisions on water prices meet the following Condition [4] derived in the second stage:

Condition [4] $p_d{}^{NE} - p_u{}^{NE} + p_e\alpha_e > 2c_w w$

Condition [4] presents that the utility company allocates all the water to the downstream farmer since the water price from the downstream farmer is significantly higher than that from the upstream farmer. So, the downstream farmer is a winner who takes all the water from the utility company in the price-bidding for water allocation in the second stage.

Proposition 2-1. When $p_d{}^{NE}$ and $p_u{}^{NE}$ meet Condition [2] of Domain [2], Nash equilibrium of the decisions of the utility company in the second stage and the farmers in the third stage, and their profits are as follows:

(1) Best decisions of the utility company in the second stage:

$$(w_d)^{NE_1} = \frac{B_d - B_u + p_e\alpha_e + 6c_w w}{12c_w} = \frac{(p_e\alpha_e + 6c_w w)\left(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2\right) + b_c\alpha_c{}^2\delta_s(s_d - s_u)}{12c_w(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$(w_u)^{NE_1} = \frac{-B_d + B_u - p_e\alpha_e + 6c_w w}{12c_w} = \frac{(-p_e\alpha_e + 6c_w w)(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2) - b_c\alpha_c{}^2\delta_s(s_d - s_u)}{12c_w(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

(2) Best decisions of the farmers in the third stage:

$$(q_d^c)^{NE_1} = \frac{4c_w[\delta_s{}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s)] + b_s\alpha_s{}^2\delta_c(B_d - B_u + p_e\alpha_e + 6c_w w) - 4c_w\delta_c\delta_s(2s_d - s_u)}{12c_w(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$(q_d^s)^{NE_1} = \frac{-4c_w[\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + \delta_c{}^2(a_s\alpha_s - c_s\alpha_s)] + b_c\alpha_c{}^2\delta_s(B_d - B_u + p_e\alpha_e + 6c_w w) + 4c_w\delta_c{}^2(2s_d - s_u)}{12c_w(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$(q_u^c)^{NE_1} = \frac{4c_w[\delta_s{}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s)] + b_s\alpha_s{}^2\delta_c(-B_d + B_u - p_e\alpha_e + 6c_w w) + 4c_w\delta_c\delta_s(s_d - 2s_u)}{12c_w(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$(q_u^s)^{NE_1} = \frac{-4c_w[\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) - \delta_c{}^2(a_s\alpha_s - c_s\alpha_s)] + b_c\alpha_c{}^2\delta_s(-B_d + B_u - p_e\alpha_e + 6c_w w) - 4c_w\delta_c{}^2(s_d - 2s_u)}{12c_w(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

where

$$A = \frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c\alpha_c{}^2\delta_s(a_s\alpha_s - c_s\alpha_s) - b_c b_s\alpha_c{}^2\alpha_s{}^2 w}{(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)},$$

$$B_d = \frac{b_c\alpha_c{}^2\delta_s s_d}{(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)},$$

and $B_u = \frac{b_c\alpha_c{}^2\delta_s s_u}{(b_s\alpha_s{}^2\delta_c{}^2 + b_c\alpha_c{}^2\delta_s{}^2)}$.

(3) Profit of the farmers and the utility company:

$$\pi_d{}^{[1]} = [a_c - b_c\alpha_c\{(q_d^c)^{NE_1} + (q_u^c)^{NE_1}\}]\alpha_c(q_d^c)^{NE_1} + [a_s - b_s\alpha_s\{(q_d^s)^{NE_1} + (q_u^s)^{NE_1}\}]\alpha_s(q_d^s)^{NE_1} - c_c\alpha_c(q_d^c)^{NE_1} - c_s\alpha_s(q_d^s)^{NE_1} + s_d(q_d^s)^{NE_1} - w_d{}^{NE_1}p_d{}^{NE}$$

$$= [(a_c\alpha_c - c_c\alpha_c) - b_c\alpha_c{}^2\{(q_d^c)^{NE_1} + (q_u^c)^{NE_1}\}](q_d^c)^{NE_1} + [(a_s\alpha_s - c_s\alpha_s) - b_s\alpha_s{}^2\{(q_d^s)^{NE_1} + (q_u^s)^{NE_1}\}](q_d^s)^{NE_1} + s_d(q_d^s)^{NE_1} - w_d{}^{NE_1}p_d{}^{NE}$$

$$\pi_u{}^{[1]} = [a_c - b_c\alpha_c\{(q_d^c)^{NE_1} + (q_u^c)^{NE_1}\}]\alpha_c(q_u^c)^{NE_1} + [a_s - b_s\alpha_s\{(q_d^s)^{NE_1} + (q_u^s)^{NE_1}\}]\alpha_s(q_u^s)^{NE_1} - c_c\alpha_c(q_u^c)^{NE_1} - c_s\alpha_s(q_u^s)^{NE_1} + s_u(q_u^s)^{NE_1} - w_u{}^{NE_1}p_u{}^{NE}$$

$$= [(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2\{(q_d^c)^{NE_1} + (q_u^c)^{NE_1}\}](q_u^c)^{NE_1} + [(a_s\alpha_s - c_s\alpha_s) -$$

$$b_s{\alpha_s}^2\{(q_d^s)^{NE_1} + (q_u^s)^{NE_1}\}](q_u^s)^{NE_1} + s_u(q_u^s)^{NE_1} - {w_u}^{NE_1}{p_u}^{NE}$$

$${\pi_h}^{[1]} = ({p_d}^{NE} + p_e\alpha_e){w_d}^{NE_1} + {p_u}^{NE}{w_u}^{NE_1} - c_w[({w_d}^{NE_1})^2 + ({w_u}^{NE_1})^2]$$

The calculation and the proof of deriving the Nash equilibrium for Proposition 1-1 are shown in Appendix F.1.

Proposition 2-2. When ${p_d}^{NE}$ and ${p_u}^{NE}$ meet Condition [3] of Domain [3], Nash equilibrium of the decisions of the utility company in the second stage and the farmers in the third stage, and their profits are as follows:

(1) Best decisions of the utility company in the second stage:

$$(w_d)^{NE_2} = 0$$

$$(w_u)^{NE_2} = w > 0$$

(2) Best decisions of the farmers in the third stage:

$$(q_d^c)^{NE_2} = 0$$

$$(q_d^s)^{NE_2} = 0$$

$$(q_u^c)^{NE_2} = \frac{{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 2b_s{\alpha_s}^2\delta_c w - \delta_c\delta_s s_u}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)} > 0$$

$$(q_u^s)^{NE_2} = \frac{-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + {\delta_c}^2(a_s\alpha_s - c_s\alpha_s) + 2b_c{\alpha_c}^2\delta_s w + {\delta_c}^2 s_u}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)} > 0$$

(3) Profit of the farmers and the utility company:

$${\pi_d}^{[2]} = 0$$

$${\pi_u}^{[2]} = [a_c - b_c\alpha_c(q_u^c)^{NE_2}]\alpha_c(q_u^c)^{NE_2} + [a_s - b_s\alpha_s(q_u^s)^{NE_2}]\alpha_s(q_u^s)^{NE_2} -$$

$$c_c\alpha_c(q_u^c)^{NE_2} - c_s\alpha_s(q_u^s)^{NE_2} + s_u(q_u^s)^{NE_2} - {w_u}^{NE_2}{p_u}^{NE}$$

$$= [(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2(q_u^c)^{NE_2}](q_u^c)^{NE_2} + [(a_s\alpha_s - c_s\alpha_s) -$$

$$b_s{\alpha_s}^2(q_u^s)^{NE_2}](q_u^s)^{NE_2} + s_u(q_u^s)^{NE_2} - {w_u}^{NE_2}{p_u}^{NE}$$

$${\pi_h}^{[2]} = {p_u}^{NE}w - c_w w^2$$

where

$$B_d = \frac{b_c{\alpha_c}^2\delta_s s_d}{(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)},$$

and $B_u = \frac{b_c{\alpha_c}^2\delta_s s_u}{(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)}$.

The calculation and the proof of deriving the Nash equilibrium for Proposition 1-2 are shown in Appendix F.2.

Proposition 2-3. When $p_d{}^{NE}$ and $p_u{}^{NE}$ meet Condition [4] of Domain [4], Nash equilibrium of the decisions of the utility company in the second stage and the farmers in the third stage, and their profits are as follows:

(1) Best decisions of the utility company in the second stage:

$$(w_d)^{NE_3} = w > 0$$

$$(w_u)^{NE_3} = 0$$

(2) Best decisions of the farmers in the third stage:

$$(q_d^c)^{NE_3} = \frac{\delta_s{}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 2b_s\alpha_s{}^2\delta_c w - \delta_c\delta_s s_d}{2\left(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2\right)} > 0$$

$$(q_d^s)^{NE_3} = \frac{-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + \delta_c{}^2(a_s\alpha_s - c_s\alpha_s) + 2b_c\alpha_c{}^2\delta_s w + \delta_c{}^2 s_d}{2\left(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2\right)} > 0$$

$$(q_u^c)^{NE_3} = 0$$

$$(q_u^s)^{NE_3} = 0$$

(3) Profit of the farmers and the utility company:

$$\pi_d{}^{[3]} = [a_c - b_c\alpha_c(q_d^c)^{NE_3}]\alpha_c(q_d^c)^{NE_3} + [a_s - b_s\alpha_s(q_d^s)^{NE_3}]\alpha_s(q_d^s)^{NE_3} - c_c\alpha_c(q_d^c)^{NE_3} - c_s\alpha_s(q_d^s)^{NE_3} + s_d(q_d^s)^{NE_3} - w_d{}^{NE_3}p_d{}^{NE}$$

$$= [(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2(q_d^c)^{NE_3}](q_d^c)^{NE_3} + [(a_s\alpha_s - c_s\alpha_s) -$$

$$b_s{\alpha_s}^2(q_d^s)^{NE_3}](q_d^s)^{NE_3} + s_d(q_d^s)^{NE_3} - {w_d}^{NE_3}{p_d}^{NE}$$

$${\pi_u}^{[3]} = 0$$

$${\pi_h}^{[3]} = ({p_d}^{NE} + p_e\alpha_e){w_d}^{NE_3} - c_w({w_d}^{NE_3})^2$$

$$= ({p_d}^{NE} + p_e\alpha_e)w - c_w w^2$$

, where

$$B_d = \frac{b_c{\alpha_c}^2\delta_s s_d}{(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)},$$

and $B_u = \frac{b_c{\alpha_c}^2\delta_s s_u}{(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)}$.

The calculation and the proof of deriving the Nash equilibrium for Proposition 1-3 are shown in Appendix F.3.

3.2.3.3. Analysis

Depending on the government subsidy, the Nash equilibrium of the water prices $({p_d}^{NE}$, ${p_u}^{NE})$ can meet one of Condition [2], Condition [3], and Condition [4] in the first stage. Then, based on the satisfied condition, the Nash equilibrium of the water allocation $({w_d}^{NE}, {w_d}^{NE})$ is determined among Domain [2], Domain [3], and Domain [4] in the second stage. Then, based on the determined domain of the water allocation, the Nash equilibrium of the land uses is determined among Case [2], Case [3], and Case [4] in the third stage. In other words, when the Nash equilibrium of the water prices meets Conditions [2], the Nash equilibrium of the water allocation corresponds to Domain [2] and the Nash equilibrium of the land uses corresponds to Case [2], Case [3], or Case [4]. When the Nash equilibrium of the water prices meets Conditions [3], the Nash equilibrium of the water allocation corresponds to Domain [3] and the Nash equilibrium of the land uses corresponds to Case [3]. When the Nash equilibrium of the water prices meets Conditions [4], the Nash equilibrium of the water allocation corresponds to Domain [4] and the Nash equilibrium of the land uses corresponds to Case [4].

In Chapter 4, we use the results in this chapter for a policy analysis to study the effect of the government subsidy on the decisions and outputs of the BSC. In the policy analysis, we consider that the Nash equilibrium of the water prices $({p_d}^{NE},\ {p_u}^{NE})$ meets Condition [2] of Domain [2] since this case is the most representative case in our research. In this case, depending on the government subsidy, the downstream and upstream farmers may produce both corn and switchgrass after both farmers receive water from the utility company.

4. POLICY ANALYSIS

In Chapter 3, we presented mathematical models of the BSC, where two farmers purchase water from a utility company, produce two kinds of biomass, corn and switchgrass, and sell the crops at a corn market and a switchgrass market. We derived the best decisions of the farmers and the utility company at each stage and found the Nash equilibrium of the BSC.

