

December 2019

A Data-Driven Predictive Model of Reliability Estimation Using State-Space Stochastic Degradation Model

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A DATA-DRIVEN PREDICTIVE MODEL OF RELIABILITY ESTIMATION
USING STATE-SPACE STOCHASTIC DEGRADATION MODEL

by

Farhad Balali

A Dissertation Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

in Engineering

at

University of Wisconsin-Milwaukee

December 2019

ABSTRACT

A DATA-DRIVEN PREDICTIVE MODEL OF RELIABILITY ESTIMATION USING STATE-SPACE STOCHASTIC DEGRADATION MODEL

by

Farhad Balali

The University of Wisconsin-Milwaukee, 2019
Under the Supervision of Professor Hamid Seifoddini (Advisor)
Under the Supervision of Professor Adel Nasiri (Co-Advisor)

ABSTRACT: The concept of the Industrial Internet of Things (IIoT) provides the foundation to apply data-driven methodologies. The data-driven predictive models of reliability estimation can become a major tool in increasing the life of assets, lowering capital cost, and reducing operating and maintenance costs. Classical models of reliability assessment mainly rely on lifetime data. Failure data may not be easily obtainable for highly reliable assets. Furthermore, the collected historical lifetime data may not be able to accurately describe the behavior of the asset in a unique application or environment. Therefore, it is not an optimal approach anymore to conduct a reliability estimation based on classical models. Fortunately, most of the industrial assets have performance characteristics whose degradation or decay over the operating time can be related to their reliability estimates. The application of the degradation methods has been recently increasing due to their ability to keep track of the dynamic conditions of the system over time. The main purpose of this study is to develop a data-driven predictive model of reliability assessment based on real-time data using a state-space stochastic degradation model to predict the critical time for initiating maintenance actions in order to enhance the value and prolonging the life of assets. The new degradation model developed in this thesis is introducing a new mapping function for the General Path Model based on series

of Gamma Processes degradation models in the state-space environment by considering Poisson distributed weights for each of the Gamma processes. The application of the developed algorithm is illustrated for the distributed electrical systems as a generic use case. A data-driven algorithm is developed in order to estimate the parameters of the new degradation model. Once the estimates of the parameters are available, distribution of the failure time, time-dependent distribution of the degradation, and reliability based on the current estimate of the degradation can be obtained.

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TABLE OF CONTENTS

1. Chapter 1: Introduction.....	1
1.1 Introduction	1
1.2 Problem Statement	5
1.3 Motivations	7
1.4 Purpose and Significance	8
1.5 Research Questions and Objectives	10
1.6 Definition of Terms	11
1.7 Researcher Assumptions	13
1.8 Summary.....	15
2. Chapter 2: Background and Literature Review	18
2.1 Background Review	18
2.2 Review of Concepts Relevant to Research Questions.....	30
2.2.1 Reliability	30
2.2.2 Asset Management.....	36
2.2.3 Maintenance Policies.....	37
2.2.3.1 Challenges of Classical Models	38
2.2.4 Prognosis and Health Management (PHM)	39
2.2.5 Degradation Mechanism.....	41
2.3 Review of Theories and Methods Relevant to Research Questions	45
2.3.1 Monte Carlo Simulation	45
2.3.2 Distribution Fitting.....	46
2.3.3 Poisson Distribution	47
2.3.4 Gamma Distribution	48
2.3.5 Mixture Distributions	49
2.3.6 Truncated Distributions.....	50
2.3.7 Convolutional Models	50
2.3.8 Maximum Likelihood Estimation	51
2.3.9 Expectation-Maximization Optimization	52
2.3.10 Clustering Algorithms.....	53
2.3.11 K-means Clustering	55
2.3.12 Gaussian Mixture Model Clustering	56

3.	Chapter 3: Generic System Under Study	58
3.1	Distributed Generation (DG).....	62
3.1.1	Scale of the Distributed Generation	63
3.1.2	Distributed Generation Data Flow.....	64
3.1.3	Distributed Generation – Smart Systems.....	65
3.2	Critical Component of the System Under Study	67
3.2.2	Transformers	67
3.2.2	Natural Gas (NG) Generators	69
3.2.3	Inverters	70
3.2.4	Batteries	72
3.3	Generic System under Study	74
3.4	Concluding Remarks.....	75
4.	Chapter 4: Methodology	76
4.1	Research Design.....	76
4.2	Procedures.....	80
4.2.1	Degradation Model	80
4.2.2	Degradation Model Properties	87
4.2.2.1	Damage Events	87
4.2.2.2	Damage Amount	89
4.2.2.3	Degradation Threshold.....	91
4.3	Degradation Model Parameters for Generic Use Case Under Study	91
4.4	Degradation Path Generation.....	94
5.	Chapter 5: Simulation-Based Analysis	96
5.1	Methodologies.....	98
5.1.1	Time-To-Failure Distribution	98
5.1.2	Time-Dependent Degradation Distribution.....	99
5.1.3	Probability Distribution Fitting	100
5.2	Results	102
5.2.1	Transformer	103
5.2.1.1	Time To Failure Distribution	103
5.2.1.2	Reliability.....	104
5.2.1.3	Time-Dependent Degradation Distribution	104

5.2.2	NG Generator	105
5.2.2.1	Time To Failure Distribution	105
5.2.2.2	Reliability.....	105
5.2.2.3	Time-Dependent Degradation Distribution	106
5.2.3	Inverter	106
5.2.3.1	Time To Failure Distribution	106
5.2.3.2	Reliability.....	107
5.2.3.3	Time-Dependent Degradation Distribution	107
5.2.4	Battery	108
5.2.4.1	Time To Failure Distribution	108
5.2.4.2	Reliability.....	108
5.2.4.3	Time-Dependent Degradation Distribution	109
5.3	Concluding Remarks.....	109
6	Chapter 6: Analytical Formulation	112
6.1	Methodologies.....	112
6.2	Results	120
6.3	Concluding Remarks.....	123
7	Chapter 7: Data-Driven Estimate of the Parameters	124
7.1	Methodologies.....	124
7.2	Motivations for Developing a Data-Driven Parameter Estimator	125
7.3	Algorithm Design.....	128
7.4	Results	130
7.4.1	Results of each Clustering Method.....	130
7.4.1.1	Transformer	130
7.4.1.2	NG Generator	132
7.4.1.3	Inverter.....	133
7.4.1.4	Battery.....	134
7.5	Concluding Remarks.....	135
8	Chapter 8: Future Works and Conclusion	136
8.1	Future Works.....	136
8.1.1	Developing a Time-Varying Poisson parameter as a Function of In-Service Time	136

8.1.2	Developing a Data-Driven Parameter Estimator in order to estimate the time-varying Poisson parameter.....	139
8.1.3	Developing a Time-Varying parameter for the distribution of the destruction amount.	140
8.1.4	Considering Higher Orders for the State-Space Stochastic Degradation Model	140
8.2	Conclusion	140
	References.....	144
	Appendix	162
	Appendix A: Distribution Fitting – Matlab Code	162
	Appendix B: Generate Degradation Profiles	166
	Appendix C: Parameter Estimation Based on Gaussian Mixture Models	168
	Appendix D: Parameter Estimation Based on K-Means Clustering Algorithm	170
	Appendix E: Evaluate the Performance of the Parameter Estimation Algorithms....	172
	Curriculum Vitae	174

LIST OF FIGURES

Figure 1: An example of a degradation profile	26
Figure 2: Bathtub hazard rate (failure function).	35
Figure 3: Data-Driven Asset Management.	37
Figure 4: Optimal time interval for performing maintenance actions.....	38
Figure 5: Big picture of degradation process.....	42
Figure 6: Major types of degradation estimates.....	43
Figure 7: Categories of degradation prediction.	44
Figure 8: A schematic view of k-mean clustering algorithm.....	55
Figure 9: A schematic view of the Gaussian Mixture Model clustering algorithm.	57
Figure 10: A schematic picture of the energy network including distributed generations.....	62
Figure 11: A big picture of the electrical network diagram presenting the role of the distributed generations.	63
Figure 12: Various scales of distributed generation.....	64
Figure 13: Schematic view of the data flow through a distributed generation network.	65
Figure 14: Bidirectional flow of power and information.	66
Figure 15: Schematic view of the smart distributed generation.....	67
Figure 16: Main parts of an electrical transformer. (Copyright reserved for “engineeringworldchannel”).....	68
Figure 17: Main parts of natural gas generators. (Copyright reserved for “Cregsedits ; Atlanta ; 2011”).....	70
Figure 18: Off-Grid (1 MW) micro-grid with deferrable and non-deferrable loads.....	75
Figure 19: An overview of the relationship between the Bernoulli, Binomial, and Poisson distributions.	88
Figure 20: Generated degradation paths for critical components of the system based on the predefined parameter of the degradation model.	95
Figure 21: An example of a degradation profile given the known model parameters.	96
Figure 22: An example of a degradation profile given the known model parameters.	97
Figure 23: An example of failure points of 20 degradation profiles generated given the same set of parameters	98
Figure 24: Time-Dependent Degradation Estimates of 20 Degradation Profiles Given Same Parameters.	99
Figure 25: Failure points of 20 degradation profiles generated given the same set of parameters and schematic fitted distribution function to the obtained failure time.	101
Figure 26: Time-Dependent Degradation Distribution of 20 Degradation Profiles Given Same Parameters.....	102
Figure 27: Fitted distribution of the time to failure for the transformer under study.....	103
Figure 28: Obtained reliability function for the transformer under study.....	104
Figure 29: Time-dependent degradation distribution for the transformer under study.....	104
Figure 30: Fitted distribution of the time to failure for the NG generator under study.....	105
Figure 31: Obtained reliability function for the NG generator under study.....	105
Figure 32: Time-dependent degradation distribution for the NG generator under study.....	106
Figure 33: Fitted distribution of the time to failure for the inverter under study.	106
Figure 34: Obtained reliability function for the inverter under study.....	107
Figure 35: Time-dependent degradation distribution for the inverter under study.	107
Figure 36: Fitted distribution of the time to failure for the battery under study.	108
Figure 37: Obtained reliability function for the battery under study.	108
Figure 38: Time-dependent degradation distribution for the battery under study.	109
Figure 39: Probability density of the Gamma distributions for the destruction amount based on the number of damages.	116
Figure 40: Probability density of the truncated Gamma distributions for the destruction amount based on the number of damages.....	117
Figure 41: An illustrative example of the time independency assumption of the analytical approach.....	121
Figure 42: Degradation-base reliability estimation for transformer under study.....	121
Figure 43: Degradation-base reliability estimation for NG Generator under study.....	122
Figure 44: Degradation-base reliability estimation for inverter under study.....	122
Figure 45: Degradation-base reliability estimation for battery under study	122

Figure 46: Detail of the degradation profile for the transformer under study.....	130
Figure 47: Detail of the degradation profile for the NG generator under study.....	132
Figure 48: Detail of the degradation profile for the inverter under study.....	133
Figure 49: Detail of the degradation profile for the battery under study.	134
Figure 50: Effect of considering the in-service time of the asset into the reliability and probability of failure estimations.....	137
Figure 51: Obtained distribution of time-to-soft-failure	138
Figure 52: Obtained time-dependent distribution of the degradation estimates.	138
Figure 53: Comparison between the reliability function with time-varying and time-invariant parameter.	139
Figure 54: Schematic view of data-driven parameter estimator in order to estimate the time-varying Poisson parameter.	139

LIST OF TABLES

Table 1: The detail of the cumulative damage amount.....	90
Table 2: Assumed degradation model parameters for considered degradation mechanism of the transformer under study.	94
Table 3: Detail of an illustrative example for the reason of truncation.....	115
Table 4: Estimated degradation model parameters for transformer based on the Gaussian Mixture Models (GMM) clustering algorithm with and without outlier detection method.	131
Table 5: Estimated degradation model parameters for transformer based on the k-means clustering algorithm with and without outlier detection method.	131
Table 6: Estimated degradation model parameters for NG generator based on the Gaussian Mixture Models (GMM) clustering algorithm with and without outlier detection method.	132
Table 7: Estimated degradation model parameters for NG generator based on the K-Means clustering algorithm with and without outlier detection method.	132
Table 8: Estimated degradation model parameters for inverter based on the Gaussian Mixture Models (GMM) clustering algorithm with and without outlier detection method.	133
Table 9: Estimated degradation model parameters for inverter based on the K-Means clustering algorithm with and without outlier detection method.	133
Table 10: Estimated degradation model parameters for battery based on the Gaussian Mixture Models (GMM) clustering algorithm with and without outlier detection method.	134
Table 11: Estimated degradation model parameters for battery based on the K-Means clustering algorithm with and without outlier detection method.	134

LIST OF ABBREVIATIONS

IloT: Industrial Internet of Things
AM: Asset Management
DG: Distributed Generation
DES: Distributed Electrical System
PHM: Prognosis and Health Management
RES: Renewable Energy Resources
ESD: Energy Storage Device
RUL: Remaining Useful Life
NG: Natural Gas
NDDA: Non-Destructive Degradation Analysis
DDA: Destructive Degradation Analysis
CDF: Cumulative Distribution Function
PDF: Probability Distribution Function
PACF: Partial Auto-Correlation Function
PF: Probability of Failure
LL: Log-Likelihood
AIC: Akaike Information Criterion
NLME: Non-Linear Mixed-Effects models
ML: Machine Learning
EM: Expectation-Maximization
MLE: Maximum Likelihood Estimate
GMM: Gaussian Mixture Model

ACKNOWLEDGMENTS

I would like to express my deepest appreciation to my advisors, Dr. Hamid Seifoddini and Dr. Adel Nasiri who have the attitude and the substance of a genius. They continually and convincingly conveyed a spirit of adventure with regard to my research. Without their guidance and persistent help, this dissertation would not have been possible.

Besides my advisor, I would like to thank my thesis committee member, Dr. Mathew Petering, Dr. Jeijin Jang, and Dr. Xiaohang Yue for their encouragement and insightful comments. Working with them was a great honor and opportunity for me that taught me valuable lessons that will be a precious asset for me throughout my academic career.

I cannot begin to express my thanks to my colleague, co-author, and friend, Dr. Emad Omrani, who has always inspired, supported and nurtured me through all these years.

I would like to express my deepest appreciation to Dr. Ehsan Soofi for all his unwavering supports during my study.

My sincere thanks also go to Mrs. Betty Warras for offering me help and support for all graduate matters.

I must express my very profound gratitude to my beloved wife and my best colleague, Jessie, for providing me with unfailing support and continuous encouragement throughout my years of study.

Last but not least, I would like to thank my parents and sister who are thousands of miles away, for supporting me through all these years. This accomplishment would not have been possible without them.

DEDICATION

To My Beloved Family

1. Chapter 1: Introduction

1.1 Introduction

The concept of Industrial Internet of Things (IIoT) such as new types of assets, data, sensor networks, data analytics, and processing power can provide the foundation to apply data-driven methodologies. The data-driven predictive models of reliability estimation can become a major tool in increasing the life of assets, lowering capital cost, and reducing operating and maintenance costs. Indeed, the predictive model of reliability estimation becoming a critical factor in the efficiency of capital-intensive corporations. The reliability of industrial systems, such as energy and water network, significantly impacts customers as well as providers' bottom line. For instance, the reliability of the electrical power generation network significantly impacts customers as well as energy providers' bottom line. In addition to that, the new types of assets such as distributed generations and smart loads have been emerged, which have motivated the researchers to develop a more robust predictive model of reliability estimation. In addition to that, connectivity between the various sectors of the electrical network has been expressively increased due to the high penetration of the new smart hardware and software tools. Therefore, an accurate predictive model of reliability estimation is necessary in order to optimize various types of decision such as maintenance policy, lifetime analysis, risk management, etc.

The importance of the reliability estimation algorithm is not only limited to the electrical systems. A robust methodology may be applicable to other applications, which failure events may cause circumstances, with some modifications in defining the parameters governing the equipment or system under study. In this thesis, the application of the

developed algorithm is presented for the Distributed Electrical Systems (DESS) as the main generic use case. Continuous demand for electricity and a large number of applications and individuals which may be affected due to the electrical shortage states the reason for choosing DES as the case study. Furthermore, high penetration of the DESSs may offer economic and environmental benefits, which are becoming the main concerns of the societies. Connectivity between the various sectors of the electrical network has been expressively increased due to the high penetration of the new smart hardware and software tools. This connectivity proposes a new potential era in the field of reliability assessment by providing real-time status of the equipment. Traditionally, the reliability of the equipment was estimated mainly based on the offline algorithms, which are independent of the real operational and environmental status of the system. Consequently, the accuracy of the developed algorithms may be affected due to this independency. High penetration of smart devices can provide real-time data regarding the status of the systems. Therefore, a data-driven predictive model of reliability estimation can be developed in order to optimize the maintenance dispatches, replacement schedules, availability, and reliability of the.

Classical models of reliability estimation mainly rely on historical failure data. It should be considered that obtaining lifetime data in a timely manner is one of the current challenges. Failure data may not be easily obtainable for highly reliable assets. Furthermore, the collected historical lifetime data may not be able to accurately describe the behavior of the asset in a unique application or environment. For instance, if the lifetime data are collected based on the experimental tests given specific environmental and operational conditions, there is no guarantee that the asset behavior remains

unchanged in other conditions during its lifetime. Therefore, it is not an optimal approach anymore to estimate reliability based on classical models.

Fortunately, most of the industrial assets have performance characteristics whose degradation or decay over the operating time can be related to their reliability estimates. Degradation indicates the process of lowering the rank, status, or grade, which leads to a less effective level of performance. Degradation based analysis is one of the valuable approaches of condition-based maintenance algorithms in order to obtain reliability information especially for highly reliable systems, critical assets, and recently developed products. The application of the degradation methods has been recently increasing due to their ability to keep track of the dynamic conditions of the system over time. The main purpose of the degradation-based models is to predict the future condition of the asset and perform the maintenance actions in an optimized time window before the actual failure of the system occurs. Since the degradation-based analysis defines the failure events based on the predefined threshold, the failure is said to have occurred as a soft failure. This indicates that the asset under the study is considered as a failed unit when the degradation profile hits the threshold for the first time.

Inaccurate modeling of the degradation phenomenon leads to inaccurate estimation of reliability, maintenance policy, risk, lifetime prediction, etc. In this thesis, a wide variety of the currently developed models of degradation are studied in detail. Degradation models based on the Gamma process and General Path Model have been applied in various studies conducted by other researchers. The main purpose of this study is to develop a predictive model of reliability estimation based on a state-space stochastic degradation model to predict the critical time for initiating the maintenance actions in order

to enhance the value of the assets. Indeed, the new degradation model developed in this study extends the General Path Model based on a series of Gamma Processes degradation models in the state-space environment. Poisson distributed weights are considered for each of the Gamma processes. Therefore, the main scientific contribution of the new degradation model is extending the General Path Model based on Series of Gamma Processes degradation models in the state-space environment by considering the Poisson distributed weights for each of the Gamma processes.

Furthermore, a new data-driven algorithm is developed in order to estimate the parameters of the developed degradation model. The developed parameter estimator in this study in an alternative methodology to the “two-step parameter estimation approach” applied in the General Path degradation model. Once the estimates of the parameters are available, distribution of the failure time, time-dependent distribution of the degradation, and reliability based on the current estimate of the degradation can be obtained.

To sum up, the main scientific contribution of this study are (1) developed a new state-space stochastic degradation model to accurately capture the dynamic behavior of assets., (2) applied simulation techniques to estimate reliability of assets over time and estimate the critical failure time using the new developed model, (3) estimated the reliability based on analytical formulation for degradation prediction model, and (4) developed a new data-driven parameter estimation algorithm based on the new degradation model.

1.2 Problem Statement

Classical models of reliability estimation mainly rely on historical failure data. It should be considered that obtaining lifetime data in a timely manner is one of the current challenges. Failure data may not be easily obtainable for highly reliable assets. Furthermore, the collected historical lifetime data may not be able to accurately describe the behavior of the asset in a unique application or operating environment. For instance, if the lifetime data are collected based on the experimental tests given specific environmental and operational conditions, it is not guaranteed that the asset behavior remains unchanged in other conditions during its lifetime. Most of the classical methods of reliability estimation have considered several assumptions regarding the evolution of the model parameters in order to be able to provide an estimate of the reliability, availability, probability of failure, etc. In the next steps, some assumptions regarding the distribution of the parameters can be made in order to obtain the reliability estimates. For instance, in the area of reliability estimation, there are several developed algorithms that assume the rate of failure is known prior. Furthermore, the failure time or time between each failure event can be estimated based on the Poisson distribution. In most of the cases, the value of the λ is determined based on the engineering insights, laboratory experiments, standards, or history of the component. It should be noted that the same asset may behave completely different in each application. As an example, the optimized maintenance orders may not be similar for two transformers, which have been manufactured the same but have been installed in two different applications. Therefore, it is not an optimal approach anymore to estimate reliability based on classical models.

There are other types of classical algorithms of reliability estimation which seek to estimate the reliability based on the degradation models based on multiple observation of the degradation mechanism for the same asset. Although these types of analysis may seem more robust compared to the classical model of reliability estimation, they are not still an optimized solution in order to predict the future condition of an asset. The main concern is still the same. There is no guarantee that the same assets perform exactly similar under the same operating and environmental conditions. Indeed, a more robust algorithm of reliability estimation is needed to estimate the future condition of each asset based on the real-time status of that specific asset. Furthermore, obtaining the degradation observations may not be always possible in a timely manner at a reasonable cost especially for the destructive degradation tests.

Fortunately, most of the industrial assets have performance characteristics whose degradation or decay over the operating time can be related to their reliability estimates. Degradation indicates the process of lowering the rank, status, or grade, which leads to a less effective level of performance. Degradation based analysis is one of the valuable approaches of condition-based maintenance algorithms in order to obtain reliability information especially for highly reliable systems, critical assets, and recently developed products. The application of the degradation methods has been recently increasing due to their ability to keep track of the dynamic conditions of the system over time. The main purpose of the degradation-based models is to predict the future condition of the asset and perform the maintenance actions in an optimized time window before the actual failure of the system occurs. Since the degradation-based analysis defines the failure events based on the predefined threshold, the failure is said to have occurred as a soft

failure. This indicates that the asset under the study is considered as a failed unit the degradation profile hits the threshold for the first time. Inaccurate modeling of the degradation phenomenon leads to inaccurate estimation of reliability, maintenance policy, risk, lifetime prediction, etc.

Therefore, the main scientific contributions of this study are (1) developed a new state-space stochastic degradation model to accurately capture the dynamic behavior of assets., (2) applied simulation techniques to estimate reliability of assets over time and estimate the critical failure time using the new degradation model, (3) estimated the reliability based on analytical formulation for degradation prediction model, and (4) developed a new data-driven parameter estimation algorithm based on the new degradation model.

1.3 Motivations

The concept of Industrial Internet of Things (IIoT) such as new types of assets, data, sensor networks, data analytics, and processing power can provide the foundation to apply data-driven methodologies. The data-driven predictive models of reliability estimation can become a major tool in increasing the life of assets, lowering capital cost, and reducing operating and maintenance costs. Indeed, the predictive Model of reliability estimation becoming a critical factor in the efficiency of capital-intensive corporations. The reliability of industrial systems, such as energy and water network, significantly impacts customers as well as providers' bottom line. For instance, the reliability of the electrical power generation network significantly impacts customers as well as energy providers' bottom line. In addition to that, the new types of assets such as distributed generations and smart loads have been emerged, which have motivated the researchers to develop

a more robust predictive model of reliability estimation. In addition to that, connectivity between the various sectors of the electrical network has been expressively increased due to the high penetration of the new smart hardware and software tools. Therefore, an accurate predictive model of reliability estimation is necessary in order to optimize various types of decision such as maintenance policy, lifetime analysis, risk management, etc.

1.4 Purpose and Significance

Traditionally, the lifetime of an asset could be determined using manufacturers' suggestions, laboratory results, or well-defined standards. Obtaining life-time data is not always possible due to various reasons. For instance, technological development might lead to zero or a few failures during the test periods. Although an accelerated condition might increase the chance of observing the failure event, it should be considered that some of these tests are destructive which is not desirable especially for the expensive units. It should be noted that these models might not be able to reflect the actual behavior of the asset during its lifetime. In addition to that, the same asset might behave totally different in each application.

Maintenance dispatches and replacement schedules are determined based on the lifetime analysis of each asset. The scheduled maintenance could be categorized as follow based on the time of the maintenance actions and failure event.

- i. Too often, more than actual needs.
- ii. Too rare, less than actual needs which leads to failure.
- iii. Within an optimized interval which enhances the overall life-cycle cost of the network.

The need for applying a more robust model of reliability estimation became obvious during the last few years. The wide applications of the smart devices which lead to more connectivity in the network could significantly support the idea. An optimized methodology should consider the real-time health status of an asset as well as its history in order to obtain optimized network reliability. As a result, the main purpose of this study is to develop a data-driven predictive model of reliability estimation based on real-time data to predict the critical time for initiating maintenance actions in order to enhance the value and prolong the life of assets.

It should be considered that collecting several measurements might not be able to reveal the most beneficial information. Therefore, the raw measurements of asset characteristics should be mapped to a healthy score, such as degradation value, in order to enhance the ability of the analytics to detect the upcoming failures or unreliable events. Determining the degradation estimates may need technical knowledge regarding the physics of the assets under study. Although the degradation models are developed in this study, the main purpose of this dissertation is not limited to the development of the degradation models.

The primary purpose of this research is to develop a predictive model of reliability based on real-time data using a state-space stochastic degradation model to predict the critical time for initiating maintenance actions in order to enhance the value and prolong the life of assets. The new degradation model developed in this thesis is extending the General Path Model based on a series of Gamma Processes degradation models in the state-space environment by considering the Poisson distributed weights for each of the Gamma processes. Furthermore, the new parameter estimation model developed in this

study is an alternative methodology to the “two-step parameter estimation approach” applied in the General Path degradation model.

1.5 Research Questions and Objectives

As mentioned earlier, the classical models of reliability estimation are not providing an optimum estimate of the reliability since they are not able to reflect the actual status of the assets. High penetration of smart devices such as sensors and actuators provides time-series monitoring data which enables the analyst to obtain accurate real-time insight regarding how the assets perform. Since the raw measurements might not be able to reveal very beneficial information, health indicators should be clearly defined for the system under study. Most of the industrial assets have performance characteristics whose degradation or decay over the operating time can be related to their reliability estimates. Degradation indicates the process of lowering the rank, status, or grade, which leads to a less effective level of performance. The main purpose of the degradation-based models is to predict the future condition of the asset and perform the maintenance actions in an optimized time window before the actual failure of the system occurs. Since the degradation-based analysis defines the failure events based on the predefined threshold, the failure is said to have occurred as a soft failure. This indicates that the asset under the study is considered as a failed unit when the degradation profile hits the threshold for the first time.

The definition of the failure in the degradation-based models is completely different than classical models. Collected lifetime data for the classical methodologies are representing the actual physical failure of the asset. In the degradation-based models, failure events are said to be occurred since facing the actual physical failure of the system

is not desirable. Failure events are usually defined as a point of time when the degradation profile hits the critical limit or threshold defined in advanced. Therefore, predictive algorithms must be able to predict the critical failure time based on the first hitting time models to initiate maintenance before the failure occurs. It should be noted that synthetic data are used in this dissertation in order to verify the robustness of the developed models. The followings were the primary objective of this study.

- I. Developed a new discrete-time state-space stochastic degradation model to accurately capture the dynamic behavior of assets.
- II. Estimated the reliability of an asset in the time domain and estimated the critical soft failure time using the new degradation model based on simulation techniques.
- III. Estimated the reliability of an asset in the degradation domain based on the analytical formulation of the new degradation model.
- IV. Developed a data-driven parameter estimation algorithm based on the new degradation model.

1.6 Definition of Terms

- **Degradation profile:**

Degradation profile indicates a time-series estimates of the health condition of the asset under the study.

- **Degradation model:**

The degradation model specifies the model which is able to describe the behavior of the degradation profile. Degradation model may be obtained by fitting techniques based on the observed degradation values, and projection

methodologies in order to predict the future condition of an asset. In other words, a valid degradation model must be tuned in parameters by appropriate training, verification and validation approach based on past information. The obtained model can be adopted for predicting future values.

- **Data-Driven degradation model:**

The parameters of the data-driven degradation model may be defined based on the actual data representing the system under study.

- **Degradation threshold:**

Degradation threshold is a point of degradation estimates in which the asset is considered as a failed unit if the degradation estimator reaches that level.

- **Failure event (soft):**

In degradation-based analyses failure event refers to a point in which the values of the degradation estimates hit the degradation threshold for the first time.

- **Damage event (hazard increment):**

Damage event is defined as an incident of changes in the degradation estimator values. If the degradation value does not change for a few cycles, it means that no destructive event has occurred during this time interval.

- **Damage amount:**

Damage amount refers to the expected amount of damage due to a single destructive event. Since the uncertainty is considerable in defining this value, it has been assumed that the behavior of destruction amount can be explained by a statistical distribution.

- **Cumulative damage amount:**

Cumulative damage amount denotes the changes in the values of degradation estimates from one cycle to another. It should be considered that the total destruction amount is a convolution of single destruction amounts.

- **Unreliability:**

Unreliability denotes a probability which the asset under study is not able to perform its intended functions for a given period of time under the specific working conditions. In this study, unreliability is defined based on each time interval. It means that reliability at time k , R_k , is the probability that the asset under study is not able to perform its intended functions during the next upcoming cycle under the specific working conditions.

1.7 Researcher Assumptions

- The predictive model of reliability estimation provides the reliability estimates based on a time-series degradation profile. In this study, it has been assumed that the overall condition of the asset over time can be explained by a single degradation profile. The same methodology can be applied for the cases in which the asset has more than one degradation profile. In those cases, the estimated reliability is only reflecting the reliability of the asset with respect to the specific degradation profile. For instance, the overall degradation of an electrical transformer might be affected by both oil quality and vibration. In this case, the developed methodology in this study provides two estimates of the reliability based on the oil and vibration degradation profile. More detail of the

post-analysis for assets with multiple degradation profile is presented in the future work of this study.

- It has been assumed that the frequency and intensity of changes in degradation estimates, known as damage events, are independent than each other. It has been assumed that the occurrence of damage events in each cycle, time horizon, will not affect the other cycles.
- The frequency of damage events is assumed to follow a Poisson distribution. Poisson distribution expresses the probability of a given number of events occurring in a fixed interval of time if these events occur with a known rate (λ).
- The robustness of the model can be affected by the approaches regarding the definition of the model parameters. In this study, it has been assumed that the parameters of the new degradation models are time-invariant.
- The value of the degradation estimator is expected to change each time in which a damage event occurs. In this study, it has been assumed that the cumulative damage amount occurred by a variable number of damages in each interval is explainable by statistical distributions. The parameters of the statistical distribution can be affected by the number of damage events. In this thesis, it has been assumed that the cumulative damage amount is obtained by linear summation of damages that occurred by each damage event.
- Degradation profiles are monotonically increasing. It means that the occurred destructions cannot be healed without performing the maintenance or replacement actions.

- For the state-space degradation model considered in this study, the essence of the threshold is similar to the damage amount. In other words, the threshold value can be considered as the maximum value of the destructive amount, which if its occurrence leads to the failure. Therefore, it can be concluded that the statistical distribution of the threshold is similar to the distribution of the destructive amount.
- It has been assumed that the degradation estimates cannot be greater than the threshold.
- In this study, it has been assumed that the degradation estimator is providing the estimates within a certain interval. For instance, the estimates of the degradation estimator may always fall into $[0, 1]$ or $[0, 100]$.

1.8 Summary

The concept of Industrial Internet of Things (IIoT) such as new types of assets, data, sensor networks, data analytics, and processing power can provide the foundation to apply data-driven methodologies. The data-driven predictive models of reliability estimation can become a major tool in increasing the life of assets, lowering capital cost, and reducing operating and maintenance costs. Indeed, the predictive Model of reliability estimation becoming a critical factor in the efficiency of capital-intensive corporations. The reliability of industrial systems, such as energy and water network, significantly impacts customers as well as providers' bottom line. An accurate predictive model of reliability estimation is necessary in order to optimize various types of decision such as maintenance policy, lifetime analysis, risk management, etc.

Classical models of reliability mainly rely on historical failure data. It should be considered that obtaining lifetime data in a timely manner is one of the current challenges. Failure data may not be easily obtainable for highly reliable assets. Furthermore, the collected historical lifetime data may not be able to accurately describe the behavior of the asset in a unique application or environment. For instance, if the lifetime data are collected based on the experimental tests given specific environmental and operational conditions, there is no guarantee that the asset behavior remains unchanged in other conditions during its lifetime. Therefore, it is not an optimal approach anymore to estimate reliability based on classical models.

Fortunately, most of the industrial assets have performance characteristics whose degradation or decay over the operating time can be related to their reliability estimates. The main purpose of the degradation-based models is to predict the future condition of the asset and perform the maintenance actions in an optimized time window before the actual failure of the system occurs. Inaccurate modeling of the degradation phenomenon leads to inaccurate estimation of reliability, maintenance policy, risk, lifetime prediction, etc.

In this thesis, a wide variety of the currently developed models of degradation are studied in detail. Degradation models based on the Gamma process and General Path Model have been applied in various studies. The main purpose of this study is to develop a data-driven predictive model of reliability based on real-time data using a state-space stochastic degradation model to predict the critical time for initiating maintenance actions in order to enhance the value and prolong the life of assets. Indeed, the new degradation model developed in this study extends the General Path Model based on a series of

Gamma Processes degradation models in the state-space environment. Poisson distributed weights are considered for each of the Gamma processes. The application of the developed algorithm is illustrated for the distributed electrical systems as a generic use case.

A new data-driven algorithm is developed in order to estimate the parameters of the developed degradation model. The developed parameter estimator in this study is an alternative methodology to the “two-step parameter estimation approach” applied in the General Path degradation model. Once the estimates of the parameters are available, distribution of the failure time, time-dependent distribution of the degradation, and reliability based on the current estimate of the degradation can be obtained.

To sum up, the main scientific contributions of this study are (1) developed a new state-space stochastic degradation model to accurately capture the dynamic behavior of assets., (2) applied simulation techniques to estimate reliability of assets over time and estimate the critical failure time using the new developed model, (3) estimated the reliability based on analytical formulation for degradation prediction model, and (4) developed a new data-driven parameter estimation algorithm based on the new degradation model.

2. Chapter 2: Background and Literature Review

2.1 Background Review

Predictive models of maintenance determine the optimized schedule of the maintenance actions based on the principles of condition-based maintenance (CBM) [1]. Predictive models are in charge to make a decision using condition monitoring information to optimize the availability of operating plants. CBM empowers the early detection of faults or failures, which leads to reducing the downtime and operating costs, simplifying proactive responses, and enhancing the productivity, reliability, availability, maintainability, and safety (RAMS) of assets [2] [3]. Predictive maintenance models seek to estimate the Remaining Useful Life (RUL) and predict the future health conditions of the assets. It obtains information about the asset's current condition and historical data from the same class of assets [4]. An efficient predictive model of maintenance must be able to predict defect evolution over time and offer enough time for maintenance operation. Most parts of the literature emphasizes on the application of RUL prediction to make reliability assessments [5]. For products that are highly reliable, assessing reliability based on the lifetime data is challenging. Few or perhaps no failures may occur during the monitoring time. Consequently, most of the observations are censored data, which cannot provide very beneficial information about the failed proportion of products [6].

Recently, degradation data have considered being a superior alternative to lifetime data since they are usually more informative compared with lifetime data [7]. Most failures arise from a degradation mechanism, such as the crack length of the filters. Most of the industrial assets have characteristics that degrade or grow over time [8]. These

characteristics may or may not be directly observable. To conduct a degradation-based analysis, one has to prespecify a threshold level of degradation, obtain degradation data over time, and define that failure occurs when the amount of degradation exceeds that level. For instance, the crack grows over time can be directly observed [9] [10]. Failure is defined to occur when the crack reaches a specified length. Another example is the brightness of fluorescent lights that decreases over time. Its failure is defined to occur when the light's luminosity degrades to 60% or less of its luminosity at 100 hours of use [11]. These types of failures are referred to as "soft" failures because the units are still working, but their performance has become unacceptable. On the other hand, degradation of the characteristics may not be directly observable. For instance, air filter clogging may be mapped into the degradation of the characteristics based on the air mass flow, which may be calculated based on the sensor measurements at different points of the system and thermodynamics principles [12]. If the physics of the failure is well-known and accurately studied, mathematical equations, such as the Arrhenius equation in the evaluation of the kinetic degradation, can be applied. Accordingly, degradation data may provide some useful information to assess reliability even when failures do not occur during the monitoring period. Several failure mechanisms can be drawn to an underlying degradation process and the failure time distribution can be estimated earlier in degradation analysis. There is no need to wait for the actual failure point for degradation-based analysis [13] [14].