In this chapter, we analyze the effect of the government subsidy on the BSC, with the Nash equilibrium of the farmers' and the utility company's decisions $({p_d}^{NE}, {p_u}^{NE}, {w_d}^{NE}, {w_u}^{NE}, (q_d^c)^{NE}, (q_d^s)^{NE}, (q_u^c)^{NE}, (q_u^s)^{NE})$ which meets Condition [2] of Domain [2]. The government subsidy affects the decisions and profits of the downstream farmers, the upstream farmers and the utility company $(\pi_d, \pi_u, \text{and } \pi_h)$, the producer surplus (PS_π), the consumer surplus (CS) in a corn market (CS_c) and a switchgrass market (CS_s), and total social welfare (SW) in the BSC. Producer surplus is the difference between the price the producers are willing to supply their product for and the actual price of the product at a market. Consumer surplus is the difference between the price consumers are willing to pay for a product and the actual price of the product at a market. At a market with a linear inverse demand function, $P(Q) = a - bQ$, where Q is the total quantity of the product sold at the market, producer surplus is the sum of the profits of the producers and consumer surplus can be obtained by $\frac{bQ^2}{2}$. Government expenditure (S_{gov}) is the sum of the subsidies provided to the two farmers and is $s_d q_d^c + s_u q_u^c$. In a market analysis, economic welfare (social welfare) at an equilibrium is the sum of producer surplus and consumer surplus. In our model, we estimate economic performance of the subsidy policy through subtracting the government expenditure on the subsidy from the social welfare.

$PS_\pi = \pi_d + \pi_u + \pi_h$: Producer surplus in the BSC

$CS_c = \frac{b_c(q_d^c + q_u^c)^2}{2}$: Consumer surplus at a corn market in the BSC

$CS_s = \frac{b_s(q_d^s + q_u^s)^2}{2}$: Consumer surplus at a switchgrass market in the BSC

$S_{gov} = s_d q_d^c + s_u q_u^c$: Government expenditure on the subsidy

$SW = PS_\pi + CS_c + CS_s$: Social welfare in the BSC

$\Delta_{SW} = SW - S_{gov}$: Social welfare subtracted by the government expenditure

In this research, we study two forms of subsidy policy to the two farmers: [1] Different amounts of subsidies per unit output quantity to the farmers (subsidy discrimination), [2] Equal amounts of subsidies per unit output quantity to the farmers (subsidy equalization). Subsidy equalization is a special case of subsidy discrimination.

Furthermore, we conduct a parametric analysis for the two forms of the subsidy policy by using realistic parameters from literature. The effect of the subsidy amount on the profits of the players and the social welfare subtracted by the government expenditure of the BSC is analyzed. We analyze efficiencies of the two subsidy forms under the same budget limit. The values used in the parameter analysis are shown in Table 5.

Table 5. Parameters values from literature reviews

	Value	Unit	Source
p_e	0.010	\$/kWh	Uria-Martinez et al (2018)
c_w	3.5×10^{-10}	\$/gal^2	Wichelns (2010)
α_c	6-10	t/ha	Liska et al. (2009), Pordesimo et al. (2004), U.S. Department of Energy (2011)
α_s	14-22	t/ha	Spatari et al. (2005)
a_c	176	\$/t	Bai et al. (2012)
b_c	6.4×10^{-5}	\$/t^2	Bai et al. (2012)
c_c	76-88	\$/t	Purdue Crop Cost and Return Guide (2011), U.S. Department of Energy (2011)
c_s	38-48	\$/t	Kumar and Sokhansanj (2007), U.S. Department of Energy (2011)
δ_c	1,260,100	gal/ha	Hamilton et al. (2015)
δ_s	1,439,737	gal/ha	Hamilton et al. (2015)

In addition to the data set in Table 5, we need more information about values of two parameters, α_e and c_w. Since there are many hydroelectric dams with a range of properties, the amount of electricity generated varies much. Water flow, height of the dam, turbine efficiency, and other factors affect hydroelectricity generation. In case of the Hoover Dam, one cubic foot (7.48 gallon) of water falling 8.81 feet per second generates one horsepower (0.7457 kilowatt) at 100 percent efficiency. Average head (the vertical distance water travels) the turbine operates at is 510 to 530 feet (Colorado River and Hoover Dam, 2017). For example, when a gallon of water falls 530 feet, it generates 6.00 kilowatt-seconds which can be approximately converted to 0.0017 kWh/gal ($\alpha_e = 1.7 \cdot 10^{-3}$).

Given the multiplicity of water rights, allocations, and contractual arrangements that characterize irrigation in the Unites States, there is considerable variation in the prices paid for irrigation water. Some farmers with riparian water rights or exchange agreements with the federal government receive water at very low cost [\$5 to \$10 per $1{,}000\ \mathrm{m}^3$(264,172 gallon)], while other farmers with less favorable contracts or those who purchase water from some state-level irrigation agencies pay much higher prices ranging from \$20 to more than \$100 per $1{,}000\ \mathrm{m}^3$ (Wichelns, 2010). The government is willing to charge \$5 per $1{,}000\ \mathrm{m}^3$, approximately $\$1.89 \times 10^{-5}$ per gallon as water price, and this value can be converted to

$\$3.5 \cdot 10^{-10}/\text{gal}^2$. We assume this cost to be the water supply cost in our model ($c_w = 3.5 \cdot 10^{-10}$).

In parameter analysis, we use the following parameters: $p_e = 0.01, c_w = 3.5 \cdot 10^{-10}, \alpha_c = 10, \alpha_s = 22, \alpha_e = 1.7 \cdot 10^{-3}, a_c = 176, b_c = 6.4 \cdot 10^{-5}, c_c = 76, c_s = 48, c_w = 3.5 \cdot 10^{-10},\ \delta_c = 1{,}260{,}100, \delta_s = 1{,}439{,}737, w = 8 \cdot 10^5,\ a_s = 180, b_s = 8$.

Moreover, for policy analysis in this chapter, we only consider a situation, where the utility company allocates its water to both farmer (Domain [2]) in the second stage, and the two farmers compete in both corn and switchgrass markets (Case [2]) in the third stage. The situation satisfies following conditions:

(1) $p_d + p_e\alpha_e + p_u \geq 2c_w w$

(2) $p_d + p_e\alpha_e - p_u \geq -2c_w w$

(3) $p_d + p_e\alpha_e - p_u \leq 2c_w w$

(4) ${\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 3b_s{\alpha_s}^2\delta_c w_d - \delta_c\delta_s(2s_d - s_u) \geq 0$

(5) $-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + {\delta_c}^2(a_s\alpha_s - c_s\alpha_s) + 3b_c{\alpha_c}^2\delta_s w_d + {\delta_c}^2(2s_d - s_u) \geq 0$

(6) ${\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 3b_s{\alpha_s}^2\delta_c w_u - \delta_c\delta_s(-s_d + 2s_u) \geq 0$

(7) $-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + {\delta_c}^2(a_s\alpha_s - c_s\alpha_s) + 3b_c{\alpha_c}^2\delta_s w_u + {\delta_c}^2(-s_d + 2s_u) \geq 0$

(8) $b_s\alpha_s{}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c\alpha_c{}^2\delta_s(a_s\alpha_s - c_s\alpha_s) - b_cb_s\alpha_c{}^2\alpha_s{}^2(2w_d + w_u) +$

$b_c\alpha_c{}^2\delta_s s_d \geq 0$

(9) $b_s\alpha_s{}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c\alpha_c{}^2\delta_s(a_s\alpha_s - c_s\alpha_s) - b_cb_s\alpha_c{}^2\alpha_s{}^2(w_d + 2w_u) +$

$b_c\alpha_c{}^2\delta_s s_u \geq 0$

Conditions (1 ~ 3) imply Domain [2] in the second stage and Conditions (4 ~ 9) imply Case [2] in the third stage. Through these conditions, we could find the effect of the government subsidy on the three-stage BSC with two duopolistic markets. These conditions are applied to Proposition 3, Proposition 4, Corollary 3, Corollary 4, Corollary 5, and Corollary 6, which we discuss throughout this chapter.

4.1. Subsidy Discrimination

In this section, we analyze the effect of the discriminated government subsidy on the BSC. The government makes the decision to provide different subsidies to recipients who have different conditions. The policymakers may like to consider discriminating the subsidies depending upon the recipients' background and the holistic perspectives on the systems in order to ensure efficient results, better output from less income. In this research, the downstream farmer may have an advantage, in that the utility company can earn additional profit from hydroelectricity generation by water release to the downstream.

Corollary 3. If six times the supply cost per unit water from the utility company is greater than the revenue from sale of hydroelectricity generated by unit water $(6c_w w > p_e \alpha_e)$, the following holds for the effect of the policies on the supply chain's decision variables:

(1) $\frac{\partial p_d}{\partial s_d} = \frac{\partial p_u}{\partial s_u} = 2 \cdot \frac{\partial p_u}{\partial s_d} = 2 \cdot \frac{\partial p_d}{\partial s_u} > 0$ (2) $\frac{\partial w_d}{\partial s_d} = \frac{\partial w_u}{\partial s_u} = -\frac{\partial w_u}{\partial s_d} = -\frac{\partial w_d}{\partial s_u} > 0$

(3) $\frac{\partial q_d^c}{\partial s_d} = \frac{\partial q_u^c}{\partial s_u} = -2 \cdot \frac{\partial q_u^c}{\partial s_d} = -2 \cdot \frac{\partial q_d^c}{\partial s_u} < 0$ (4) $\frac{\partial q_d^s}{\partial s_d} = \frac{\partial q_u^s}{\partial s_u} = -2 \cdot \frac{\partial q_u^s}{\partial s_d} = -2 \cdot \frac{\partial q_d^s}{\partial s_u} > 0$

The proof of Corollary 3 is shown in Appendix G.

Corollary 3 reveals the reaction of the farmers and the utility company to the discriminated government subsidies per unit output quantity to the downstream farmer (s_d) and the upstream farmer (s_u). (1) The farmers would announce a higher price of water for more amounts of subsidies per unit output quantity. A farmer in particular would announce a higher price when subsidy to that farmer increases than when subsidy to the other farmer increases. (2) The utility company would increase water allocation to a farmer when subsidy to that farmer increases, whereas the water allocation to a farmer would decrease when subsidy to the other farmer increases. (3) A farmer would decrease corn production when subsidy to that farmer increases, whereas a farmer would increase corn production when subsidy to the other farmer increases. (4) On the other hand, a farmer would increase switchgrass production when subsidy to that farmer increases, whereas a farmer would decrease switchgrass production when subsidy to the other farmer increases.

Proposition 3. Under the discriminated government subsidies to each farmer $(s_d \neq s_u)$, the Nash equilibrium of the BSC is as follows:

For decision variables of the BSC,

$$p_d{}^{NE} = 1.769205591 \cdot 10^{-4} + 4.995691093 \cdot 10^{-13}(2s_d + s_u)$$

$$p_u{}^{NE} = 2.902538925 \cdot 10^{-4} + 4.995691093 \cdot 10^{-13}(s_d + 2s_u)$$

$$(w_d)^{NE_1} = 4.404761904 \cdot 10^{5} + 3.568350781 \cdot 10^{-4}(s_d - s_u)$$

$$(w_u)^{NE_1} = 3.595238096 \cdot 10^{5} - 3.568350781 \cdot 10^{-4}(s_d - s_u)$$

$$(q_d^c)^{NE_1} = 1.762994846 \cdot 10^{-1} - 9.836019102 \cdot 10^{-5}(2s_d - s_u)$$

$$(q_d^s)^{NE_1} = 1.516396465 \cdot 10^{-1} + 8.608796850 \cdot 10^{-5}(2s_d - s_u)$$

$$(q_u^c)^{NE_1} = 1.120568006 \cdot 10^{-1} + 9.836019102 \cdot 10^{-5}(s_d - 2s_u)$$

$$(q_u^s)^{NE_1} = 1.516393456 \cdot 10^{-1} - 8.608796850 \cdot 10^{-5}(s_d - 2s_u)$$

For outcomes of the BSC,

$$\pi_d = 3.606615295 \cdot 10^2 + 2.869598951 \cdot 10^{-5}(4{s_d}^2 + {s_u}^2) + 1.010931575 \cdot 10^{-1}(2s_d - s_u) - 1.147839580 \cdot 10^{-4} s_d s_u$$

$$\pi_u = 2.699945523 \cdot 10^2 + 2.86959895 \cdot 10^{-5}({s_d}^2 + 136{s_u}^2) - 1.010930167 \cdot 10^{-1}(s_d - 35s_u) - 2.008719264 \cdot 10^{-3} s_d s_u$$

$$\pi_h = 2.571634314 \cdot 10^2 + 1.782637822 \cdot 10^{-16}(s_d - s_u)^2 + 6.399242400 \cdot 10^{-7} s_d + 5.590416224 \cdot 10^{-7} s_u$$

$$CS_c = 3.2 \cdot 10^{-5}[2.883562852 - 9.836047418 \cdot 10^{-4}(s_d + s_u)]^2$$

$$CS_s = 4 \cdot [6.672137826 + 1.893935307 \cdot 10^{-3}(s_d + s_u)]^2$$

$$\mathrm{S}_{gov} = 1.721759370 \cdot 10^{-4}({s_d}^2 + {s_u}^2 - s_d s_u) + 1.516396465 \cdot 10^{-1} s_d + 1.516393456 \cdot 10^{-1} s_u$$

$$\Delta_{SW} = 1065.889472 - 1.43479947 \cdot 10^{-5}{s_d}^2 + 3.773522620 \cdot 10^{-3}{s_u}^2 + 5.05464494 \cdot 10^{-2} s_d + 3.386611930 s_u - 1.922631296 \cdot 10^{-3} s_d s_u$$

Proposition 3 reveals the effect of the subsidies on the BSC. We could analyze the decisions on water allocations of the utility company and the land allocations of the farmers.