There are several important references that have used degradation data to assess reliability. Nelson [15] presented a survey of the degradation modeling methods until the 1990s. Nelson (1981) studied a situation in which the degradation measurement is

destructive which means only one measurement could be made on each item. This indicates that obtaining several degradation profiles based on the actual tests on the asset may not have economic justification since the asset cannot stay in service after obtaining the degradation data. Nelson (1990) [16] reviewed the degradation literature, surveyed applications, described basic principles of degradation-base analysis. In the literature of degradation models, there are two major types of modeling for degradation data. One approach assumes that degradation is completely a random process in time. Doksum (1991) [17] applied a Wiener process model to analyze degradation data. Wiener process is a stochastic process that randomly projects the degradation based on the drift and shift parameter of the Wiener process over time. Tang & Chang (1995) [18] modeled nondestructive accelerated degradation data for the power supply units as a collection of stochastic processes. Whitmore & Schenkelberg (1997) [19] studied the degradation process based on a Wiener diffusion process with a time scale transformation. The performance of their developed model is demonstrated for self-regulating heating cables [20].

An alternative approach to model the degradation process is to consider more general statistical models. In that case, degradation can be modeled by a function of time and some possibly multidimensional random variables. Lu & Meeker (1993) [21] [22] developed statistical methods, called the General Path model, using degradation measures to estimate a time-to-failure distribution for a broad class of degradation models. However, this model is based on the fact that a population of “identical” components has a common degradation form. They considered a linear and nonlinear mixed-effects model and developed a two-stage method to obtain estimates of the

parameter. It should be considered that a wide variety of the degradation models can be explained by the General Path model based on the unique definition of the function, which maps the time and explanatory variables into the degradation estimate.

Tseng, Hamada & Chiao (1995) [23] presented a case study that used degradation data and a fractional factorial design to enhance the reliability assessment of fluorescent lamps. It should be considered that their model is only applicable to the cases in which the degradation process is observable. Yacout, Salvatores & Orechwa (1996) [24], used degradation data of metallic Integral Fast reactor fuel pins irradiated in Experimental Breeder Reactor II to estimate the time-to-failure distribution. The time-to-failure distribution was obtained based on a fixed threshold failure model and the two-stage estimation approach proposed by Lu & Meeker (1993) [25]. Lu et al. (1997) [26] proposed a linear regression model with random regression coefficients and a standard-deviation function for investigating linear degradation data from semiconductors. Su et al. (1999) [27] considered a random coefficient degradation model with random sample size and used Maximum Likelihood Estimation (MLE) to estimate the parameters. They developed their model based on a data set from a semiconductor application to illustrate the use case of their methods [28]. One of the applicable models which have been used by several researchers in the various application was degradation path models developed by Meeker & Escobar (1998) [29]. Indeed, the developed model was an extension to the original General Path model by considering the effect of explanatory variables.

Wu & Tsai (2000) [30] presented a fuzzy-weighted estimation method to modify the two-stage procedure proposed by Lu & Meeker (1993) [31]. The developed model was based on the degradation data of the metal film resistor of Wu & Shao (1999) [32]. The

former seemed to reduce the affection of different patterns of degradation paths and improve the estimation results of time-to-failure distribution providing much tighter confidence intervals [33]. Crk (2000) [34] proposed a model that covers many of the developed degradation models by that time and considers a component or a system performance degradation function whose parameters may be random, correlated and stress-dependent in the case of accelerated degradation tests. Jiang & Zhang (2002) [35] presented a dynamic model of degradation based on the data of fatigue crack growth.

In the literature, degradation models can be categorized into two main categories as a stochastic process and the general degradation path model [36]. In the first approach, degradation is assumed to follow a stochastic process. This method requires selecting a probability distribution to represent the measurements at each observation time. Two main steps are involved in the process as (1) estimating the distribution parameters for each time and (2) fitting the estimates as functions of time [37]. The distribution can be obtained based on the principles of the probability distribution fitting. Another method includes considering the degradation as an independent increment process, such as the Wiener or Gamma process [38]. In these methods, the distribution of the failure time is generally estimated by maximizing the likelihood function. The main benefit of the stochastic process model is that the degradation process is fundamentally a continuous state stochastic process [22]. A Wiener process has found application as a degradation model in other studies, for example, Doksum and Høyland (1992) [39] and Lu (1995) [40] [41].

The degradation path approach is based on the profile of degradation over time. Two methods are widely applied in the literature [42]. One is based on the mixed-effects model,

which consists of two main steps as (1) selecting an appropriate relationship between degradation and time such as linear relationship, among which the parameters can be estimated for each item individually; and (2) estimating the distribution function of parameters for all items [43]. The other method is an approximate method also called the pseudo-failure-time-based method. In the approximate method, the pseudo-failure times are analyzed as a complete sample of failure times [44]. The advantage of the degradation path model is considering the unit-to-unit variation.

The procedure that Lu and Meeker (1993) and Meeker and Escobar (1998) [42] [21] [45] developed to estimate reliability using a degradation measure can be summarized as follow:

1. Fit a general path model. Least squares estimation can be used to estimate the parameters for each path.
2. Determine the statistical distribution of each of the random parameters from the general path model.
3. Use the resulting distributions to solve for the time to failure distribution $F_T(t)$ if a closed-form expression exists.
4. If no closed-form expression for $F_T(t)$ exists, use the parameter distributions from (2) to simulate a large number N of random degradation paths.
5. To estimate $F_T(t)$, compute the proportion of random paths generated in (3) that cross a pre-determined critical level (which defines failure) before time t . That proportion is the estimate of $F_T(t)$.

$$\begin{cases} Y_{ij} = \eta_{ij} + \varepsilon_{ij} = \eta(t_j, \phi, \theta_i) + \varepsilon_{ij} & i = 1, 2, \dots, n \quad ; \quad j = 1, 2, \dots, m & \text{Equation(1)} \\ T_i = \min \left\{ MLT_i, \min \left\{ t_j : \eta(t_j, \phi, \theta_i) \geq \eta_{Fi} \right\} \right\} & & \text{Equation(2)} \end{cases}$$

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

(θ_i) and (ε_{ij}) are independent.

$$\begin{cases} i & : \text{Critical assets index} \\ j & : \text{Observation index} \\ Y_{ij} & : \text{Degradation value (observed based on health indicator) for unit } i \text{ at observation } j \\ \eta_{ij} & : \text{Degradation path model for unit } i \text{ at observation } j \\ \varepsilon_{ij} & : \text{Error term for unit } i \text{ at observation } j \\ t_j & : \text{Time of } j^{\text{th}} \text{ observation} \\ \phi & : \text{Fixed-effect parameter (same for all units)} \\ \theta_i & : \text{Time-effect parameter for unit } i \\ \sigma_\varepsilon^2 & : \text{Variance of error} \\ T_i & : \text{Failure time for unit } i \\ MLT_i & : \text{Maximum Life-Time for unit } i \\ \eta_{Fi} & : \text{Defined failure level for unit } i \end{cases}$$

Distribution function of T, failure time, can be written as follow:

$$p(T_i \leq t) = F_T(t) = F_T(t_j, \phi, \theta_i, \eta_{Fi}) \quad \text{Equation (3)}$$

For simple path models, the distribution function $F(T)$ can be expressed in a closed-form. For many path models, however, this is not always possible. Lu & Meeker (1993) [22] proposed a two-stage method of estimation for the case where the vector of random effects or some appropriate reparameterization follows a Multivariate Normal Distribution [46] [47]. Since full maximum likelihood estimation of random-effect parameters in general, algebraically intractable and computationally intensive when they appear nonlinearly in the path model, the authors proposed this two-stage method as an

alternative to the computationally intensive ones [48]. Simulation studies showed that the method compared well with the more computationally intensive methods [49].

Pinheiro & Bates (1995) [50] [51] used Lindstrom and Bates's method (Lindstrom & Bates, 1990) [52] to obtain an approximated maximum likelihood estimate of the parameters. The LME (linear mixed-effects models) and NLME (nonlinear mixed-effects models) functions, were developed to attain this goal (Pinheiro & Bates, 2000) [53]. In other words, these functions were developed for the specific case where the random effects follow a Multivariate Normal Distribution. Meeker & Escobar (1998) [29] used the numerical method with the NLME function developed by Pinheiro & Bates (1995, 2000) [54] in a number of examples. In all of them, the failure time distribution $F(T)$ was estimated numerically using Monte Carlo simulation. In addition, the authors presented two other methods of degradation data analysis, namely the approximate and the analytical method [55]. Both of them are difficult to apply when the degradation path model is nonlinear and has more than one random parameter. The methods described so far rely on maximum likelihood or least-squares estimation of the model parameters and Monte Carlo simulation [56] [57].

Figure 1 shows an example of a degradation profile where T is the time when the degradation estimates reach the critical value, D , for the first time [58]. As mentioned before, degradation-based model failure is said to be occurred [59] [60]. It means that the asset might be still working above the threshold value but, from the engineering point of view reliability has been degraded enough to be called a failed unit [61] [62].

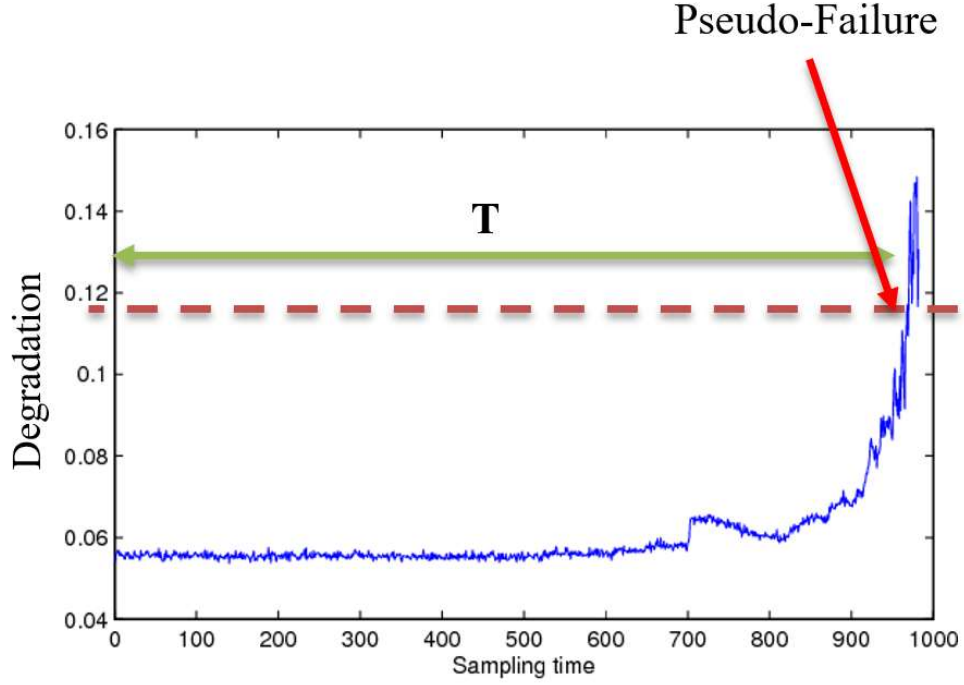


Figure 1: An example of a degradation profile

Suppose that the actual degradation process can be modeled as follow:

$$\eta_i = \phi + \theta_i \cdot t_j \quad ; \quad \text{Equation (2.4)}$$

$$\eta_{Fi} = \phi + \theta_i \cdot T_i \Rightarrow T_i = \frac{\eta_{Fi} - \phi}{\theta_i} \Rightarrow T_i = \tau(\phi, \theta_i, \eta_{Fi}) \quad \text{Equation (2.5)}$$

$$\left\{ \begin{array}{l} \text{if } \theta_i \sim \text{Weibull}(\alpha, \beta) \Rightarrow F_T(t) = p(T < t) = \exp\left[-\left(\frac{\eta_{Fi} - \phi}{\alpha t}\right)^\beta\right] \\ \text{if } \theta_i \sim N(\mu, \sigma^2) \Rightarrow F_T(t) = p(T < t) = \Phi\left(\frac{t - ((\eta_{Fi} - \phi) / \mu)}{\sigma t / \mu}\right) \\ \text{if } \theta_i \sim LN(\mu, \sigma^2) \Rightarrow F_T(t) = p(T < t) = \Phi\left(\frac{\log(t) - [\log(\eta_{Fi} - \phi) - \mu]}{\sigma}\right) \end{array} \right.$$

The stochastic gamma process has been widely used to model uncertain degradation in engineering systems and structures [63]. The gamma process is an example of an analytically tractable stochastic cumulative process that is widely used to model degradation processes, such as corrosion, creep and wear, in engineering systems, structures and components [64]. The theory of gamma processes provides an analytical framework for predicting the reliability and estimating the maintenance cost, including costs of inspection, repair or replacements, and consequences of the failure [65] [66]. This probabilistic model can be subsequently used for cost optimization by appropriately choosing the inspection interval and preventive maintenance (PM) criterion. The periodic model of Condition-Based Maintenance (CBM) of components subjected to Gamma process degradation was presented by Abdel-Hameed (1987) [67] [68] and Park (1988) [69]. Other applications include recession of coastal cliffs, deterioration of coating on steel structures and concrete structure degradation are presented by Meadowcroft (2002) [70], Noortwijk (2007) [71] [72], and Frangopol (2004) [73], respectively. The model of non-periodic CBM was presented by Grall et al. (2002) [74] and that of imperfect inspection by Kallen and van Noortwijk (2005) [75]. Castanier et al. (2003) [76] studied such a maintenance policy in which both the future maintenance and the inspection schedule depend on the magnitude of degradation. Optimization of inspection and repair for the Wiener and Gamma processes of degradations was discussed by Dagg (2004) [77]. A comprehensive review of the Gamma process model and its applications can be found in a recent review article of van Noortwijk (2009) [78] [79].

Based on the references in the field, the Gamma process was first applied by Moran (1954) [80] [81] in a series of papers and a book published in the fifties of the last century

to model water flow into a dam. Abdel-Hameed (1975) [82] [83] proposed to apply the Gamma process as a model for a deterioration occurring randomly in time. During the last few decades, Gamma processes were pleasingly fitted to data on creep of concrete (Baz̃ant et al., 1977) [84] [85], fatigue crack growth (Crowder et al., 2004) [86], corroded steel gates (Noortwijk et al., 2004) [87], thinning due to corrosion (MJ et al., 2005) [88], and chloride ingress into concrete (JD et al., 2004) [89]. Statistical estimation methods that were established contain the maximum-likelihood method and method of moments (Baz̃ant et al., 1977) [84], as well as the Bayesian method with perfect inspection (Dufresne et al., 1991) [90], and imperfect inspection (MJ et al., 2005) [91]. A method for estimating a gamma process by means of expert judgment is proposed by (Nicolai et al., 2016) [92]. On the basis of the gamma deterioration processes, case studies have been performed to determine optimal dike heightening (Speijker LJP et al., 2000), optimal sand nourishment sizes (Noortwijk et al., 2000) [93], optimal maintenance decisions for steel coatings (Heutink A et al., 2004) [94], and optimal inspection intervals for high-speed railway tracks (Meier-Hirmer et al., 2005) [95], berm breakwaters (Noortwijk et al., 1996) [96] [97], and automobile brake pads (Crowder M et al., 2007) [98].

In order for the stochastic deterioration process to be monotonic, we can best consider it as a gamma process. In words, a gamma process is a stochastic process with independent, non-negative increments (e.g., the increments of the crest-level decline of a dike) having a gamma distribution with an identical scale parameter. In the case of a gamma deterioration, dikes can only decrease in height due to crest-level decline. Abdel-Hameed (1975) [99] was the first to propose the gamma process as a proper model for a deterioration occurring randomly in time. In his two-page paper, he called this stochastic

process the “gamma wear process”. An advantage of modeling deterioration processes through gamma processes is that the required mathematical calculations are relatively straightforward [100]. The gamma process is suitable to model gradual damage monotonically accumulating over time in a sequence of tiny increments, such as wear, fatigue, corrosion, crack growth, erosion, consumption, creep, swell, degrading health index, etc. In mathematical terms, the gamma process is defined as follows. Recall that a random quantity X has a gamma distribution with shape parameter α and scale parameter β if its probability density function is given by Equation (6) [101] .

$$Gamma(x|\alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad \text{Equation (6)}$$

where α and β are the shape and scale parameters, respectively. The Gamma function for $\alpha > 0$ is $\Gamma(\alpha) = \int_{z=0}^{\infty} z^{\alpha-1} e^{-z} dz$. Furthermore, let $\alpha(t)$ be a non-decreasing, right-continuous, real-valued function for $t \geq 0$, with $\alpha(0) \equiv 0$. The Gamma process with α and β as shape and scale parameter is a continuous-time stochastic process with the following properties:

- $X(0) = 0$ with probability one
- $X(\tau) - X(t) \sim Gamma(\alpha(\tau) - \alpha(t), \beta)$ for all $\tau > t \geq 0$
- $X(t)$ has independent increments

Mean and variance of the Gamma process can be calculated as the following equations present.

$$E(X(t)) = \frac{\alpha(t)}{\beta} \quad \text{Equation (7)}$$

$$\text{Var}(X(t)) = \frac{\alpha(t)}{\beta^2} \quad \text{Equation (8)}$$

2.2 Review of Concepts Relevant to Research

Questions

2.2.1 Reliability

Degradation data provide a useful resource for obtaining reliability information for some highly reliable products and systems [102]. In addition to product/system degradation measurements, it is common nowadays to dynamically record product/system usage as well as other life-affecting environmental variables such as load, amount of use, temperature, and humidity. Often it is difficult to assess component reliability using traditional methods due to a lack of observed failures [103] [104]. For many such components, degradation measures, recorded over time, will contain important information about performance and reliability. These measures can be used to predict the remaining time to failure and to estimate the overall reliability distribution for that component. Degradation measures are inherent in situations where failure occurs due to a process of accumulation of damage [105].

Reliability can have various definitions based on each application and level of analysis. Reliability analyses can be performed at either component or system level. Based on the ISO 8402, reliability can be defined as “the ability of an item to perform a required function, under given environmental and operational conditions for a stated period of time” [106]. The term ‘item’ might refer to a component, unit, or a complex system. Reliability can also define as “the probability that a product performs its intended

function without failure under specified conditions for a specified period of time”. Therefore, the reliability of a system is the likelihood that it will perform its required functions under stated conditions for a specified period of time [107]. Thus, unavoidably, engineering judgment is required in defining essential concepts such as “required functions,” “stated conditions,” and “specified period of time”. Most of the definitions consider three main elements for reliability function as follow [108]:

- The intended function might be a single or a combination of several functions in order to provide a predefined level of service. It should be considered that each component or system might have the ability to perform various functions. Each of these functions might be active or passive depending on the definition of the required service for estimating the reliability. In some cases, a product might be still in service while the performance of its functions might be deprived enough to be considered as an unreliable product.
- Specified period of time is generally offered by manufacturers or service companies as lifetime or warrantee period. It should be noted that the suggested times are generally an estimate of the expected value for the lifetime. Therefore, in the case of customer dissatisfaction during these periods, the manufacturer or service company is in charge of repair or replacement.
- Specified conditions are generally considered as a basis to evaluate the system performance and reliability of the equipment given those specific conditions. Various environmental, operational, and usage conditions lead to different reliability estimates for the same system or product. Sever conditions might cause immediate failures. Accelerated reliability experiments are based on exposing the

product in severe conditions to evaluate its performance and reliability. Therefore, reliability is highly tight to the specific conditions during the reliability function development. These environmental or operating working conditions are not always controllable or predictable in real-world applications.

The North American Electric Reliability Council (NERC) [109] has introduced a more comprehensive definition of reliability. NERC defines the reliability of a system in terms of two basic functional aspects:

- Adequacy which is always the ability of the systems to supply the aggregate demand and requirements of customers, taking into account scheduled and reasonably expected unscheduled outages of system elements.
- Security is the ability of the systems to withstand sudden disturbances such as unanticipated loss of system elements.

One of the purposes of reliability analysis is quantifying the probability by any attempt to measure it involving probabilistic and statistical methods. Reliability analysis incorporates activities to identify potential failure modes and mechanisms, to make reliability predictions, and to quantify risks for critical components in order to optimize the life-cycle costs for a product [110]. Prior experience and history can be helpful in this analysis. The data used to make reliability predictions may be historical, from the previous testing of similar products, or from the reported field failures of similar products. Reliability can be estimated from the test data using parametric or nonparametric techniques. In parametric estimation, the distribution of the test data should be known or assumed. Parametric techniques provide an inaccurate estimation if the assumptions are incorrect [111]. The parameters are the constants that describe the distribution. Nonparametric

estimates do not assume that the data belong to a given probability distribution. Generally, nonparametric estimates make fewer assumptions than parametric estimates.

Reliability engineering tries to ensure that the unit is reliable during the operation in a specific condition by avoiding any failure. In other words, the purpose of reliability engineering is maximizing reliability while minimizing failure effects. The main purpose of the reliability modeling is to find the probability of failure at any instance of time-based on the ability of the components to perform their intended functions for a specific period (lifetime) under the specific conditions (environmental and operational). The main purposes of reliability modeling can be summarized as following [112] [113]:

- Estimating the Probability of Failure
- Predicting the future condition of the assets in order to prevent physical failure.
- Providing advanced warning of failures.
- Reducing life-cycle cost.
- Monitoring the risk associated with each decision.
- Minimizing unscheduled maintenances.
- Improving the customer's satisfaction.

Performance is usually associated with the functionality of a product. Performance is related to the question, "How well does a product work?" Reliability is associated with the ability of a product to perform as intended without failure and within specified performance limits for a specified time in its lifecycle. Based on the ISO 8402, quality is defined as "The totality of features and characteristics of a product or service that bear on its ability to satisfy stated or implied needs". Quality is also sometimes defined as

“conformance to specifications” [114]. “To measure quality, we make a judgment about a product today. To measure reliability, we make judgments about what the product will be like in the future”. Reliability is, therefore, an extension of quality into the time domain. Product quality can impact product reliability but, it should be considered that a high-quality product may or may not have high reliability [115].

Reliability function or survival function or Cumulative Distribution Function (CDF) are representing the probability that a random variable falls above a certain level. Indeed, this indicates the probability of observing an event that did not happen before the time t [116].

$$\begin{aligned}\bar{F}(t) &= R(t) = 1 - F(t) && \text{Equation (9)} \\ R(t) &= \text{Probability (Product Life} > t) \\ R(t) &= \int_t^{\infty} f(\tau) d\tau\end{aligned}$$

As Figure 2 presents, the failure rate function is implying the frequency of the failure over the unit of time [117].

- Infant mortality (Decreasing): It rarely happens in the real-world unless for the stable state of the failure after the early stages. Most of the time, the units fail with higher probability as getting more age. It can be possible that a system has this behavior at least for a finite time. This step is also called burn-in. The evolution of software programs can be considered as an example of this type [118].
- Useful Life (Constant): In many cases, for simplification of the problem it is assumed to have a constant value over time. In this case, the distribution family function is Exponential, and the calculations need less effort compared with other

scenarios. Usually, the product population reaches its lowest hazard rate during this period [119].

- Wear out (Increasing): This indicates the end of useful life and the start of the wear-out phase. In this case, the failure rate has an increasing trend over time. This step illustrates the aging stage. When the hazard rate becomes too high, replacement or repair of the population of products should be conducted [120].
- Combination of all the cases (bathtub curve): Useful life of an asset usually refers to the time between the worn-out and start-up points which mostly indicates a stable behavior.

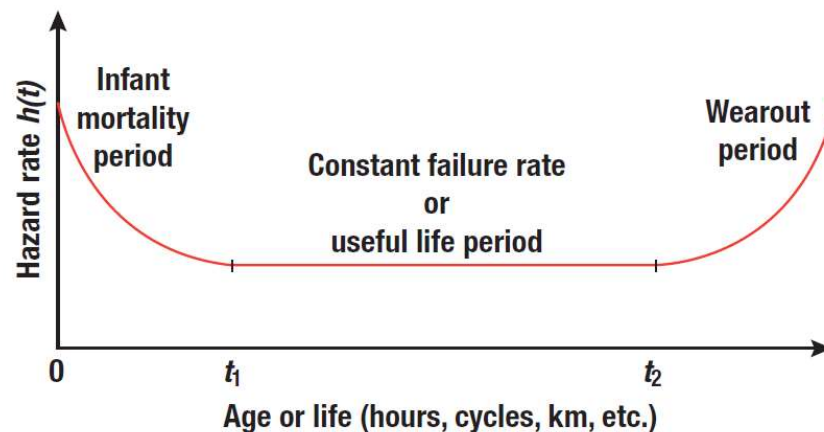


Figure 2: Bathtub hazard rate (failure function).

Optimizing reliability must involve the consideration of the actual life-cycle periods. The actual hazard rate curve will be more complex in shape and may not even exhibit all of the three periods. For reliability prediction, moment-based parameters, such as the mean and variance of a lifetime, are often not of primary interest. Rather, engineers may be more interested in estimating quantiles of the lifetime or (similarly) failure probabilities for a given (fixed) mission lengths [121]. The choice of distribution to fit often involves the

phase of life that is of interest, as determined by the shape of the hazard function, and many techniques have been developed that address modeling the hazard rate directly as a linear or polynomial function.

The mean or expected value of T , a measure of the central tendency of the random variable, also known as the first moment. $MTTF$ is the expected life $E(T)$ of a non-repairable product [122].

$$MTTF = \int_0^{\infty} t f(t) dt = \int_0^{\infty} R(t) dt \quad \text{Equation (10)}$$

This is also called the mean time between failures (MTBF) (mostly for repairable products) when the product exhibits a constant hazard rate; that is, the failure probability density function is an exponential [123] [124]. The MTTF should be used only when the failure distribution function is specified because the value of the reliability function at a given MTTF depends on the probability distribution function used to model the failure data. Furthermore, different failure distributions can have the same MTTF while having very different reliability functions [125].

2.2.2 Asset Management

An asset is an item or entity that has potential or actual value to an organization. Assets are a resource with economic value that in the future can generate cash flow, reduce expenses, and improve sales [126]. Asset Management (AM) is an ongoing process of maintaining, upgrading, acquiring and operating the assets cost-effectively, based on a continuous condition assessment. AM is responsible to balance the costs,

opportunities, and risks against the desired performance of the assets. Data-driven AM activities can be viewed from different points of view [127].

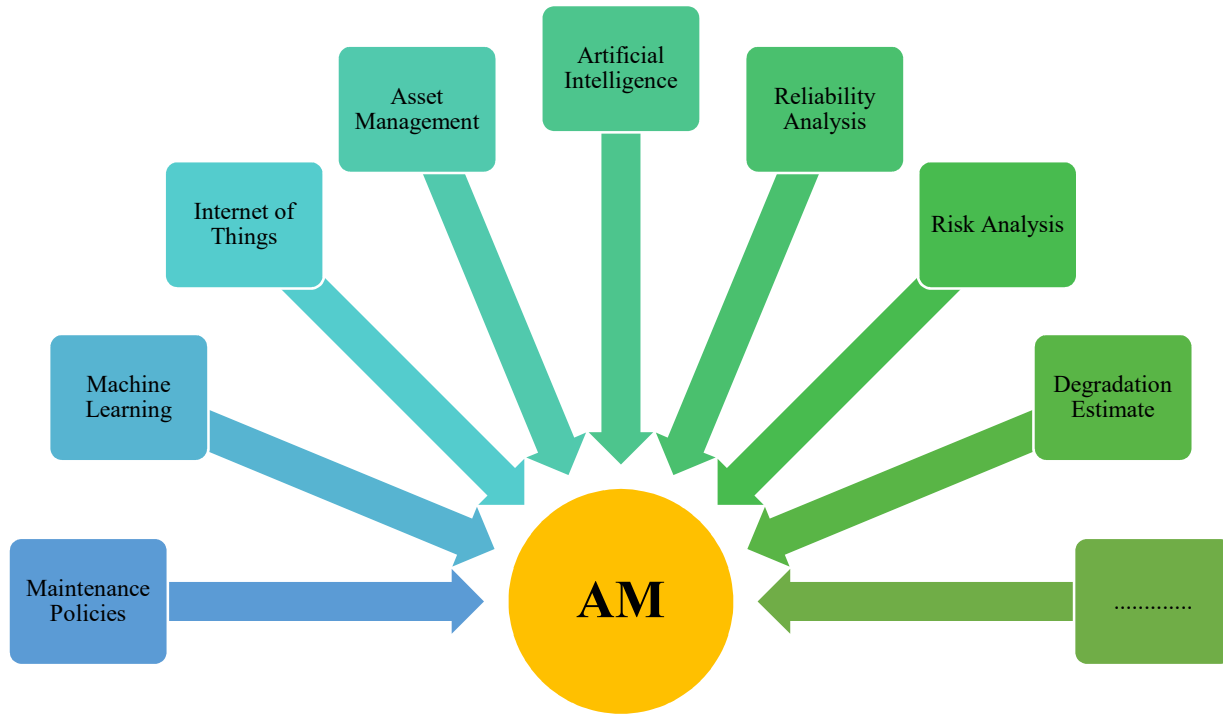


Figure 3: Data-Driven Asset Management.

2.2.3 Maintenance Policies

Maintenance policies can be categorized from different points of view. In general, there are three main categories for performing maintenance [128].

- i. Reactive Maintenance
- ii. Preventive Maintenance
- iii. Predictive Maintenance

Reactive maintenance is mainly based on the run-to-failure strategy which lets the assets fail and then, schedule the maintenance. Preventive maintenance is regularly performed to reduce failure frequency and downtime [129]. In general, 80% of the failures

are planned and 20% are completely unexpected. Predictive maintenance is performed based on condition-based strategies [130]. Data-driven predictive maintenance approaches rely on the condition of equipment by providing full visibility of the asset's status. Predictive algorithms are responsible to predict when an asset may fail before it happens [131]. Indeed, predictive maintenance is an intelligent health monitoring approach that tries to avoid future failures and optimize the maintenance schedule. Figure 8 presents the optimal time interval for performing maintenance actions. If the maintenance is scheduled more too often, the life-cycle cost of the unit will increase. In some cases, too often maintenance might degrade the asset more quickly. On the other hand, if the maintenance is scheduled too late, the actual physical failure might happen. Therefore, finding an optimal interval for performing the maintenance actions is among one of the most important decisions [132].



Figure 4: Optimal time interval for performing maintenance actions.

2.2.3.1 Challenges of Classical Models

The most important challenges of the traditional models are as follow:

- Lifetime data cannot be easily obtained.

- Deterministic models are not able to express the real behavior of the assets.
- Technological developments lead to a few or zero failure data-sets as the result of the analysis.
- Some of the laboratory tests and accelerating monitoring actions are expensive.

2.2.4 Prognosis and Health Management (PHM)

PHM is an intelligent condition monitoring approach to:

- Predict the future units' condition.
- Predict event which system no longer performs its intended functions.
- Estimate time to failure.

PHM is an allowing discipline of technologies and methods with the potentials of enhancing reliability estimation that has been revealed due to complexities in environmental and operational usage conditions as well as the effects of maintenance actions [133]. Over the last few decades, several types of research have been conducted in PHM of information and electronics-rich systems as a means to provide advance warnings of failure, enable predictive maintenance, improve system identification, extend system life, and diagnose intermittent failures that can lead to field failure return exhibiting no-fault-found symptoms [134].

Extremely high operational availability of information regarding the behavior of the systems has been historically difficult to achieve [135]. The main reason is because of the lack of understanding of the interactions between the various covariates, application environments, and their effect on system degradation over time. Consequently, there is

a pressing need to develop new methods that apply in-situ system operational and environmental conditions to detect performance degradation. The most promising discipline of methods with the potential of enhancing the reliability, availability, and maintainability prediction is called PHM [136]. Traditionally, PHM has been implemented using methods that are either model-based or data-driven. The model-based approaches consider the physical processes and interactions between components in the system. The data-driven approaches use statistical pattern recognition and Machine Learning (ML) to detect changes in parameter data, thereby enabling diagnostic and prognostic measures to be calculated [137].

Data-driven techniques are utilized to learn from the data and intelligently provide valued information to enhance the decision-making process. They assume that the statistical characteristics of the system remain relatively stable until a fault arises in the system [138]. Anomalies and trends or patterns are detected in data collected by in situ monitoring to determine the health state of a system. The trends are then beneficial to predict the time to failure of the system [139].

Health assessment is carried out in real-time using the in-situ data and anomaly detection techniques. Knowledge regarding the physical processes in the system and steady-state conditions can help to select the appropriate data analytic techniques. ML is one of the methods to implement anomaly detection techniques, in which the monitored data are compared in real-time against a healthy baseline to check for possible anomalies. This is the semi-supervised learning approach wherein data representing all the possible healthy states of the system are assumed to be available a priori. The healthy baseline consists of a collection of parameter data that represent all the possible

variations of the healthy operating states of a system. The baseline data is collected during various combinations of operating states and loading conditions when the system is known to be functioning normally [140]. The baseline can also consist of threshold values based on specifications and standards. It is important that the baseline data should not contain any operational anomalies. The presence of anomalies in the base-line affects the definition of healthy system behavior and hence causes the misclassification of data.

2.2.5 Degradation Mechanism

There is a growing demand to validate asset reliability relatively quickly with minimal testing. As mentioned before, it might be possible that the output of the analysis includes zero or a few failure events due to technological developments. Therefore, it might not be ideal to assess the reliability based on the traditional approaches. Alternatively, degradation measures will contain beneficial information regarding asset performance and reliability. Degradation measures are applied to estimate the unit condition over time by:

- Preventing an unexpected failure.
- Enhancing the service life/prognosis and monitoring.
- Optimizing the reliability, availability, and risk.
- Analyzing the component before the actual failure point.

Figure 5 depicts a big picture of the degradation process. Consider that the goal is to obtain the degradation measures of an asset over time based on the history of the assets for the most critical internal and external covariates. It has been assumed that the acceptable range for the critical covariates is known prior. Therefore, any deviation from

the acceptable area might be an indication for upcoming failure or unpleasable events. As mentioned before, raw data might not reveal very beneficial information. They can convert to the health indicator statistics in order to extract the most helpful information. As an example, suppose that an electrical transformer is under the monitoring process. Based on initial analyses, a few covariates such as voltage, current, temperature, and etc. have been determined as critical covariates which directly affect the degradation measures. In the next step, the raw measurements can be turned into the health indicator statistics such as efficiency. Based on the Steady-State conditions, the efficiency of the transformer can be collected and monitored over time. Any abnormal trend in the health indicator estimates might be an indication of forthcoming failure.

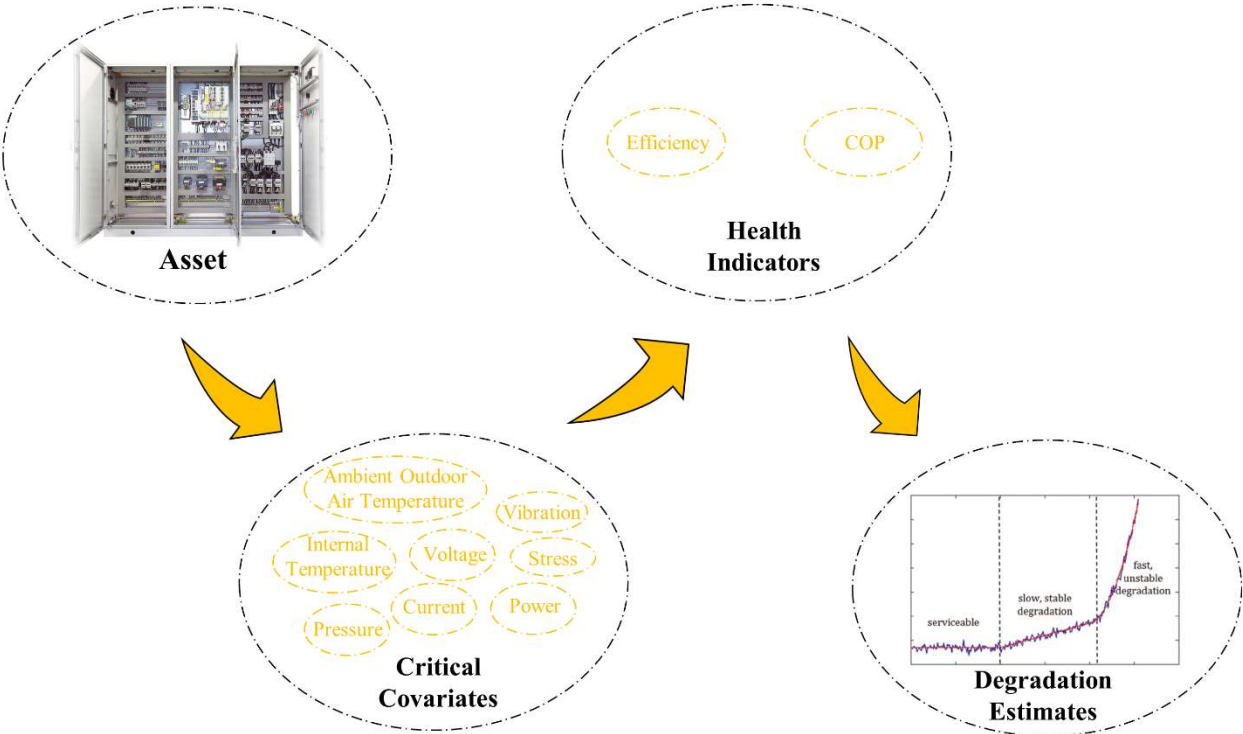


Figure 5: Big picture of degradation process.

As Figure 6 presents, degradation measures can be classified as transitory or permanent degradation. Transitory degradation measures are a part of the degradation process which can be restored by performing the needed maintenances. In some cases, maintenance power is not able to restore the asset to the as good as new condition. The difference between the degradation estimates at time zero, when the unit is new, and degradation right after performing the maintenance can be considered as the permanent degradation. Permanent degradation can be used as an indication of replacing the asset over its lifetime.

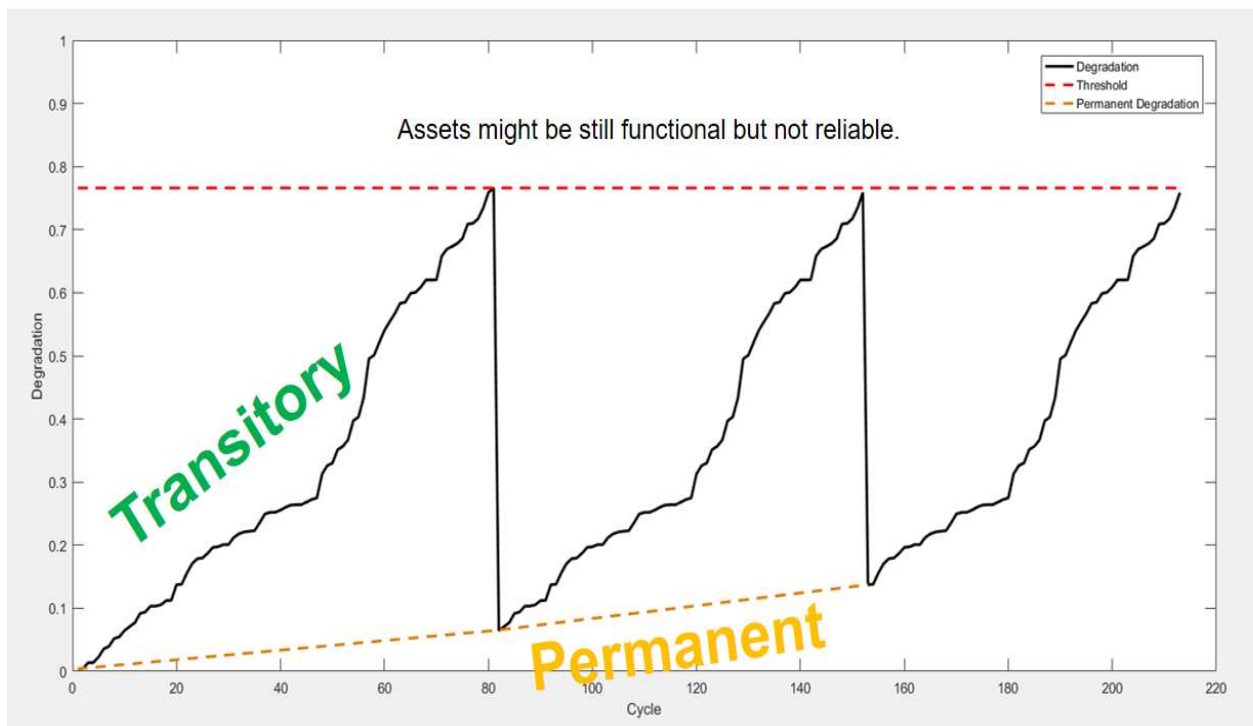


Figure 6: Major types of degradation estimates.

Degradation models can be categorized as Model-Driven, Data-Driven, and Hybrid or Fusion models. Model-Driven algorithms are mostly based on the physics governing the system. Therefore, mathematical models are needed in order to reach a reliable

result. Data-Driven algorithms are very applicable when the system is complex or the physics of the failure governing the system is unknown. Data-Driven algorithms rely on the data-sets which present the system behaviors. ML algorithms are very applicable in order to perform statistical data analyses and extend outcomes over time. Hybrid or Fusion models are based on a combination of Model and Data-Driven algorithms. It means that the mathematical models are able to present the physics of the failure for some parts of the system. Data-Driven algorithms are in charge of the complex parts of the system.

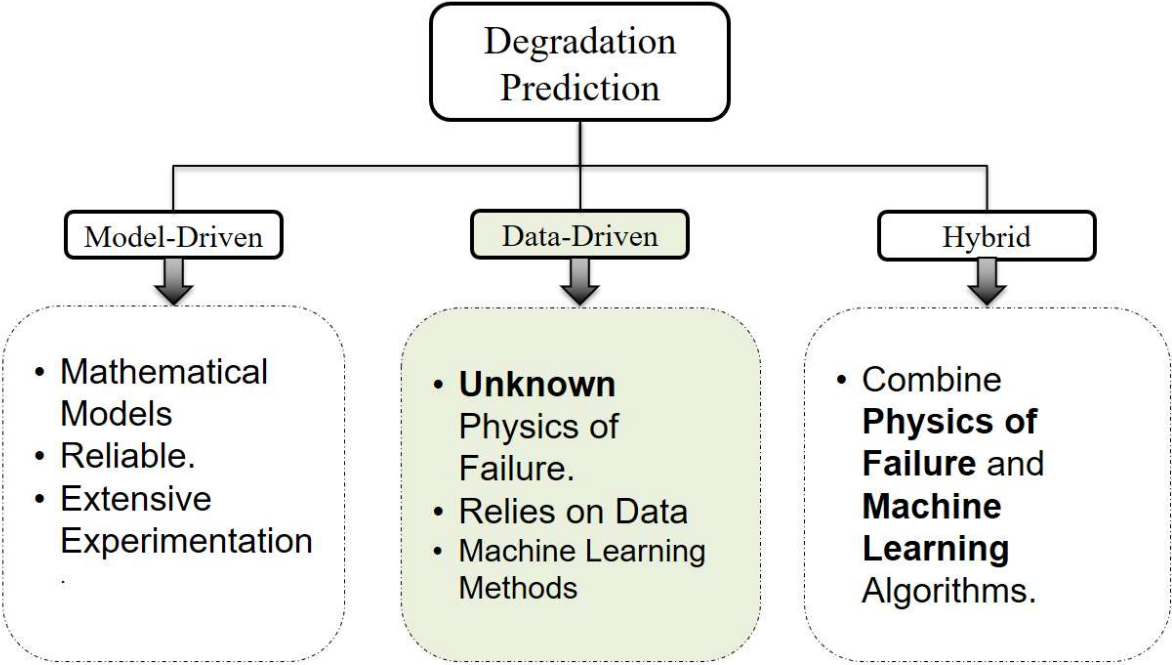


Figure 7: Categories of degradation prediction.

2.3 Review of Theories and Methods Relevant to Research Questions

2.3.1 Monte Carlo Simulation

Monte Carlo methods are an extensive class of computational algorithms that depend on repeated random sampling to acquire numerical results. The fundamental concept is to apply randomness to solve problems that might be deterministic [141]. They are often applied in physical and mathematical problems and are most applicable when it is problematic or impossible to use other approaches. Monte Carlo methods are mainly used in three problem categories as optimization, numerical integration, and generating draws from a probability distribution [142].

In application to systems engineering problems, Monte Carlo-based predictions are developed in order to predict the upcoming failures. In other problems, the objective is generating draws from a sequence of probability distributions satisfying a nonlinear evolution equation [143]. These streams of probability distributions can always be interpreted as the distributions of the random states of a Markov process whose transition probabilities rest on the distributions of the current random states. Monte Carlo methods vary, but lean towards to follow a particular class as follow:

1. Define a domain of possible inputs
2. Generate inputs randomly from a probability distribution over the domain
3. Perform a deterministic computation on the inputs
4. Aggregate the results

Monte Carlo simulation is drawing a large number of pseudo-random uniform variables at one time, or once at numerous different times. The main idea behind this method is that the results are computed based on repeated random sampling and statistical analysis. Indeed, Monte Carlo simulation is random experimentations, in the case that, the results of these experiments are not well known. Monte Carlo simulations are characteristically characterized by several unknown parameters, many of which are difficult to find experimentally [144]. Monte Carlo simulations sample from a probability distribution for each variable to generate hundreds or thousands of possible outcomes. The results are analyzed to get probabilities of diverse outcomes occurring. Monte Carlo methods are especially useful for simulating phenomena with substantial uncertainty in inputs and systems with numerous degrees of freedom.

2.3.2 Distribution Fitting

Probability distribution fitting is the fitting of a probability distribution to a series of data regarding the repeated measurement of a random variable. The purpose of distribution fitting is to predict the probability or to forecast the frequency of incidence of the magnitude of the phenomenon in a certain interval. There are several probability distributions of which some can be fitted more closely to the observed data than others, depending on the features of the phenomenon and of the distribution [145]. The distribution giving a close fit is supposed to lead to accurate predictions. Therefore, the main purpose of distribution fitting is to select a distribution that suits the data well. Parametric methods of distribution fitting are the method of moments, maximum spacing estimation, method of moments, and, maximum likelihood estimation. Predictions of

incidence based on fitted probability distributions are subject to uncertainty, which may arise from the following situations [146]:

- The true probability distribution of events may depart from the fitted distribution, as the observed data series may not be completely representative of the real probability of occurrence of the phenomenon based on a random error.
- The occurrence of events in another situation may depart from the fitted distribution as this occurrence can also be dependent on random error.
- A change of environmental and operational conditions may origin a change in the probability of occurrence of the phenomenon.

Statistical techniques are applied to estimate the parameters of the various distributions. These parameters describe the distribution. There are four major parameters used in distribution fitting as location, scale, shape, and threshold. Not all parameters necessary exist for each distribution [147]. Distribution fitting includes estimating the parameters that define the various distributions.

2.3.3 Poisson Distribution

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that indicates the probability of a given number of events occurring in a fixed interval of time if these events occur with a known constant rate and independently of the time since the last event. The Poisson distribution is a discrete probability distribution of the number of events occurring in a given time period, given the average number of times the event occurs over that time period [148] [149].

For instance, an individual may keep track of the car passing a signal light by the means of Poisson distribution. A Poisson process is a model for a sequence of discrete events where the average time between events is known, but the precise timing of events is random. The arrival of an event is independent of the event before. The Poisson distribution is a suitable model if the following assumptions are held [150].

- k is the number of times an event occurs in an interval and k can take integer values.
- The occurrence of one event does not affect the probability that another event will occur. That indicates that events happen independently.
- The average rate at which events occur is assumed to be a constant value.
- Two events cannot occur at exactly the same instant.

Equation (11) presents the general formula for the probability distribution function of X , as a random variable, in which the rate of the events is assumed to be a constant value equal to λ .

$$p(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad \text{Equation (11)}$$

2.3.4 Gamma Distribution

In probability theory and statistics, the gamma distribution is a two-parameter family of continuous probability distributions. A gamma distribution is a general type of statistical distribution that is related to the beta distribution and arises naturally in processes for which the waiting times between Poisson distributed events are relevant [151]. In other words, The Gamma distribution is a continuous, positive-only, unimodal distribution that

encodes the time required for alpha events to occur in a Poisson process with a mean arrival time of beta. The gamma distribution is a family of right-skewed, continuous probability distributions. These distributions are useful in real-life where something has a natural minimum of 0. For example, it is commonly used in finance, for elapsed times, or during Poisson processes. The exponential distribution, Erlang distribution, and chi-squared distribution are special cases of the gamma distribution. The gamma distribution is another widely applied distribution. Its standing is mainly due to its relation to exponential and normal distributions. Equation (2.12) presents the general formula for the probability density function of the gamma distribution, where α and β are shape and scale parameters, respectively [152].

$$f(x; \alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad \text{for } x > 0, \beta > 0 \quad \text{Equation(12)}$$

2.3.5 Mixture Distributions

A mixture distribution is a mixture of two or more probability distributions. Random variables are drawn from more than one main population to generate a new distribution, which is a mixture of several distributions. All the distributions involved in the mixture model should either be all discrete probability distributions or all continuous probability distributions [153]. The distributions can be originated from diverse distributions (e.g. a normal distribution and a t-distribution) or they can be made up of the same distribution with different parameters [154]. Mixture distributions are a suitable way to demonstrate how variables can be differently distributed. An example of when mixture distribution might be applied is when there is not sufficient idea regarding the possible outcomes. A mixture distribution, sometimes also named a mixture density, is a distribution formed

from the weighted combination of two or more component distributions. The component distributions can be univariate or multivariate [155]. Assume you have a population, where each individual in the population belongs to exactly one of several groups. If you can estimate the distribution of some quantity for each group, the distribution for the population as a whole is obtained as a mixture of these, with each component weighted as the fraction of the total population represented by that group [156].

2.3.6 Truncated Distributions

A truncated distribution is a conditional distribution that consequences from restricting the domain of probability distribution. Truncated distributions arise in applied statistics in cases where the ability to record, or even to know about, occurrences are limited to values that lie within a specified range [157]. Where sampling is such as to retain the knowledge of items that fall outside the required range, without recording the actual values, this is known as censoring, as opposed to the truncation here. A reason for applying a truncated distribution may be that there is no interest beyond the truncation point. Another reason for truncation is that the distribution is not valid beyond the truncation point. For example, the damage amount above a certain level may not be a valid outcome given a statistical distribution function. A truncated distribution has its domain limited to a certain range of values [158].

2.3.7 Convolutional Models

The convolution of probability distributions arises in probability theory and statistics as the operation in terms of probability distributions that resemble the addition of independent random variables and, by extension, to creating linear combinations of

random variables [159]. The probability distribution of the sum of two or more independent random variables is the convolution of their individual distributions. The term is inspired by the fact that the probability density function of a sum of random variables is the convolution of their corresponding probability density functions respectively. In mathematics, convolution is a mathematical operation on two or more functions that produce another function stating how the shape of one is modified by the other. It is defined as the integral of the product of the two functions after one is reversed and shifted. The convolution of two vectors represents the area of overlap under the points as one slides across the other one. A convolution is integral that expresses the amount of overlap of one function as it is shifted over another function. The main application of convolution models in engineering is in describing the output of a linear, time-invariant system [160].

2.3.8 Maximum Likelihood Estimation

Maximum Likelihood Estimation (MLE) is a method of estimating the parameters of a probability distribution by maximizing a likelihood function so that under the assumed statistical model the observed data is most possible [161]. The point in the parameter space that maximizes the likelihood function is called the maximum likelihood estimate. If the likelihood function is differentiable, the derivative test for determining maxima can be applied. In some cases, the first-order conditions of the likelihood function can be solved explicitly. As an example, the ordinary least squares estimator maximizes the likelihood of the linear regression model. Under some conditions, numerical methods will be essential to find the maximum of the likelihood function. From a statistical standpoint, a given set of observations are a random sample from an unknown population. The goal of maximum likelihood estimation is to make inferences about the population that is most

likely to have generated the sample. Maximum likelihood estimation is a method that determines values for the parameters of a model. The parameter values are found such that they maximize the likelihood that the process described by the model produced the data that were actually observed [162].

2.3.9 Expectation-Maximization Optimization

Expectation-Maximization (EM) algorithm is an iterative method to find maximum likelihood or maximum posterior estimates of parameters in statistical models, where the model depends on unobserved latent variables. The EM iteration alternates between the execution of an expectation (E) step, which creates a function for the expectation of the log-likelihood evaluated using the current estimate for the parameters, and a maximization (M) step, which computes parameters maximizing the expected log-likelihood found on the E step [163]. These parameter-estimates are then applied to determine the distribution of the latent variables in the next E step. The EM algorithm is used to find local maximum likelihood parameters of a statistical model in cases where the equations cannot be solved directly. Finding a maximum likelihood solution typically requires taking the derivatives of the likelihood function with respect to all the unknown values, the parameters, and the latent variables, and simultaneously solving the resulting equations. In some statistical models, the result may be a set of interlocking equations in which the solution to the parameters requires the values of the latent variables and vice versa but substituting one set of equations into the other produces an unsolvable equation [164].

The EM algorithm proceeds from the observation that there is a way to solve these two sets of equations numerically. One can simply pick random values for one of the two sets

of unknowns, use them to estimate the second set, then use these new values to find an improved estimate of the first set, and then keep alternating between the two until the resulting values both converge to fixed points. No guarantee exists that the sequence converges to a maximum likelihood estimator. For multimodal distributions, this means that an EM algorithm may converge to a local maximum of the observed data likelihood function, depending on starting values [165].

2.3.10 Clustering Algorithms

Clustering is a Machine Learning (ML) technique that includes the grouping of data points. Given a set of data, a clustering algorithm can be applied to classify each data point into a specific group. Ideally, data points that are in the same group should have similar properties, while data points in different groups should have highly dissimilar properties. Clustering is one of the most commonly used methods of unsupervised learning for statistical data analysis. Clustering methods can be applied to gain some valuable insights from the data by investigating what groups the data points fall into when we apply a clustering algorithm [166]. Therefore, the clustering task refers to grouping a set of data in such a way that data points in the same group, known as a cluster, are more similar, in some sense, to each other than to those in other groups. Clustering analysis is not itself one specific algorithm. Clusters can be obtained by various algorithms that differ significantly in terms of understanding of what establishes a cluster and how to efficiently the clusters can be constituted [167]. Clustering can, therefore, be formulated as a multi-objective optimization problem. In general, clustering algorithms can be categorized as following [168]:

- **Connectivity-based clustering or hierarchical clustering**

The core idea of objects being more related to nearby objects than to objects farther away. These algorithms connect "objects" to form "clusters" based on their distance. (Ex. Linkage clustering)

- **Centroid-based clustering**

In centroid-based clustering, clusters are represented by a central vector, which may not necessarily be a member of the data set. When the number of clusters is fixed to k , k -means clustering gives a formal definition as an optimization problem: find the k cluster centers and assign the objects to the nearest cluster center, such that the squared distances from the cluster are minimized. (Ex. K-means clustering)

- **Distribution-based clustering**

The clustering model most closely related to statistics is based on distribution models. Clusters can then easily be defined as objects belonging most likely to the same distribution. A convenient property of this approach is that this closely resembles the way artificial data sets are generated: by sampling random objects from a distribution. (Ex. Gaussian Mixture Model (GMM) clustering)

- **Density-based clustering**

In density-based clustering, clusters are defined as areas of higher density than the remainder of the data set. Objects in these sparse areas that are required to separate clusters are usually considered to be noise and border points. (Ex. Density-based spatial clustering of applications with noise (DBSCAN))

Clustering algorithms can be divided into two subgroups as the following present.

- Hard Clustering: In hard clustering, each data point either belongs to a cluster completely or not.
- Soft Clustering: In soft clustering, instead of putting each data point into a separate cluster, a probability or likelihood of that data point to be in those clusters is assigned.

2.3.11 K-means Clustering

k-means clustering is a method of vector quantization, originally from signal processing, that is popular for cluster analysis in data mining. Partition the n observations, $d = \{d_1, d_2, \dots, d_n\}$ into K clusters ($K \leq n$) such that the sets minimize the within-cluster sum of squares. where μ_i is mean of the points in the cluster S_i

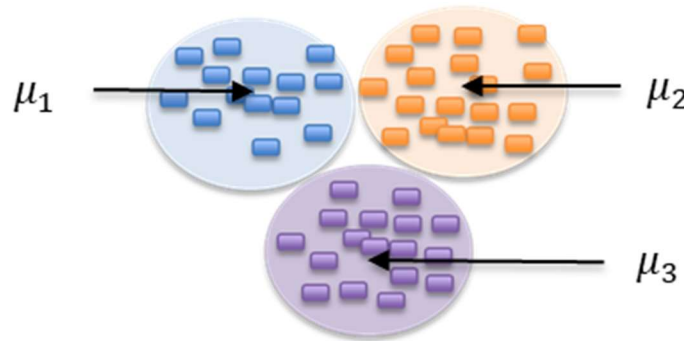


Figure 8: A schematic view of k-mean clustering algorithm.

$$\arg \min_S \sum_{i=1}^K \sum_{x_j \in S_i} \|x_j - \mu_i\|^2 \quad \text{Equation (12)}$$

The problem is computationally difficult, NP-hard; however, efficient heuristic algorithms converge quickly to a local optimum. Some iterative methods such as Expectation-Maximization (EM) can be applied in order to find a local optimum point [169]. The algorithm has a loose relationship with the k-nearest neighbor classifier, a

widespread machine learning technique for classification that is often confused with k-means due to the name. K-means clustering is one of the most common algorithms which uses an iterative refinement technique. It is also referred to as Lloyd's algorithm. The algorithm has converged when the assignments no longer change. The algorithm does not guarantee to reach the optimum point. The algorithm is often offered as assigning objects to the nearest cluster by distance. Applying a diverse distance function other than (squared) Euclidean distance may stop the algorithm from converging [170].

2.3.12 Gaussian Mixture Model Clustering

Gaussian mixture models can be used to cluster unlabeled data in much the same way as k-means. There are, however, a couple of advantages to using Gaussian mixture models over k-means. First and foremost, k-means does not account for variance. By variance, we are referring to the width of the bell shape curve. One way to consider the k-means model is that it places a circle at the center of each cluster, with a radius defined by the most distant point in the cluster. In contrast, Gaussian mixture models can handle even very oblong clusters [171]. The second difference between k-means and Gaussian mixture models is that the former performs hard classification whereas the latter performs soft classification. In other words, k-means tell us what data point belongs to which cluster but cannot provide the probabilities that a given data point belongs to each of the possible clusters.

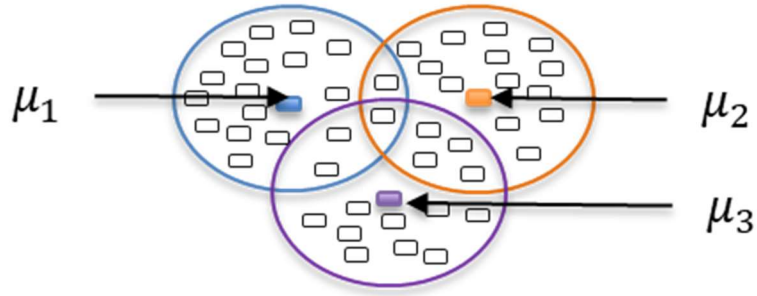


Figure 9: A schematic view of the Gaussian Mixture Model clustering algorithm.

$$p(x) = \sum_{i=1}^K p(x|y=i) \cdot p(y=i) \quad \text{Equation (13)}$$

3. Chapter 3: Generic System Under Study

Although the developed predictive model of reliability assessment in this thesis can be applied to various applications, the main motivations for conducting this research have been initially raised based on the reliability analysis of the electrical energy network. One of the primary concerns of the electrical energy networks is the ability to enhance the reliability of the assets and consequently, the overall electrical system. In general, a reliable asset is able to perform its intended function under the specific working conditions over the predefined cycles, known as the asset lifetime. The intended functions of an asset might include various roles. As a result, an asset might be still working while being considered as an unreliable asset. Specific working conditions are usually provided by the manufacturers or can be obtained as the results of laboratory tests. Specific working conditions are usually the nominal conditions that enable the owners to obtain more value of the asset by enhancing its life-cycle characters. Furthermore, an expected lifetime for an asset is provided by the manufacturers or estimated by the decision-makers. It should be considered that an asset might face completely a unique condition over its upcoming cycles. On the other hand, an asset might behave entirely different in each application. Therefore, a more realistic measurement should consequently estimate the lifetime of the assets in order to optimize the assets' value by keeping the reliability of the network above a certain level.

Energy network reliability significantly impacts customer as well as energy providers' bottom line. Obviously, any shortage in demand due to unreliable assets is not desirable from the energy providers' point of view since, it leads to the loss of income and in some

cases, reputation. On the other hand, customers would like to always have access to a reliable electrical network in order to perform their desired functions.

During the last decades, new types of assets such as distributed generations and smart loads have emerged. Some of these technologies such as Solar Photovoltaic (PV) panels and Wind turbines have existed for a few decades but, their applications are constantly increasing. There is a constant growth in energy consumption and consequently energy generation around the world. During recent decades, renewable energy sources took heed of scientists and policymakers as a remedy for substituting traditional sources. Wind and Solar are among the least reliable sources because of their dependence on wind speed and irradiance and therefore their intermittent nature. Energy storage systems are usually coupled with these sources to increase the reliability of the hybrid system. As a result, inherent unreliability is always attached to the wind and solar energy resources.

Several studies have focused on the predictive models to enhance the reliability of the electrical network including the renewable facilities. It should be considered that these types of reliability assessment are mainly dealing with the uncertainty of energy resources. In order to enhance the overall reliability of the electrical network, the reliability of the components of the systems should be studied carefully. As an example, it might be possible that solar PV panels be able to convert the irradiance to the electrical energy but, the electrical inverters may not be able to perform their intended functions. Consequently, studying the uncertainty of the energy resources coupled with uncertainty in the future condition of the assets will enhance the overall reliability of the electrical energy networks.

New Hardware and Software tools for connectivity have been developed more than any time before. Decision-makers and analysts are usually able to have an insight regarding the asset performance close to real-time. Consequently, an optimized asset management policy can significantly enhance the reliability of the components and the overall network. Therefore, RA becomes a critical factor in the efficiency of capital-intensive corporations including the distributed electric power generation. The following are a summary of the primary motivations for conducting this research study.

- i. Energy network reliability significantly impacts customer as well as energy providers' bottom line. Therefore, an accurate predictive model of RA must be applied in order to maintain the reliability of the electrical energy networks above a certain level.
- ii. Reliability Assessment is becoming a critical factor in the efficiency of capital-intensive corporations including electrical power generation.
- iii. The new types of assets such as distributed generations and smart loads have been emerged. New H/W and S/W tools for connectivity have been integrated. Therefore, more information regarding the real-time condition of the assets must be available. This information is mapped into the degradation estimates as a health indicator., which can enhance the accuracy of the predictive model of RA based on the degradation models.

The application of the proposed methodology in Chapter 3 is illustrated for an electrical Distributed Energy System (DES). Asset Management (AM) related decisions are very critical for DES since interruption in electrical generation is not desirable either from energy providers or consumers' point of view. The main purpose of this chapter is to provide information regarding the generic electrical system under study, especially for the critical components which are constantly subject to fail.

The penetration of renewable energy resources is considerable in DES. Since the source of renewable energies is intermittent in nature, there is some level of uncertainty in the amount of forecasted energy generation. It should be considered that a DES should be reliable enough to be integrated into the energy supply chain of the upper stream entities. Failure of the critical assets of a DES might cause interruption to meet the electrical demand, which in some cases is not tolerable by the main utility company. Therefore, DESs should apply a robust predictive model of AM in order to guarantee the continuous generation as promised by Demand Response (DR) algorithms.

It should be noted that this chapter does not seek to study the electrical components of the system from a technical point of view. In other words, this chapter briefly describes the most important feature of the critical components of the generic system under study. The provided information in this chapter might be helpful to define a robust health or degradation criteria. For instance, the self-discharge rate can be considered as one of the health indicators which illustrate the degradation process. Degradation estimator definition can highly affect the outcome of the other algorithms since most of the data-driven algorithms rely on degradation estimates in order to assess the performance of the asset. In this study, it has been assumed that the time-series degradation estimates are

available. Indeed, health indicator identification is out of the scope of this study. The rest of this chapter briefly explains the main features and common causes of failure for critical components of a DES.

3.1 Distributed Generation (DG)

The main purpose of the DGs are bringing the generation point closer to the customer to enhance the overall system efficiency and reliability. The sources of energy generation could be either from non-renewable or renewable resources but, there are more potentials for high penetration of renewable energy resources such as solar PV panels and wind turbines [172]. Figure 10 presents a schematic picture of the energy network including distributed generations. It should be noted that the utility grid is still playing an important role in the system to meet any remaining part of the loads. As a result, the overall reliability of the system can still remain high while relying more on distributed generation facilities [173]. Figure 11 shows a big picture of the electrical network diagram presenting the role of the DGs.



Figure 10: A schematic picture of the energy network including distributed generations.

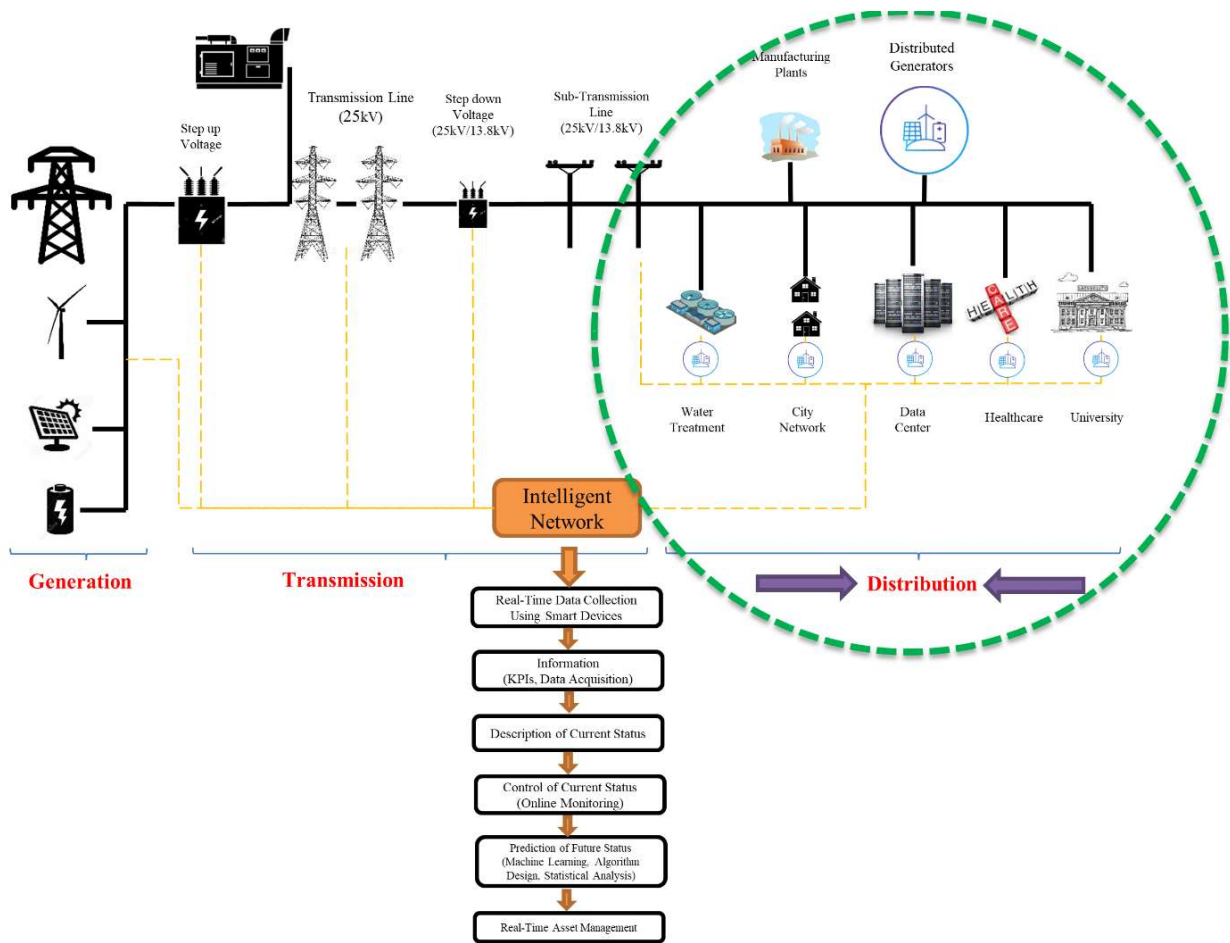


Figure 11: A big picture of the electrical network diagram presenting the role of the distributed generations.

3.1.1 Scale of the Distributed Generation

Figure 12 depicts a big picture of various scales of DGs for an electrical network. The application of the DGs could be at various points of the network either in large or small-scale.

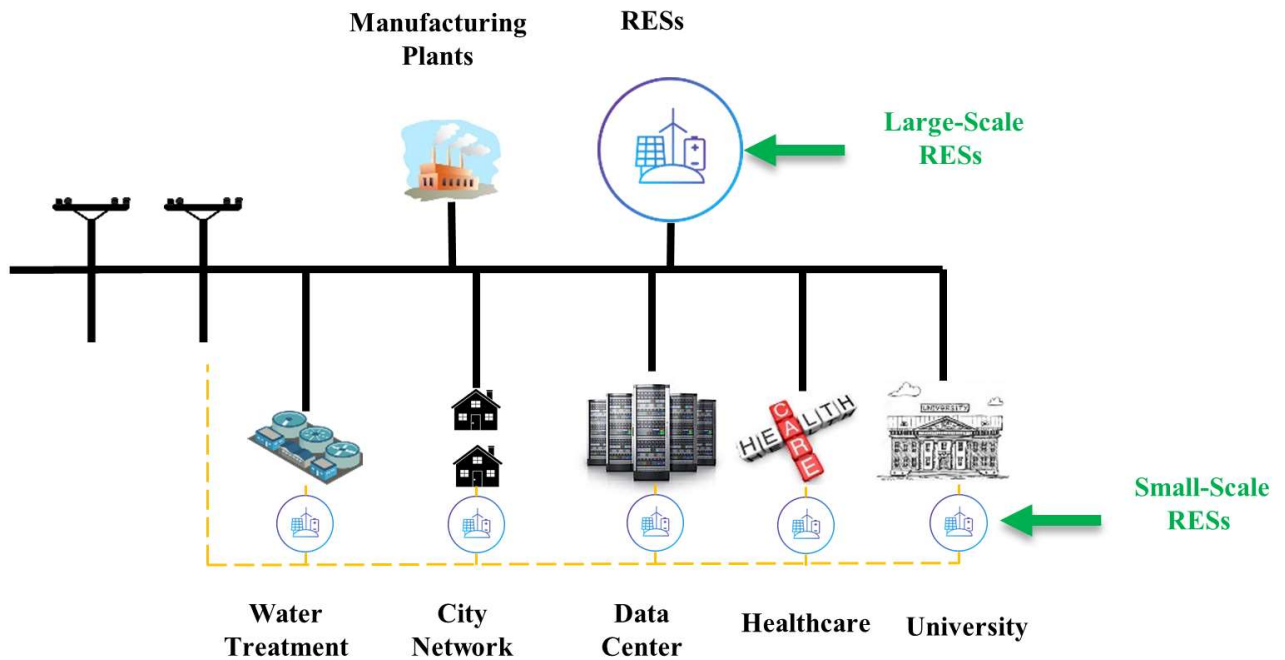


Figure 12: Various scales of distributed generation.

3.1.2 Distributed Generation Data Flow

Figure 13 presents a schematic view of the data flow through a distributed generation network. The data analytics can be executed either at the component or system level. Fog and Edge analytics are primarily focusing on the component and a series of components respectively. Cloud analysis is more toward the high-level decision regarding the system with respect to the obtained insight of components. Cloud level decisions might not be close to the real-time decision due to data latency, bandwidth limitation, and etc.

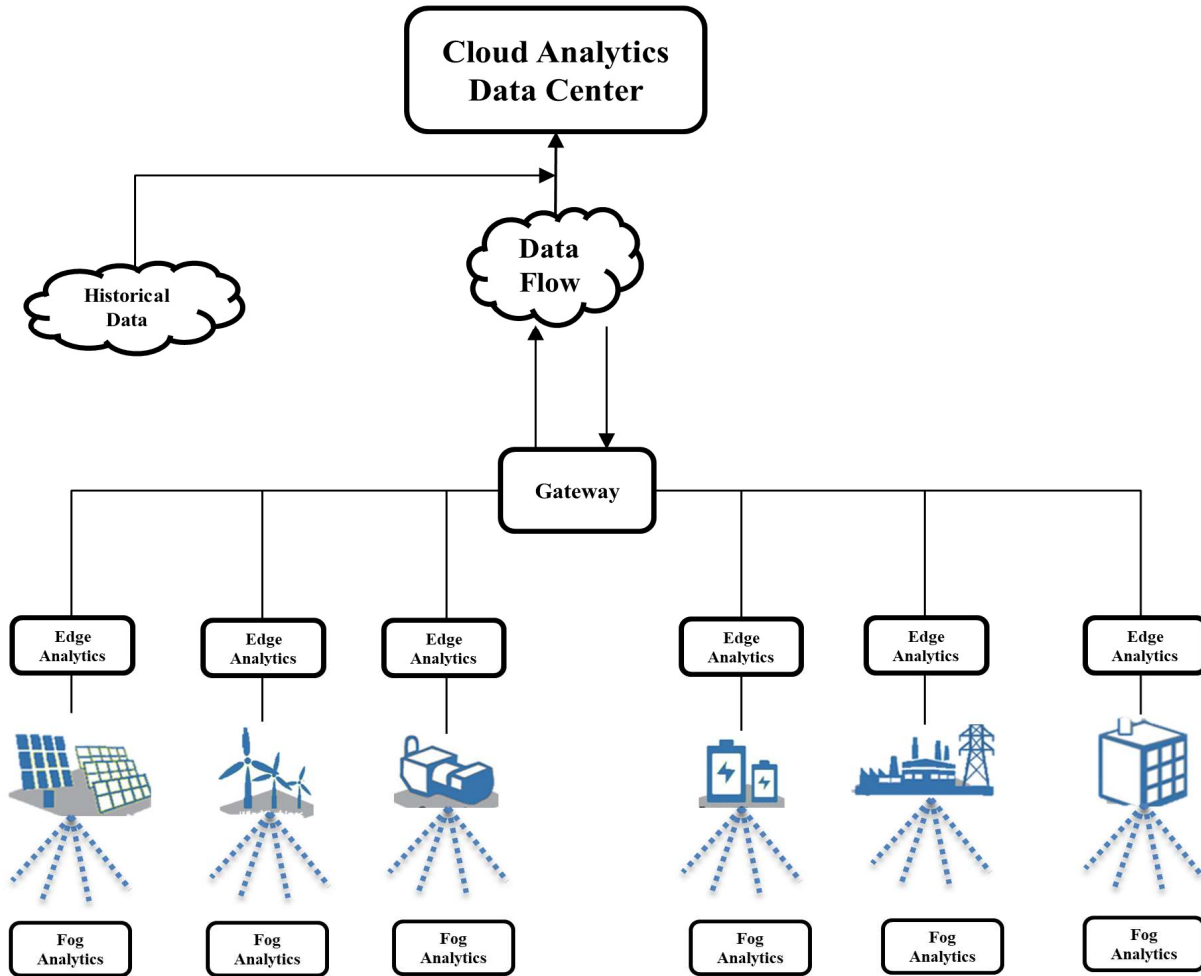


Figure 13: Schematic view of the data flow through a distributed generation network.