For the utility company's water allocation, the utility company allocates the entire water to the upstream farmer with $(s_d - s_u) \leq -1.234397113 \cdot 10^9$, while the utility company allocates the entire water to the downstream farmer with $(s_d - s_u) \geq 1.007534941 \cdot 10^9$. The utility company would allocate water to both the downstream farmer and the upstream farmer in case of $-1.234397113 \cdot 10^9 < (s_d - s_u) < 1.007534941 \cdot 10^9$.

For the downstream farmer's land allocation, the farmer produces only corn with $(2s_d - s_u) \leq -1.761449935 \cdot 10^3$, while the farmer produces only switchgrass with $(2s_d - s_u) \geq 1.792386562 \cdot 10^3$. The downstream farmer would produce both corn and switchgrass in case of $-1.761449935 \cdot 10^3 < (2s_d - s_u) < 1.792386562 \cdot 10^3$.

For the upstream farmer's land allocation, the farmer produces only corn with $(s_d - 2s_u) \geq 1.761446440 \cdot 10^3$, while the farmer produces only switchgrass with $(s_d - 2s_u) \leq -1.139249522 \cdot 10^3$. The upstream farmer would produce both corn and switchgrass in case of $-1.139249522 \cdot 10^3 < (s_d - 2s_u) < 1.761446440 \cdot 10^3$.

Corollary 4. In the case of the discriminated government subsidy to the farmers, the following holds for the effect of the policies on the outcome of the supply chain:

$$\frac{\partial \pi_d}{\partial s_d} = 2.021858750 \cdot 10^{-1} + 1.147839580 \cdot 10^{-4}(2s_d - s_u)$$

$$\frac{\partial \pi_d}{\partial s_u} = -1.010931575 \cdot 10^{-1} - 1.147839580 \cdot 10^{-4}(2s_d - s_u)$$

$$\frac{\partial \pi_u}{\partial s_d} = -1.010930167 \cdot 10^{-1} + 5.7391979 \cdot 10^{-5}(s_d - 35s_u)$$

$$\frac{\partial \pi_u}{\partial s_u} = 3.538251276 - 5.7391979 \cdot 10^{-5}(35s_d - 136s_u)$$

$$\frac{\partial \pi_h}{\partial s_d} = 6.399242400 \cdot 10^{-7} + 3.565275644 \cdot 10^{-16}(s_d - s_u)$$

$$\frac{\partial \pi_h}{\partial s_u} = 5.590416224 \cdot 10^{-7} - 3.565275644 \cdot 10^{-16}(s_d - s_u)$$

$$\frac{\partial \Delta_{SW}}{\partial s_d} = 5.05464494 \cdot 10^{-2} - 2.86959894 \cdot 10^{-5}(s_d + 67s_u)$$

$$\frac{\partial \Delta_{SW}}{\partial s_u} = 3.38661193 - 1.922631296 \cdot 10^{-3}(s_d - 3.925373s_u)$$

Corollary 4 reveals the effect of the government subsidies on the profits of the players, and on the total social welfare in the BSC. We find that the effect has to do with the relation between the subsidy to the downstream farmer (s_d) and the upstream farmer (s_u).

Increasing s_d in case of $(2s_d - s_u) > -1.761447144 \cdot 10^3$ and increasing s_u in case of $(2s_d - s_u) < -8.80725488 \cdot 10^2$ would improve the downstream farmer's profit. Increasing s_d in case of $(s_d - 35s_u) > 1.76144852 \cdot 10^3$ and increasing s_u in case of $(35s_d - 136s_u) < 6.16506232 \cdot 10^4$ would improve the upstream farmer's profit. Increasing s_d in case of $(s_d - s_u) > -1.79488013 \cdot 10^9$, and increasing s_u in case of $(s_d - s_u) < 1.56801795 \cdot 10^9$ would improve the utility company's profit. Increasing s_d in case of $(s_d + 67s_u) < 1.76144648 \cdot 10^3$ and increasing s_u in case of $(s_d - 3.925373s_u) < 1.76144638 \cdot 10^3$ would improve social welfare of the BSC.

4.2. Subsidy Equalization

In this section, we analyze the effect of equalized government subsidy on the BSC. The government makes the decision to provide equal subsidies to the recipients under subsidy equalization which is a special case of subsidy discrimination. Subsidy equalization could be considered when policy equity is prioritized over policy efficiency, or when the advantage of the discrimination is regarded as being insignificant. In this research, subsidy equalization could be the justifiable policy, since both farmers produce homogeneous final products (corn and switchgrass) through identical technology. Note that, under this policy, if a farmer receives subsidy, the other farmer also receives the same amount of subsidy for the same output quantity. Here, we set $s_s = s_u = s$.

Corollary 5. If six times the supply cost per unit water from the utility company is greater than the revenue from sale of hydroelectricity generated by unit water $(6c_w w > p_e \alpha_e)$, the following holds for the effect of the policies on the supply chain's decision variables:

(1) $\frac{\partial p_d}{\partial s} = \frac{\partial p_u}{\partial s} > 0$ (2) $\frac{\partial w_d}{\partial s} = \frac{\partial w_u}{\partial s} = 0$

(3) $\frac{\partial q_d^c}{\partial s} = \frac{\partial q_u^c}{\partial s} < 0$ (4) $\frac{\partial q_d^s}{\partial s} = \frac{\partial q_u^s}{\partial s} > 0$

The proof of Corollary 5 is shown in Appendix H.

Corollary 5 reveals the reaction of the farmers and the utility company to the equalized government subsidies to both farmers (s). (1) The farmers would announce a higher price of water for more subsidies. (2) The change in value of the equalized subsidy does not affect the water allocation of the utility company to the farmers. (3,4) Both farmers produce less corn and more switchgrass for a higher subsidy.

Proposition 4. Under equalized government subsidies to both farmers $(s_d = s_u = s)$, the Nash equilibrium of the BSC is as follows:

For decision variables of the BSC,

$$p_d{}^{NE} = 1.769205591 \cdot 10^{-4} + 1.498707328 \cdot 10^{-12} s$$

$$p_u{}^{NE} = 2.902538925 \cdot 10^{-4} + 1.498707328 \cdot 10^{-12} s$$

$$(w_d)^{NE_1} = 4.404761904 \cdot 10^{5}$$

$$(w_u)^{NE_1} = 3.595238096 \cdot 10^{5}$$

$$(q_d^c)^{NE_1} = 1.762994846 \cdot 10^{-1} - 9.836047419 \cdot 10^{-5} s$$

$$(q_d^s)^{NE_1} = 1.516396465 \cdot 10^{-1} + 8.608796850 \cdot 10^{-5} s$$

$$(q_u^c)^{NE_1} = 1.120568006 \cdot 10^{-1} - 9.836047419 \cdot 10^{-5} s$$

$$(q_u^s)^{NE_1} = 1.516393456 \cdot 10^{-1} + 8.608796850 \cdot 10^{-5} s$$

For outcomes of the BSC,

$$\pi_d = 3.606615295 \cdot 10^{2} + 1.010927174 \cdot 10^{-1} s + 2.86959895 \cdot 10^{-5} s^2$$

$$\pi_u = 2.699945523 \cdot 10^{2} + 3.437158260 \cdot 10^{0} s + 1.922631297 \cdot 10^{-3} s^2$$

$$\pi_h = 2.571634314 \cdot 10^2 + 1.198965862 \cdot 10^{-6} s$$

$$CS_c = 1.238372209 \cdot 10^{-10}(1465.813822 - s)^2$$

$$CS_s = 5.739185516 \cdot 10^{-7}(1761.448187 + s)^2$$

$$\mathrm{S}_{gov} = 1.721759370 \cdot 10^{-4}(\mathrm{s} + 1761.448187)\mathrm{s}$$

$$\Delta_{SW} = 1065.889472 + 3.437158379 s + 1.836543328 \cdot 10^{-3} s^2$$

Proposition 4 reveals the effect of equalized subsidy on the BSC. We can analyze the decisions on water allocation of the utility company and the land allocation of the farmers. Both the downstream farmer and the upstream farmer cultivate corn with $0 \leq s < 1.139246242 \cdot 10^3$, while no farmer cultivates corn with $s \geq 1.792381402 \cdot 10^3$. In the case of $1.139246242 \cdot 10^3 \leq s < 1.792381402 \cdot 10^3$, the upstream farmer does not cultivate corn while the downstream farmer still cultivates corn.

Corollary 6. In the case of government subsidy of equal value to the farmers, the following holds for the effect of the policy on the outcome of the supply chain:

(1) $\frac{\partial \pi_d}{\partial s} = 1.010927174 \cdot 10^{-1} + 5.7391979 \cdot 10^{-5} s$

(2) $\frac{\partial \pi_u}{\partial s} = 3.437158260 \cdot 10^{0} + 3.845262594 \cdot 10^{-3} s$

(3) $\frac{\partial \pi_h}{\partial s} = 1.198965862 \cdot 10^{-6}$

(4) $\frac{\partial \Delta_{SW}}{\partial s} = 3.437158379 + 3.673086656 \cdot 10^{-3} s$

Corollary 6 reveals the effect of the equalized subsidy on the profits of the supply chain, customer surplus in switchgrass markets, and total social welfare. According as the amount of the equalized subsidy increases, the social welfare subtracted by the government expenditure, and the profits of the farmers and the utility company would increase.

4.3. Results

In Chapter 4, we analyzed the effect of government subsidies on the BSC. The subsidies are classified into two forms: (1) discriminated subsidies to the farmers (subsidy discrimination) and (2) equal subsidies to the farmers (subsidy equalization).

Under subsidy discrimination, the results show that the government subsidy increases farmers' water prices, their switchgrass production, and the utility company's water allocation to a farmer who receives higher subsidy than the other farmer. Depending on the relation between the two discriminated subsidies, the subsidy policy can increase or decrease the players' profits and the social welfare in the BSC. Also, an excessive amount of subsidy to a farmer can cause disappearance of the corn market or monopoly of the corn market and the switchgrass market.

Under subsidy equalization, the results show that the government subsidy to the farmers increases farmers' water prices and switchgrass production, and the social welfare subtracted by the government expenditure, whereas increasing subsidy decreases corn production and does not affect water allocation by the utility company. Compared to the discriminated subsidies, a higher equalized subsidy would improve the farmers' profits and the social welfare subtracted by the government expenditure, in any case. However, excessive amount of subsidy could cause disappearance or monopoly of a corn market.

Through parametric analysis, we found the effect of the government subsidy on the social welfare subtracted by the government expenditure under subsidy equalization, as shown in Figure 3, and under subsidy discrimination, as shown in Figure 4. In Figure 3, a solid line, Curve (1), represents the change in social welfare subtracted by government expenditure according as the amount of the equalized subsidy to the farmers increases from 0 to 400. In Figure 4, a dotted line, Curve (2), represents the change in social welfare subtracted by government expenditure according as the amount of the discriminated subsidy to the upstream farmer increases from 0 to 800.