3.1.3 Distributed Generation – Smart Systems

Figure 14 presents a bidirectional flow of power and information due to the wide application of the DGs. High penetration of the RESs and DGs would let some customers meet their demand based on their own local generation. In addition to that, they can sell the extra generated electricity back to the utility using the smart meters and electrical components [174].

It should be considered that Energy Storage Devices (ESDs) are able to expressively cut the electricity bills of the customer. Customers are able to store the generated energy for their own future consumption or sell the extra stored amount back to the utility. The proposer size of the ESDs significantly affects the Return on Investment (RoI) values.



Figure 14: Bidirectional flow of power and information.

Figure 15 depicts a schematic view of the smart distributed generation. Connectivity is the core of the smart DGs. A central control data center is constantly optimizing the schedule of the electrical equipment with respect to the various parameters such as the predicted generation, loads, and price of the energy resources. It should be noted the application of the ESDs and variable profile for the energy price is offering the potential for the customers in order to cut their electricity bills. In addition to the energy-saving opportunities for the customers, there are some potentials for the energy providers as well. For instance, high penetration of the DGs make the energy providers able to reduce their peak capacity.

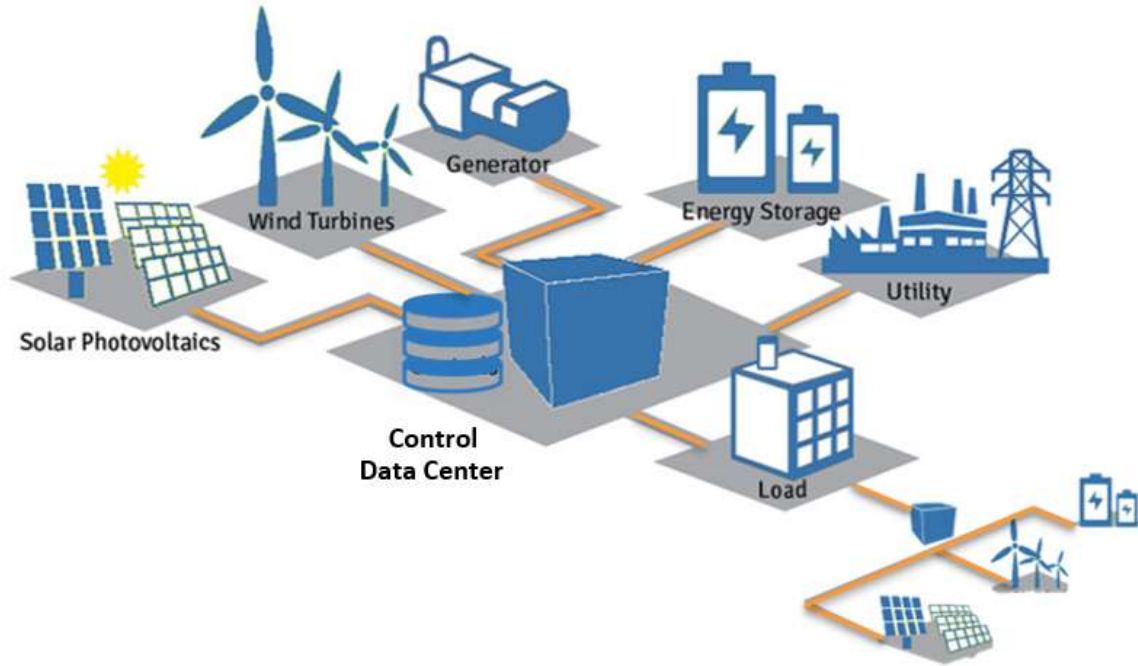


Figure 15: Schematic view of the smart distributed generation.

3.2 Critical Component of the System Under Study

3.2.2 Transformers

Transformers are one of the most critical components of the electrical distribution network. Transformers are designed in order to step down and step up the voltage as needed at any point of the point from generation, transmission, and distribution sectors. Transformers usually connect the subsystem of a system and deliver the electricity to the end customers. The main application of the transformers is in the electrical power transmission and distribution network. Based on the application of the devices, the size of the unit might vary. Generally, transformers are among the cost-expensive assets which are constantly subject to fail. It should be considered that the transformer failures can cause serious interruption for downstream customers. Therefore, an optimized

approach for monitoring and maintaining the transformers are essentials in order to enhance network reliability [175]. Figure 16 depicts the main parts of an electrical transformer.

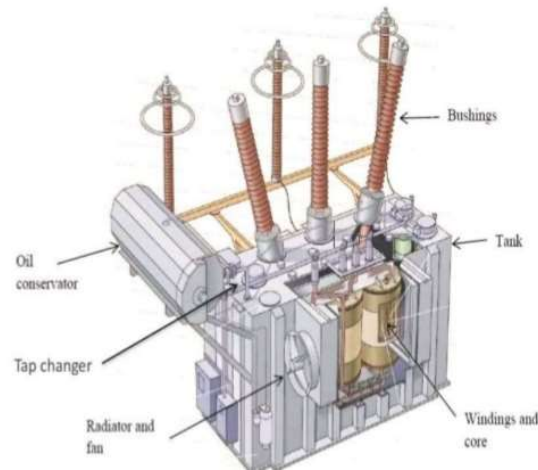


Figure 16: Main parts of an electrical transformer. (Copyright reserved for “engineeringworldchannel”)

Although there are various reasons for a transformer to be failed, this study mainly focuses on the most common failures of a transformer as follow [176]:

- High ambient temperature (Over-loading): If the temperatures are substantially higher than expected and the unit is not exposed to direct sunlight, then there's most likely a problem with overloading and cooling.
- High current and voltage (Insulation breakdown): High current and voltage which are high above the rated values are the main reasons. The major reasons for the insulation breakdown are, aging of insulation, Partial discharges in the insulation, Transient overvoltages due to lightning or switching in the network, Current forces on the windings due to external faults with high current. Insulation Breakdown of the windings will cause short circuits and /or earth

faults. These faults cause severe damaging to the windings and the transformer core.

- Sudden high voltages (Partial discharge): Bushing failure usually occurs over time; Loosening of conductors is caused by transformer vibrations which result in overheating. Sudden high fault voltages causes' partial discharge (the breakdown of solid/liquid electrical insulators) which damage the bushes and causes its degeneration and complete breakdown within hours.
- Polluted oil or oil leakage (Internal over-flashing): Not replacing old oil over a long time or its deficiency due to leakage causes internal over-flashing.
- Direct contact between core and winding (Over-heating): The over-heating reaches the core surface which is in direct contact with the windings.

3.2.2 Natural Gas (NG) Generators

The main function of the NG generators is generating the electricity by consuming natural gas as the fuel. The main application of the NG generators is for the situation in which access to the grid is limited or a back-up power source is essential. Since the system under study in this research is in island mode without any tie to the utility grid, the reliability of the NG generators should be high in order to meet the loads with the minimum shortage [177]. NG generator maintenance schedule should be carefully planned since too many maintenances accelerate the degradation of the unit and too rare maintenances would lead to facing the failure and interruption in the system. Figure 17 presents the main parts of natural gas generators.

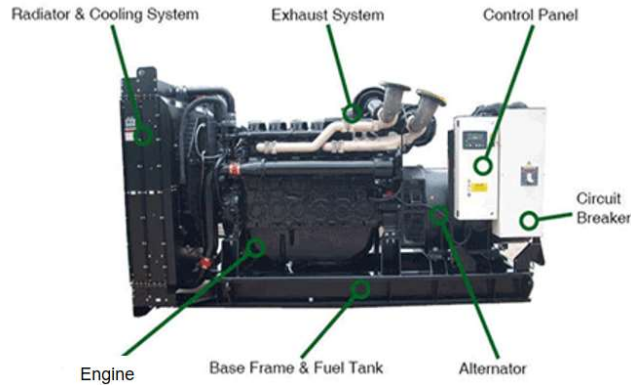


Figure 17: Main parts of natural gas generators. (Copyright reserved for “Cregsedits ; Atlanta ; 2011”)

Although there are various reasons for an NG generator to be failed, this study mainly focuses on the most common failures of an NG generator as follow :

- Low ambient temperature.
- Low coolant temperature.
- Air in the fuel system.
- Fuel filter clogging.
- Oil, fuel, or coolant leaks.

3.2.3 Inverters

The main role of the electrical inverters is changing the DC to AC power. In this study, we assume the solar PV panels and wind turbines are always reliable and the components which are subject to fail are the inverter. This unit is a cost-expensive unit that needs a careful maintenance schedule. The most important features of an inverter are input voltage, output voltage, output power, and output frequency [178]. The following are the most common application of electrical inverters.

- DC power source usage
- Uninterruptible Power Supply (UPS)
- Electrical power transmission and distribution
- Solar and Wind
- Battery
- Electric motor speed control

Although there are various reasons for an inverter to be failed, this study mainly focuses on the most common failures of an inverter as follow [179]:

- High temperature caused by high current value: Inverters are made up of electronic components, and therefore sensitive to temperatures. High temperatures will lead to a significant reduction in production and can even result in a production stop if the maximum operating temperature is reached. An assessment must, therefore, be made as early as the design stage to determine whether the proposed cooling technology is adequate and whether it has sufficient capacity.
- Over and under-voltage: If either current or voltage increases to a level that the inverter is not rated for, it can cause damage to components in the device, most frequently the inverter bridge. Often this damage will be caused by the excess heat generated by the spike in voltage or current. Over-current can be avoided with fuses or circuit breakers but avoiding over-voltage can be tricky.

- Electro-mechanical wear on capacitors: Inverters rely on capacitors to provide a smooth power output at varying levels of current; however electrolytic capacitors have a limited lifespan and age faster than dry components. Capacitors are also extremely temperature-sensitive. Temperatures over the stated operating temperature, often caused by high current, can reduce the life of the component. However, as the electrolytes evaporate faster at higher temperatures, capacitor life increases when they are running at lower than operating temperature.
- Isolation fault due to the short circuit: Another common problem is the “isolation fault”. This fault occurs as a result of a short circuit between various parts of the circuit, and the inverter will then report an “isolation alarm”. The short-circuit is usually the result of a combination of moisture and damage to the sleeve on the cabling, faulty installation, poor connection of the DC cables to the panel, or moisture in the connection part of the PV module. In the event of an isolation fault, the inverter will stop working completely or continue to work at the minimum “required” isolation level.

3.2.4 Batteries

Storing electrical energy means absorbing the electricity, storing for a period of time and then releasing it to the energy suppliers or power services. In this process, energy storage devices (ESDs) can be a temporal time bridge or covering a geographical gap between energy supply and demand [180]. Energy storage systems mediate between variable sources and variable loads. Energy storage systems can implement in large or

small scale from generation to final delivery to the customers. Reliability is one of the most important parameters in the electrical networks and ESDs increase the total reliability of the network. ESDs charge when the demand is lower than other periods and discharge during peak demand periods to smooth the generation system and prevent any shortage in the network [181]. ESDs can also be utilized in order to take some economic benefits. ESDs can be charged when the price of electricity is lower, usually off-peak periods, and the discharge to the system when the prices are higher. In addition to that, high penetration of the DGs significantly affects the application of the ESDs. For instance, for a small-scale DG which is supposed to meet its own demand, a battery can be coupled to the system in order to save the generated electricity from the solar PV panels for night time periods. It should also be considered that the peak generation of solar and wind energy resources are complementary to each other. The followings are the most important applications of the batteries [182].

- Load leveling
- Peak shaving
- Frequency regulation
- Spinning reserve
- Capacity firming
- Power quality

Although there are various reasons for a battery to be failed, this study mainly focuses on the most common failures of a battery as follow:

- Elevated Temperatures

- Repeated Cycling
- Excessive DC Ripple Current
- Over-charging / Under-charging/ Over-discharge
- Vibration

3.3 Generic System under Study

Figure 18 depicts a generic system under the study in this research as an off-grid (1 MW) micro-grid with deferrable and non-deferrable loads. This system is an off-grid system which means there is no connection to the utility grid and the loads should be met based on an island mode. The priority of the smart loads of the network is critical, essential, and normal loads. Normal or hotel loads are flexible to be met later. It has been assumed that the critical components of the systems are known prior as a result of engineering insights toward the system. Furthermore, the rest of the systems are assumed to be always reliable to perform their intended functions satisfactorily. In the next section, the most critical components of the system under study are briefly explained in terms of the functions, applications, main features, and common failures.

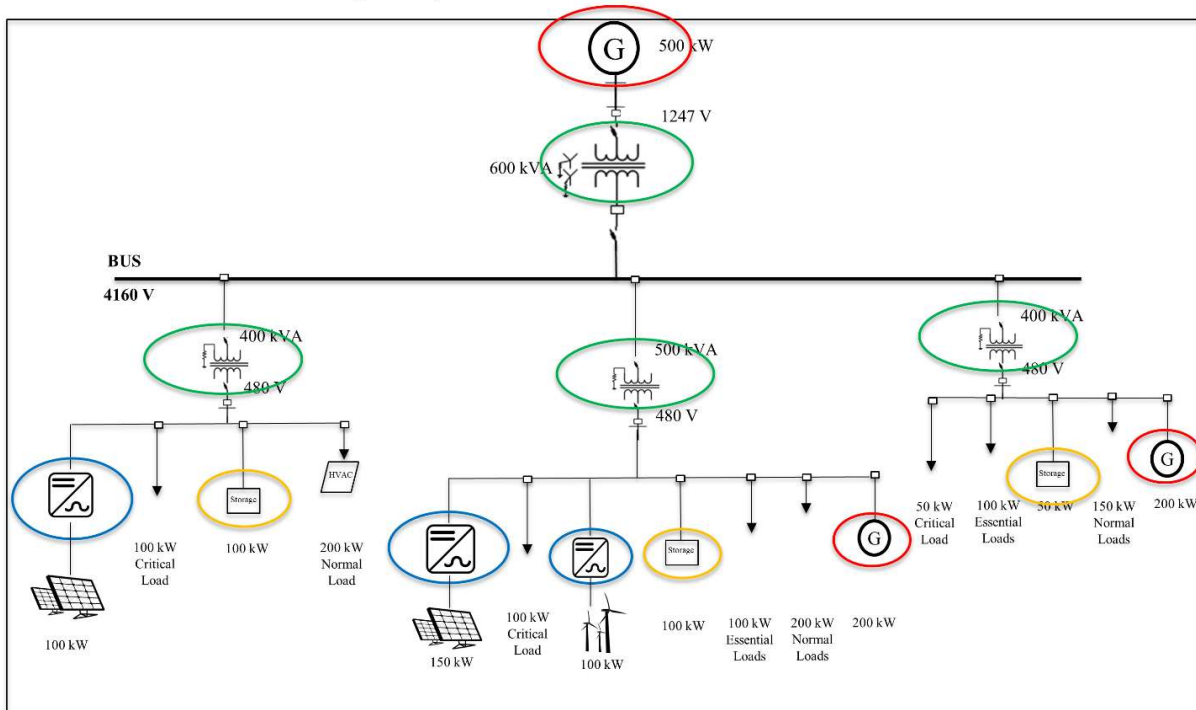


Figure 18: Off-Grid (1 MW) micro-grid with deferrable and non-deferrable loads.

3.4 Concluding Remarks

This main contribution of this study is the application of the data-driven degradation based first hitting time models for the distributed electrical systems by considering the time and critical covariate effects. As mentioned before, the new era is highly relying on the real-time status of the units rather than the historical lifetime data. It should be noted that the core of the analysis is the degradation model predictions since it highly affects the reliability, availability, risk, and probability of failure for each asset and overall system. Therefore, any attempt to enhance the degradation model performance in order to be able to accurately describe the asset behavior in real-time would certainly enhance the upstream statistics such as reliability and risk.

4. Chapter 4: Methodology

4.1 Research Design

As mentioned in Chapter 1, the main purpose of this thesis is to develop a predictive model of reliability estimation based on real-time data using a discrete-time state-space stochastic degradation model to prevent actual failure by predicting the time to maintenance (soft failure) in order to enhance the value and to prolong the life of assets. The main scientific contributions of this study are (1) developed a new state-space stochastic degradation model to accurately capture the dynamic behavior of assets, (2) applied simulation techniques to estimate reliability of assets over time and estimate the critical failure time using the new degradation model, (3) estimated the reliability based on analytical formulation for degradation prediction model, and (4) developed a new data-driven parameter estimation algorithm based on the new degradation model.

Classical models of reliability estimation mainly rely on historical failure data. It should be considered that obtaining lifetime data in a timely manner is one of the current challenges. Failure data may not be easily obtainable for highly reliable assets. Furthermore, the collected historical lifetime data may not be able to accurately describe the behavior of the asset in a unique application or environment. For instance, if the lifetime data are collected based on the experimental tests given specific environmental and operational conditions, there is no guarantee that the asset behavior remains unchanged in other conditions during its lifetime. Therefore, it is not an optimal approach anymore to estimate reliability based on classical models.

Fortunately, most of the industrial assets have performance characteristics whose degradation or decay over the operating time can be related to their reliability estimates. Degradation indicates the process of lowering the rank, status, or grade, which leads to a less effective level of performance. The application of the degradation methods has been recently increasing due to their ability to keep track of the dynamic conditions of the system over time. The main purpose of the degradation-based models is to predict the future condition of the asset and perform the maintenance actions in an optimized time window before the actual failure of the system occurs. Since the degradation-based analysis defines the failure events based on the predefined threshold, the failure is said to have occurred as a soft failure. This indicates that the asset under the study is considered as a failed unit when the degradation profile hits the threshold for the first time.

Inaccurate modeling of the degradation phenomenon leads to inaccurate assessment of reliability, maintenance policy, risk, lifetime prediction, etc. In this thesis, a wide variety of the currently developed models of degradation are studied in detail. Degradation models based on the Gamma process and General Path Model have been applied in various studies. The main purpose of this study is to develop a new data-driven predictive model of reliability estimation based on real-time data using a state-space stochastic degradation model to predict the critical time for initiating maintenance actions in order to enhance the value and prolonging the life of assets. Indeed, the new degradation model developed in this study extended the General Path Model based on a series of Gamma Processes degradation models in the state-space environment. Poisson distributed weights are considered for each of the Gamma processes. Therefore,

the new degradation model developed in this thesis is based on the General Path Model which considers a series of Gamma Processes in the state-space environment by considering Poisson distributed weights for each of the Gamma processes.

Monte Carlo simulation is applied in order to estimate the distribution of the time to soft failure based on the simulated degradation profiles. In addition to that, at each point of the time, the distributions of the degradation observations are also obtainable based on the generated degradation profiles. Therefore, one of the purposes is to develop predictive algorithms that are able to predict the critical failure time based on the first hitting time models to initiate maintenance before the failure occurs. It should be considered that degradation estimates might not be directly obtainable. In the next step of this study, an exact formulation of an analytical approach of reliability estimation is presented. The main benefit of the presented analytical approach is its dependency on the real-time status of the asset since the analyses are not based on the obtained distribution functions. The developed model in this step can be considered as a data-driven algorithm due to the dependency of the degradation estimates to the real-time monitoring data. The followings are the primary steps of this thesis.

- I. Developed a new stochastic state-space degradation model, in the state-space environment or based on the first order Autoregressive models (AR) to accurately capture the dynamic behavior of assets.

Based on the first-order AR model, or state-space model, considered in this thesis, the health condition of the asset in the next cycle highly depends on the current status of the system as well as the behavior of the stochastic process happening during that period of time. It should be considered that the health condition of the

asset is represented by time-series degradation estimates. Later, this stochastic process is introduced as the changes in the degradation estimator, which indicates the cumulative occurred damage.

- II. Applied Monte-Carlo simulation technique to estimate the reliability of assets over time. In addition to that, the critical soft failure time using the new degradation model is estimated based on the simulated degradation profiles.

In this step, the main purpose is to estimate the distribution of the critical failure time based on the first hitting time degradation models to initiate the maintenance before the failure occurs. It should be noted that in degradation-based analysis failure occurs when the degradation estimate hits the predefined threshold for the first time. Several degradation profiles can be generated given the same set of model parameters based on the Monte-Carlo simulation. Therefore, the distribution of the critical failure time can be obtained based on the statistical inference. In addition to that, the time-dependent distribution of degradation can be also obtained based on simulated degradation profiles.

- III. Estimated the reliability based on an analytical formulation of reliability based on the new degradation model developed in this thesis.

In this step, the probability that the degradation estimate reaches a predefined variable threshold defines as unreliability is analytically obtained based on the statistical tools. Indeed, unreliability and reliability can be estimated given the current degradation estimate at each point of time. The main motivation for the analytical formulation of reliability based on the degradation profile is to achieve an automatic, quick-responding, accurate, and robust scheme to estimate the

reliability of an asset based on the real-time monitoring data. For most of the applications, the threshold cannot be estimated without any uncertainty. Therefore, it has been considered that the threshold is a random variable following the same distribution as the destructive amount. It should be considered that the parameters of the model are assumed to be known prior. In the next step, unsupervised clustering techniques are deployed to introduce a data-driven algorithm to estimate the parameters of the new degradation model based on the observed time-series degradation estimates.

- IV. Developed an algorithmic data-driven parameter estimation model for the new degradation model.

Parameters of the new degradation model are estimated based on the unsupervised clustering techniques. The main purpose of this step is to apply the clustering algorithms as an unsupervised learning technique to define the clusters in which the hazard, or damage, events occurred zero and only one time. Therefore, the parameters of the new degradation model can be estimated based on the share or weight, centroid, and variance of each cluster.

4.2 Procedures

4.2.1 Degradation Model

In recent decades, highly reliable products are more often being designed and industrialized in a shorter amount of time. Consequently, it is not usually possible to assess the new designs to failure under normal operating and environmental conditions. It may be possible to infer the reliability behavior of un-failed samples with only the

accumulated test time information. In these cases, some assumptions are made about the distribution of the parameters. However, this may end to a large level of uncertainty in the results. An alternative option in this situation is the application of degradation models. Degradation analysis involves the measurement of performance data that can be directly or indirectly translated into the health indicator in order to obtain information regarding the presumed level of failure. Therefore, the main purpose of the degradation models is to associate failure or damage mechanisms into the degradation estimates. The degradation-based analysis permits the analyst to extrapolate to an assumed failure time, defines as degradation or failure threshold, based on the measurements of degradation over time.

Degradation estimates may be obtained either directly or indirectly. For some applications, it is possible to directly measure the degradation of a physical characteristic over time. For instance, wear of brake pads, propagation of crack size, the voltage of a battery, and flux of an LED bulb can be directly measured. If the physical characteristic can be measured directly, analysis refers to a Non-Destructive Degradation Analysis (NDDA) category. In other cases, direct measurement of degradation might not be possible without destructive measurement techniques that would directly affect the performance of the product. Consequently, only one degradation measurement is possible for each product. For instance, the measurement of corrosion in a chemical container or the strength measurement of an adhesive bond can be measured only once. These cases fit into the Destructive Degradation Analysis (DDA) category. It should be noted that the obtained time-dependent distributions of the degradation estimates are very applicable for assets in DDA category. In addition to that, the obtained distribution of

the time to soft failure can be more applicable for assets in the Non-Destructive Degradation Analysis (NDDA) category.

As mentioned in Chapter 2, degradation models can be categorized into two main categories as physic-base and statistical-based models. The developed model in this thesis is based on the principles of the statistical-based models. From now on, the degradation model refers to the statistical-based degradation model. The followings are the main reason for selecting the statistical-based model.

- It may be very time consuming to identify the physics of the degradation phenomenon. In some cases, it may not be possible to accurately obtain the model. There are more challenges to detect this relationship for new products.
- There is no general physics-based model that can be easily adapted to various applications of the same product. The dynamic behavior of the system makes it impossible to develop a general degradation model.
- Reliability estimation may not be accurately derived since the physic-base model may contain several parameters, which are indeed random variables in real-world applications.

Statistical-based degradation models are more general which has led to the development of several statistical approaches for modeling degradation data. In this study, degradation models are developed based on the state-space or Autoregressive (AR) models.

Equation (14) and (15) present the General Path Model developed by Lue and Meeker (1993). The detail of the General Path Model is presented in Chapter 2. The developed

degradation model in this study is primarily based on the General Path model with a unique definition for the path models or mapping function in the state-space environment. As mentioned in Chapter 2, a wide variety of the degradation models can be explained by General Path Model with a specific definition of the path model. The considered path model in this study is based on a first-order state-space stochastic model. The detail of the developed model is discussed in this chapter.

$$\begin{cases} Y_{ij} = \eta_{ij} + \varepsilon_{ij} = \eta(t_j, \phi, \theta_i) + \varepsilon_{ij} & i = 1, 2, \dots, n \quad ; \quad j = 1, 2, \dots, m \end{cases} \quad \text{Equation (14)}$$

$$\begin{cases} T_i = \min \left\{ MLT_i, \min \left\{ t_j : \eta(t_j, \phi, \theta_i) \geq \eta_{Fi} \right\} \right\} \end{cases} \quad \text{Equation (15)}$$

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

(θ_i) and (ε_{ij}) are independent.

- i : Critical assets index
- j : Observation index
- Y_{ij} : Degradation value (observed based on health indicator) for unit i at observation j
- η_{ij} : Degradation path model for unit i at observation j
- ε_{ij} : Error term for unit i at observation j
- t_j : Time of j^{th} observation
- ϕ : Fixed-effect parameter (same for all units)
- θ_i : Time-effect parameter for unit i
- σ_ε^2 : Variance of error
- T_i : Failure time for unit i
- MLT_i : Maximum Life-Time for unit i
- η_{Fi} : Defined failure level for unit i

So far, it has been mentioned that a first-order state-space stochastic model is considered as a unique mapping function for the General Path Model to accurately model the time-series degradation data. Properties of the stochastic process embedded in the

degradation model highly affect the accuracy and robustness of the new degradation model. The gamma process is one of the most commonly applied degradation models for various applications. In this thesis, the stochastic nature of the degradation model is explained based on the main principles of Gamma processes. In each increment of time, the variable number of damage or hazard events may occur. The weight of each event is explained by a Poisson distribution. The amount of changes in the degradation estimates due to the stochastic process is assumed to follow a Gamma process. It has been also assumed that the number of damage events linearly affects the amount of damage. Therefore, the main scientific contribution of the new degradation model developed in this thesis is to define a new mapping function for the General Path Model based on a series of Gamma Processes degradation models in the state-space environment by considering Poisson distributed weights for each of the Gamma processes. The detail of the developed model is explained in this chapter.

The developed degradation model in this thesis is based on the state-space models, which mainly rely on the past observed values of the process under study. Equation (16) presents a general form of degradation models applied in this study, while d_t is the degradation value at time t , p is the order of the AR model, η is a function which maps the previous degradation values into the degradation estimate at time t . In other words, the presented model in Equation (17) is an $AR(p)$, which represents an Autoregressive model of order p , which ε_t are assumed independent and following a $N(0, \sigma^2)$ distribution. This indicates that it is expected to predict the future value of the degradation based on the obtained knowledge regarding the observed degradation values.

$$d_t = \eta(d_{t-p}:d_{t-1}) + \varepsilon_t \quad (\text{Equation 16})$$

The $AR(p)$ can be presented Equation (18), which considers a linear mapping function of past values into the estimate at time t , where $\varphi = (\varphi_1, \varphi_2, \dots, \varphi_p)'$ is the vector of model coefficients.

$$d_t = \sum_{j=1}^p \varphi_j \cdot d_{t-j} + \varepsilon_t \quad (\text{Equation 17})$$

Yule Walker's equations establish Equation (18).

$$d_t = \varphi_1 \cdot d_{t-1} + \varphi_2 \cdot d_{t-2} + \dots + \varphi_p \cdot d_{t-p} + \varepsilon_t \quad (\text{Equation 18})$$

$\varphi = (\varphi_1, \varphi_2, \dots, \varphi_p)'$ can be obtained if we write these equations for $j = 1, 2, \dots, p$.

The first possibility is to form a set of direct inversions when $p = 1$.

$$d_t = \varphi_1 \cdot d_{t-1} + \varepsilon_t \quad (\text{Equation 19})$$

Equation (20) forms the over-determined system which can be readily solved using the usual least-squares estimator as the Equation (21) presents. The overall concepts will remain the same for higher orders of the model.

$$\underbrace{\begin{pmatrix} d_2 \\ \dots \\ d_t \end{pmatrix}}_{\mathbf{b}} = \underbrace{\begin{pmatrix} d_1 \\ \dots \\ d_{t-1} \end{pmatrix}}_{\mathbf{A}} \cdot \varphi_1 \quad (\text{Equation 20})$$

$$\hat{\varphi}_1 = (A^T A)^{-1} A^T b \quad (\text{Equation 21})$$

In time series analysis, the partial autocorrelation function (PACF) gives the partial correlation of a stationary time series with its particular lagged values, regressed the values of the time series at all shorter lags. Given a time series d_t , the partial autocorrelation of lag p , is the autocorrelation between d_t and d_{t+p} with the linear dependence of d_t on d_{t+1} through d_{t+p-1} removed. Partial autocorrelation plots such as Box and Jenkins, are a commonly used tool for identifying the order of an autoregressive model.

In this thesis, a first-order AR model is considered with a unity coefficient matrix in order to predict the deterministic part of the model. However, a stochastic model is in charge to capture the changes in the values of degradation estimates in each time interval. The stochastic model has its own parameters in order to define the stochastic damage mechanism. Time-invariant parameters are considered for the parameters of the new degradation model. In the first part of this thesis, it has been assumed that sufficient engineering insight is available regarding the model parameters. In the next sections, a new data-driven algorithmic model is developed based on the unsupervised clustering techniques in order to estimate the model parameters.

Equation (22) states the degradation model considered in this study where Δ_t is a stochastic process of the cumulative amount of damage that occurred at time t .

$$\hat{d}_t = d_{t-1} + \Delta_t \quad (\text{Equation 22})$$

As mentioned before, at each point of the time, various numbers of damage events may occur with a specific weight which is explained by a Poisson distribution. In general, it can be stated that the changes in degradation estimate from time $t - 1$ to time t can be

estimated based on the expected value for the stochastic process, which represents the cumulative amount of damage. The expected value of the stochastic process can be evolved over time by considering the time-variant parameters for the developed degradation model.

$$\hat{d}_t = d_{t-1} + E(\delta_t) \quad (\text{Equation 23})$$

Equation (24) presents the degradation estimate at time t , d_t , as a function of a known number of damage events occurred at time t , D , probability of observing X event, $p(X)$, and damage amount due to X events occurred at time t , $\delta_{X,t}$.

$$\hat{d}_t = d_{t-1} + \sum_{X=0}^D p(X) \cdot \delta_{X,t} \quad (\text{Equation 24})$$

4.2.2 Degradation Model Properties

In this section, the main properties of the presented degradation model are explained in detail. The main properties are (1) rate of observing the damage or hazard events and (2) damage amount occurred due to a single event, and (3) cumulation damage amount occurred in each cycle.

4.2.2.1 Damage Events

In probability theory and statistics, the binomial distribution with parameters n and p is the discrete probability distribution of the number of successes in a sequence of n independent experiments. Each of these experiments asking a yes or no questions, and each with its own Boolean-valued outcome as a success (with probability p) or failure/no/false/zero (with probability $q = 1 - p$). A single success/failure experiment is

also called a Bernoulli trial and a sequence of outcomes is named a Bernoulli process. The actual distribution of the Poisson is also given by Bernoulli distribution. Poisson distribution is a limiting version of the Binomial distribution which n becomes large and np approaches a value called rate of event, λ . Figure (19) presents an overview of the relationship between the Bernoulli, Binomial, and Poisson distributions.

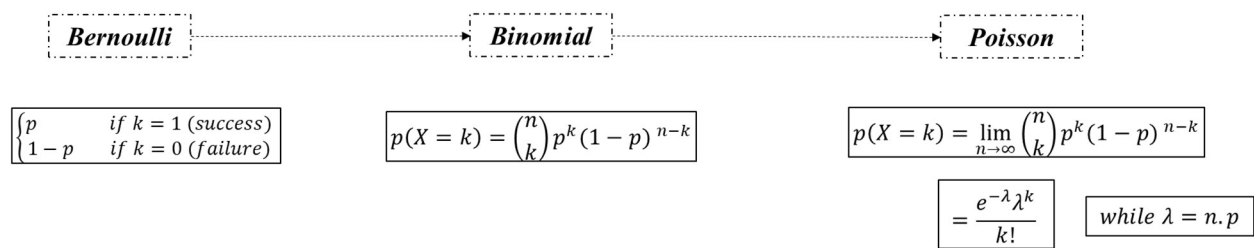


Figure 19: An overview of the relationship between the Bernoulli, Binomial, and Poisson distributions.

In this thesis, it has been assumed that the rate of damage event is following a Poisson distribution. Poisson distribution is a discrete probability distribution that presents the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant rate, known as Poisson parameter, and independently of the time since the last event. The Poisson distribution is very popular for its properties to model the number of times an event occurs in an interval of time or space. The occurrence of one event does not affect the probability that another event will occur or not. This indicates that destructive events occur independently. In general, the average rate of Poisson events is constant unless the Poisson distribution is defined as a variable or function. For instance, the Poisson parameter can be defined as a function of time in order to indicate a higher rate of events over time. In addition to that, two events cannot occur at the same time. It means that there should be a subinterval in which the probability

of observing more than one event is zero. This assumption clearly presents the relationship between the Bernoulli and Poisson distributions. In this case, the probability of observing x destructive event, $p(x)$, is following a Poisson distribution with the rate of λ , which can be a function of the age of the asset under study. This implies that destructive events will increase as the asset gets aged. Equation (25) presents the Probability Density Function (PDF) of a Poisson distribution. The following assumptions are considered in order to obtain the degradation estimate at each point of time.

$$p(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad \text{Equation (25)}$$

4.2.2.2 Damage Amount

The Gamma distribution is another commonly used statistical distribution. Its reputation is largely due to its relation to Exponential and Normal distributions. The Gamma distribution is a two-parameter family of continuous probability distributions. The exponential distribution, Erlang distribution, and chi-squared distribution are special cases of the gamma distribution. Equation (26) presents the probability density function (PDF) of the Gamma distribution.

$$f(x; \alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \quad \text{for } x > 0, \beta > 0 \quad \text{Equation (26)}$$

In this study, it has been assumed that damage amount due to a single damage event, $\delta_{1,t}$, is following a Gamma distribution with (α, β) parameters as the shape and scale parameters, respectively. The main reason for considering the Gamma distribution over the Normal distribution is that the amount of damage cannot be a negative value.

It should be noted that the number of occurred damages or destructive events in each period is assumed to follow a Poisson distribution with a constant rate. In this study, it has been assumed that destruction amount due to a single destructive event, $\delta_{1,t}$, is following a Gamma distribution with (α, β) parameters as the shape and scale parameters, respectively. Furthermore, it has been considered that the damage amount has a linear effect with the number of damage events. The following table may clearly explain this assumption.

Table 1: The detail of the cumulative damage amount.

<i>Number of Destructive Events (i)</i>	<i>Distribution of the Destruction Amount</i>	<i>Mean of Distribution of the Destruction Amount</i>	<i>Variance of Distribution of the Destruction Amount</i>
0	N/A	N/A	N/A
1	$Gamma(\alpha, \beta)$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
2	$Gamma(2\alpha, \beta)$	$\frac{2\alpha}{\beta}$	$\frac{2\alpha}{\beta^2}$
....
D	$Gamma(D\alpha, \beta)$	$\frac{D\alpha}{\beta}$	$\frac{D\alpha}{\beta^2}$

In Section 3.2.1, the detail of the degradation model considered in this thesis is explained in detail. The rest of the analyses are mainly based on the new degradation model. In Section 3.2.2, a simulation-based analysis is developed based on the Monte-Carlo simulation techniques in order to obtain the distribution of time to failure and degradation estimates at each point of the time.