In Figure 1, Point [A] addresses the social welfare without any subsidy. Point [B] represents social welfare under subsidy equalization with $s_d = s_u = 400$. In Figure 2, Point [C], Point [D], and Point [E] represent social welfare under subsidy discrimination with the budget of 800 for the sum of unit subsidy to the two farmers $(s_d + s_u = 800)$. Point [C] shows a case where the government provides subsidy only to the downstream farmer $(s_d = 800, s_u = 0)$, while Point [E] shows the opposite case where the government provides subsidy only to the upstream farmer $(s_d = 800, s_u = 0)$. Point [D] shows a case where the government provides an equal amount of subsidy to both farmers $(s_d = 400, s_u = 400)$, which is also represented by Point [B] in Figure 1.

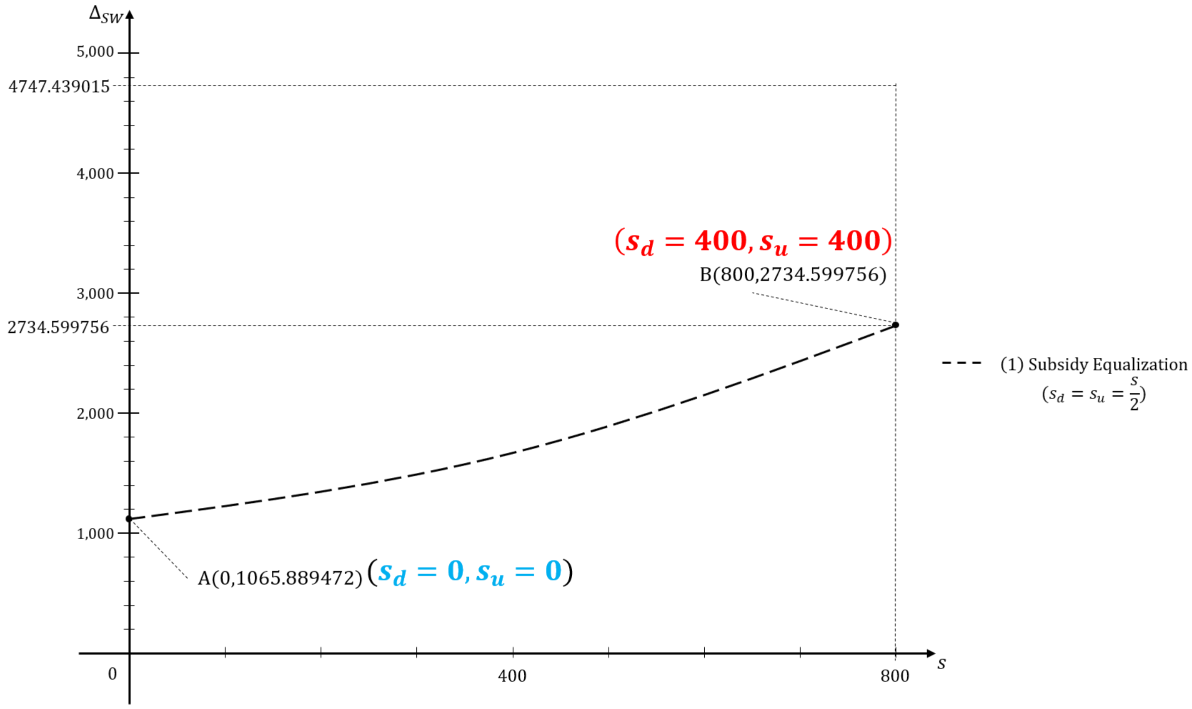

Figure 3. Social welfare subtracted by government expenditure under subsidy equalization

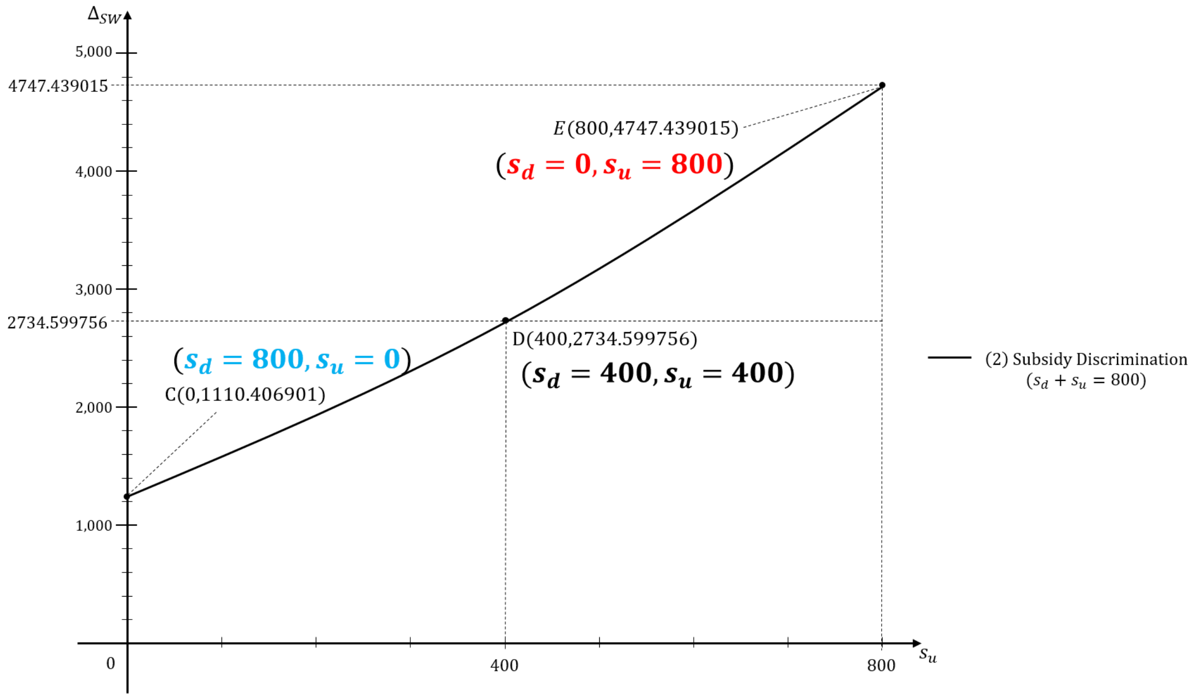

Figure 4. Social welfare subtracted by government expenditure under subsidy discrimination

From the parametric analysis, we found that both forms of subsidy policy improve social welfare. From the perspective of the sum of the unit subsidies to the farmers, increasing the sum of the subsidies induces higher social welfare. Under a constrained budget for the sum of unit subsidy to the two farmers, subsidy discrimination shows higher social welfare in case of $s_d < s_u$ and lower social welfare in case of $s_d > s_u$ compared to subsidy equalization $(s_d = s_u)$. Increasing the subsidy to the upstream farmer is more efficient in improving social welfare than increasing the subsidy to the downstream farmer.

Therefore, the government needs to decide the amount of the subsidies while also considering the dilemma between efficiency and equity of the policy. The effect of the subsidy on the BSC could vary depending on policy priorities.

5. CONCLUSIONS AND FUTURE RESEARCH

We proposed a game theoretical model of a biofuel supply chain (BSC) where two farmers, located in the downstream and the upstream, and a utility company make decisions throughout a three-stage leader-follower Stackelberg game. In the first stage, the two farmers compete on water price under Bertrand competition. In the second stage, the utility company allocates its water to the two farmers based on the quoted water price by the farmers. In the third stage, the two farmers compete on corn quantity and switchgrass quantity at a corn market and a switchgrass market under Cournot competition. We solved the problem by using backward induction method after deriving the KKT optimality conditions of the subproblems in each stage.

We found the equilibrium of the decisions in the BSC and conducted sensitivity analysis to study the effect of the government subsidy on the BSC. Furthermore, we performed parametric analysis using realistic values from the literature. The result shows that the subsidies could improve social welfare unless they exceed certain limit that causes negative profits for farmers. Subsidy discrimination, especially, with higher subsidy to the upstream farmer than the downstream farmer is more efficient in improving social welfare than subsidy equalization.

Although our model considers many realistic conditions, there are a possibility of making the model even closer to real-world problems. In future, we could replace the corn market and the switchgrass market with corn refineries and switchgrass refineries that are considered as players in the game of the BSC. Water consumption in biofuel production could affect the water supply and biomass prices. Through these extensions, we could analyze the reaction of the refineries to the decisions of the farmers and the utility company. Moreover, future research could consider the land type, yield amount, precipitation along with uncertainty in crop production for a multi-period model. This problem can be modeled through stochastic programming which is rarely used in research on the BSC.

REFERENCES

Allevi, E., Conejo, A. J., Oggioni, G., Riccardi, R., Ruiz, C. (2018). Evaluating the strategic behavior of cement producers: An equilibrium problem with equilibrium constraints, European Journal of Operational Research, 264(2), 717-731.

Assila, B., Kobbane, A., Elmachkour, M., Koutbi, M. E. (2017). A dynamic stackelberg-cournot game for competitive content caching in 5G networks. 2017 International Conference on Wireless Networks and Mobile Communications (WINCOM), Rabat, 1-6. doi: 10.1109/WINCOM.2017.8238184

Awudu, I., & Zhang, J. (2013). Stochastic production planning for a biofuel supply chain under demand and price uncertainties. Applied Energy, 103, 189-196.

Azadeh, A., Arani, H. V., Dashti, H. (2014). A stochastic programming approach towards optimization of biofuel supply chain. Energy, 76(1), 513-525.

Bai, Y., Luo, L., Voet, E. (2010). Life Cycle Assessment of Switchgrass-derived Ethanol as Transport Fuel. The International Journal of Life Cycle Assessment, 15 (5), 468-477.

Bai, Y., Ouyang, Y., Pang, J. (2012). Biofuel supply chain design under competitive agricultural land use and feedstock market equilibrium. Energy Economics, 34(5), 1623-1633.

Bai, Y., Ouyang, Y., Pang, J. (2016). Enhanced models and improved solution for competitive biofuel supply chain under land use constraints. European Journal of Operations Research, 249(1), 281-297.

Bajgiran, A. H. (2018), A Biofuel Supply Chain Equilibrium Analysis with Subsidy Consideration, Ph.D. Dissertation. University of Wisconsin Milwaukee.

Bard, J. F. (1998). Practical bilevel optimization: algorithm and applications. Springer US, 30.

Bizikova, L., Crawford, E., Nijnik, M., Swart, R. (2014). Climate change adaptation planning in agriculture: processes, experiences and lessons learned from early adapters. Mitigation and Adaption Strategies for Global Change, 19(4), 411-430. https://doi.org/10.1007/s11027-012-9440-0

Bizikova, L., Roy, D., Venema, H., McCandless, M., Swanson, D., Khachtryan, A., Borden, C., Zubrycki, K. (2014). Water – Energy – Food Nexus and Agricultural Investment: A Sustainable Development Guidebook. International Institute for Sustainable Development, Canada.

Bracken, J., McGill, J. T. (1973) Mathematical problem with optimization problems in the constraints. Operations Research, 21(1), 37-44.

Caldentey, R., Haugh, M. (2017). A Cournot-Stackelberg Model of Supply Contracts with Financial Hedging and Identical Retailers. Foundations and Trends(R) in Technology, Information and Operations Management, 11(1-2), 124-143. http://dx.doi.org/10.1561/0200000075

Chu, Y., You, F. (2014). Integrated scheduling and dynamic optimization by Stackelberg game: bilevel model formulation and efficient solution algorithm. Industrial & Engineering Chemistry Research, 53(13), 5564-5581.

Chue, Y., You, F., Wassick, J. M., Agarwal, A. (2015). Integrated planning and scheduling under production uncertainties: bi-level model formulation and hybrid solution method. Computer & Chemical Engineering, 72(2), 255-272.

Cobuloglu, H. I., Büyüktahtakın, İ. E. (2014). A mixed-integer optimization model for the economic and environmental analysis of biomass production. Biomass and Bioenergy, 67, 8-23. doi: 10.1016/j.biombioe.2014.03.025

Cobuloglu, H. I., Büyüktahtakın, İ. E. (2015). Food vs. biofuel: An optimization approach to the spatio-temporal analysis of land-use competition and environmental impact. Applied Energy, 140, 418-434.

Cobuloglu, H. I., Büyüktahtakın, İ. E. (2017). A two-stage stochastic mixed-integer programming approach to the competition of biofuel and food production. Computer & Industrial Engineering, 107, 251-263.

Colson, B., Marcotte, P., Savard, G. (2007). An overview of bilevel optimization. Annals of Operations Research, 153(1), 235-256.

Dal-Mas, M., Giarola, S., Zamboni, A., & Bezzo, F. (2011). Strategic design and investment capacity planning of the ethanol supply chain under price uncertainty. Biomass and Bioenergy, 35(5), 2059-2071.