4.2.2.3 Degradation Threshold

Degradation threshold is a point of degradation profile in which failure is said to occur when a degradation profile crosses that limit for the first time. For the state-space degradation model presented in Section 3.2.1, the essence of the threshold is similar to the damage amount. In other words, the threshold value can be considered as the maximum value of the destructive amount, which its occurrence leads to soft failure. Therefore, it can be concluded that the statistical distribution of the threshold is similar to the distribution of the damage amount. In the case of Gamma distribution, it has been assumed that the scale parameters of the Gamma distributions are the same for threshold and damage amount. Threshold properties are usually defined when the asset under study is working as good as new without any sign of degradation. It should be noted that the degradation threshold may have a unique definition for the same unit in different applications.

4.3 Degradation Model Parameters for Generic Use

Case Under Study

As mentioned before, a degradation model parameter estimator is ultimately providing data-driven estimates of the parameters based on the unsupervised clustering algorithm. Before integrating the parameter estimator into the overall algorithm, it can be assumed that engineering insight is available in order to determine the properties of the degradation model. In this case, the robustness of the overall algorithm can also be verified since the true value of the model parameters is known prior. Therefore, it can be concluded that

the overall algorithm may offer a robust predictive model of reliability estimation when the time series degradation estimates are available.

The technical detail of the degradation models applied in this study has been discussed in detail in Chapter 2 and Chapter 3. The degradation mechanism may behave differently. For some properties, degradation may arise gradually over time, while some other cases only face a few critical periods. It is expected that the new data-driven predictive model of reliability estimation performs acceptably for both of the cases.

It should be noted that the new algorithm estimates the reliability based on the time-series degradation profile. Some studies may have assumed that the overall health status of a single unit must be presented by a single health indicator, while others believe each degradation mechanism may have its own separate health indicator. Although the main principles of degradation estimators have been discussed in this study, the technical detail of the degradation estimator development is out of the scope of this study. As mentioned in Chapter 1, the main objective of this thesis is to develop a robust algorithm in order to map the time-series degradation data into the reliability estimate. The next sections of this chapter provide analysis based on the predetermined value of the degradation model parameters. The actual value of the model parameters may not be even obtainable for real-world scenarios. Indeed, it was among one of the main motivations for conducting this study. The assumed predetermined value for the degradation model parameters has been obtained based on several studies, laboratory tests, experimental analysis, standard, etc., which have been previously conducted by other researchers. The actual value of the parameters is not the point of concern in this study since the robustness of the overall algorithm is independent than the initial value of

the model parameters. Furthermore, it has been tried to cover most of the cases for the degradation paths in order to validate the robustness of the designed algorithm. The following Tables present the predefined value of the degradation model parameters for the critical equipment of the Generic system under study. At this point in the study, the focus is merely on obtaining the degradation profiles for the critical components of the system. In the next step, reliability estimation is conducted for each component by taking the independency between the common causes of failure into account. In this thesis, it has been assumed that the overall health condition of the critical components can be presented by a single time-series degradation profile. The same methodology can be applied for the cases which presenting the overall health condition of an asset by a single time-series degradation profile might not be easily possible. For instance, more detail analysis can be conducted for the most common causes of failure, provided in Section 4.2, for the most critical components of the system. In the next step of the detailed analysis, covariance analysis can reveal the correlation between each degradation profile either with or without considering some exogenous variables, which mainly affect the health status of several degradation mechanisms. Contrary to this method, it can be also assumed that degradation profiles are completely independent than each other, which may not be always true. Finally, reliability estimation can be performed for each single time-series degradation profile either representing a single degradation mechanism or overall degradation status of the asset. It should be noted that the main objective of this thesis is to map the time-series degradation estimates into the reliability estimates. Table 4 presents the predetermined degradation model parameters for the critical components of the generic system under study. It should be considered that the initial estimate of

these values might not be available or obtainable but, this study seeks to integrate an algorithmic data-driven parameter estimator into the algorithm to estimate the values of each of the model parameters. The performance of the degradation model parameter estimator can be evaluated since degradation profiles are generated based on the known values of the parameters.

Table 2: Assumed degradation model parameters for considered degradation mechanism of the transformer under study.

Component \ Parameters	Poisson Parameter for destructive events $\hat{\lambda}$	Gamma Shape Parameter for destruction amount $\hat{\alpha}$	Gamma Scale Parameter for destruction amount $\hat{\beta}$	Gamma Shape Parameter based on maximum destructive events
Transformer	0.1	40	20	30
NG Generator	0.4	15	20	100
Inverter	1.4	10	50	300
Battery	0.35	20	50	150

4.4 Degradation Path Generation

Figure (20) presents the generated degradation profiles for the critical components of the system based on the predefined parameter of the degradation model. As the figure shows, transformer damage events occur less frequently but, with higher damage amount. In this thesis, it has been tried to cover the most possible forms of degradation profiles in terms of being either more toward discrete or continuous events. As mentioned

earlier, the value of the degradation model parameters will not be a concerning point if a robust degradation parameter estimator can be integrated into the overall algorithm.

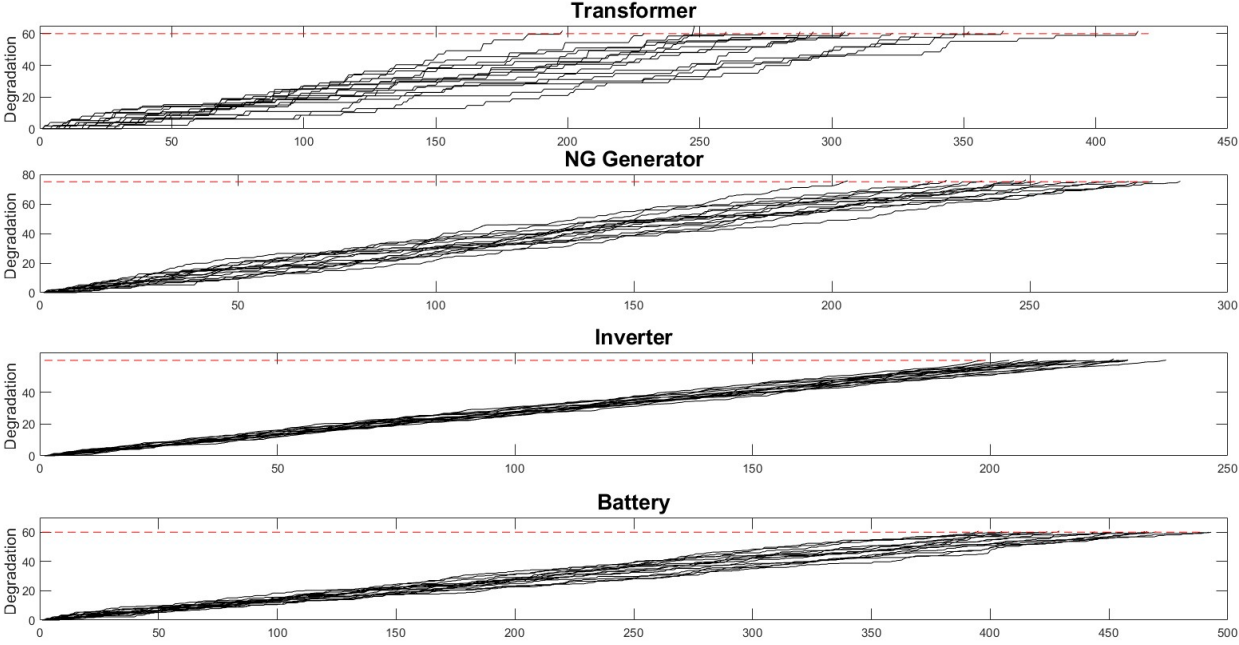


Figure 20: Generated degradation paths for critical components of the system based on the predefined parameter of the degradation model.

In the next chapters, the application of the developed algorithm is illustrated for the critical components of the Generic System under study, which represents a Distributed Electrical System (DES). The main purpose of this chapter is to present the robustness of the developed algorithm. Simulation results are also provided in order to state the main motivations for developing a predictive model of Asset Management (AM) based on the exact analytical formulation. Furthermore, a data-driven method of estimation is also presented in order to state the robustness of the model for various applications.

5. Chapter 5: Simulation-Based Analysis

As mentioned in Chapter 1, it has been assumed that sufficient engineering insight regarding the destruction frequency and intensity is available in the first step of the analysis. Therefore, degradation profiles can be generated based on the presented degradation model in Equation (8) and given the known model parameters, $\theta = \{\lambda, \alpha, \beta\}$. Figure (21) and Figure (22) present two examples of the degradation profiles given the known model parameters. These two figures present degradation profiles generated based on two different concepts based on the model parameters. Figure (21) states a continuous case in which destructive events occur frequently but, the destruction amount is relatively small compared with the second case. Figure (22) presents a discrete case in which events occur less frequently but, the destruction properties are larger.

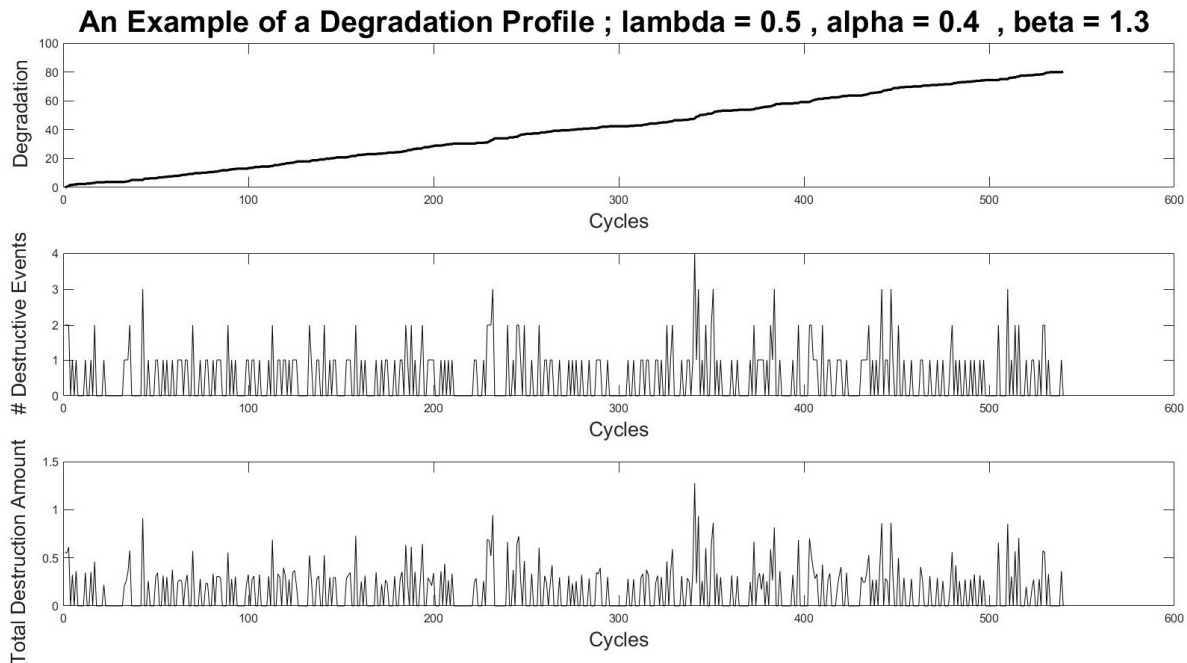


Figure 21: An example of a degradation profile given the known model parameters.

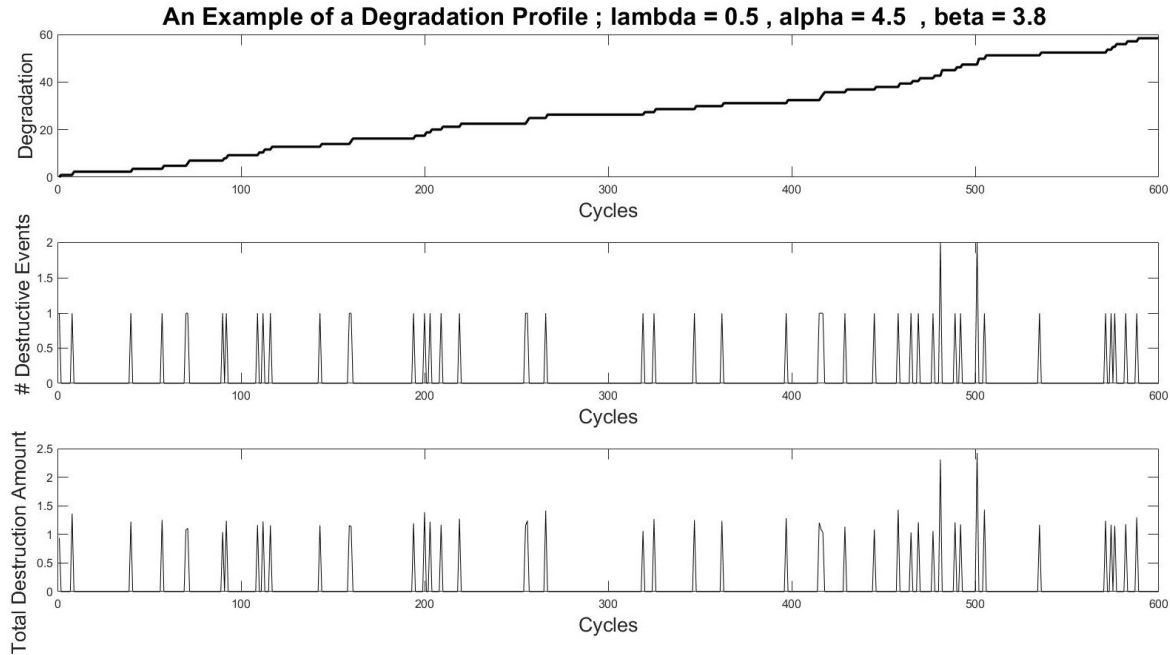


Figure 22: An example of a degradation profile given the known model parameters.

Degradation estimates can be generated based on the given values of the parameters of the model, which has been presented in Section 3.2.1. The robustness of the model is sufficient to cover the vast majority of the degradation mechanisms. Therefore, several degradation profiles can be generated for a given set of model parameters by applying the principles of the Monte-Carlo Simulation. As mentioned in Chapter 1, failure occurs when the degradation profile hits a predefined variable threshold for the first time. In this study, it has been assumed that the distribution of the threshold is as same as the distribution of the damage amount and its parameters can be determined prior based on the engineering insight. In addition to that, it can be stated that the result of the analysis highly depends on the considered value for the degradation threshold.

As Section 3.2.2 presented, soft failure times can be obtained as the time which degradation profile hits the degradation threshold for the first time. In the next step of the analysis, the distribution of the time to failure and time-dependent degradation can be

obtained based on the statistical inferences. Sections 3.2.2.1 and 3.2.2.2 explain the detail of the mythology in detail.

5.1 Methodologies

5.1.1 Time-To-Failure Distribution

As mentioned in Section 3.2.2, degradation profiles can be generated for a given set of model parameters based on the principles of the Monte-Carlo simulation. In the next step, the vector of failure times can be obtained as the point of time in which the degradation profile hits the threshold value. Figure (23) presents an overview of the approach in order to obtain the statistical distribution function of the time to failure. Only 20 degradation profiles have been presented in Figure (23) to prevent visual confusion. The number of degradation profiles are needed in order to obtain the distribution function of the time to failure, shown by red circles.

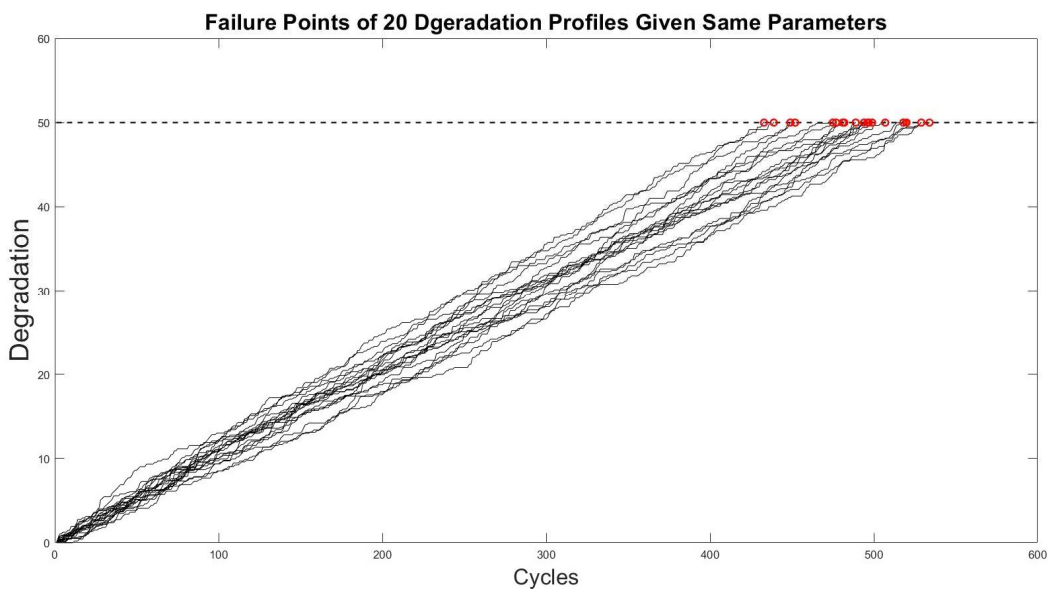


Figure 23: An example of failure points of 20 degradation profiles generated given the same set of parameters

5.1.2 Time-Dependent Degradation Distribution

Degradation profiles evolve over time. In some applications, it might be possible that evolution starts after a certain point in time. In other words, degradation estimators might indicate completely healthy conditions up to a point of time, which degradation mechanism starts to evolve. As mentioned in Section 3.2.1, several degradation profiles can be obtained given the same set of model parameters due to the nature of the considered stochastic process. At each instant of time, distribution of the estimated or measured degradation profiles can be determined. The main application of this approach is for the cases in which the tests are destructive, and analysts cannot wait until the degradation estimator reaches its failure threshold. Figure 24 shows a schematic view of a few time-dependent degradation estimates of 20 degradation profiles, which have occurred given the same set of parameters. It should be considered that as the asset in-service time increases, it is expected to observe more uncertainty in the degradation estimates at each instant of time.

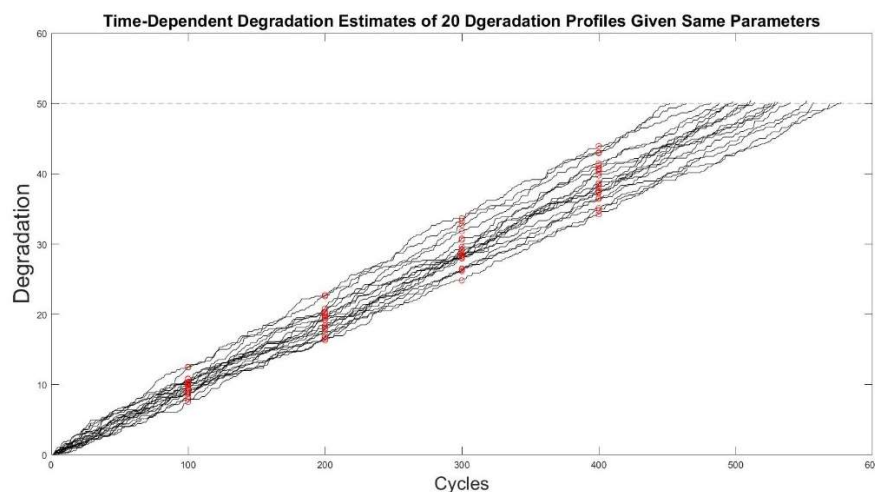


Figure 24: Time-Dependent Degradation Estimates of 20 Degradation Profiles Given Same Parameters.

5.1.3 Probability Distribution Fitting

Probability distribution fitting refers to fitting a series of data concerning repeated measurements to a set of statistical distributions. The main purpose of distribution fitting is to predict the probability or to forecast the frequency of occurrence of the phenomenon in a certain interval. There are wide varieties of discrete and continuous probability distributions that can be fitted into the observed data. Probability distribution selection depends on the characteristics of the phenomenon and probability distribution. The distribution which closely fits the data is expected to provide reliable predictions if the characteristics of the population under the study remain as same as periods in which samples are collected. As a result of probability distribution fitting, distribution should be selected which suits the data well given the selection criteria.

In this section, a framework is designed in order to fit the most applicable statistical distribution into the obtained failure times and degradation estimates at each instant of time. It should be considered that the number of failure times might be less than the number of degradation profiles since some of them might not cross the threshold during the considered horizon. In this study, the following continuous distributions are considered.

- Normal
- Exponential
- Gamma
- Logistic
- T location-scale
- Uniform
- Rayleigh

- Beta
- Inverse-Gaussian
- Log-logistic
- Lognormal
- Weibull'

Two selection criteria as “Log-Likelihood (LL)” and “Akaike Information Criterion (AIC)” are available in the designed framework. In this thesis, LL is mainly considered as the distribution function selection criteria. The detail of the Matlab code is presented in Appendix A.

Figure 25 and Figure 26 present a schematic view of the distributions of failure times and degradation estimates at each instant of time for 20 degradation profiles, which have been generated given the same set of parameters.

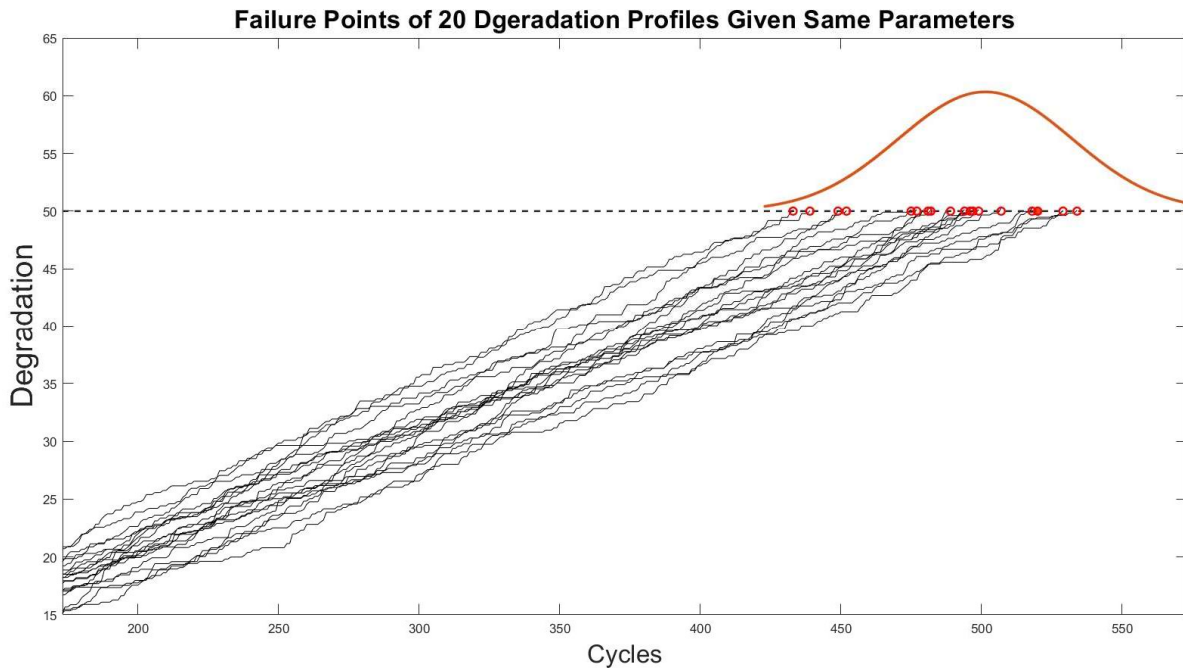


Figure 25: Failure points of 20 degradation profiles generated given the same set of parameters and schematic fitted distribution function to the obtained failure time.

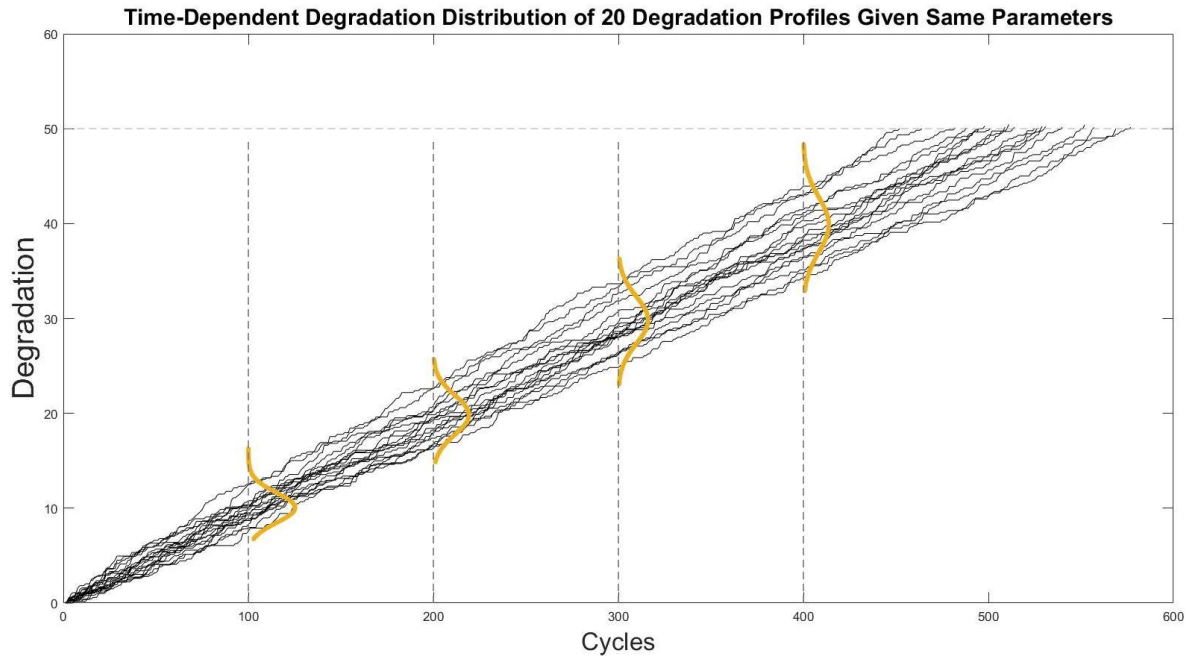


Figure 26: Time-Dependent Degradation Distribution of 20 Degradation Profiles Given Same Parameters.

5.2 Results

In this section of the thesis, the main purpose is to obtain the distribution of the failure time and time-dependent degradation at each point of the time. Obviously, a single distribution can present the best fitted distribution to the obtained failure times and degradation data at each point of time.

In this thesis, the overall approach of the Monte-Carlo simulation seeks to generate the degradation profiles for several times. It should be considered that all the degradation profiles must be generated based on the same set of degradation model parameters and the threshold value. In this study, Monte-Carlo iteration is set to 5000 times for each of the critical components. This indicates the statistical distributions are obtained based on

the analysis of 5000 degradation profiles. It should be noted that the number of failure times might be less than 5000 since some of the profiles might not hit the threshold during the considered horizon. The ability to generate several degradation profiles in a timely and costly manner is one of the main benefits of the proposed methodology.

5.2.1 Transformer

5.2.1.1 Time To Failure Distribution

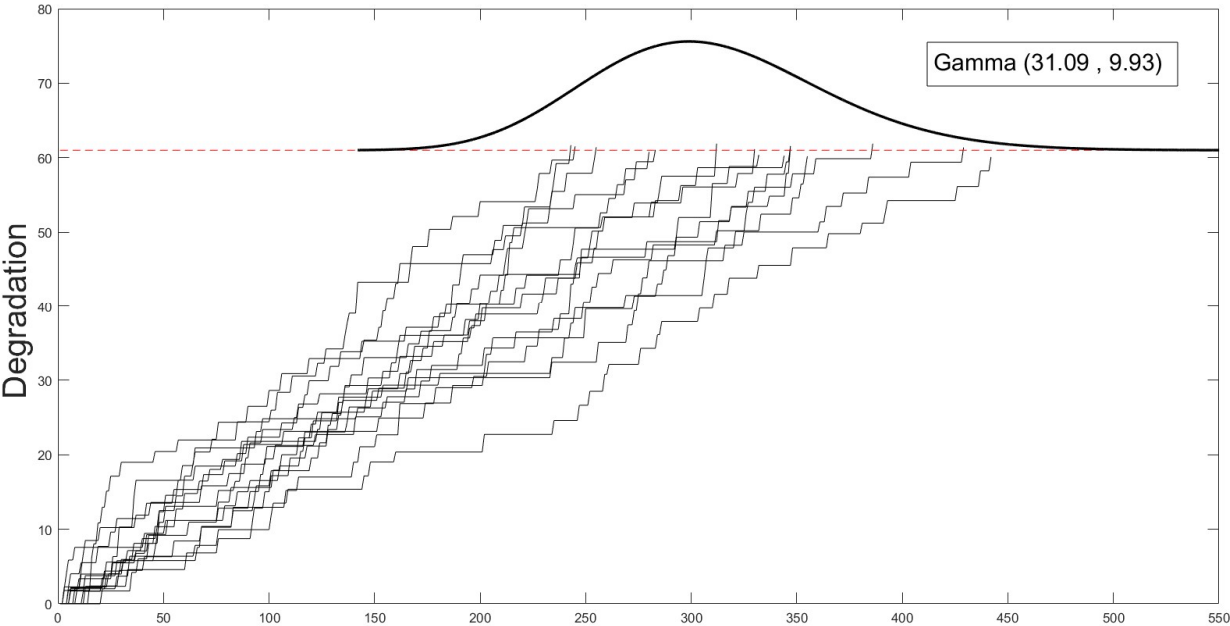


Figure 27: Fitted distribution of the time to failure for the transformer under study.

5.2.1.2 Reliability

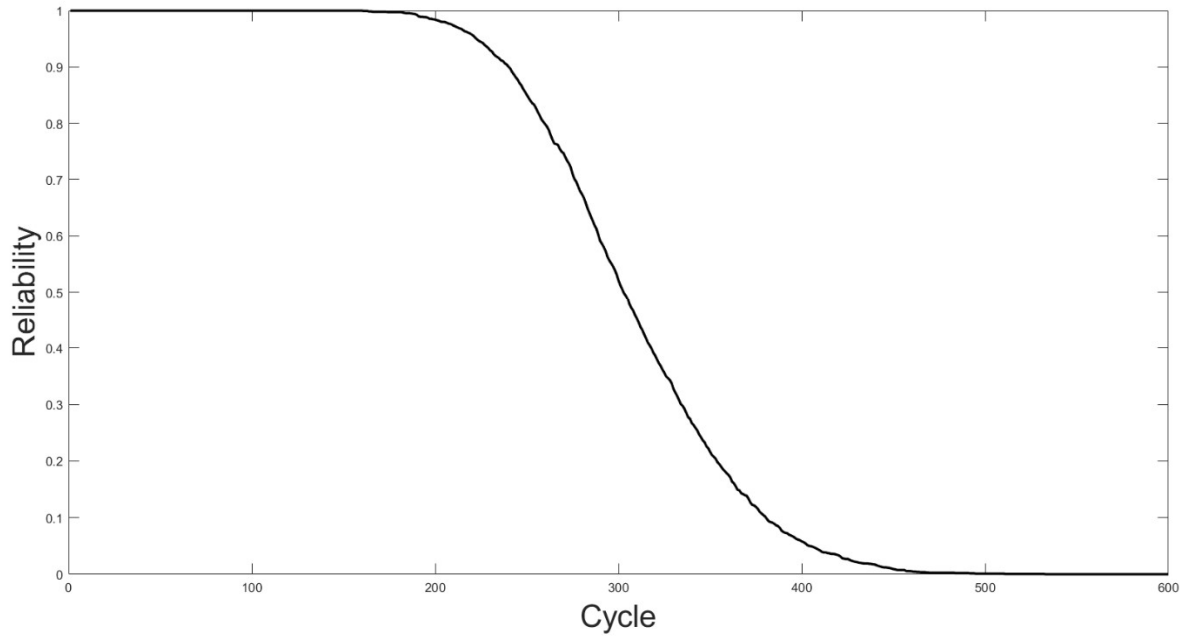


Figure 28: Obtained reliability function for the transformer under study.

5.2.1.3 Time-Dependent Degradation Distribution

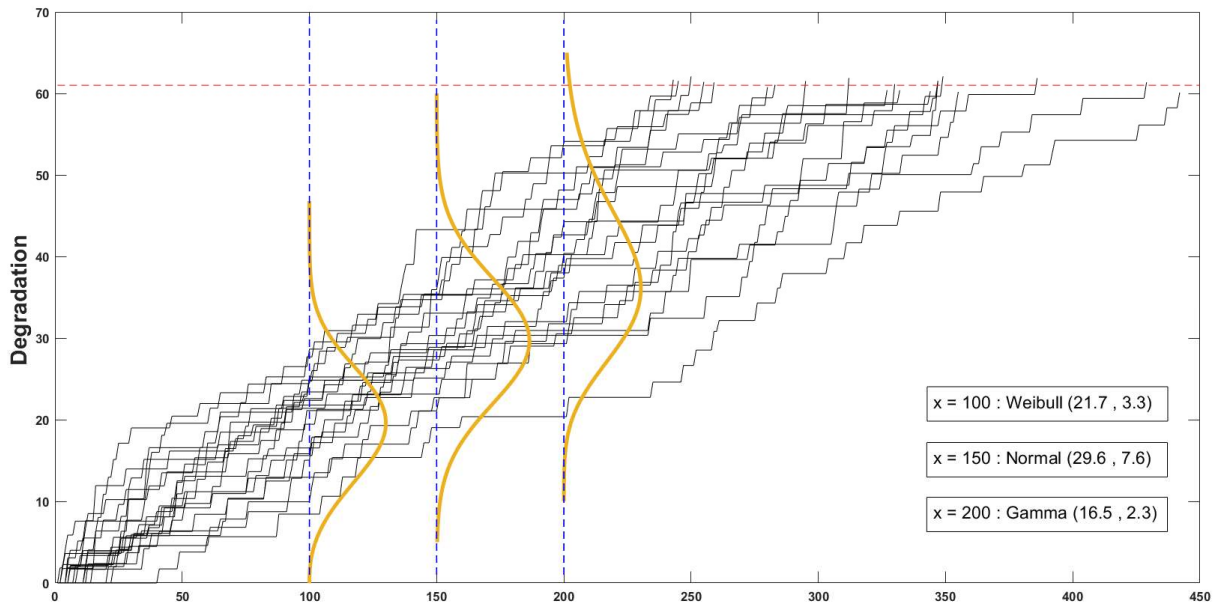


Figure 29: Time-dependent degradation distribution for the transformer under study.

5.2.2 NG Generator

5.2.2.1 Time To Failure Distribution

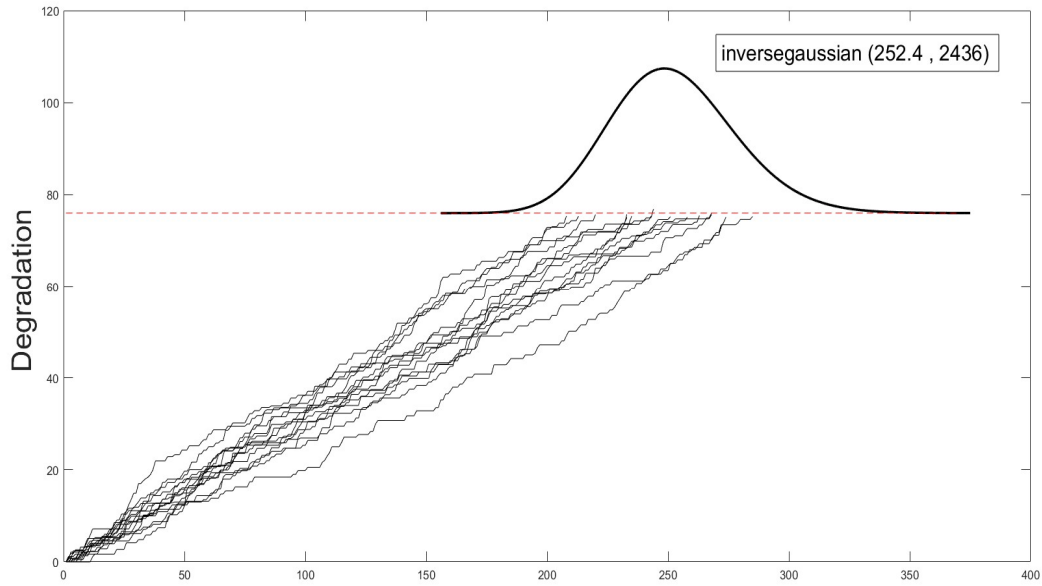


Figure 30: Fitted distribution of the time to failure for the NG generator under study.

5.2.2.2 Reliability

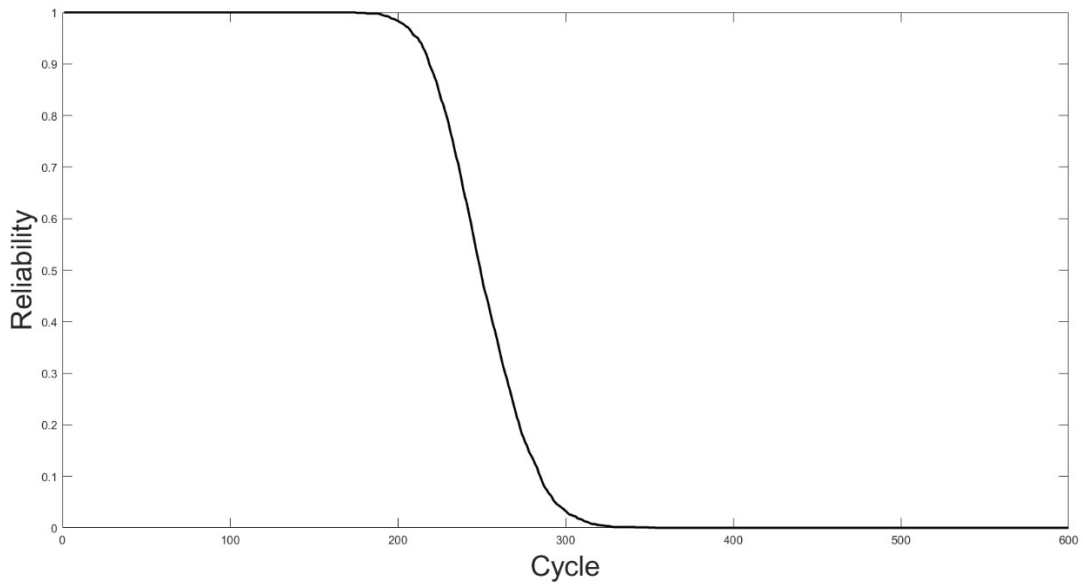


Figure 31: Obtained reliability function for the NG generator under study.

5.2.2.3 Time-Dependent Degradation Distribution

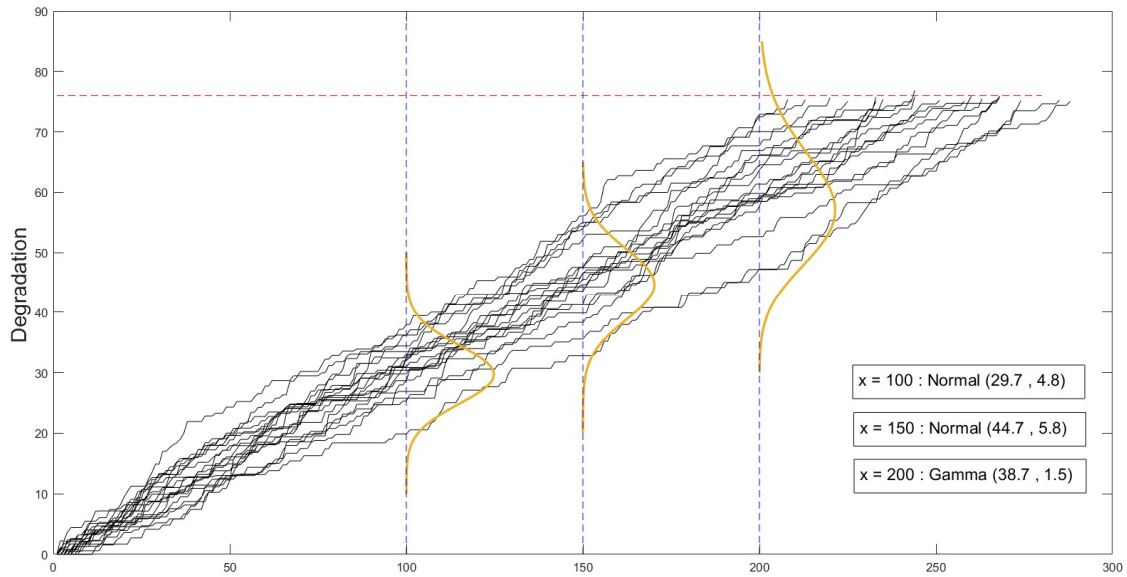


Figure 32: Time-dependent degradation distribution for the NG generator under study.

5.2.3 Inverter

5.2.3.1 Time To Failure Distribution

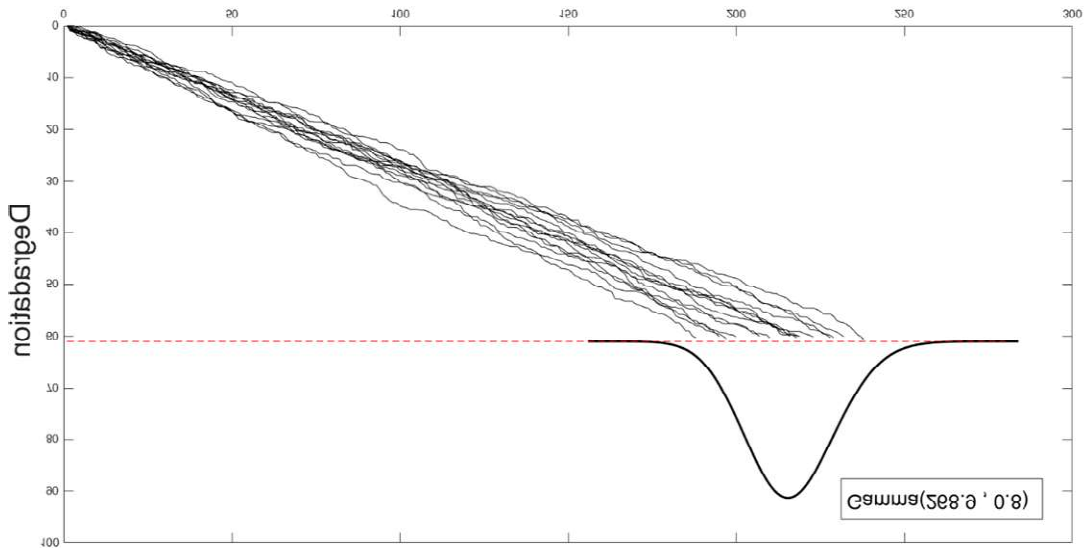


Figure 33: Fitted distribution of the time to failure for the inverter under study.

5.2.3.2 Reliability

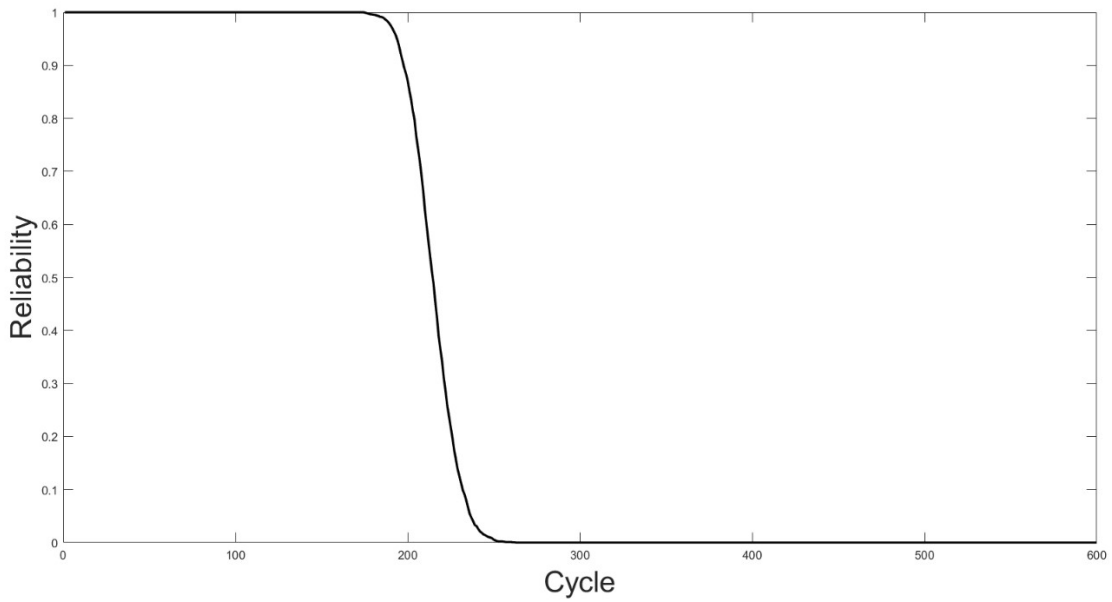


Figure 34: Obtained reliability function for the inverter under study.

5.2.3.3 Time-Dependent Degradation Distribution

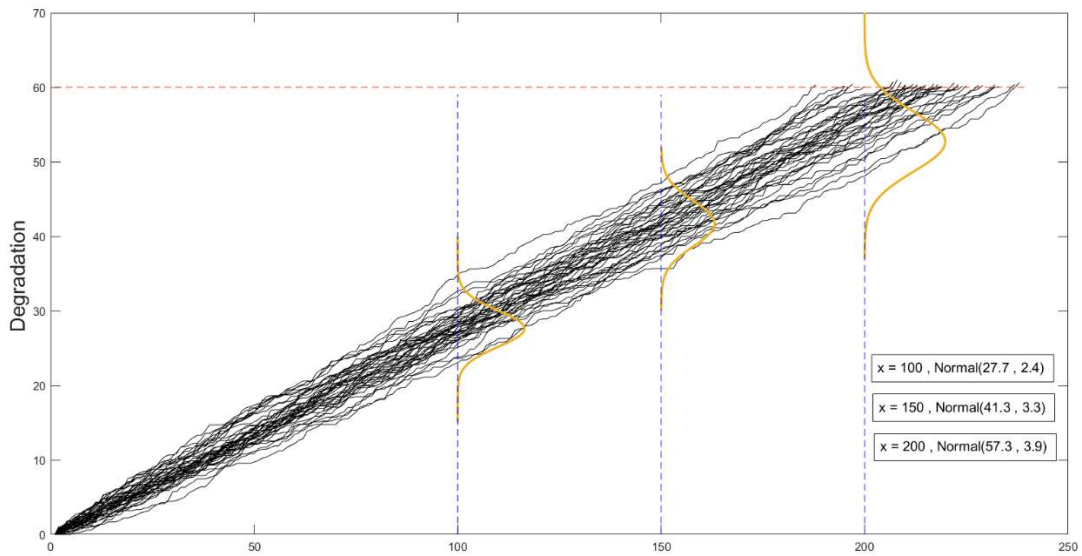


Figure 35: Time-dependent degradation distribution for the inverter under study.

5.2.4 Battery

5.2.4.1 Time To Failure Distribution

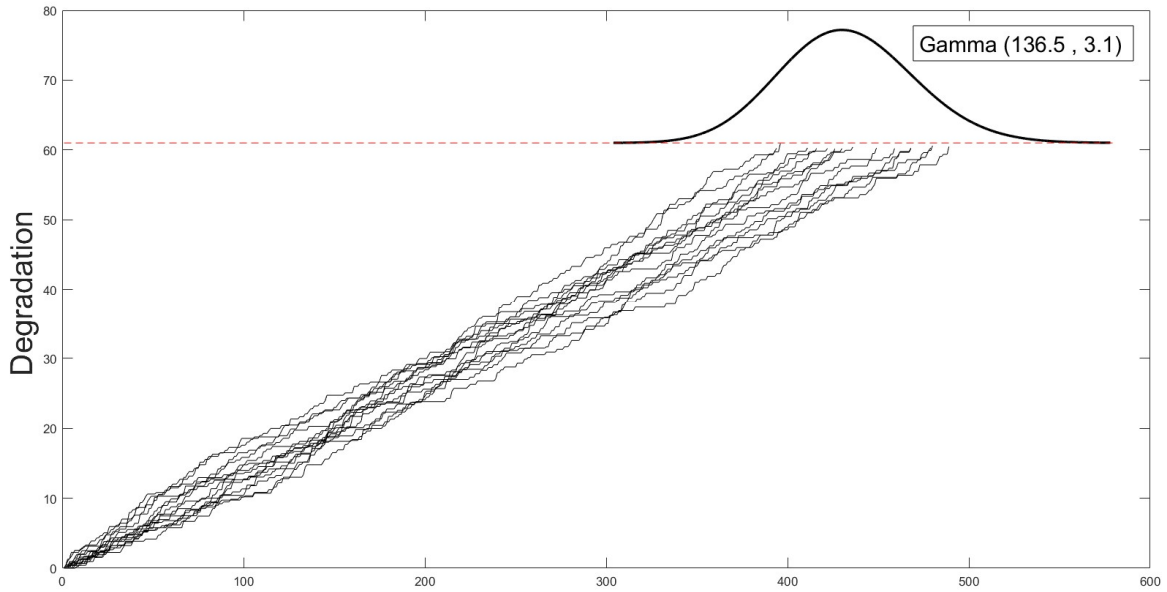


Figure 36: Fitted distribution of the time to failure for the battery under study.

5.2.4.2 Reliability

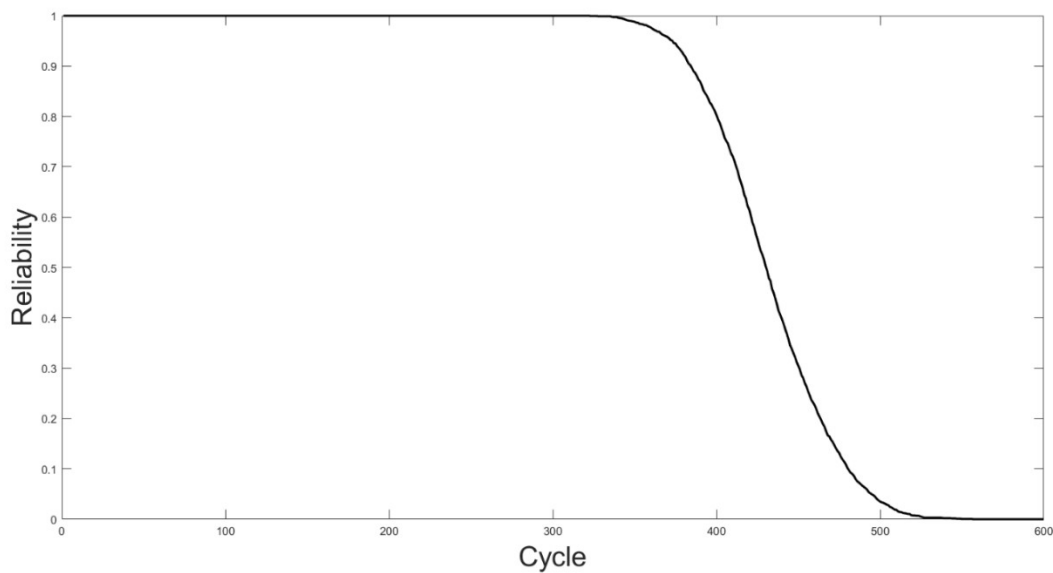


Figure 37: Obtained reliability function for the battery under study.

5.2.4.3 Time-Dependent Degradation Distribution

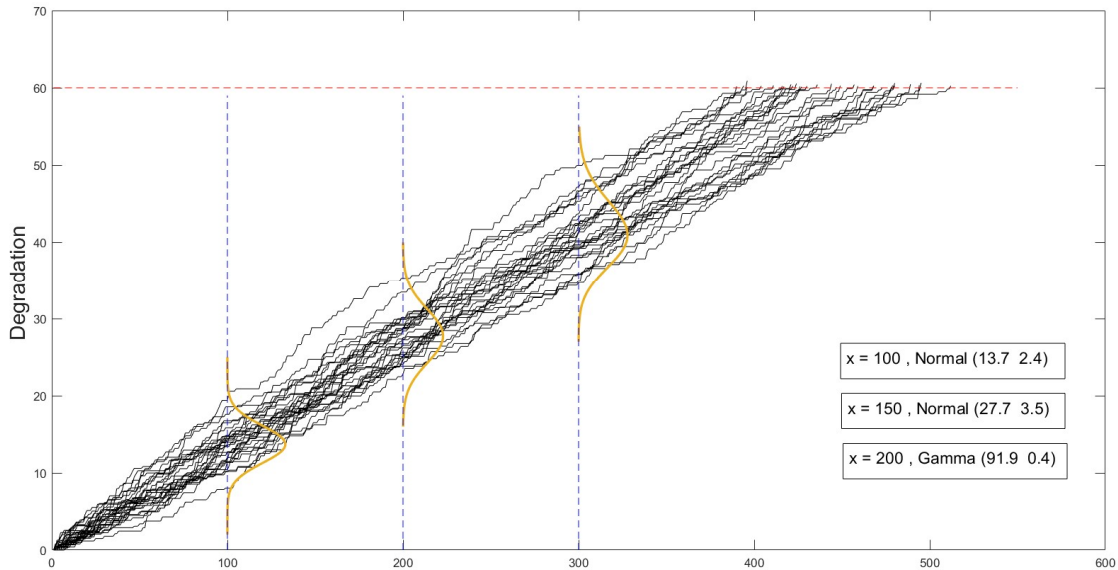


Figure 38: Time-dependent degradation distribution for the battery under study.

5.3 Concluding Remarks

As mentioned in Chapter 1, the main purpose of this thesis is to develop a predictive model of reliability assessment based on real-time data using a discrete-time state-space stochastic degradation model to prevent actual failure by predicting the time to maintenance (soft failure) in order to enhance the value and to prolong the life of assets. Simulation techniques are applied to calculate the reliability of assets over time and estimate the critical failure time using the developed model

Classical models of reliability assessment mainly rely on historical failure data. It should be considered that obtaining lifetime data in a timely manner is one of the current challenges. Failure data may not be easily obtainable for highly reliable assets. Furthermore, the collected historical lifetime data may not be able to accurately describe the behavior of the asset in a unique application or environment. For instance, if the

lifetime data are collected based on the experimental tests given specific environmental and operational conditions, there is no guarantee that the asset behavior remains unchanged in other conditions during its lifetime. Therefore, it is not an optimal approach anymore to conduct a reliability assessment based on classical models.

Fortunately, most of the industrial assets have performance characteristics whose degradation or decay over the operating time can be related to their reliability estimates. Degradation indicates the process of lowering the rank, status, or grade, which leads to a less effective level of performance. The application of the degradation methods has been recently increasing due to their ability to keep track of the dynamic conditions of the system over time. The main purpose of the degradation-based models is to predict the future condition of the asset and perform the maintenance actions in an optimized time window before the actual failure of the system occurs. Since the degradation-based analysis defines the failure events based on the predefined threshold, the failure is said to be occurred as a soft failure. This indicates that the asset under the study is considered as a failed unit when the degradation profile hits the threshold for the first time.

Monte Carlo simulation is applied in order to find the distribution of the time to failure based on the simulated degradation profiles. In addition to that, at each point of the time, the distributions of the degradation observations are also obtainable. Therefore, one of the purposes is to develop predictive algorithms which are able to predict the critical failure time based on the first hitting time models to initiate the maintenance before the failure occurs. In this section, the main steps of obtaining reliability based on the time, cycle, or in-service life, distribution of critical time to failure, and time-variant degradation

distribution are explained in detail. The results of the analyses are presented for the generic use case of this thesis.

6 Chapter 6: Analytical Formulation

6.1 Methodologies

In this chapter, reliability is estimated based on an analytical approach for the new degradation model developed in this study. The developed degradation model in this study offers a new mapping function for the General Path Model based on a series of Gamma Processes degradation models in the state-space environment. Poisson distributed weights are considered for each of the Gamma processes. The detail of the concepts, which the developed degradation model relies on is explained in Chapter 4.

The main purpose of this section of the study is to obtain the reliability given the estimate of degradation. In this case, finding the statistical distribution of the time to failure or degradation is not the point of interest. In other words, predictive models are not providing reliability estimates by fitting a statistical distribution into the observed or generated lifetime data. As mentioned before, it has been assumed that the data-driven time-series degradation estimates are available. Equation (27) and (28) are the considered degradation models presented in Section 3.2.1.

$$d_t = d_{t-1} + \Delta_t \quad \text{Equation (27)}$$

$$\hat{d}_t = d_{t-1} + \sum_{X=0}^D p(X) \cdot \delta_{X,t} \quad \text{Equation (28)}$$

Probability of failure, or unreliability, at time t , PF_t can be defined as Equation (29) presents, while d_F is the degradation threshold in which failure is said to occur.

$$PF_t = p(d_t \geq d_F) \quad \text{Equation (29)}$$

$$PF_t = p\left(d_{t-1} + \sum_{X=0}^D p(X) \cdot \delta_{X,t} \geq d_F\right) \quad \text{Equation (30)}$$

$$PF_t = p\left(d_F - \sum_{X=0}^D p(X) \cdot \delta_{X,t} \leq d_{t-1}\right) \quad \text{Equation (31)}$$

$$\text{if } d_F - \sum_{X=0}^D p(X) \cdot \delta_{X,t} = \Phi \quad \text{then } PF_t = CDF_{\Phi}(d_{t-1}) \quad \text{Equation (32)}$$

It should be considered that the value of d_{t-1} has been observed in the last period. At the current period, the main purpose is to estimate the PF_t given the observed value of degradation in the previous period. Therefore, PF_t can be estimated if the CDF of $d_F - \sum_{X=0}^D p(X) \cdot \delta_{X,t}$ is obtainable.

For the state-space degradation model presented in Section 3.2.1, the essence of the threshold is similar to the destruction amount. In other words, the threshold value can be considered as the maximum value of the destructive amount, which if its occurrence leads to the failure. Therefore, it can be concluded that the statistical distribution of the threshold is similar to the distribution of the destructive amount. In the case of Gamma distribution, it has been assumed that the scale parameters of the Gamma distributions are the same for threshold and destruction amount.

$$\Phi = d_F - \sum_{X=0}^D p(X) \cdot \delta_{X,t} \quad \text{Equation (33)}$$

while

$$\begin{cases} p(X) \sim \text{Poisson}(\lambda) \\ \delta_{X,t} \sim \text{Gamma}(\alpha, \beta) \\ d_F \sim \text{Gamma}(\alpha_F, \beta) \end{cases} \quad (\text{Poisson Case})$$

As mentioned in Section 1.7, the proposed degradation model in this study is always within a certain interval as $[d_{min}, d_{max}]$. This indicates that the estimates of the degradation estimator are always within $[d_{min}, d_{max}]$. Since this study does not seek to discuss the approaches for obtaining the degradation estimates, it can be assumed that the degradation estimates are normalized within $[d_{min}, d_{max}]$, as a percentage value. Consequently, the failure threshold is also must be within the $[d_{min}, d_{max}]$, since the threshold and destruction amount have a similar essence.

$$0 \leq d_F \leq d_{max} \quad \text{Equation (34)}$$

$$0 \leq \sum_{X=0}^D p(X) \cdot \delta_{X,t} \leq d_{max} \quad \text{Equation (35)}$$

Equation (36) defines the boundaries of the Φ by considering that the failure threshold cannot be lower than degradation estimates.

$$0 \leq d_F - \sum_{X=0}^D p(X) \cdot \delta_{X,t} \leq d_{max} \quad \text{Equation (36)}$$

In statistics and probability theory, a truncated distribution is a conditional distribution that results from limiting the domain of some other probability distribution. In practice, truncated distributions arise in cases in which the information regarding the occurrences is limited to values that are above or below a given threshold or within a specified range. Suppose that X is a random variable distributed according to some PDF, $f(x)$, with CDF

$F(x)$ both of which have infinite support. Equation (37) presents the PDF of the truncated distribution, $Tr(x)$, after restricting the domain to be between two constants.

$$f(x|a \leq X \leq b) = \frac{f(x)}{F(b) - F(a)} \quad \text{Equation (37)}$$

Table 3 presents the detail of an illustrative example for the reason for considering the truncation functions. It has been assumed that the maximum number of destructive events that might occur in a cycle is 10. Figure 6 also presents the probability density of the Gamma distributions for the destruction amount based on the number of damages. It should be noted that the probability of observing more damages exponentially decreases.

Table 3: Detail of an illustrative example for the reason of truncation.

<i>Number of Destructive Events (i)</i>	(α, β)	<i>Distribution of the Destruction Amount</i>
0	N/A	N/A
1	(5, 2)	<i>Gamma</i> (α, β)
2	(2*5, 2)	<i>Gamma</i> ($2\alpha, \beta$)
....
10	(10*5, 2)	<i>Gamma</i> ($10\alpha, \beta$)

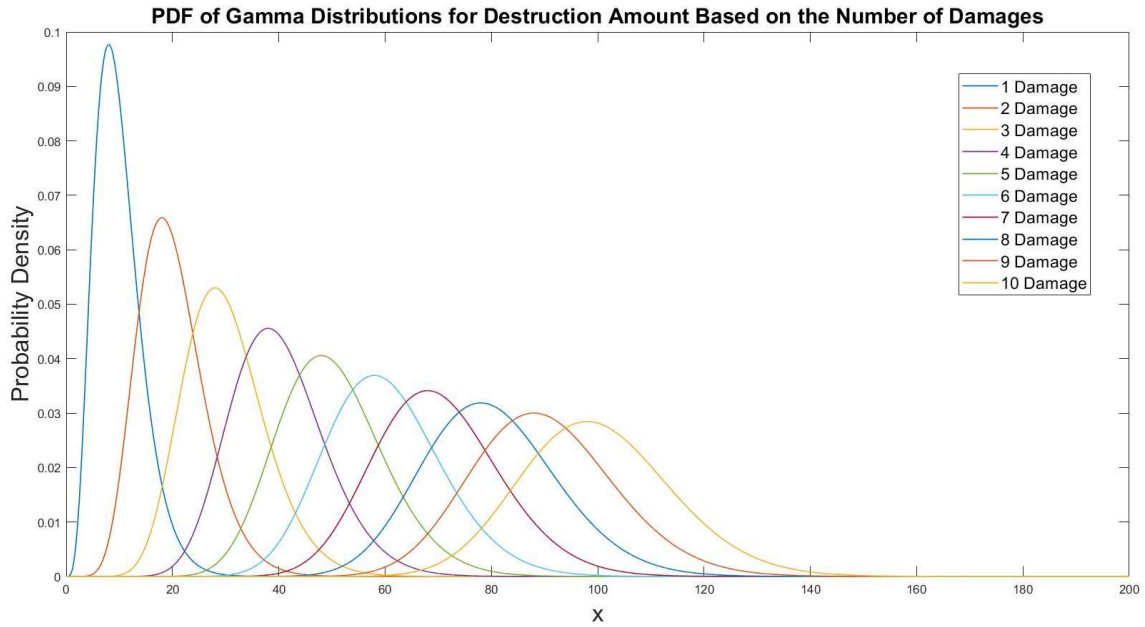


Figure 39: Probability density of the Gamma distributions for the destruction amount based on the number of damages.

Assume that the degradation estimator is normalized to always provide estimates between $[0, 100]$. Therefore, the domain of the threshold and total destruction amount occurred in each cycle should also have the same range as the degradation estimates. Equation (38) and (39) state the restriction of the domain of threshold and total destructive amount.

$$\text{Assume } 0 \leq d_t \leq 100 \quad \text{Equation (38)}$$

$$\text{Therefore, } \begin{cases} 0 \leq d_F \leq 100 \\ 0 \leq \sum_{X=0}^{10} p(X) \cdot \delta_{X,t} \leq 100 \end{cases} \quad \text{Equation (39)}$$

Figure (40) presents the probability density of the truncated Gamma distributions for the destruction amount based on the number of damages of the illustrative example. It

should be noted that the properties of the truncated distributions might not be as same as the original distribution. For instance, the distribution of the total destruction amounts due to the 8, 9, and 10 damages might not be the Gamma.

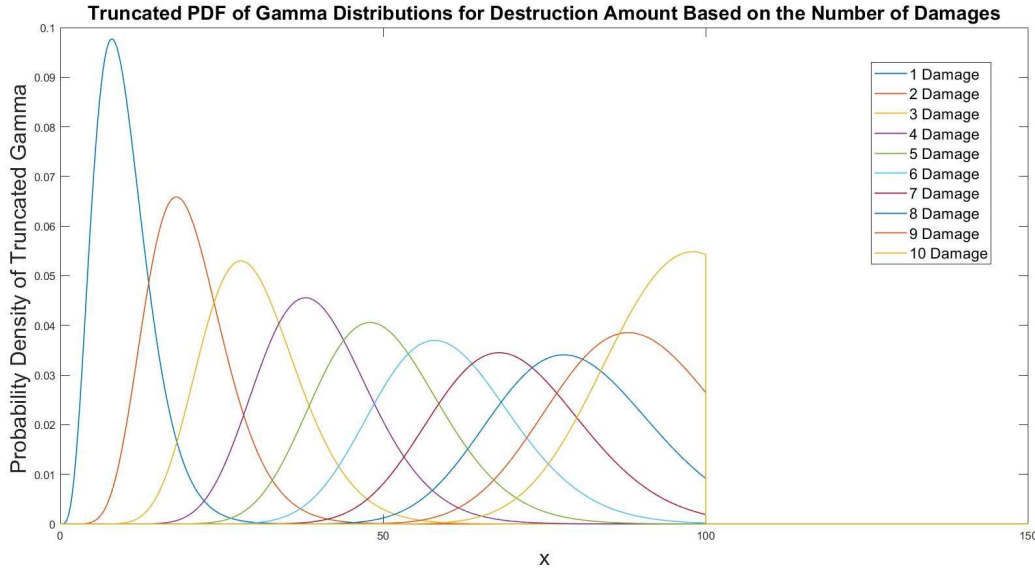


Figure 40: Probability density of the truncated Gamma distributions for the destruction amount based on the number of damages.

Assume that finding the distribution of the $Y = X_1 + X_2$ is the point of interest where random variables are following $X_1 \sim \text{Gamma}(\alpha_1, \beta)$ and $X_2 \sim \text{Gamma}(\alpha_2, \beta)$. Based on the principles of the Moment Generating Functions, it can be proved that $Y \sim \text{Gamma}(\alpha_1 + \alpha_2, \beta)$ by assuming the independent variables. The following equations present the detail.

$$E(e^{(X_1+X_2)t}) = E(e^{X_1t}e^{X_2t}) = E(e^{X_1t}).E(e^{X_2t}) \quad X_1 \text{ and } X_2 \text{ are independent} \quad \text{Equation (40)}$$

$$M(t; \alpha, \beta) = E(e^{Xt}) = \int_0^{+\infty} e^{xt} f(x; \alpha, \beta) dx \quad \text{Equation (41)}$$

$$\int_0^{+\infty} e^{xt} \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} dx = \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^{+\infty} x^{\alpha-1} e^{-(\beta-t)x} dx \quad \text{Equation (42)}$$

$$\frac{\beta^\alpha}{\Gamma(\alpha)} \cdot \frac{\Gamma(\alpha)}{(\beta - t)^\alpha} = \frac{1}{\left(1 - \frac{t}{\beta}\right)^\alpha} \quad \text{Equation (43)}$$

By using the property of independent random variables:

$$M_{(X_1+X_2)t} = M_{X_1t} \cdot M_{X_2t} \quad \text{Equation (44)}$$

$$M_{(X_1+X_2)t} = \frac{1}{\left(1 - \frac{t}{\beta}\right)^{\alpha_1}} \cdot \frac{1}{\left(1 - \frac{t}{\beta}\right)^{\alpha_2}} = \frac{1}{\left(1 - \frac{t}{\beta}\right)^{\alpha_1 + \alpha_2}} \quad \text{Equation (45)}$$

Therefore, it can be concluded that $Y \sim \text{Gamma}(\alpha_1 + \alpha_2, \beta)$ by assuming the independent variables.

The problem at this phase is regarding the assumptions of the individual random and independent Gamma variables after truncations. First of all, it cannot be easily assumed that the distribution of the damage amount is still a Gamma distribution after truncation. Secondly, there is no guarantee that the scale parameters remain the same in order to be able to apply the properties of the Gamma distribution for several random independent variables. Consequently, convolutional models need to be applied in order to find the distribution of the total destruction amount.

The convolution of probability distributions arises in probability theory and statistics as the operation in terms of probability distributions that correspond to the addition of independent random variables and, by extension, to forming linear combinations of random variables. The general formula for the distribution of the sum $Z = X + Y$ of two independent continuously distributed random variables with density functions f, g is as the following shows.

$$h(z) = (f * g)(z) = \int_{-\infty}^{+\infty} f(z-t)g(t)dt \quad \text{Equation (46)}$$

Therefore, the PDF of the $\Phi = d_F(\theta_F) - \sum_{i=0}^D p(i) \cdot \delta_{i,t}(\theta)$ can be obtained by applying the convolutional model on the truncated distributions as the Equation (51) presents.

$$\Phi' = Tr(d_F) - Tr\left(\sum_{X=0}^D p(X) \cdot \delta_{X,t}\right) \quad \text{Equation (47)}$$

$$\text{let } X = Tr(d_F) \quad \text{and} \quad Y = Tr\left(\sum_{X=0}^D p(X) \cdot \delta_{X,t}\right)$$

$$\text{let } X \sim f_X(x) \quad \text{and} \quad Y \sim f_Y(y) \quad , \quad Z = X + (-Y)$$

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x)f_Y(z-x)dx \quad \text{Equation (48)}$$

$$\text{since } f_{-Y}(z-x) = f_Y(x-z) \quad \text{Equation (49)}$$

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(x)f_Y(x-z)dx \quad \text{Equation (50)}$$

Therefore,

$$f(\Phi) = Conv\left(Tr(d_F \sim \text{Gamma}(\alpha_F, \beta)), -Tr\left(\sum_{x=0}^D \frac{e^{-\lambda}\lambda^x}{x!} \cdot \text{Gamma}(x\alpha, \beta)\right)\right) \quad \text{Equation (51)}$$

Therefore, PF_t , unreliability, and R_t , reliability can be estimated given the degradation estimate, in degradation domain, at last cycle as the Equation (52) presents.

$$PF_t = CDF_{\Phi}(d_{t-1}) = \int_{-\infty}^{d_{t-1}} f(\Phi)d(d) \quad \text{Equation (52)}$$

$$R_t = 1 - PF_t$$

Equation (53)

6.2 Results

The results of Monte-Carlo simulation, presented in Chapter 5, are obtained based on the statistical distribution fitting principles over the cycles. This indicates that the estimates of reliability and probability of failure are based on the behavior of the obtained distribution over the cycles. One of the main concerns is the robustness of the approach. It may be possible that the same asset under study behaves completely unique after a point of time. In this scenario, the results of the analysis based on the statistical distribution may not be able to reveal the actual health status of the asset. Therefore, an exact analytical formulation of a data-driven predictive model of Asset Management (AM) is presented in this chapter. It is expected to reach a more robust approach since the estimates are not completely based on what occurred in the past.

In the first steps of the analysis, I have tried to enhance the performance of the simulation-based analysis by assigning higher weights to the most recent failure data points. The main problem, which is difficulties in obtaining the real-time estimates of several degradation profiles, still exists. Therefore, the reliance on the real-time estimates of several degradation profiles is the main motivation to develop an exact formulation of a data-driven predictive model of reliability estimation.

It should be noted that in Chapter 5, the estimates of the reliability and probability of failure are based on the cycles. In Chapter 6, all the estimates are based on the current estimate of the degradation regardless of the point of the time which degradation

estimator reaches to that level. It is expected to obtain more robust and realistic results since the degradation profiles are mainly based on the real-time data collected from smart devices and sensors. Figure 41 presents an illustrative example of the time independency assumption of the analytical approach.

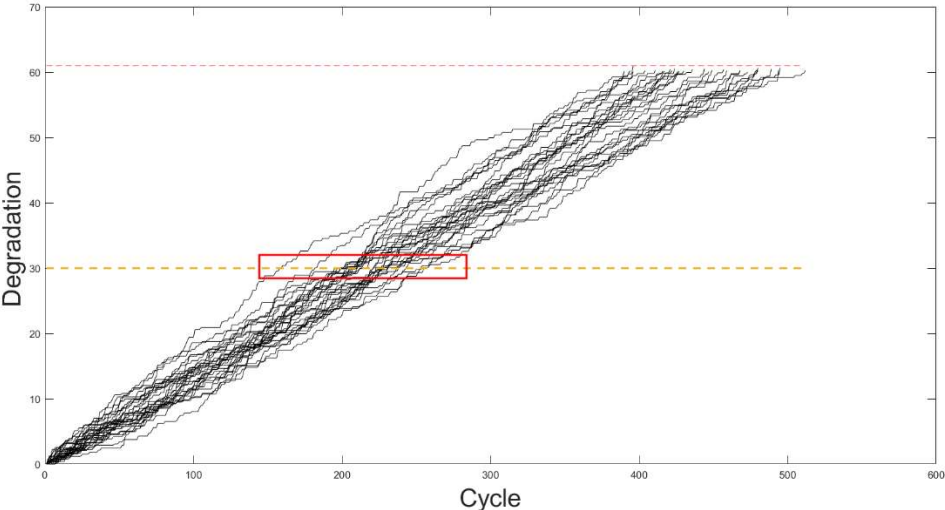


Figure 41: An illustrative example of the time independency assumption of the analytical approach.