Davis, A. S., Hill, J. D., Chase, C. A., Johanns, A. M., Liebman, M. (2012). Increasing Cropping Systems Diversity Balances Productivity. Profitability and Environmental Health, PLoS ONE,7(10). doi:10.1371/journal.pone.0047149

Fulton, M., Karp, L. (1989). Estimating the Objective of a Public Firm in a Natural Resource Industry. Journal of Environmental Economics and Management, 78(1), 233-255.

Ghaderi, H., Pishvaee, M. S., Moini, A., (2016). Biomass supply chain network design: An optimization-oriented review and analysis, Industrial Crops and Products, 94, 972-1000.

GRACE. (2014). Meet the Nexus: How Food, Water and Energy are Connected. GRACE Communications Foundation.

Hamilton, S. K., Hussain, M. Z., Bhardwaj, A. K., Basso, B., Robertson, G. (2015). Comparative water use by maize, perennial crops, restored prairie, and poplar trees in the US Midwest. Environmental Research Letters, 10(4).

Hightower, M., Pierce, S. A. (2008). The energy challenge. Nature, 452(7185), 285-286.

Hoff, H. (2011). Understanding the Nexus. Background paper for the Bonn 2011 Nexus Conference: The Water, Energy and Food Security Nexus. Stockholm Environment Institute, Stockholm.

Holahan, W. L. (2010). Microeconomics of Reliable Urban Water Supply: The Comparative Economic Advantage of Great Lake Cities. Center for Economic Development Publication, 47. https://dc.uwm.edu/ced_pubs/47

IEA (2012). Technology Roadmap: Hydropower. International Energy Agency, Paris, France.

IRENA (2018). Renewable Power Generation Costs in 2017. International Renewable Energy Agency, Abu Dhabi.

Kim, J., Realff, M. J., Lee, J. H., Whittaker, C., Furtner, L. (2011). Design of biomass processing network for biofuel production using an MILP model. Biomass and Bioenergy, 35(2), 853-871. https://doi.org/10.1016/j.biombioe.2010.11.008

Koh, A. (2012). An evolutionary algorithm based on Nash Dominance for Equilibrium Problems with Equilibrium Constraints. Applied Soft Computing, 12(1), 161-173.

Kumar and Sokhansanj (2007). Switchgrass (Panicum Vigratum, L.) delivery to a biorefinery using integrated biomass supply analysis and logistics (IBSAL) Model. Bioresource Technology, 98(5), 1033-1044. doi: 10.1016/j.biortech.2006.04.027

Lacombe, G., Douangsavanh, S., Baker, J., Hoanh, C. T., Bartlett, R., Jeuland, M., Phongpachith, C. (2014). Are hydropower and irrigation development complements or substitute? The example of the Nam Ngum River in the Mekong Basin. Water International, 39(5), 649-670. doi:10.1080/02508060.2014.956205

Larson, E. (2006). A Review of Life-cycle Analysis Studies on Liquid Biofuel Systems for the Transport Sector. Energy for Sustainable Development, 10 (2), 109-126.

Liska, A. J., Yang, H.S., Bremer, V. R., Klopfenstein, T. J., Walters, D. T., Erickson, G. E., Cassman, K. G. (2009). Improvements in life cycle energy efficiency and greenhouse gas emissions of corn-ethanol. Journal of Industrial Ecology, 13(1), 58-74. https://doi.org/10.1111/j.1530-9290.2008.00105.x

Luo, Y., Miller, S. (2013). A game theory analysis of market incentives for US switchgrass ethanol. Ecological Economics, 93, 42-56.

Luo, Z., Pang, J., Ralph, D. (1996). Mathematical Programs with Equilibrium Constraints. Cambridge: Cambridge University Press. doi:10.1017/CBO9780511983658.

Ma, J., Li, Q. (2014). The Complex Dynamics of Bertrand-Stackelberg Pricing Models in a Risk-Averse Supply Chain. Discrete Dynamics in Nature and Society, 2014(4). 1-14. doi:10.1155/2014/749769.

Madani, K. (2010). Game theory and water resources. Journal of Hydrology, 381(3-4), 225-238. https://doi.org/10.1016/j.jhydrol.2009.11.045

Marufuzzaman M., Eksioglu S. D., Hernandez R., 2014, Environmentally Friendly Supply Chain Planning and Design for Biodiesel Production via Wastewater Sludge, Journal of Transportation Science

Papapostolou, C., Kondili, E., Kaldellis, J. K. (2011). Development and implementation of an optimization model for biofuels supply chain. Energy, 36(10), 6019-6026.

Pimentel, D., Berger, B., Filiberto, D., Newton, M., Wolfe, B., Karabinakis, E., Clark, S., Poon, E., Abbett, E., Nandagopal, S. (2004). Water Resources: Agricultural and Environmental Issues. BioScience, 54(10), 909–918. https://doi.org/10.1641/0006-3568(2004)054[0909:WRAAEI]2.0.CO;2

Pordesimo, L. O., Edens, W. C., Sokhansanj, S. (2004). Distribution of aboveground biomass in corn stover. Biomass and Bioenergy, 26(4), 337-343.

Pozo, D., Contreas, J. (2011). Finding Multiple Nash Equilibrium in Pool Based Market: A Stochastic EPEC Approach, IEEE Transactions on Power Systems, 26(3), 1744-1752, doi: 10.1109/TPWRS.2010.2098425

Prokop, J., Ramsza, M., Wiśnicki, B. (2015). A Note on Bertrand Competition under Quadratic Cost Functions, Gospodarka Narodowa, Warsaw School of Economics, issue 2, 5-14.

Purdue extension ID-166-W, (2011). Purdue crop cost & return guide. https://ag.purdue.edu/agecon/Documents/2011%20Purdue%20Crop%20Cost%20and%20Return%20Guide.pdf

Ruiz-Hernández, D., Elizalde, J., Delgado-Gómez, D. (2017). Cournot-Stackelberg games in competitive delocation. Annals of Operations Research, Springer, 256(1), 149-170. DOI: 10.1007/s10479-016-2288-z

Schaible, G., Aillery, M. (2012). Water Conservation in Irrigated Agriculture: Trends and Challenges in the Face of Emerging Demands. SSRN Electronic Journal. doi:10.2139/ssrn.2186555

Sexton, S., Zilberman, D., Rajagopal, D., Hochman, G. (2009). The Role of Biotechnology in a Sustainable Biofuel Future. AgBioForum, 12.

Siddiqui, S., Christensen, A. (2016). Determining energy and climate market policy using multiobjective programs with equilibrium constraints, Energy, 94(C), 316-325.

Siriruk, P. (2009). Cournot competition under uncertainty in power market, Ph.D. Dissertation. Auburn University.

Spatari, S., Zhang, Y., MacLean, H. L. (2005). Life Cycle Assessment of Switchgrass- and Stover-Derived Ethanol-Fueled Automobiles. Environmental Science & Technology, 39(24), 9750-9758.

Stackelberg, H., Bazin, D., Hill, R., Urch, L. (2011). Market structure and equilibrium. Springer-Verlag Berlin Berlin Heidelberg. doi: 10.1007/978-3-642-12586-7

U.S. Department of Energy (2011). U.S. Billion-ton Update: Biomass Supply for a Bioenergy and Bioproducts Industry. U.S. Department of Energy. doi:10.2172/1219219

U.S. Department of Energy (2014). The Water-Energy Nexus: Challenges and Opportunities. U.S. Department of Energy, DOE/EPSA-0002.

Uria-Martinez, R., Johnson, M., Oconnor, P. (2018). 2017 Hydropower Market Report. doi:10.2172/1513459

Varian, H. R. (2006). Intermediate Microeconomics, Seventh Edition, W.W. Norton and Company.

Water Resources and Environment Administration, Hydrological Analysis of Development Scenarios, Nam Ngum River Basin, Agence Française de Dévelopement and Asian Developing Bank, Vientiane, 2008.

Wichelns, D. (2010). Agricultural water pricing: United States. https://www.oecd.org/unitedstates/45016437.pdf

Wright, L., Turhollow, A., 2010. Switchgrass Selection as a 'Model' Bioenergy Crop: A History of the Process. Biomass and Bioenergy, 34 (6), 851-868.

Xie, F., Huang, Y., Eksioglu, S. (2014). Integrating multimodal transport into cellulosic biofuel supply chain design under feedstock seasonality with a case study based on California, Bioresource Technology, 152, 15-23. https://doi.org/10.1016/j.biortech.2013.10.074.

Yue, D., You, F., (2014). Game-theoretic modeling and optimization of multi-echelon supply chain design and operation under Stackelberg game and market equilibrium, 71(4), 347-361.

Zhang, C., Gu, B., Yamori, K., Xu, S., Tanaka, Y. (2015). A Novel Stackelberg-Bertrand Game Model for Pricing Content Provider. EAI Endorsed Transactions on Collaborative Computing. 1. doi:10.4108/icst.mobimedia.2015.259082

Zhang, X., Li, H., Deng, Z., Ringler, C., Gao, Y., Hejazi, M., Leung, R. (2018). Impacts of climate change, policy and Water-Energy-Food nexus on hydropower development. Renewable Energy, 116 (A), 827-834. https://doi.org/10.1016/j.renene.2017.10.030

Zhang, X., Vesselinov, V. V., (2016). Energy-water nexus: Balancing the tradeoffs between two level decision makers. Applied Energy, 183, 77-87. doi:10.1016/j.apenergy.2016.08.156

APPENDICES

Appendix A.

Appendix A shows the calculation and the proof for derivation used in Section 3.2.1.3.

Appendix A.1.

Appendix A.1. represents proof of Case [2] in Table 3.

$$(a_c\alpha_c - c_c\alpha_c - b_c{\alpha_c}^2\boldsymbol{q_u^c}) - 2b_c{\alpha_c}^2\boldsymbol{q_d^c} - \delta_c\lambda_1 = 0 \quad \text{(A-1-1)}$$

$$(a_s\alpha_s - c_s\alpha_s - b_s{\alpha_s}^2\boldsymbol{q_u^s} + s_d) - 2b_s{\alpha_s}^2\boldsymbol{q_d^s} - \delta_s\lambda_1 = 0 \quad \text{(A-1-2)}$$

$$(a_c\alpha_c - c_c\alpha_c - b_c{\alpha_c}^2\boldsymbol{q_d^c}) - 2b_c{\alpha_c}^2\boldsymbol{q_u^c} - \delta_c\lambda_4 = 0 \quad \text{(A-1-3)}$$

$$(a_s\alpha_s - c_s\alpha_s - b_s{\alpha_s}^2\boldsymbol{q_d^s} + s_u) - 2b_s{\alpha_s}^2\boldsymbol{q_u^s} - \delta_s\lambda_4 = 0 \quad \text{(A-1-4)}$$

$$w_d - \delta_c\boldsymbol{q_d^c} - \delta_s\boldsymbol{q_d^s} = 0 \quad \text{(A-1-5)}$$

$$w_u - \delta_c\boldsymbol{q_u^c} - \delta_s\boldsymbol{q_u^s} = 0 \quad \text{(A-1-6)}$$

From Equation (A-1-1), Equation (A-1-3), Equation (A-1-2), and Equation (A-1-4), we derive outputs from the duopolistic corn market and the duopolistic switchgrass market, respectively.

$$q_d^c = \frac{(a_c\alpha_c - c_c\alpha_c)}{3b_c{\alpha_c}^2} + \frac{\delta_c(-2\lambda_1 + \lambda_4)}{3b_c{\alpha_c}^2} \qquad q_u^c = \frac{(a_c\alpha_c - c_c\alpha_c)}{3b_c{\alpha_c}^2} + \frac{\delta_c(\lambda_1 - 2\lambda_4)}{3b_c{\alpha_c}^2}$$

$$q_d^s = \frac{(a_s\alpha_s - c_s\alpha_s)}{3b_s{\alpha_s}^2} + \frac{(2s_d - s_u)}{3b_s{\alpha_s}^2} + \frac{\delta_s(-2\lambda_1 + \lambda_4)}{3b_s{\alpha_s}^2} \qquad q_u^s = \frac{(a_s\alpha_s - c_s\alpha_s)}{3b_s{\alpha_s}^2} + \frac{(-s_d + 2s_u)}{3b_s{\alpha_s}^2} + \frac{\delta_s(\lambda_1 - 2\lambda_4)}{3b_s{\alpha_s}^2}$$

Then we derive λ_2 and λ_4 by putting the above outputs into Equation (A-1-5) and Equation (A-1-6) as follows:

$$\lambda_1 = \frac{b_s{\alpha_s}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c{\alpha_c}^2\delta_s(a_s\alpha_s - c_s\alpha_s)}{(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2)} - \frac{b_c{\alpha_c}^2 b_s{\alpha_s}^2(2w_d + w_u)}{(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2)} + \frac{b_c{\alpha_c}^2\delta_s s_d}{(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2)}$$

$$\lambda_4 = \frac{b_s{\alpha_s}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c{\alpha_c}^2\delta_s(a_s\alpha_s - c_s\alpha_s)}{(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2)} - \frac{b_c{\alpha_c}^2 b_s{\alpha_s}^2(w_d + 2w_u)}{(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2)} + \frac{b_c{\alpha_c}^2\delta_s s_u}{(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2)}$$

Then the best responses are follows:

$$(q_d^c)^{BR} = \frac{\delta_s{}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s)}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{3b_s\alpha_s{}^2\delta_c w_d}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{\delta_c\delta_s(-2s_d + s_u)}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)}$$

$$(q_d^s)^{BR} = \frac{-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + \delta_c{}^2(a_s\alpha_s - c_s\alpha_s)}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{3b_c\alpha_c{}^2\delta_s w_d}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{\delta_c{}^2(2s_d - s_u)}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)}$$

$$(q_u^c)^{BR} = \frac{\delta_s{}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s)}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{3b_s\alpha_s{}^2\delta_c w_u}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{\delta_c\delta_s(s_d - 2s_u)}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)}$$

$$(q_u^s)^{BR} = \frac{-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + \delta_c{}^2(a_s\alpha_s - c_s\alpha_s)}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{3b_c\alpha_c{}^2\delta_s w_u}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{\delta_c{}^2(-s_d + 2s_u)}{3(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)}$$

Appendix A.2.