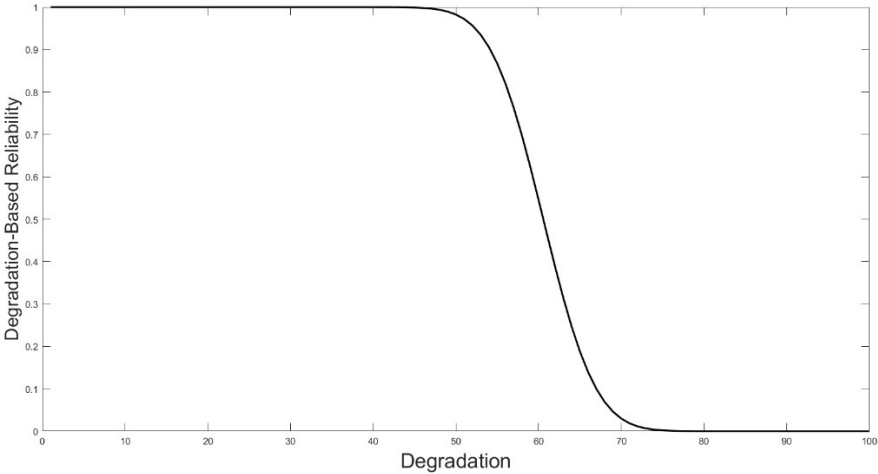


Figure 42: Degradation-base reliability estimation for transformer under study.

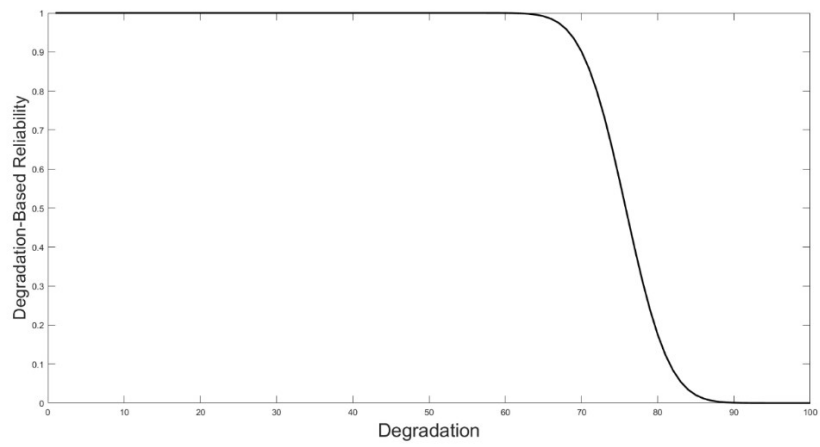


Figure 43: Degradation-base reliability estimation for NG Generator under study

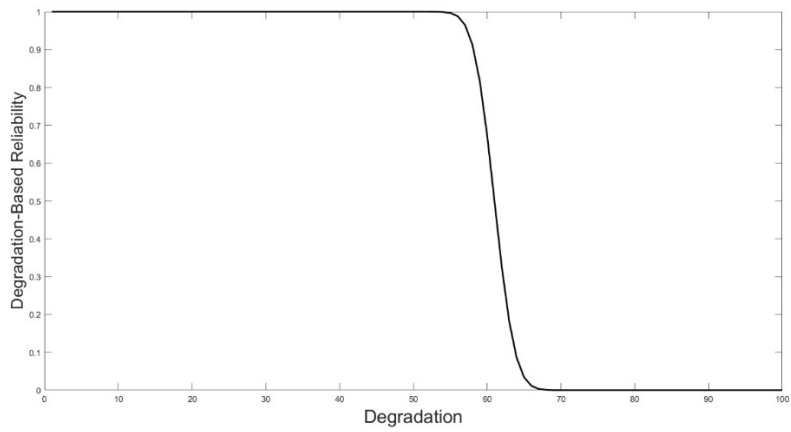


Figure 44: Degradation-base reliability estimation for inverter under study

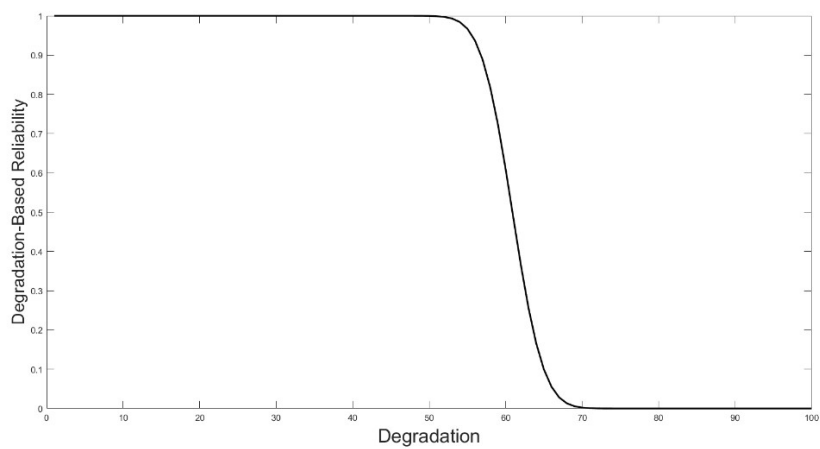


Figure 45: Degradation-base reliability estimation for battery under study

6.3 Concluding Remarks

In this chapter, reliability is estimated based on an analytical formulation of the reliability with respect to the new degradation model developed in this study. The new degradation model in this study presented a new mapping function for the General Path Model based on a series of Gamma Processes degradation models in the state-space environment. Poisson distributed weights were considered for each of the Gamma processes. The main purpose of this chapter was to obtain the degradation-base reliability estimate based on an analytical formulation. The developed reliability estimation framework can be implemented to estimate the reliability based on the real-time degradation data, which is able to perform automatic, quick-responding, accurate, and robust. Predictive models based on the analytical formulation are not relying on fitting the statistical distributions into the observed or generated data, which may not be able to accurately describe the behavior of the critical time to failure. The formulated model of degradation-based reliability is computationally faster than the simulation-based analysis. The accuracy of the reliability assessment highly ties to the accuracy of the approaches for obtaining the time-series degradation profile. This indicates that reliability may be inaccurately estimated if the time-series degradation data cannot accurately describe the real status of the asset under study. The detail of the algorithm is presented in the appendix of this study based on Matlab.

7 Chapter 7: Data-Driven Estimate of the Parameters

7.1 Methodologies

So far, it has been assumed that the engineering insights are sufficient to define the properties of the degradation model, known as model parameters. These parameters are expected to have a unique value for each piece of equipment. In real-world applications, these parameters might not be the same for completely similar assets. On the other hand, these values are usually determined by experimental tests in laboratories or standards, which typically provide the same values for similar assets. For this reason, the output of the proposed methodology may not be able to precisely reflect the actual status of the equipment. In addition to that, the proposed models may not be efficiently applied to the other applications if the values of the parameters are not known prior. Therefore, the effectiveness of the overall framework of the proposed model can be enhanced by integrating a robust model to estimate the degradation model parameters.

This section presents a new data-driven algorithm in order to estimate the parameters of the new degradation model developed in this thesis. The proposed algorithm is mainly based on clustering principles as unsupervised learning. Once the parameters of the degradation are estimated based on the time series degradation data, it can be concluded that the overall algorithms proposed in this thesis are data-driven. Therefore, the main goal of this thesis, which is providing a data-driven predictive model reliability estimation, is reached based on the data-driven state-space stochastic models.

7.2 Motivations for Developing a Data-Driven Parameter Estimator

Maximum Likelihood Estimation (MLE) is a method of estimating the parameters of distribution by maximizing its likelihood function. If the likelihood function is differentiable, the derivative assessment for determining maxima can be applied. For some cases, such as the linear regression model, the first-order conditions of the likelihood function can be solved explicitly. In the case of the linear regression model, the ordinary least squares estimator maximizes the likelihood of the linear regression model. Under most circumstances, numerical methods will be necessary to find the maximum of the likelihood function. The detail of the MLE method is presented in Chapter 2.

In order to find the optimal value of the parameters of the proposed degradation model based on the MLE approach, it is necessary to derive the likelihood function and maximize its value by defining the derivative of the logarithmic equations with respect to the parameters. Following Equations state the MLE equations, while $x = \{x_1, x_2, \dots, x_n\}$ are random observations drawn from an unknown population, θ is the vector of unknown parameters, $L(\theta ; x)$ is the likelihood function, and k is the number of unknown parameters.

$$L(\theta ; x) = \prod_{i=1}^n f(x_i, \theta) \quad \text{Equation (54)}$$

$$\iota(\theta ; x) = \ln L(\theta ; x) \quad \text{Equation (55)}$$

$$\frac{\partial l(\theta ; y)}{\partial \theta_1} = 0 \quad ; \quad \frac{\partial l(\theta ; y)}{\partial \theta_2} = 0 \quad ; \dots ; \quad \frac{\partial l(\theta ; y)}{\partial \theta_k} = 0 \quad \text{Equation (56)}$$

For instance, for the case of Poisson rate of destructive events;

$$L(\lambda, \alpha, \beta ; x) = \prod_{n=1}^N f(\Phi) \quad \text{Equation (57)}$$

$$L(\lambda, \alpha, \beta ; x) = \prod_{i=1}^N \text{Conv} \left(\text{Tr}(d_F \sim \text{Gamma}(\alpha_F, \beta)), -\text{Tr} \left(\sum_{x=0}^D \frac{e^{-\lambda} \lambda^{x_i}}{x_i!} \cdot \text{Gamma}(x_i, \alpha, \beta) \right) \right) \quad \text{Equation (58)}$$

There is a logarithm that is affecting the second summation, which in this case is the main issue for calculating the derivative of this expression and then solving for the parameters. Therefore, the MLE approach is going to be computationally very tough. An iterative method should be applied to estimate the unknown parameters of the degradation model.

As mentioned in Chapter 2, the Expectation-Maximization (EM) algorithm is used to find local maximum likelihood parameters of a statistical model in cases where the equations cannot be solved directly. In statistics, an EM algorithm is an iterative method to find maximum likelihood or maximum a posteriori estimate of parameters in statistical models, where the model depends on unobserved latent variables. The EM iteration alternates between performing an expectation (E) step, which creates a function for the expectation of the log-likelihood evaluated using the current estimate for the parameters, and a maximization (M) step, which computes parameters maximizing the expected log-likelihood found on the E step. These parameter-estimates are then used to determine the distribution of the latent variables in the next E step.

Equation (59) and (60) can be achieved based on the Bayes Rule, where $p(C_c|x_n)$ is the probability that x_n generated by cluster C_c and π_c is the share or probability of the cluster c .

$$p(C_c|x_n) = \frac{p(x_n | C_c) \cdot p(C_c)}{\sum_{i=1}^C p(x_n | C_i) \cdot p(C_i)} \quad \text{Equation (59)}$$

$$p(C_c|x_n) = \frac{\pi_c p(x_n | C_c)}{\sum_{i=1}^C \pi_j p(x_n | C_i)} \quad \text{Equation (60)}$$

Steps of the EM algorithm can be summarized as follow:

- **Initialization Step:**

- Randomly assign samples without replacement from the dataset $X = \{x_1, \dots, x_N\}$.

- Randomly assign mean to the component sample mean. For example, for $k = 2$, $\hat{\mu}_1 = x_{10}$ and $\hat{\mu}_2 = x_{25}$.

- Set all component variance estimates to the sample variance.

$$\hat{\sigma}_1^2 = \dots = \hat{\sigma}_K^2 = \frac{1}{N} \sum_{n=1}^N (x_n - \bar{x})^2 \quad \text{where} \quad \bar{x} = \frac{1}{N} \sum_{n=1}^N x_n$$

- Set all component distribution prior estimates to the uniform distribution.

$$\hat{\pi}_1 = \dots = \hat{\pi}_C = \frac{1}{C} \quad \text{Equation (61)}$$

- **Expectation Step:**

$$\hat{\gamma}_{n,c} = \frac{\pi_c p(x_n | \hat{\mu}_c, \hat{\sigma}_c)}{\sum_{j=1}^C \pi_j p(x_n | \hat{\mu}_j, \hat{\sigma}_j)} \quad \forall n, k \quad \text{Equation (62)}$$

- **Maximization Step:** Using the $\hat{\gamma}_{n,k}$ calculated in the Expectation step, calculate the following in that order $\forall k$.

$$\circ \hat{\pi}_c = \sum_{n=1}^N \frac{\hat{\gamma}_{n,c}}{N} \quad \text{Equation (63)}$$

$$\circ \hat{\mu}_c = \frac{\sum_{n=1}^N \hat{\gamma}_{n,c} \cdot x_n}{\sum_{n=1}^N \hat{\gamma}_{n,c}} \quad \text{Equation (64)}$$

$$\circ \hat{\sigma}_c^2 = \frac{\sum_{n=1}^N \hat{\gamma}_{n,c} \cdot (x_n - \hat{\mu}_c)^2}{\sum_{n=1}^N \hat{\gamma}_{n,c}} \quad \text{Equation (65)}$$

When the number of clusters C is not known a priori, it is typical to guess the number of components and fit that model to the data using the EM algorithm. There are various criteria for selecting the optimal number of clusters. More detail of the optimal number of clusters will be discussed in section 2.4.2. Finally, the probability of observing destructive events can be estimated based on the filtered degradation data.

For the case of Poisson damage events, the quality of the clustering task can highly affect the accuracy of the parameter estimation process. In this case, the analyst may define the degradation changes over time. It should be noted that it might be possible that more than one destructive event had happened during a specific week. Therefore, a robust clustering method is needed in order to accurately detect the cycles, which only one destructive event has occurred. The rate of the destructive event is expected to be estimated based on the information provided by the cluster which represents cycles with no destructive events.

7.3 Algorithm Design

Clustering is a Machine Learning (ML) technique that includes the grouping of data points. Given a set of data, a clustering algorithm can be applied to classify each data

point into a specific group. Ideally, data points that are in the same group should have similar properties, while data points in different groups should have highly dissimilar properties. Clustering is one of the most commonly used methods of unsupervised learning for statistical data analysis. Clustering methods can be applied to gain some valuable insights from the data by investigating what groups the data points fall into when we apply a clustering algorithm. Therefore, the clustering task refers to grouping a set of data in such a way that data points in the same group, known as a cluster, are more similar, in some sense, to each other than to those in other groups.

More detail of the clustering algorithms is presented in Chapter 2. In this study, K-means and Gaussian Mixture Models (GMM) clustering algorithms are applied in order to detect the cluster which represents the degradation data for the cycle which only one destructive event had occurred. The performance of each of these clustering methods is also compared in order to obtain a robust clustering approach.

In this thesis, the performance of the following clustering approaches is evaluated.

- K-Mean Clustering
- Neural Network
- Gaussian Mixture Models

It should be considered that the number of clusters may highly affect the quality of the clustering task. There is not a single approach that performs well for all the datasets in order to define the optimal number of clusters. In addition to that, the type of the application and the motivations for performing a clustering task may highly affect the

optimal number of the clusters. In this study, it has been assumed that the optimal number of clusters is determined based on the “Davies Bouldin” criteria.

Since there is not a single clustering approach that may perform best on all the datasets, a more robust clustering algorithm is going to be considered. Indeed, an ensemble of all the considered clustering approaches is going to be studied rather than a single clustering approach. Given the same set of time-series degradation estimates, it is expected to obtain consistent results if the clustering task is performed several times. Therefore, the integration of an outlier detector may enhance the performance of the ensembles of clustering approaches. In this study, the Generalized Extreme Studentized Deviate (GESD) outlier detection method is applied to test for outliers.

7.4 Results

7.4.1 Results of each Clustering Method

7.4.1.1 Transformer

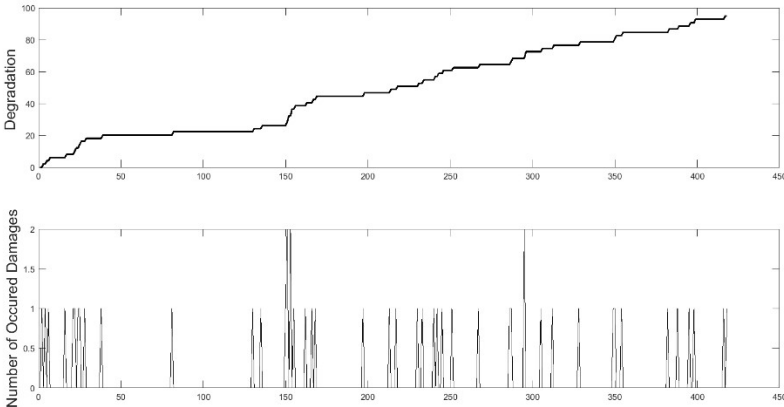


Figure 46: Detail of the degradation profile for the transformer under study.

Table 4: Estimated degradation model parameters for transformer based on the Gaussian Mixture Models (GMM) clustering algorithm with and without outlier detection method.

	Actual Values	Mean Estimated Values WITH outlier detection (n = 200)	Mean Percentage Error WITH outlier detection (n = 200)
$\hat{\lambda}$	0.1	0.1013	1.25 %
$\hat{\alpha}$	40	32.13	- 19.67 %
$\hat{\beta}$	20	15.26	- 23.7 %

Table 5: Estimated degradation model parameters for transformer based on the k-means clustering algorithm with and without outlier detection method.

	Actual Values	Mean Estimated Values WITH outlier detection (n = 200)	Mean Percentage Error WITH outlier detection (n = 200)
$\hat{\lambda}$	0.1	0.0982	- 1.82 %
$\hat{\alpha}$	40	35.21	- 11.97 %
$\hat{\beta}$	20	16.85	- 15.75 %

7.4.1.2 NG Generator

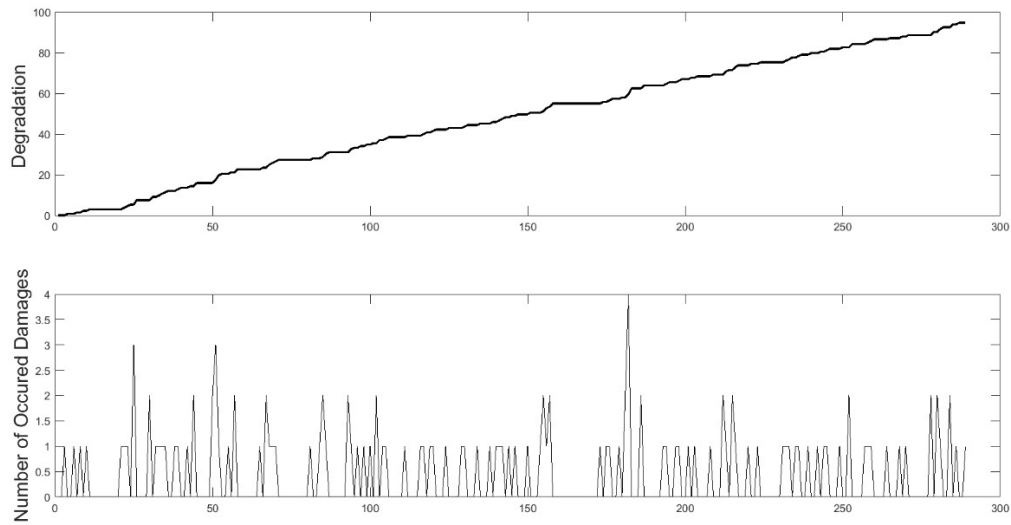


Figure 47: Detail of the degradation profile for the NG generator under study.

Table 6: Estimated degradation model parameters for NG generator based on the Gaussian Mixture Models (GMM) clustering algorithm with and without outlier detection method.

	Actual Values	Mean Estimated Values WITH outlier detection (n = 200)	Mean Percentage Error WITH outlier detection (n = 200)
$\hat{\lambda}$	0.4	0.4014	0.35 %
$\hat{\alpha}$	15	13.15	12.33 %
$\hat{\beta}$	20	17.15	14.25 %

Table 7: Estimated degradation model parameters for NG generator based on the K-Means clustering algorithm with and without outlier detection method.

	Actual Values	Mean Estimated Values WITH outlier detection (n = 200)	Mean Percentage Error WITH outlier detection (n = 200)
$\hat{\lambda}$	0.4	0.3750	-6.25 %
$\hat{\alpha}$	15	15.51	3.44 %
$\hat{\beta}$	20	21.06	5.33 %

7.4.1.3 Inverter

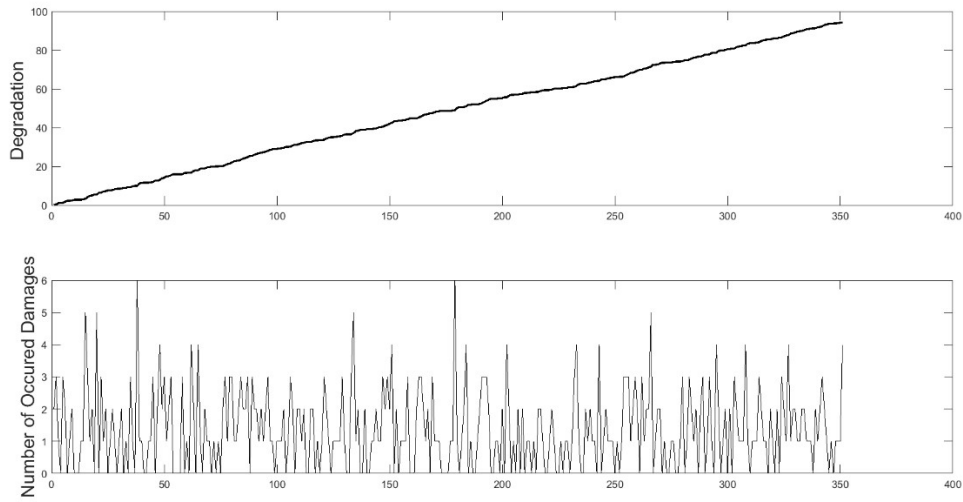


Figure 48: Detail of the degradation profile for the inverter under study.

Table 8: Estimated degradation model parameters for inverter based on the Gaussian Mixture Models (GMM) clustering algorithm with and without outlier detection method.

	Actual Values	Mean Estimated Values WITH outlier detection (n = 200)	Mean Percentage Error WITH outlier detection (n = 200)
$\hat{\lambda}$	1.4	1.39	- 0.53 %
$\hat{\alpha}$	10	10.88	8.87 %
$\hat{\beta}$	50	52.76	5.52 %

Table 9: Estimated degradation model parameters for inverter based on the K-Means clustering algorithm with and without outlier detection method.

	Actual Values	Mean Estimated Values WITH outlier detection (n = 200)	Mean Percentage Error WITH outlier detection (n = 200)
$\hat{\lambda}$	1.4	1.35	- 3.57 %
$\hat{\alpha}$	10	12.12	21.26 %
$\hat{\beta}$	50	62.26	24.52 %

7.4.1.4 Battery

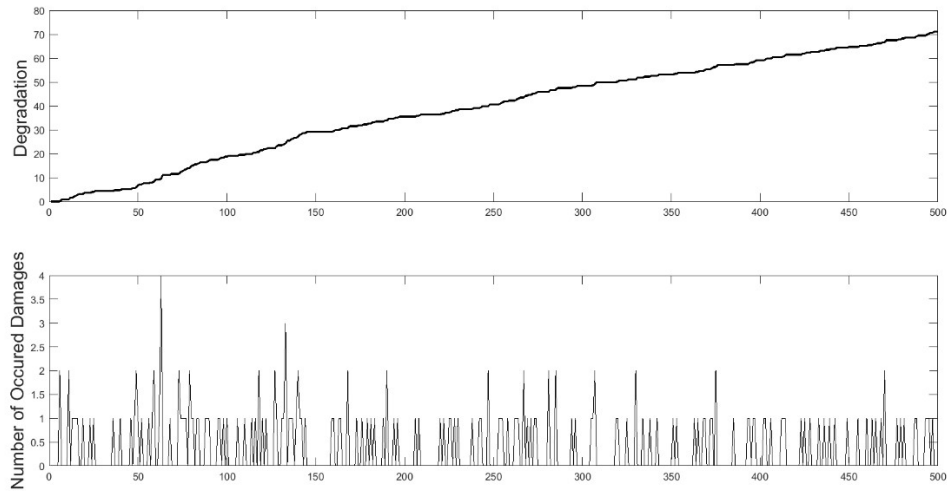


Figure 49: Detail of the degradation profile for the battery under study.

Table 10: Estimated degradation model parameters for battery based on the Gaussian Mixture Models (GMM) clustering algorithm with and without outlier detection method.

	Actual Values	Mean Estimated Values WITH outlier detection (n = 200)	Mean Percentage Error WITH outlier detection (n = 200)
$\hat{\lambda}$	0.35	0.3520	0.56 %
$\hat{\alpha}$	20	22.20	11.04 %
$\hat{\beta}$	50	54.62	9.24 %

Table 11: Estimated degradation model parameters for battery based on the K-Means clustering algorithm with and without outlier detection method.

	Actual Values	Mean Estimated Values WITH outlier detection (n = 200)	Mean Percentage Error WITH outlier detection (n = 200)
$\hat{\lambda}$	0.35	0.3297	- 5.78 %
$\hat{\alpha}$	20	22.85	14.27 %
$\hat{\beta}$	50	57.02	14.04 %

7.5 Concluding Remarks

The parameters of the new degradation model developed in this study are expected to have a unique value for each piece of equipment. In real-world applications, these parameters might not be the same for completely similar assets. On the other hand, these values are usually determined by experimental tests in laboratories or standards, which typically provide the same values for similar assets. For this reason, the output of the proposed methodology may not be able to precisely reflect the actual status of the equipment. In addition to that, the proposed models may not be efficiently applied to the other applications if the values of the parameters are not known prior. Therefore, the effectiveness of the overall framework of the proposed model can be enhanced by integrating a robust data-driven algorithm to estimate the parameters of the new degradation model. This section presented a new data-driven algorithm in order to estimate the parameters of the new degradation model developed in this thesis. The proposed algorithm is mainly based on clustering principles as unsupervised learning. Once the parameters of the degradation are estimated based on the time series degradation data, it can be concluded that the overall algorithms proposed in this thesis are data-driven. Therefore, the main goal of this thesis, which is providing a data-driven predictive model reliability estimation, is reached based on the data-driven state-space stochastic models of degradation.

8 Chapter 8: Future Works and Conclusion

8.1 Future Works

8.1.1 Developing a Time-Varying Poisson parameter as a Function of In-Service Time

The results of the developed data-driven predictive model of AM are presented in section 6.2. The obtained results are merely based on the degradation estimate, which is supposed to accurately describe the real-time health status of the asset under study. One of the main advantages of the developed model is its independence of the several historical degradation profiles. In addition to that, the analytical approach is computationally faster than the simulation-based analysis. Furthermore, the results of the analysis do not depend on any statistical distribution, which is fitted based on several historical or generated degradation profiles. The robustness of the analytical approach highly depends on the robustness of the degradation estimator. This indicates that upon the accurate estimate of the degradation, the developed analytical approach should be robust enough to provide an estimate of reliability and probability of failure. In the first step of this thesis, it has been assumed that sufficient engineering insight is available to provide the values of the degradation model parameters. Indeed, this assumption is not always true for real-world applications. Therefore, it can be concluded that the developed methodology may perform more effectiveness by integrating an algorithm, which is able to provide an estimate of the applied degradation model parameter. Chapter 7 presents the detail of the developed parameter estimator in order to enhance the robustness of the

overall approaches for various real-world applications. It should be noted that the parameter estimator is applicable for both simulation-based and analytical analysis.

As mentioned in Chapter 1 of this thesis, the rates of destructive events are assumed to be constant over the monitoring period. This indicates that the probability of failure and reliability estimates do not consider the “*in-service time*” of the asset under study. It means that given the same degradation estimate, the probabilities of failure are the same for an asset which is somehow new or old. Therefore, in-service time is another factor that may enhance the robustness of the developed model. In order to consider the effect of in-service time or age of the asset, a time-dependent rate of the destructive event may be applied. For instance, it can be assumed that in-service time of the asset can be mapped into the rate of destructive events as the Equation () presents. Therefore, Figure () shows the effect of considering the in-service time of the asset into the reliability and probability of failure estimations.

$$\lambda_t = \lambda_0 + \left(\frac{\text{in-service time}}{\text{maximum lifetime}} \right) \cdot \lambda_0 \quad \text{Equation (66)}$$

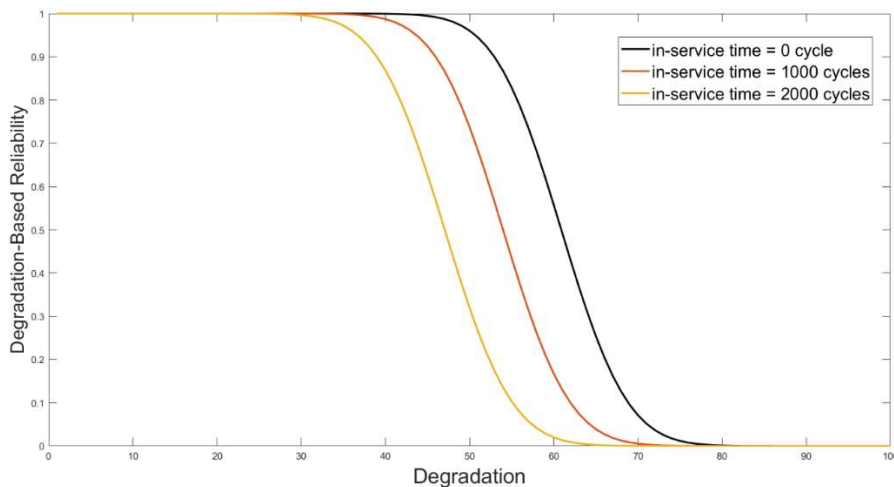


Figure 50: Effect of considering the in-service time of the asset into the reliability and probability of failure estimations

$$\lambda = 0.1 + 0.001 * age$$

Equation (67)

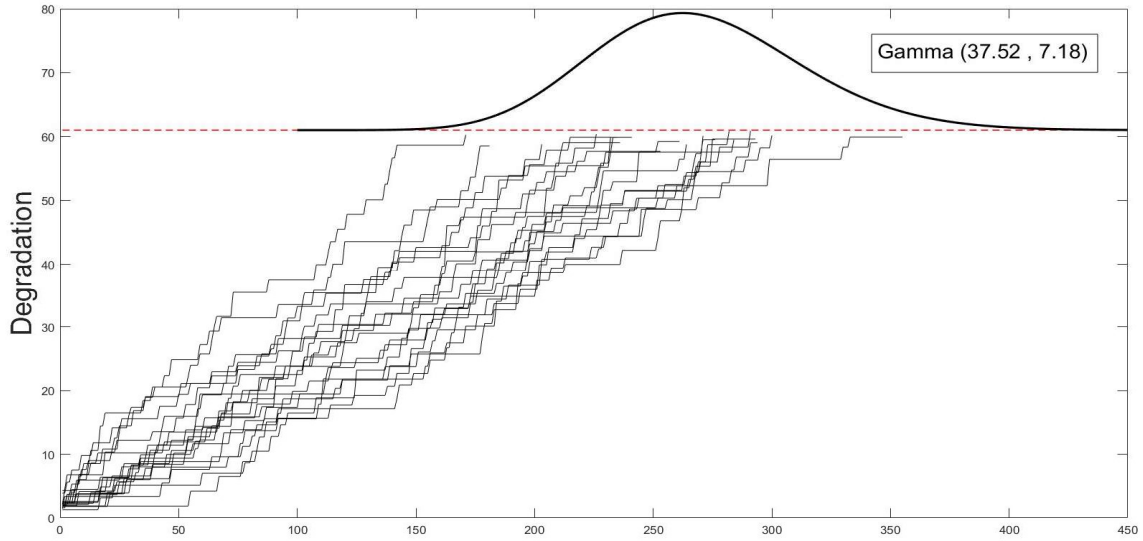


Figure 51: Obtained distribution of time-to-soft-failure

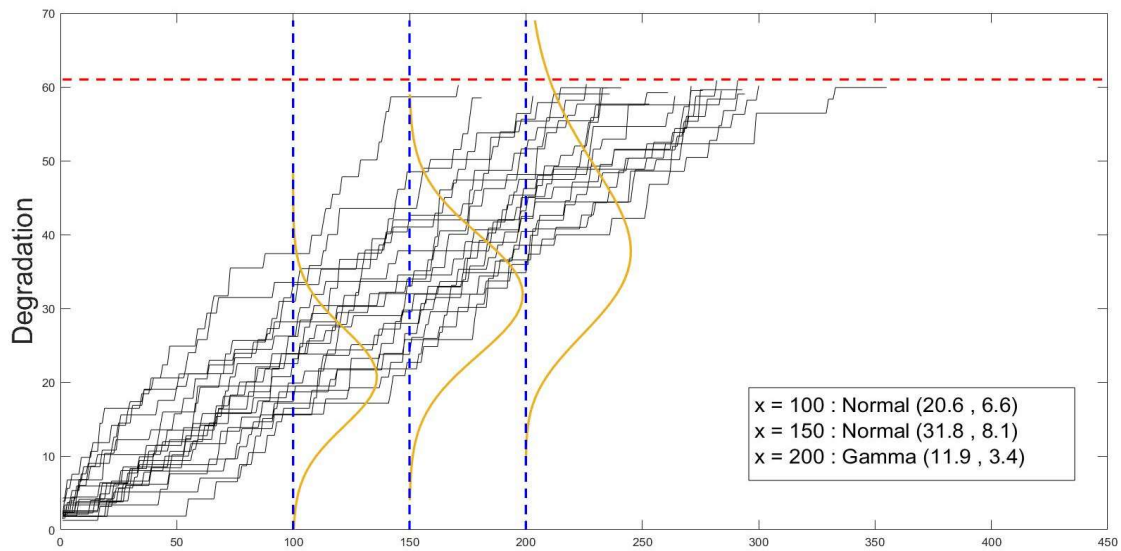


Figure 52: Obtained time-dependent distribution of the degradation estimates.

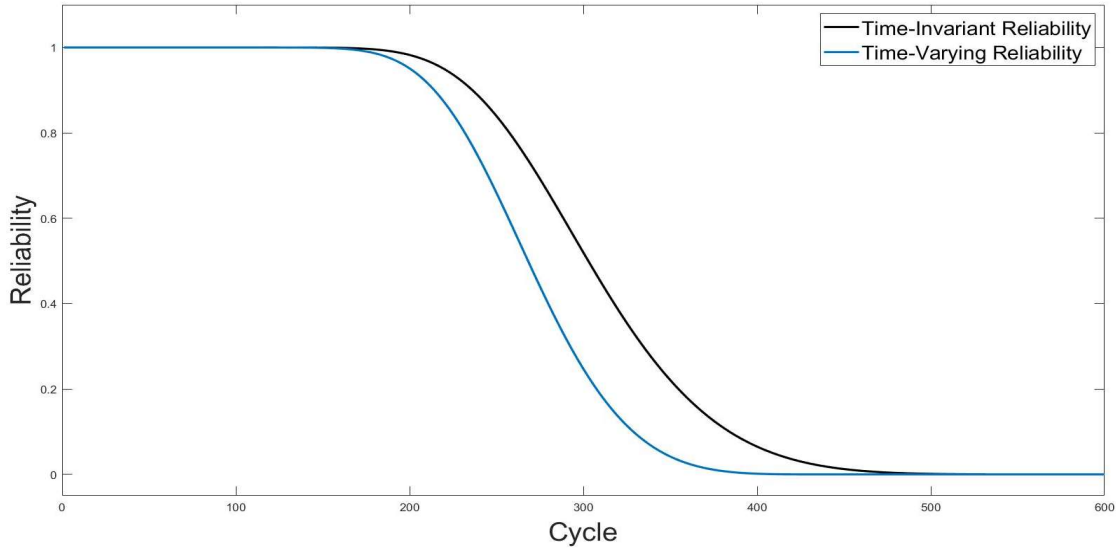


Figure 53: Comparison between the reliability function with time-varying and time-invariant parameter.

8.1.2 Developing a Data-Driven Parameter Estimator in order to estimate the time-varying Poisson parameter.

$$\lambda = 0.1 + 0.001 * age \quad \text{Equation (68)}$$

$$\hat{\lambda} = 0.098 + 0.0013 * age \quad \text{Equation (69)}$$

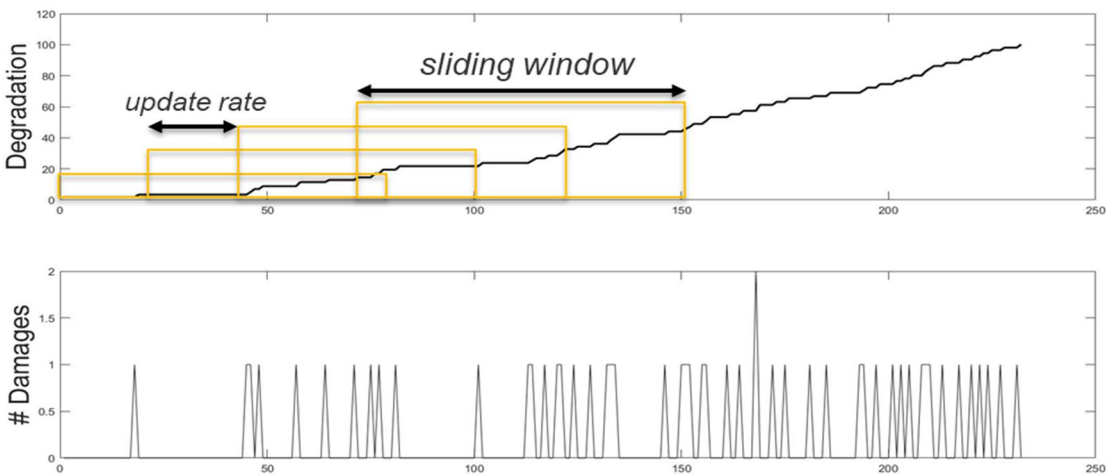


Figure 54: Schematic view of data-driven parameter estimator in order to estimate the time-varying Poisson parameter.