Appendix A.2. represents proof of Case [3] in Table 3.

$$(a_c\alpha_c - c_c\alpha_c) - 2b_c\alpha_c{}^2\boldsymbol{q_u^c} - \delta_c\lambda_4 = 0 \quad \text{(A-2-1)}$$

$$(a_s\alpha_s - c_s\alpha_s + s_u) - 2b_s\alpha_s{}^2\boldsymbol{q_u^s} - \delta_s\lambda_4 = 0 \quad \text{(A-2-2)}$$

$$w_u - \delta_c\boldsymbol{q_u^c} - \delta_s\boldsymbol{q_u^s} = 0 \quad \text{(A-2-3)}$$

From Equation (A-2-1) and Equation (A-2-2), we derive a new equation as follows:

$$b_c\alpha_c{}^2\delta_s\boldsymbol{q_u^c} - b_s\alpha_s{}^2\delta_c\boldsymbol{q_u^s} = \frac{\delta_s(a_c\alpha_c - c_c\alpha_c) - \delta_c(a_s\alpha_s - c_s\alpha_s + s_u)}{2} \quad \text{(A-2-4)}$$

q_u^c, q_u^s, and λ_4 are derived by solving the simultaneous equations, Equation (A-2-3) and Equation (A-2-4), as follows:

$$\boldsymbol{q_u^c} = \frac{\delta_s{}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s)}{2(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{2b_s\alpha_s{}^2\delta_c w_u}{2(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{-\delta_c\delta_s s_u}{2(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)}$$

$$\boldsymbol{q_u^s} = \frac{-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + \delta_c{}^2(a_s\alpha_s - c_s\alpha_s)}{2(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{2b_c\alpha_c{}^2\delta_s w_u}{2(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{\delta_c{}^2 s_u}{2(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)}$$

$$\lambda_4 = \frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c\alpha_c{}^2\delta_s(a_s\alpha_s - c_s\alpha_s)}{(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{-2b_c\alpha_c{}^2 b_s\alpha_s{}^2 w_u}{(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)} + \frac{b_c\alpha_c{}^2\delta_s s_u}{(b_c\alpha_c{}^2\delta_s{}^2 + b_s\alpha_s{}^2\delta_c{}^2)}$$

Appendix A.3.

Appendix A.3. represents proof of Case [4] in Table 3.

$$(a_c\alpha_c - c_c\alpha_c - b_c{\alpha_c}^2\boldsymbol{q_u^c}) - 2b_c{\alpha_c}^2\boldsymbol{q_d^c} - \delta_c\lambda_1 = 0 \quad \text{(A-3-1)}$$

$$(a_s\alpha_s - c_s\alpha_s - b_s{\alpha_s}^2\boldsymbol{q_u^s} + s_d) - 2b_s{\alpha_s}^2\boldsymbol{q_d^s} - \delta_s\lambda_1 = 0 \quad \text{(A-3-2)}$$

$$w_d - \delta_c\boldsymbol{q_d^c} - \delta_s\boldsymbol{q_d^s} = 0 \quad \text{(A-3-3)}$$

From Equation (A-3-1) and Equation (A-3-2), we derive a new equation as follows:

$$b_c{\alpha_c}^2\delta_s\boldsymbol{q_u^c} - b_s{\alpha_s}^2\delta_c\boldsymbol{q_u^s} = \frac{\delta_s(a_c\alpha_c - c_c\alpha_c) - \delta_c(a_s\alpha_s - c_s\alpha_s + s_u)}{2} \quad \text{(A-3-4)}$$

$q_u^c, q_u^s,$ and λ_4 are derived by solving the simultaneous equations, Equation (A-3-3) and Equation (A-3-4), as follows:

$$\boldsymbol{q_d^c} = \frac{{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s)}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)} + \frac{2b_s{\alpha_s}^2\delta_c w_d}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)} + \frac{-\delta_c\delta_s s_d}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)}$$

$$\boldsymbol{q_d^s} = \frac{-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + {\delta_c}^2(a_s\alpha_s - c_s\alpha_s)}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)} + \frac{2b_c{\alpha_c}^2\delta_s w_d}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)} + \frac{{\delta_c}^2 s_d}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)}$$

$$\lambda_1 = \frac{b_s{\alpha_s}^2\delta_c(a_c\alpha_c - c_c\alpha_c) + b_c{\alpha_c}^2\delta_s(a_s\alpha_s - c_s\alpha_s)}{\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)} + \frac{-2b_c{\alpha_c}^2 b_s{\alpha_s}^2 w_d}{\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)} + \frac{b_c{\alpha_c}^2\delta_s s_d}{\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)}$$

Appendix B.

Appendix B shows the calculation and the proof for derivation used in Corollary 1 in Subsection 3.2.1.4.

$$\frac{\partial\left(q_d^c\right)^{BR}}{\partial w_d} = \frac{b_s{\alpha_s}^2\delta_c}{\left(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2\right)} > 0$$

$$\frac{\partial\left(q_d^s\right)^{BR}}{\partial w_d} = \frac{b_c{\alpha_c}^2\delta_s}{\left(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2\right)} > 0$$

$$\frac{\partial(q_u^c)^{BR}}{\partial w_d} = 0$$

$$\frac{\partial(q_u^s)^{BR}}{\partial w_d} = 0$$

$$\frac{\partial\left(q_d^c\right)^{BR}}{\partial w_u} = 0$$

$$\frac{\partial (q_d^s)^{BR}}{\partial w_u} = 0$$

$$\frac{\partial (q_u^c)^{BR}}{\partial w_u} = \frac{b_s {\alpha_s}^2 \delta_c}{\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} > 0$$

$$\frac{\partial (q_u^s)^{BR}}{\partial w_u} = \frac{b_c {\alpha_c}^2 \delta_s w_u}{\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} > 0$$

The above results are only applied to a case of:

$$\frac{{\delta_s}^2 (a_c \alpha_c - c_c \alpha_c) - \delta_c \delta_s (a_s \alpha_s - c_s \alpha_s) + 3 b_s {\alpha_s}^2 \delta_c w_d - \delta_c \delta_s (2 s_d - s_u)}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} \geq 0$$

$$\frac{-\delta_c \delta_s (a_c \alpha_c - c_c \alpha_c) + {\delta_c}^2 (a_s \alpha_s - c_s \alpha_s) + 3 b_c {\alpha_c}^2 \delta_s w_d + {\delta_c}^2 (2 s_d - s_u)}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} \geq 0$$

$$\frac{{\delta_s}^2 (a_c \alpha_c - c_c \alpha_c) - \delta_c \delta_s (a_s \alpha_s - c_s \alpha_s) + 3 b_s {\alpha_s}^2 \delta_c w_u - \delta_c \delta_s (-s_d + 2 s_u)}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} \geq 0$$

$$\frac{-\delta_c \delta_s (a_c \alpha_c - c_c \alpha_c) + {\delta_c}^2 (a_s \alpha_s - c_s \alpha_s) + 3 b_c {\alpha_c}^2 \delta_s w_u + {\delta_c}^2 (-s_d + 2 s_u)}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} \geq 0$$

$$\frac{b_s {\alpha_s}^2 \delta_c (a_c \alpha_c - c_c \alpha_c) + b_c {\alpha_c}^2 \delta_s (a_s \alpha_s - c_s \alpha_s) - b_c b_s {\alpha_c}^2 {\alpha_s}^2 (2 w_d + w_u) + b_c {\alpha_c}^2 \delta_s s_d}{\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} \geq 0$$

$$\frac{b_s {\alpha_s}^2 \delta_c (a_c \alpha_c - c_c \alpha_c) + b_c {\alpha_c}^2 \delta_s (a_s \alpha_s - c_s \alpha_s) - b_c b_s {\alpha_c}^2 {\alpha_s}^2 (w_d + 2 w_u) + b_c {\alpha_c}^2 \delta_s s_u}{\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} \geq 0$$

Appendix C.

Appendix C shows the calculation and the proof for derivation used in Table 4 in Subsection 3.2.2.4. Best responses of the utility company in the second stage could be derived by using the KKT conditions of the utility company's problem as follows:

$$\mathcal{L}_3 = \pi_h + \lambda_7 (w - \boldsymbol{w_d} - \boldsymbol{w_u}) + \lambda_8 \boldsymbol{w_d} + \lambda_9 \boldsymbol{w_u} \quad \text{(C-1)}$$

$$\frac{\partial \mathcal{L}_3}{\partial \boldsymbol{w_d}} = p_d + p_e \alpha_e - 2 c_w \boldsymbol{w_d} - \lambda_7 + \lambda_8 = 0 \quad \text{(C-2)}$$

$$\frac{\partial \mathcal{L}_3}{\partial \boldsymbol{w_u}} = p_u - 2 c_w \boldsymbol{w_u} - \lambda_7 + \lambda_9 = 0 \quad \text{(C-3)}$$

$$0 \leq \lambda_7 \perp [w - \boldsymbol{w_d} - \boldsymbol{w_u}] \geq 0 \quad \text{(C-4)}$$

$0 \leq \lambda_8 \perp [\boldsymbol{w_d}] \geq 0$ (C-5)

$0 \leq \lambda_9 \perp [\boldsymbol{w_u}] \geq 0$ (C-6)

We have eight cases for positivity of $\lambda_7, \lambda_8,$ and λ_8, since each lambda can be equal to or greater than zero.

(1) $\lambda_7 = 0, \lambda_8 = 0, \lambda_9 = 0$

$p_d + p_e\alpha_e - 2c_w\boldsymbol{w_d} = 0$ (C-1-2)

$p_u - 2c_w\boldsymbol{w_u} = 0$ (C-1-3)

$w - \boldsymbol{w_d} - \boldsymbol{w_u} \geq 0$ (C-1-4)

$\boldsymbol{w_d} \geq 0$ (C-1-5)

$\boldsymbol{w_u} \geq 0$ (C-1-6)

$$\boldsymbol{w_d} = \frac{p_d + p_e\alpha_e}{2c_w}, \boldsymbol{w_u} = \frac{p_u}{2c_w}, \lambda_7 = 0, \lambda_8 = 0, \lambda_9 = 0$$

(2) $\lambda_7 = 0, \lambda_8 = 0, \lambda_9 > 0$

$p_d + p_e\alpha_e - 2c_w\boldsymbol{w_d} = 0$ (C-2-2)

$p_u - 2c_w\boldsymbol{w_u} + \lambda_9 = 0$ (C-2-3)

$w - \boldsymbol{w_d} - \boldsymbol{w_u} \geq 0$ (C-2-4)

$\boldsymbol{w_d} \geq 0$ (C-3-5)

$\boldsymbol{w_u} = 0$ (C-2-6)

$$\boldsymbol{w_d} = \frac{p_d + p_e\alpha_e}{2c_w}, \boldsymbol{w_u} = 0, \lambda_7 = 0, \lambda_8 = 0, \lambda_9 = -p_u$$

This solution is infeasible, since $p_u > 0$ results in $\lambda_9 < 0$.

(3) $\lambda_7 = 0, \lambda_8 > 0, \lambda_9 = 0$

$$p_d + p_e\alpha_e - 2c_w\boldsymbol{w_d} + \lambda_8 = 0 \quad \text{(C-3-2)}$$

$$p_u - 2c_w\boldsymbol{w_u} = 0 \quad \text{(C-3-3)}$$

$$w - \boldsymbol{w_d} - \boldsymbol{w_u} \geq 0 \quad \text{(C-3-4)}$$

$$\boldsymbol{w_d} = 0 \quad \text{(C-3-5)}$$

$$\boldsymbol{w_u} \geq 0 \quad \text{(C-3-6)}$$

$$\boldsymbol{w_d} = 0, \boldsymbol{w_u} = \frac{p_u}{2c_w}, \lambda_7 = 0, \lambda_8 = -p_d - p_e\alpha_e, \lambda_9 = 0$$

This solution is infeasible, since $p_d > 0$ and $p_e\alpha_e > 0$ result in $\lambda_8 < 0$.

(4) $\lambda_7 > 0, \lambda_8 = 0, \lambda_9 = 0$

$$p_d + p_e\alpha_e - 2c_w\boldsymbol{w_d} - \lambda_7 = 0 \quad \text{(C-4-2)}$$

$$p_u - 2c_w\boldsymbol{w_u} - \lambda_7 = 0 \quad \text{(C-4-3)}$$

$$w - \boldsymbol{w_d} - \boldsymbol{w_u} = 0 \quad \text{(C-4-4)}$$

$$\boldsymbol{w_d} \geq 0 \quad \text{(C-4-5)}$$

$$\boldsymbol{w_u} \geq 0 \quad \text{(C-4-6)}$$

$$\boldsymbol{w_d} = \frac{p_d + p_e\alpha_e - p_u + 2c_w w}{4c_w}, \boldsymbol{w_u} = \frac{-p_d - p_e\alpha_e + p_u + 2c_w w}{4c_w}, \lambda_7 = \frac{p_d + p_e\alpha_e + p_u - 2c_w w}{2}, \lambda_8 = 0, \lambda_9 = 0$$

(5) $\lambda_7 = 0, \lambda_8 > 0, \lambda_9 > 0$

$$p_d + p_e\alpha_e - 2c_w\boldsymbol{w_d} + \lambda_8 = 0 \quad \text{(C-5-2)}$$

$$p_u - 2c_w\boldsymbol{w_u} + \lambda_9 = 0 \quad \text{(C-5-3)}$$

$$w - \boldsymbol{w_d} - \boldsymbol{w_u} \geq 0 \quad \text{(C-5-4)}$$

$$\boldsymbol{w_d} = 0 \quad \text{(C-5-5)}$$

$\boldsymbol{w_u} = 0$ (C-5-6)

$$\boldsymbol{w_d} = 0, \boldsymbol{w_u} = 0, \lambda_7 = 0, \lambda_8 = -p_d - p_e\alpha_e, \lambda_9 = -p_u$$

This solution is infeasible, since $p_d > 0, p_e\alpha_e > 0,$ and $p_u > 0$ result in $\lambda_8 < 0$ and $\lambda_9 < 0$.

(6) $\lambda_7 > 0, \lambda_8 > 0, \lambda_9 = 0$

$p_d + p_e\alpha_e - 2c_w\boldsymbol{w_d} - \lambda_7 + \lambda_8 = 0$ (C-6-2)

$p_u - 2c_w\boldsymbol{w_u} - \lambda_7 = 0$ (C-6-3)

$w - \boldsymbol{w_d} - \boldsymbol{w_u} = 0$ (C-6-4)

$\boldsymbol{w_d} = 0$ (C-6-5)

$\boldsymbol{w_u} \geq 0$ (C-6-6)

$$\boldsymbol{w_d} = 0, \boldsymbol{w_u} = w, \lambda_7 = p_u, \lambda_8 = -p_d - p_e\alpha_e + p_u, \lambda_9 = 0$$

(7) $\lambda_7 > 0, \lambda_8 = 0, \lambda_9 > 0$

$p_d + p_e\alpha_e - 2c_w\boldsymbol{w_d} - \lambda_7 = 0$ (C-7-2)

$p_u - 2c_w\boldsymbol{w_u} - \lambda_7 + \lambda_9 = 0$ (C-7-3)

$w - \boldsymbol{w_d} - \boldsymbol{w_u} = 0$ (C-7-4)

$\boldsymbol{w_d} \geq 0$ (C-7-5)

$\boldsymbol{w_u} = 0$ (C-7-6)

$$\boldsymbol{w_d} = w, \boldsymbol{w_u} = 0, \lambda_7 = p_d + p_e\alpha_e, \lambda_8 = 0, \lambda_9 = p_d + p_e\alpha_e - p_u$$

(8) $\lambda_7 > 0, \lambda_8 > 0, \lambda_9 > 0$

$p_d + p_e\alpha_e - 2c_w\boldsymbol{w_d} - \lambda_7 + \lambda_8 = 0$ (C-8-2)

$p_u - 2c_w\boldsymbol{w_u} - \lambda_7 + \lambda_9 = 0$ (C-8-3)

$$w - \boldsymbol{w_d} - \boldsymbol{w_u} = 0 \quad \text{(C-8-4)}$$

$$\boldsymbol{w_d} = 0 \quad \text{(C-8-5)}$$

$$\boldsymbol{w_u} = 0 \quad \text{(C-8-6)}$$

$$\boldsymbol{w_d} = 0, \boldsymbol{w_u} = 0$$

This solution is infeasible, since w should be positive.

Appendix D.

Appendix D shows the calculation and the proof for derivation used in Corollary 2 in Section 3.2.2.4.

$$\frac{\partial w_d^{BR}}{\partial p_d} = \frac{1}{4c_w} > 0$$

$$\frac{\partial w_u^{BR}}{\partial p_d} = \frac{-1}{4c_w} < 0$$

$$\frac{\partial w_d^{BR}}{\partial p_u} = \frac{-1}{4c_w} < 0$$

$$\frac{\partial w_u^{BR}}{\partial p_u} = \frac{1}{4c_w} > 0$$

The above results are only applied to a case of:

$p_d + p_e\alpha_e + p_u \geq 2c_w w$,

$p_d + p_e\alpha_e - p_u \geq -2c_w w$,

and

$p_d + p_e\alpha_e - p_u \leq 2c_w w$.

Appendix E.

Appendix E shows the calculation and the proof for derivation used in Proposition 1 in Section 3.2.3.3. From the farmers' KKT conditions in the first stage, Equations (53), (54), (55), and (56), we have two equations for p_d and p_u.

$$[(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2\{(q_d^c)^{BR} + (q_u^c)^{BR}\}]\frac{\partial (q_d^c)^{BR}}{\partial p_d} + [(a_s\alpha_s - c_s\alpha_s) - b_s{\alpha_s}^2\{(q_d^s)^{BR} +$$

$$(q_u^s)^{BR}\}]\frac{\partial (q_d^s)^{BR}}{\partial p_d} + s_d\frac{\partial (q_d^s)^{BR}}{\partial p_d} - {w_d}^{BR} - \frac{\partial {w_d}^{BR}}{\partial p_d}p_d = 0 \quad \text{(E-1)}$$

$$[(a_c\alpha_c - c_c\alpha_c) - b_c{\alpha_c}^2\{(q_d^c)^{BR} + (q_u^c)^{BR}\}]\frac{\partial (q_u^c)^{BR}}{\partial p_u} + [(a_s\alpha_s - c_s\alpha_s) - b_s{\alpha_s}^2\{(q_d^s)^{BR} +$$

$$(q_u^s)^{BR}\}]\frac{\partial (q_u^s)^{BR}}{\partial p_u} + s_u\frac{\partial (q_u^s)^{BR}}{\partial p_u} - {w_u}^{BR} - \frac{\partial {w_u}^{BR}}{\partial p_u}p_u = 0 \quad \text{(E-2)}$$

We have the following results from the best responses of the utility company in the second stage and the farmers in the third stage:

$$(q_d^c)^{BR} + (q_u^c)^{BR} = \frac{2{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - 2\delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 3b_s{\alpha_s}^2\delta_c w - \delta_c\delta_s(s_d + s_u)}{3(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)}$$

$$(q_d^s)^{BR} + (q_u^s)^{BR} = \frac{-2\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + 2{\delta_c}^2(a_s\alpha_s - c_s\alpha_s) + 3b_c{\alpha_c}^2\delta_s w + {\delta_c}^2(s_d + s_u)}{3(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)}$$

$$\frac{\partial (q_d^c)^{BR}}{\partial p_d} = \frac{b_s{\alpha_s}^2\delta_c}{4c_w(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)}$$

$$\frac{\partial (q_d^s)^{BR}}{\partial p_d} = \frac{b_c{\alpha_c}^2\delta_s}{4c_w(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)}$$

$$\frac{\partial (q_u^c)^{BR}}{\partial p_u} = \frac{b_s{\alpha_s}^2\delta_c}{4c_w(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)}$$

$$\frac{\partial (q_u^s)^{BR}}{\partial p_u} = \frac{b_c{\alpha_c}^2\delta_s}{4c_w(b_s{\alpha_s}^2{\delta_c}^2 + b_c{\alpha_c}^2{\delta_s}^2)}$$

$$\frac{\partial {w_d}^{BR}}{\partial p_d} = \frac{1}{4c_w}$$

$$\frac{\partial {w_u}^{BR}}{\partial p_u} = \frac{1}{4c_w}$$

We substitute the above values into Equation (E-1) and Equation (E-2) as follows:

$$\frac{b_s\alpha_s{}^2\delta_c\left[(a_c\alpha_c-c_c\alpha_c)-b_c\alpha_c{}^2\left(\frac{2\delta_s{}^2(a_c\alpha_c-c_c\alpha_c)-2\delta_c\delta_s(a_s\alpha_s-c_s\alpha_s)+3b_s\alpha_s{}^2\delta_c w-\delta_c\delta_s(s_d+s_u)}{3\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}\right)\right]}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+$$

$$\frac{b_c\alpha_c{}^2\delta_s\left[(a_s\alpha_s-c_s\alpha_s)-b_s\alpha_s{}^2\left(\frac{-2\delta_c\delta_s(a_c\alpha_c-c_c\alpha_c)+2\delta_c{}^2(a_s\alpha_s-c_s\alpha_s)+3b_c\alpha_c{}^2\delta_s w+\delta_c{}^2(s_d+s_u)}{3\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}\right)\right]}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+\frac{b_c\alpha_c{}^2\delta_s s_d}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}-$$

$$2p_d-p_e\alpha_e+p_u-2c_w w=0 \qquad \text{(E-1')}$$

$$\frac{b_s\alpha_s{}^2\delta_c\left[(a_c\alpha_c-c_c\alpha_c)-b_c\alpha_c{}^2\left(\frac{2\delta_s{}^2(a_c\alpha_c-c_c\alpha_c)-2\delta_c\delta_s(a_s\alpha_s-c_s\alpha_s)+3b_s\alpha_s{}^2\delta_c w-\delta_c\delta_s(s_d+s_u)}{3\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}\right)\right]}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+$$

$$\frac{b_c\alpha_c{}^2\delta_s\left[(a_s\alpha_s-c_s\alpha_s)-b_s\alpha_s{}^2\left(\frac{-2\delta_c\delta_s(a_c\alpha_c-c_c\alpha_c)+2\delta_c{}^2(a_s\alpha_s-c_s\alpha_s)+3b_c\alpha_c{}^2\delta_s w+\delta_c{}^2(s_d+s_u)}{3\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}\right)\right]}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+\frac{b_c\alpha_c{}^2\delta_s s_u}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+$$

$$p_d+p_e\alpha_e-2p_u-2c_w w=0 \qquad \text{(E-2')}$$

Then we can simplify Equation (E-1') and Equation (E-2') as follows:

$$\frac{b_s\alpha_s{}^2\delta_c\left[\left(3b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)(a_c\alpha_c-c_c\alpha_c)+2b_c\alpha_c{}^2\delta_c\delta_s(a_s\alpha_s-c_s\alpha_s)+b_c\alpha_c{}^2\left(-3b_s\alpha_s{}^2\delta_c w+\delta_c\delta_s(s_d+s_u)\right)\right]}{3\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)^2}+$$

$$\frac{b_c\alpha_c{}^2\delta_s\left[\left(b_s\alpha_s{}^2\delta_c{}^2+3b_c\alpha_c{}^2\delta_s{}^2\right)(a_s\alpha_s-c_s\alpha_s)+2b_s\alpha_s{}^2\delta_c\delta_s(a_c\alpha_c-c_c\alpha_c)+b_s\alpha_s{}^2\left(-3b_c\alpha_c{}^2\delta_s w-\delta_c{}^2(s_d+s_u)\right)\right]}{3\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)^2}+$$

$$\frac{b_c\alpha_c{}^2\delta_s s_d}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}-2p_d-p_e\alpha_e+p_u-2c_w w=0 \qquad \text{(E-1'')}$$

$$\frac{b_s\alpha_s{}^2\delta_c\left[\left(3b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)(a_c\alpha_c-c_c\alpha_c)+2b_c\alpha_c{}^2\delta_c\delta_s(a_s\alpha_s-c_s\alpha_s)+b_c\alpha_c{}^2\left(-3b_s\alpha_s{}^2\delta_c w+\delta_c\delta_s(s_d+s_u)\right)\right]}{3\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)^2}+$$

$$\frac{b_c\alpha_c{}^2\delta_s\left[\left(b_s\alpha_s{}^2\delta_c{}^2+3b_c\alpha_c{}^2\delta_s{}^2\right)(a_s\alpha_s-c_s\alpha_s)+2b_s\alpha_s{}^2\delta_c\delta_s(a_c\alpha_c-c_c\alpha_c)+b_s\alpha_s{}^2\left(-3b_c\alpha_c{}^2\delta_s w-\delta_c{}^2(s_d+s_u)\right)\right]}{3\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)^2}+$$

$$\frac{b_c\alpha_c{}^2\delta_s s_d}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+p_d+p_e\alpha_e-2p_u-2c_w w=0 \qquad \text{(E-2'')}$$

Then we can simplify Equation (X-1'') and Equation (X-2'') further as follows:

$$\frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c-c_c\alpha_c)+b_c\alpha_c{}^2\delta_s(a_s\alpha_s-c_s\alpha_s)-b_c\alpha_c{}^2b_s\alpha_s{}^2w}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+\frac{b_c\alpha_c{}^2\delta_s s_d}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+(-2p_d-p_e\alpha_e+p_u-$$

$$2c_w w)=0 \qquad \text{(E-1''')}$$

$\frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c-c_c\alpha_c)+b_c\alpha_c{}^2\delta_s(a_s\alpha_s-c_s\alpha_s)-b_c\alpha_c{}^2b_s\alpha_s{}^2w}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+\frac{b_c\alpha_c{}^2\delta_s s_u}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}+(p_d+p_e\alpha_e-2p_u-$

$2c_w w)=0$ (E-2''')

Then we find the equilibrium values of p_d and p_u after solving the above simultaneous equations, as follows:

$$p_d{}^{NE}=A-2c_w w+\frac{2B_d+B_u-p_e\alpha_e}{3}$$

$$p_u{}^{NE}=A-2c_w w+\frac{B_d+2B_u+p_e\alpha_e}{3}$$

, where

$$A=\frac{b_s\alpha_s{}^2\delta_c(a_c\alpha_c-c_c\alpha_c)+b_c\alpha_c{}^2\delta_s(a_s\alpha_s-c_s\alpha_s)-b_c\alpha_c{}^2b_s\alpha_s{}^2w}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)},$$

$$B_d=\frac{b_c\alpha_c{}^2\delta_s s_d}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)},$$

and $B_u=\frac{b_c\alpha_c{}^2\delta_s s_u}{\left(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2\right)}$.

Appendix F.

Appendix F shows the calculation and the proof for derivation used in Section 3.2.4.

Appendix F.1.

Appendix F.1 represents proof of Proposition 2-1 which demonstrates Equilibrium of Domain [2]. Under the conditions of (2-1) $B_d+B_u\geq 6c_w w-p_e\alpha_e-2A$, (2-2) $B_d-B_u\geq -p_e\alpha_e-6c_w w$, and (2-3) $B_d-B_u\leq -p_e\alpha_e+6c_w w$, the best responses of the utility company for the water allocation of Domain [2] in the second stage are as follows:

$$w_d^{BR}=\frac{p_d+p_e\alpha_e-p_u+2c_w w}{4c_w}$$

$$w_u^{BR}=\frac{-p_d-p_e\alpha_e+p_u+2c_w w}{4c_w}$$

In Domain [2], we find the equilibrium values of the utility company's decision on water allocation by substituting ${p_d}^{NE}$ and ${p_u}^{NE}$ into the best responses of the water allocations in the second stage, as follows:

$$(w_d)^{NE_1} = \frac{b_c {\alpha_c}^2 \delta_s (s_d - s_u)}{12c_w\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} + \frac{(p_e\alpha_e + 6c_w w)\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)}{12c_w\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)}$$

$$(w_u)^{NE_1} = -\frac{b_c {\alpha_c}^2 \delta_s (s_d - s_u)}{12c_w\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} + \frac{(-p_e\alpha_e + 6c_w w)\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)}{12c_w\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)}$$

Then, we find the equilibrium values of the farmers' decisions on land allocation by substituting $(w_d)^{NE_1}$ and $(w_u)^{NE_1}$ into the best responses of the land allocation in the third stage, as follows:

$$(q_d^c)^{NE_1} = \frac{4c_w\left[{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s)\right] + b_s {\alpha_s}^2 \delta_c (B_d - B_u + p_e\alpha_e + 6c_w w) - 4c_w\delta_c\delta_s(2s_d - s_u)}{12c_w\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)}$$

$$(q_d^s)^{NE_1} = \frac{-4c_w\left[\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + {\delta_c}^2(a_s\alpha_s - c_s\alpha_s)\right] + b_c {\alpha_c}^2 \delta_s (B_d - B_u + p_e\alpha_e + 6c_w w) + 4c_w{\delta_c}^2(2s_d - s_u)}{12c_w\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)}$$

$$(q_u^c)^{NE_1} = \frac{4c_w\left[{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s)\right] + b_s {\alpha_s}^2 \delta_c (-B_d + B_u - p_e\alpha_e + 6c_w w) + 4c_w\delta_c\delta_s(s_d - 2s_u)}{12c_w\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)}$$

$$(q_u^s)^{NE_1} = \frac{-4c_w\left[\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) - {\delta_c}^2(a_s\alpha_s - c_s\alpha_s)\right] + b_c {\alpha_c}^2 \delta_s (-B_d + B_u - p_e\alpha_e + 6c_w w) - 4c_w{\delta_c}^2(s_d - 2s_u)}{12c_w\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)}$$

Appendix F.2.

Appendix F.2 represents proof of Proposition 2-2 which demonstrates Equilibrium of Domain [3]. Under Condition (3-1) of $B_d - B_u \le -p_e\alpha_e - 6c_w w$, the equilibrium values of the utility company's decision on the water allocations of Domain [3] in the second stage are as follows:

$$(w_d)^{NE_2} = 0$$

$$(w_u)^{NE_2} = w$$

In Domain [3], the corn market and the switchgrass market are the monopoly of the upstream farmer since the downstream farmer does not produce any crop in the third stage. The equilibrium values of the farmers' decision on the land allocation of Domain [3] in the third stage are as follows:

$$(q_d^c)^{NE_2} = 0$$

$$(q_d^s)^{NE_2} = 0$$

$$(q_u^c)^{NE_2} = \frac{{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 2b_s{\alpha_s}^2\delta_c w - \delta_c\delta_s s_u}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)}$$

$$(q_u^s)^{NE_2} = \frac{-\delta_c\delta_s(a_c\alpha_c - c_c\alpha_c) + {\delta_c}^2(a_s\alpha_s - c_s\alpha_s) + 2b_c{\alpha_c}^2\delta_s w + {\delta_c}^2 s_u}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)}$$

Appendix F.3.

Appendix F.3 represents proof of Proposition 2-2 which demonstrates Equilibrium of Domain [4]. Under Condition (3-1) of $B_d - B_u \geq -p_e\alpha_e + 6c_w w$, the equilibrium values of the utility company's decision on the water allocations of Domain [4] in the second stage are as follows:

$$(w_d)^{NE_3} = w$$

$$(w_u)^{NE_3} = 0$$

In Domain [4], the corn market and the switchgrass market are the monopoly of the upstream farmer since the downstream farmer does not produce any crop in the third stage. The equilibrium values of the farmers' decision on the land allocation of Domain [4] in the third stage are as follows:

$$(q_d^c)^{NE_3} = \frac{{\delta_s}^2(a_c\alpha_c - c_c\alpha_c) - \delta_c\delta_s(a_s\alpha_s - c_s\alpha_s) + 2b_s{\alpha_s}^2\delta_c w - \delta_c\delta_s s_d}{2\left(b_c{\alpha_c}^2{\delta_s}^2 + b_s{\alpha_s}^2{\delta_c}^2\right)}$$

$$(q_d^s)^{NE_3} = \frac{-\delta_c\delta_s(a_c\alpha_c-c_c\alpha_c)+\delta_c{}^2(a_s\alpha_s-c_s\alpha_s)+2b_c\alpha_c{}^2\delta_s w+\delta_c{}^2 s_d}{2(b_c\alpha_c{}^2\delta_s{}^2+b_s\alpha_s{}^2\delta_c{}^2)}$$

$$(q_u^c)^{NE_3} = 0$$

$$(q_u^s)^{NE_3} = 0$$

Appendix G.

Appendix G shows the calculation and the proof for derivation used in Corollary 3 in Section 4.1.

$$\frac{\partial p_d}{\partial s_d} = \frac{2b_c\alpha_c{}^2\delta_s}{3(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$\frac{\partial p_u}{\partial s_d} = \frac{b_c\alpha_c{}^2\delta_s s_d}{3(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$\frac{\partial p_d}{\partial s_u} = \frac{b_c\alpha_c{}^2\delta_s}{3(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$\frac{\partial p_u}{\partial s_u} = \frac{2b_c\alpha_c{}^2\delta_s}{3(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$\frac{\partial w_d}{\partial s_d} = \frac{b_c\alpha_c{}^2\delta_s}{12c_w(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$\frac{\partial w_u}{\partial s_d} = \frac{-b_c\alpha_c{}^2\delta_s}{12c_w(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} < 0$$

$$\frac{\partial w_d}{\partial s_u} = \frac{-b_c\alpha_c{}^2\delta_s}{12c_w(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} < 0$$

$$\frac{\partial w_u}{\partial s_u} = \frac{b_c\alpha_c{}^2\delta_s}{12c_w(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$\frac{\partial q_d^c}{\partial s_d} = \frac{-2\delta_c\delta_s}{3(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} < 0$$

$$\frac{\partial q_d^s}{\partial s_d} = \frac{2\delta_c{}^2}{3(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$\frac{\partial q_u^c}{\partial s_d} = \frac{\delta_c\delta_s}{3(b_s\alpha_s{}^2\delta_c{}^2+b_c\alpha_c{}^2\delta_s{}^2)} > 0$$

$$\frac{\partial q_u^s}{\partial s_d} = \frac{-{\delta_c}^2}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} < 0$$

$$\frac{\partial q_d^c}{\partial s_u} = \frac{\delta_c \delta_s}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} > 0$$

$$\frac{\partial q_d^s}{\partial s_u} = \frac{-{\delta_c}^2}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} < 0$$

$$\frac{\partial q_u^c}{\partial s_u} = \frac{-2\delta_c \delta_s}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} < 0$$

$$\frac{\partial q_u^s}{\partial s_u} = \frac{2{\delta_c}^2}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} > 0$$

Appendix H.

Appendix H shows the calculation and the proof for derivation used in Corollary 4 in Section 4.2.

$$\frac{\partial p_d}{\partial s} = \frac{b_c {\alpha_c}^2 \delta_s}{(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2)} > 0$$

$$\frac{\partial p_u}{\partial s} = \frac{b_c {\alpha_c}^2 \delta_s}{(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2)} > 0$$

$$\frac{\partial w_d}{\partial s} = 0$$

$$\frac{\partial w_u}{\partial s} = 0$$

$$\frac{\partial q_d^c}{\partial s} = \frac{-\delta_c \delta_s}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} < 0$$

$$\frac{\partial q_d^s}{\partial s} = \frac{{\delta_c}^2}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} > 0$$

$$\frac{\partial q_u^c}{\partial s} = \frac{-\delta_c \delta_s}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} < 0$$

$$\frac{\partial q_u^s}{\partial s} = \frac{{\delta_c}^2}{3\left(b_s {\alpha_s}^2 {\delta_c}^2 + b_c {\alpha_c}^2 {\delta_s}^2\right)} > 0$$