8.1.3 Developing a Time-Varying parameter for the distribution of the destruction amount.

In this case, the new degradation model developed in this thesis is able to reflect various types of degradation mechanisms such as linear, non-linear, piecewise, etc. The parameters of the damage amount, as a random variable, are expected to present higher expected value for the damage amount as the time moves forward and asset gets old.

8.1.4 Considering Higher Orders for the State-Space Stochastic Degradation Model

Once, the historical value of the degradation estimates are collected, higher orders for the AR model may perform more effectively.

8.2 Conclusion

The concept of Industrial Internet of Things (IIoT) such as new types of assets, data, sensor networks, data analytics, and processing power can provide the foundation to apply data-driven methodologies. The data-driven predictive models of reliability assessment can become a major tool in increasing the life of assets, lowering capital cost, and reducing operating and maintenance costs. Indeed, the predictive Model of reliability assessment becoming a critical factor in the efficiency of capital-intensive corporations. An accurate predictive model of reliability assessment is necessary in order to optimize various types of decision such as maintenance policy, lifetime analysis, risk management, etc.

Classical models of reliability assessment mainly rely on historical failure data. It should be considered that obtaining lifetime data in a timely manner is one of the current challenges. Failure data may not be easily obtainable for highly reliable assets. Furthermore, the collected historical lifetime data may not be able to accurately describe the behavior of the asset in a unique application or environment. For instance, if the lifetime data are collected based on the experimental tests given specific environmental and operational conditions, there is no guarantee that the asset behavior remains unchanged in other conditions during its lifetime. Therefore, it is not an optimal approach anymore to conduct a reliability assessment based on classical models.

Fortunately, most of the industrial assets have performance characteristics whose degradation or decay over the operating time can be related to their reliability estimates. Degradation indicates the process of lowering the rank, status, or grade, which leads to a less effective level of performance. The application of the degradation methods has been recently increasing due to their ability to keep track of the dynamic conditions of the system over time. The main purpose of the degradation-based models is to predict the future condition of the asset and perform the maintenance actions in an optimized time window before the actual failure of the system occurs. Since the degradation-based analysis defines the failure events based on the predefined threshold, the failure is said to have occurred as a soft failure. This indicates that the asset under the study is considered as a failed unit when the degradation profile hits the threshold for the first time.

Inaccurate modeling of the degradation phenomenon leads to inaccurate assessment of reliability, maintenance policy, risk, lifetime prediction, etc. In this thesis, a

wide variety of the currently developed models of degradation were studied in detail. Degradation models based on the Gamma process and General Path Model had been applied in various studies. The main purpose of this study was to develop a data-driven predictive model of reliability assessment based on real-time data using a state-space stochastic degradation model to predict the critical time for initiating maintenance actions in order to enhance the value and prolonging the life of assets. Indeed, the developed degradation model in this study extended the General Path Model based on a series of Gamma Processes degradation models in the State-Space environment. Poisson distributed weights were considered for each of the Gamma processes. Therefore, the main scientific contribution of the developed degradation model was extending the General Path Model based on a series of Gamma Processes degradation models in the State-Space environment by considering Poisson distributed weights for each of the Gamma processes.

The application of the developed algorithm was illustrated for the distributed electrical systems as a generic use case. The analyses were mostly focused on the critical components of the distributed electrical systems as natural gas generators, transformers, inverters, and batteries as energy storage devices. The main motivation for applying the same methodology to various components was to consider different types of degradation profiles in terms of being more toward either discrete or continuous events. It should be noted that the developed model can be applied to any application in which its time-series degradation profile is available.

A data-driven algorithm was developed in order to estimate the parameters of the developed degradation model. The developed parameter estimator in this study was an

alternative methodology to the “two-step parameter estimation approach” applied in the General Path degradation model. Once the estimates of the parameters are available, distribution of the failure time, time-dependent distribution of the degradation, and reliability based on the current estimate of the degradation can be obtained.

To sum up, the main scientific contribution of this study were (1) developed a state-space stochastic degradation model to accurately capture the dynamic behavior of assets., (2) applied simulation techniques to calculate reliability of assets over time and estimate the critical failure time using the developed model, (3) formulated the reliability based on analytical formulation for degradation prediction model, and (4) applied a data-driven parameter estimation model based on the developed degradation model. Furthermore, the application of the developed data-driven parameter estimation model was a novel approach that has been designed for the developed degradation model in this study.

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Appendix

Appendix A: Distribution Fitting – Matlab Code

```
function F= DistributionFit(data,varargin)
% DistributionFit finds best-fitting distribution for the failure data
% F= DistributionFit(data)fits all distributions available in MATLAB's
function
% MLE to data in vector data and returns them ordered by some criterion:
% LogLikelihood or Akaike. Either 20 continuous or 5 discrete
distributions
% are used based on user input or the type of data supplied (see below)
%
% The function returns a structure array with fields:
% name: name of the distribution
% par: vector of parameter estimates (1, 2 or 3 values)
% ci: matrix of confidence interval, one column per parameter
% LL: Log-Likelihood of the data
% aic: Akaike Information Criterion
%
%
% Name Value
% 'dtype' character string that specifies if the data are continuous
% ('cont') or discrete ('disc'). If missing, the function
% decides the data are discrete if all values of data are
% natural numbers.
%
% 'ntrials' Specifies number of trials for the binomial distribution.
% NTRIALS must be either a scalar or a vector of the same
% size as data. If missing the binomial is not fitted.
%
% 'figure' Either 'on' (default), or 'off'. If 'on' a plot of the
% data and the best fitting is produced (scaled to match the
% data). Requires additional function 'plotfitdist'.
%
% 'alpha' A value between 0 and 1 specifying a confidence level
% for CI of 100*(1-alpha) (default is 0.05).
%
% 'criterion' Criterion to use to order the fits. It can be: 'LL' for
% Log-Likelihood (default), or 'AIC' for Akaike.
%
% 'output' If set to 'off' supresses output to the command window.
% Default 'on'
%
% 'pref' Preferred distribution to plot
%
% If data contains negative values, only the Normal distribution can be
fitted.
% Also, if data contains values > 1 the Beta distribution is not fitted.
If data
% contains 0 some distributions are not fitted.
%
% Statistics Toolbox is Required
```

```

warning off
% Defaults & Storage
dtype= {};
ntrials= [];
fig= 'on';
alpha= 0.05;
criterion= 'LL';
output= 'on';
F= struct('name',{},'par',[],'ci',[],'LL',[],'aic',[]);
prefdist= [];

% Arguments
for j= 1:2:length(varargin)
    string= lower(varargin{j});
    switch string(1:min(3,length(string)))
        case 'dty'
            dtype= varargin{j+1};
        case 'ntr'
            ntrials= varargin{j+1};
        case 'fig'
            fig= varargin{j+1};
        case 'alp'
            alpha= varargin{j+1};
        case 'cri'
            criterion= varargin{j+1};
        case 'out'
            output= varargin{j+1};
        case 'pre'
            prefdist= varargin{j+1}
        otherwise
            error('Unknown argument name');
    end
end

% Distributions
% remove any distribution which is not preferred
Cdist= {'normal'; 'exponential'; 'gamma'; 'logistic'; ...
        'tlocationscale';...
        'uniform'; 'ev'; 'rayleigh'; 'gev'; 'beta'; ...
        'nakagami'; 'rician'; 'inversegaussian'; 'birnbaumsaunders'; ...
        'gp'; 'loglogistic'; 'lognormal'; 'weibull'};

mustbepos= 11;
Ddist= {'binomial'; 'nbin'; 'unid'; 'geometric'; 'poisson'};

% Determine data type: Discrete or Continuous (avoid 0)
if isempty(dtype)
    if isempty(find(1- (data+1)./(fix(data)+1), 1))
        dtype= 'disc';
    else
        dtype= 'cont';
    end
end

% Fit Determined Distribution based on the defined criterion

```

```

switch dtype(1:4)
  % Continuous
  case 'cont'
    for j= 1:numel(Cdist)
      % If negative values, only fit normal
      if min(data) < 0
        [phat,pci]= mle(data, 'distribution', 'normal', 'alpha', alpha);
        F(j).name= Cdist{j};
        F(j).par= phat;
        F(j).ci= pci;
        pdfv= pdf('normal', data, F(j).par(1), F(j).par(2));
        F(j).LL= sum(log(pdfv(pdfv>0 & ~isinf(pdfv))));
        F(j).aic= 2*2- 2*F(j).LL;
        break
      % Check: if values > 1 for Beta, do nothing
      elseif strcmp('beta', Cdist{j}) && max(data) > 1
        F(j).name= 'beta';
        F(j).LL= -Inf;
        F(j).aic= Inf;
      % Check: if values > 0 for some distr. (they are sorted), do
nothing
      elseif j >= mustbepos && min(data) == 0
        F(j).name= Cdist{j};
        F(j).LL= -Inf;
        F(j).aic= Inf;
      % Any other case do the fit ...
      else
        [phat,pci]= mle(data, 'distribution', Cdist{j}, 'alpha', alpha);
        F(j).name= Cdist{j};
        F(j).par= phat;
        F(j).ci= pci;
        if numel(F(j).par) == 1
          pdfv= pdf(F(j).name, data, F(j).par(1));
        elseif numel(F(j).par) == 2
          pdfv= pdf(F(j).name, data, F(j).par(1), F(j).par(2));
        else
          pdfv=
pdf(F(j).name, data, F(j).par(1), F(j).par(2), F(j).par(3));
        end
        F(j).LL= sum(log(pdfv(pdfv>0 & ~isinf(pdfv))));
        F(j).aic= 2*numel(F(j).par)- 2*F(j).LL;
      end
    end

  % Discrete
  case 'disc'
    for j= 1:numel(Ddist)
      % Binomial needs number of trials
      if strcmp('binomial', Ddist{j})
        F(j).name= 'binomial';
        if isempty(ntrials) || (numel(ntrials) > 1 && numel(data) ~=
numel(ntrials))
          F(j).LL= -Inf;
          F(j).aic= Inf;
        else
          [phat,pci]=
mle(data, 'ntrials', ntrials, 'distribution', 'binomial', 'alpha');

```

```

        F(j).par= phat;
        F(j).ci= pci;
        pdfv= pdf('bino',data,ntrials,F(j).par(1));
        F(j).LL= sum(log(pdfv(pdfv>0 & ~isinf(pdfv))));
    end
else
    [phat,pci]= mle(data,'distribution',Ddist{j},'alpha',alpha);
    F(j).name= Ddist{j};
    F(j).par= phat;
    F(j).ci= pci;
    if numel(F(j).par) == 1
        pdfv= pdf(F(j).name,data,F(j).par(1));
    elseif numel(F(j).par) == 2
        pdfv= pdf(F(j).name,data,F(j).par(1),F(j).par(2));
    else
        pdfv=
pdf(F(j).name,data,F(j).par(1),F(j).par(2),F(j).par(3));
    end
    F(j).LL= sum(log(pdfv(pdfv>0 & ~isinf(pdfv))));
    F(j).aic= 2*numel(F(j).par)- 2*F(j).LL;
end
end
end

% Order by criterion
switch criterion
case 'LL'
    index= sortrows([(1:size(F,2))],[F.LL]'],-2);
case 'AIC'
    index= sortrows([(1:size(F,2))],[F.aic]'],2);
end
F= F(index(:,1));

```

Appendix B: Generate Degradation Profiles

```
function [degradation , failure] = generateDegradtionProfile (alpha , beta
, lambda , SimulationHorizon , InitialDegradation , maxEvent)

% This code generate the degradation profile suring the simulation horizon
and initial degradtion value
% given the set of input parameters.

% Destruction Event
% This code defines the destruction events based on the Poisson
distribution

% Destruction Amount
% This code defines the destruction amounts based on the parameters
of the gamma distribution
% Gamma parameters are mapped into mean and standard deviation to
generate dgerdation profiles

% Threshold
% This code defines the threshold based on the gamma distribution
and
% maximum number of destrctuin events which leads to failure

% Define Destruction Properties
destructionMean = alpha / beta ;
destructionStd = alpha / (beta^2) ;

% Define Threshold Properties
thresholdMean = maxEvent * alpha / beta ;
thresholdStd = maxEvent * alpha / (beta^2) ;
Threshold = min(thresholdStd*randn + thresholdMean, 100);

% Define destruction event and destruction amount variables
occuredDestructionEvent = zeros(SimulationHorizon , 1);
destructionAmount = zeros(SimulationHorizon , 1);
degradation = zeros(SimulationHorizon , 1);
degradation(1 , 1) = InitialDegradation;

% Generate Degradation Profiles
for i = 1:SimulationHorizon
    occuredDestructionEvent(i,1) = poissinv(rand, lambda);
    destructionAmount(i,1) = max(sum(
degradationStd*randn(occuredDestructionEvent(i,1), 1)+ destructionMean),
0);
    degradation(i+1 , 1) = destructionAmount(i,1) + degradation(i ,
1);
    if degradation(i,1) > Threshold
        break
    end
end

% Define Failure Point
failureTime = find(degradation > Threshold, 1, 'first');
```

```
if isempty(failureTime)
    failureTime = Input.SimulationIteration;
    degradation(end:i) = nan;
end
failure = failureTime;
end
```

Appendix C: Parameter Estimation Based on Gaussian Mixture Models

```
function Output =
DegradationModelParameterEstimator(HistoricalDegradationProfile ,
DegradationModelParameter)

%%% This code returns the estimate of the lambda, alpha, and beta
%%% based on the Gaussian Mixture Model clustering approach.

%%% Given a set of parameters, we first simulate a degradation profile
and then,
%%% try to estimate the parameters.
%%% In real-world applications, Lambda, alpha, and beta are going to be
estimated
%%% based on the Historical Time Series Degradation profile, which its
%%% properties are unknown.
%%% In this code, we try to simulate the
%%% degradation profiles with known parameters in order to test the
robustness of the model

%%% Inputs:
%%%      HistoricalDegradationProfile : Estimated Degradation Profile
%%%      DegradationModelParameter
%%%      DegradationModelParameter(1): Lambda
%%%      DegradationModelParameter(2): Alpha
%%%      DegradationModelParameter(3): Beta

    lambda = DegradationModelParameter(1);
    destructionMean = DegradationModelParameter(2) /
DegradationModelParameter(3);
    destructionStd = sqrt(DegradationModelParameter(2) /
(DegradationModelParameter(3) ^ 2));
    numberOfOccuredDestructionEvents = 0;

    if isempty (HistoricalDegradationProfile)==0
        for i=1:length(HistoricalDegradationProfile)-1
            HistoricalDegradationProfileEvaluat(1,1) = 0;
            HistoricalDegradationProfileEvaluat(i+1,1)=
HistoricalDegradationProfile(i+1,1)- HistoricalDegradationProfile(i,1);
        end
        numberOfOccuredDestructionEvents =
length(HistoricalDegradationProfileEvaluat (HistoricalDegradationProfileEva
luat>0));
    end

    if isempty (HistoricalDegradationProfile)==1 ||
numberOfOccuredDestructionEvents<50
        InitialDegradation = 0;
        degradation(1,1)=InitialDegradation;
        for k = 1:500
            damage(k,1) = poissinv(rand, lambda);
```



```

        damageAmount(k,1) =
max(sum(destructionStd*randn(damage(k,1), 1) + destructionMean), 0);
        degradation(k + 1,1) = damageAmount(k,1) +
degradation(k,1);
    end
end

if numberOfOccuredDestructionEvents > 50
    degradation = HistoricalDegradationProfile;
end

for i=1:length(degradation)-1
    damageAmountOcc(i,1)=degradation(i+1,1)-degradation(i,1);
end

MaxNumOfCluster = 10;
eva =
evalclusters(damageAmountOcc, 'gmdistribution', 'DaviesBouldin', 'KList', [1:MaxNumOfCluster]);
Optimalk = eva.OptimalK;

options = statset('MaxIter', 500);
RegularizationValue = 0.0000001;

GMM =
fitgmdist(damageAmountOcc,Optimalk, 'Options', options, 'RegularizationValue', RegularizationValue);

sortedMean = sort(GMM.mu);
destructionMeanHat = sortedMean(2);
[row] = find(GMM.mu == destructionMeanHat);
destructionStdHat = sqrt((GMM.Sigma(:, :, row)));

minMean = min(GMM.mu);
[row] = find(GMM.mu == minMean);
zeroDamageWeight = GMM.ComponentProportion(:, row);
lambdaHat = -log(zeroDamageWeight);

%%% Outputs
Output.destructionBeta= DegradationModelParameter(3);
Output.destructionBetaHat = destructionMeanHat /
(destructionStdHat^2) ;

Output.destructionAlpha = DegradationModelParameter(2);
Output.destructionAlphaHat = Output.destructionBeta *
destructionMeanHat;

Output.lambda = DegradationModelParameter(1);
Output.lambdaHat = lambdaHat;

Output.OptimalCluster = Optimalk;

end

```

Appendix D: Parameter Estimation Based on K-Means Clustering Algorithm

```
function Output = ParameterEstimatorKmean(lambda , alpha , beta)

%% This function estimates the degradation model parameter based on
%% K-Mean clustering algorithm.
%% Inputs:
%% lambda: Rate of Event
%% alpha : scale parameter of the Gamma distribution representing
the destruction amount
%% beta : shape parameter of the Gamma distribution representing the
destruction amount

gammaMean = alpha/beta;
gammaSigma = sqrt(alpha/(beta^2));

InitialDegradation = 0;
degradation(1,1)=InitialDegradation;
for k = 1:500
    destruction(k,1) = poissinv(rand, lambda);
    destructionAmount(k,1) =
max(sum(gammaSigma*randn(destruction(k,1), 1) + gammaMean), 0);
    degradation(k + 1,1) = destructionAmount(k,1) + degradation(k,1);
end

for i=1:length(degradation)-1
    if degradation(i,1)==degradation(i+1,1)
        degradation(i,2)=0; % destruction did not occure
    else
        degradation(i,2)=1; % destruction occurred
    end
end

for i=1:length(degradation)-1
    degradation(i,3)=degradation(i+1,1)-degradation(i,1);
end

destructionAmountOcc=degradation(:,3);

ActualdestructionAmountOcc=destructionAmountOcc(destructionAmountOcc>0);

MaxNumOfCluster = 10;
eva =
evalclusters(destructionAmountOcc, 'gmdistribution', 'DaviesBouldin', 'KList'
, [1:MaxNumOfCluster]);
OptimalK=eva.OptimalK;

options = statset('MaxIter',500);
```

```

[idx,C] =
kmeans(destructionAmountOcc,Optimalk,'Options',options,'EmptyAction','drop
');

sortedMean = sort(C);
gammaMeanHat = sortedMean(2);
[row] = find(C == gammaMeanHat);
gammaSigmaHat = sqrt(var(destructionAmountOcc(idx==row)));

[row2] = find(C == sortedMean(1));
lambdaHat = -log(sum(idx==row2)./length(destructionAmount));

betaHat = gammaMeanHat / (gammaSigmaHat^2);
alphaHat = gammaMeanHat * betaHat;

alphaHatPercErr = ((alphaHat-alpha)/alpha)*100;
betaHatPercErr = ((betaHat-beta)/beta)*100;
lambdaHatPercErr = ((lambdaHat-lambda)/lambda)*100;

%%% Outputs
Output.alpha = alpha;
Output.alphaHat = alphaHat;
Output.alphaHatPercErr = alphaHatPercErr;
Output.beta = beta;
Output.betaHat = betaHat;
Output.betaHatPercErr = betaHatPercErr;
Output.lambda = lambda;
Output.lambdaHat = lambdaHat;
Output.lambdaHatPercErr=lambdaHatPercErr;
Output.OptimalCluster = Optimalk;
Output.TotalNumberOfObservations = 500;
Output.NumberOfOccdestruction = length(ActualdestructionAmountOcc)-1;

end

```

Appendix E: Evaluate the Performance of the Parameter Estimation Algorithms

```
function Output = parameterEstimationEvaluation(lambda , alpha , beta ,
estimationMethod, estimationIteration , outlierDetectionFlag)
% This function evaluate the performance of the estimation algorithms.
% INPUTS:
%     lambda : rate of destructive events
%     alpha  : scale parameter of the Gamma distribution representing the
destruction amount
%     beta   : shape parameter of the Gamma distribution representing the
destruction amount
%     estimationMethod: GMM - KMeans - NN
%     estimationIteration : number of times to ignite the estimation
algorithm
%     outlierDetectionFlag : 0 (do not apply outlier detection) , 1 (
apply outlier detection)
% OUTPUTS:
%     alphaHat: MEAN estimate of alpha
%     alphaMAPE: alpha Mean Absolute Percentage Error
%     betaHat: MEAN estimate of beta
%     betaMAPE: : beta Mean Absolute Percentage Error
%     lambdaHat : MEAN estimate of lambda
%     lambdaMAPE : : lambda Mean Absolute Percentage Error

alphaHat = zeros(estimationIteration , 1);
alphaHatPercErr = zeros(estimationIteration , 1);
betaHat = zeros(estimationIteration , 1);
betaHatPercErr = zeros(estimationIteration , 1);
lambdaHat = zeros(estimationIteration , 1);
lambdaHatPercErr = zeros(estimationIteration , 1);

for i=1:estimationIteration
    if strcmp(estimationMethod , 'GMM')
        ParameterEstimator = ParEstimator(lambda , alpha , beta);
    elseif strcmp(estimationMethod , 'KMeans')
        ParameterEstimator = ParEstimatorKmean(lambda , alpha , beta);
    elseif strcmp(estimationMethod , 'NN')
        ParameterEstimator = ParEstimatorNN(lambda , alpha , beta);
    else
        error('select an existing estimation method')
    end
    alphaHat(i,1) = ParameterEstimator.alphaHat;
    alphaHatPercErr(i,1)= ParameterEstimator.alphaHatPercErr;
    betaHat(i,1) = ParameterEstimator.betaHat;
    betaHatPercErr (i,1) = ParameterEstimator.betaHatPercErr;
    lambdaHat(i,1) = ParameterEstimator.lambdaHat;
    lambdaHatPercErr(i,1) = ParameterEstimator.lambdaHatPercErr;
end

ParEst = [alphaHat, alphaHatPercErr ,betaHat, betaHatPercErr ,
lambdaHat, lambdaHatPercErr];
ParEst(any(isnan(ParEst),2), :) = [];
```

```

alphaHat = ParEst(:,1);
alphaHatPercErr = ParEst(:,2);
betaHat = ParEst(:,3);
betaHatPercErr = ParEst(:,4);
lambdaHat = ParEst(:,5);
lambdaHatPercErr = ParEst(:,6);

if outlierDetectionFlag == 1
    alphaOutlierMedian = isoutlier(alphaHat, 'median');
    alphaOutlierMean = isoutlier(alphaHat, 'mean');
    alphaOutlierQuartiles = isoutlier(alphaHat, 'quartiles');
    alphaOutlierGrubbs = isoutlier(alphaHat, 'grubbs');
    alphaOutlierGesd = isoutlier(alphaHat, 'gesd');

    betaOutlierMedian = isoutlier(betaHat, 'median');
    betaOutlierMean = isoutlier(betaHat, 'mean');
    betaOutlierQuartiles = isoutlier(betaHat, 'quartiles');
    betaOutlierGrubbs = isoutlier(betaHat, 'grubbs');
    betaOutlierGesd = isoutlier(betaHat, 'gesd');

    LambdaOutlierMedian = isoutlier(lambdaHat, 'median');
    LambdaOutlierMean = isoutlier(lambdaHat, 'mean');
    LambdaOutlierQuartiles = isoutlier(lambdaHat, 'quartiles');
    LambdaOutlierGrubbs = isoutlier(lambdaHat, 'grubbs');
    LambdaOutlierGesd = isoutlier(lambdaHat, 'gesd');

    Oulier = alphaOutlierMedian + alphaOutlierMean +
alphaOutlierQuartiles + alphaOutlierGrubbs + alphaOutlierGesd + ...
        + betaOutlierMedian + betaOutlierMean + betaOutlierQuartiles +
betaOutlierGrubbs + betaOutlierGesd + ...
        + LambdaOutlierMedian + LambdaOutlierMean +
LambdaOutlierQuartiles + LambdaOutlierGrubbs + LambdaOutlierGesd;

    alphaHat(Oulier~=0)=[];
    alphaHatPercErr(Oulier~=0) = [];
    betaHat(Oulier~=0)=[];
    betaHatPercErr(Oulier~=0) = [];
    lambdaHat(Oulier~=0)=[];
    lambdaHatPercErr(Oulier~=0)=[];

    Output.TotalOutliers = Oulier(Oulier~=0);
end

Output.alphaHat = mean(alphaHat);
Output.alphaMAPE = mean(abs(alphaHatPercErr(:,1)));
Output.betaHat = mean(betaHat);
Output.betaMAPE = mean(abs(betaHatPercErr(:,1)));
Output.lambdaHat = mean(lambdaHat);
Output.lambdaMAPE = mean(abs(lambdaHatPercErr(:,1)));

end

```

Curriculum Vitae

Farhad Balali

Result oriented and highly dedicated Industrial Data Scientist, Senior Engineer, Researcher, and Teacher with extensive experience in Data Analytics, Internet of Things, Machine Learning, Predictive Models, Statistics, Optimization, Asset Management, and Reliability Assessment with critical thinking, problem-solving and leadership. Strong communication and interpersonal skills with an engaging personality and discipline skills to deliver knowledge in a fast-paced environment.

EDUCATION

Jan 2016 – Dec 2019	University of Wisconsin-Milwaukee	Industrial Engineering (3.98/4)	Ph.D.
Jan 2014 – Dec 2015	University of Wisconsin-Milwaukee	Industrial Engineering (4/4)	M.S.
Sept 2008 – Sept 2012	Khajeh Nasir University of Technology	Industrial Engineering (16.1/20)	B.S.

SKILLS AND TECHNICAL EXPERTISE

I possess theoretical and applied expertise in the following areas:

- Data Scientist / Data Analytics / Pattern Recognition
 - Predictive Models / Model Predictive Control / Model Predictive Maintenance
 - Artificial Intelligence / Machine Learning / Cloud-Based Algorithms / Statistical Analysis
 - Internet of Things (IoT) / Data-Driven Algorithms
 - Asset Management / Reliability Assessment / Prognosis and Health Management
 - Operations Research / Optimization / Advanced Mathematical Computer Programming
 - Sustainability / Environmental Assessment / Economic Analysis
 - Renewable Energy Resources / Water-Energy-Nexus
-

PROFESSIONAL ACCOMPLISHMENTS

- Obtained experiences to apply the science and research outcomes in both industrial and academic environments.
 - Conducted research, papers, proposals, patents, literature and background reviews, presentations and reports.
 - Trained, managed, and mentored more than 300 undergraduate and graduate students.
 - Created course curriculum and designed assignments and exams accordingly.
 - Developed project status report and professional presentations.
 - Conducted critical data analysis, troubleshooting, and research on the trend emergent issues.
 - Established applied problem-solving skills to direct a project from beginning to finish by problem identification, problem structuring, group brain-storming, evaluating possible solutions, and decision making.
-

AWARDS AND HONORS

- Foxconn, Smart City Round One Competition, Milwaukee, Spring 2019
- Distinguished Graduate Student Fellow, University of Wisconsin-Milwaukee, Fall 2018
- National Interest, United States Citizenship and Immigration Services, Summer 2018
- Chancellor's Award, University of Wisconsin-Milwaukee, Spring 2017.
- Teaching and Research Assistantship, University of Wisconsin-Milwaukee, Jan 2014 – Present.
- Travel Award, University of Wisconsin-Milwaukee, Jan 2016.

- Graduation with honor, GPA 4/4, department of Industrial Engineering, University of Wisconsin-Milwaukee, December 2015.
 - Graduation with honor, ranked in the top 2% of students in the Department of Industrial Engineering, Khajeh Nasir Toosi University of Technology, Sept 2012.
-

BEHAVIORAL TRAITS

- Great passion for teamwork
 - Ability to work independently
 - High motivation with a strong work ethic
 - Good planning and leadership skills
 - Strong analytical mind with problem solving skills
 - Excellent written and verbal communication skills
 - Ability to deal with ambiguity in defining activities and direction
-

WORK EXPERIENCES

Industry

Jun 2019 - Present

- **Senior Control Algorithm Engineer, Johnson Controls Inc., Milwaukee, WI.**
 - Senior Control Algorithm Developer
 - Conducting research, papers, proposals, patents, literature and background reviews.
 - Developing test-runs, algorithms, and models in the following areas:
 - Machine Learning
 - Statistical Data Analysis / Big Data / Time-Series
 - Smart Systems / Data Collection / Data Filtration
 - Cloud-Based Algorithms
 - Condition-Based Predictive Maintenance
 - Reliability and Risk Analysis
 - Degradation Model Development
 - Central Plant Optimization
 - Predictive Models for Electrical and Thermal Loads
 - Predictive Model Development, Assessment, and Enhancement
 - Real-Time Monitoring
 - Conducting weekly presentations and work progress reports.

Academic

Teaching Assistant

Jan 2014 – May 2018

- Trained, managed, and mentored more than 300 undergraduate and graduate students at the University of Wisconsin-Milwaukee.
- Hold lectures for undergraduate and graduate students of all engineering disciplines.
- Developed presentations, handouts, assignments, course materials, quizzes, and exams.
- Provided regular office hours in addition to the flexible scheduled appointments to help the students regarding their learning process.
- Obtained average evaluation score of 4.4 out of 5.00 which is an indication of students' satisfaction during nine semesters.
- Thought the following courses at the Industrial and Manufacturing Engineering Department and Lubar School of Business at the University of Wisconsin-Milwaukee.

- o Design of Experiments
- o Engineering Methodology
- o Computer-Aided Engineering Drawing
- o Supply Chain and Business Strategies

Research Assistant

Aug 2015 – May 2018

- Center for Sustainable Electrical Energy System, University of Wisconsin-Milwaukee.
 - o Conducting research on:
 - Internet of Things (IoT)
 - Machine Learning Algorithms / Statistics / Data Analytics
 - Connected Systems
 - Algorithms Development
 - Predictive Models / Predictive Model Control / Predictive Model Maintenance
 - Optimization / Data-Driven Algorithms
 - Data Analysis / Pattern Recognition / Algorithm Development / Big Data
 - Renewable Energy / Water-Energy-Nexus Optimization / Sustainability
 - Asset Management and Reliability Assessment
 - Cost-Effective and Sensitivity analysis
 - Economic Model Development
 - o Conducting laboratory Experiments for achieving scientific data to be presented in scholarly publications.
 - o Conducting literature survey and technical reviews.
 - o Participating in research group meetings and seminars.
 - o Compiling and analyzing the research results.

Internships

- **Post Graduate Intern, Johnson Controls Inc.,** Milwaukee, WI, May 2018 – May 2019
 - o Conducting research, papers, proposals, literature and background reviews.
 - o Developing test-runs, algorithms, and models in the following areas:
 - Machine Learning
 - Statistical Data Analysis / Big Data / Time-Series
 - Smart Systems / Data Collection / Data Filtration
 - Cloud-Based Algorithms
 - Condition-Based Predictive Maintenance
 - Reliability and Risk Analysis
 - Degradation Model Development
 - Predictive Models for Electrical Power Consumption
 - Predictive Model Development, Assessment, and Enhancement
 - Real-Time Monitoring
 - o Conducting weekly presentations and work progress reports.
- **Data Analysis and Optimization,** Melli Shoe Co., Tehran, Iran, Jun 2012 – Sep 2012
 - o Conducting a data-driven statistical approach to reveal the abnormalities of the production line align controlling the quality and inventory, Melli Shoe's Central Branch

- **Optimization / Big Data Analysis**, Pars Electric Co., Tehran, Iran, Jun 2011 – Sep 2011
 - Big data analysis / Pattern recognition / Abnormalities and Relation detection
 - Significantly optimized the production cost, reducing the waste and production time by leveraging SPC control charts.
-

PUBLICATIONS

Book

- **M. H. Balali**, N. Nouri, A. Nasiri, “**Internet of Things (IoT) for Data-Intensive Industrial and Building Asset Management: Algorithms and Implementation**”, Springer, (Jan 2020).

Book Chapters

- **M. H. Balali**, N. Nouri, W. Otieno “**Environmental Analysis of Self-Lubricating Composites**”, Book Chapter of “**Advances in Self Lubricating Composites**”, Springer, (2017).

Refereed Journal Papers

- N. Nouri, **M. H. Balali**, A. Nasiri, W. Otieno. " **Water Withdrawal and Consumption Reduction for Electrical Energy Generation Systems**", *Applied Energy*, (2019).
- **M. H. Balali**, N. Nouri, E. Omrani, A. Nasiri, W. Otieno “**An Overview of Environmental, Economic, and Material Developments of Solar and Wind Sources Coupled with Energy Storage Systems**”, *International Journal of Energy Research*, (2017).
- **M. H. Balali**, N. Nouri, A. Nasiri, M. Rashidi, W. Otieno. “**A multi-predictor model to estimate solar and wind energy generations**”, *International Journal of Energy Research*, (2018).
- M. Khayami, A. Nasiri, **M. H. Balali**. “**A Novel Weak-Node Detection Algorithm in MV Network based on the Jacobian Matrix for Optimal Placement of Energy Storage Devices**”, *International Journal of Smart Grid*.
- M. Rabbani, F. Navazi, H. Farrokhi-Asl, **M. H. Balali**, “**A sustainable transportation location routing problem with the soft time windows for the distribution systems**”, *Uncertain Supply Chain Management*, (2018).
- **M. H. Balali**, N. Nouri, K. Pichka, H. Seifoddini. " **Facility Location Selection of a Manufactory in the United State of America.**" *International Research Journal of Applied & Basic Sciences (IRJABS)*, (2016).
- **M. H. Balali**, N. Nouri, and E. Pakdamanian. " **Application of the Minimax Control Chart for multivariate manufacturing process.**" *International Research Journal of Applied & Basic Sciences (IRJABS)*, (2013).

Refereed Conference Papers

- **F. Balali**, H. Seifoddini, A. Nasiri, “**Essentials to Develop Data-Driven Predictive Models of Prognostics and Health Management for Distributed Electrical Systems**” to 2019 IEEE International Conference on Prognostics and Health Management (ICPHM), *IEEE Xplore* (2019).

- M. Sabbah, M. Rashidi, A. Nasiri, A. Bani-Ahmed, **M. H. Balali**. “**Design and Implementation of a Series Resonant Solid State Transformer**”. *IEEE, ECCE* (2017).
- **M. H. Balali**, N. Nouri, A. Nasiri, H. Seifoddini, “**Development of an Economical Model for a Hybrid System of Renewable Energy and Energy Storage**”, *IEEE Xplore ICRERA* (2015).

Non-Refereed Publications

- **M. H. Balali**, “**An Economical Model Development for a Hybrid System of Grid Connected Solar PV and Electrical Storage System**”, University of Wisconsin-Milwaukee, 2015. (Master’s thesis)

PATENT

- Model Predictive Maintenance (MPM) and Model Predictive Control (MPC) Systems with Artificial Intelligence (AI) Constraints, JCI Ref. 19-0925-PRO | F&L Ref. 117277-0808. (submitted)

COMPUTER SKILLS

- MATLAB / Simulink
- R / Minitab / SPSS / SAS / JMP
- Visual C++ / CPLEX / Python
- Microsoft Azure
- Apache Hadoop
- Lindo / Lingo / Gurobi

PROFESSIONAL MEMBERSHIP AND ACTIVITIES

Editorial Member

- Internet of Things and Cloud Computing
- Progress in Energy and Fuels

Journal Reviewer

- Journal Reviewer for “*Renewable Energy*”, Elsevier, 2017, 2018.
- Journal Reviewer for “*Energy*”, Elsevier, 2017, 2018.
- Journal Reviewer for “*Clean Technologies and Environmental Policy*”, Springer, 2017, 2018.
- Journal Reviewer for “*International Journal of Energy Research*”, Wiley, 2016.
- Journal Reviewer for “*Journal of Industrial Engineering International*” (JIEI), Springer, 2017.

Conference Reviewer

- Conference Paper Reviewer for “IEEE Energy Conversion Congress and Exposition (ECCE), 2018.
- Conference Paper Reviewer for “IEEE Energy Conversion Congress and Exposition (ECCE), 2017.
- Conference Paper Reviewer for “The International Conference on Functional Materials and Biomaterials”, 2016.
- Conference Paper Reviewer for “The International Conference on Energy Engineering and Environmental Protection”, 2016.

Professional Society Membership

- Member of IEEE Society on Reliability
- Member of Institute of Industrial Engineering, IIE.
- Member of INFORMS.
- Member of IEEE Advancing Technology for Humanity.

CONFERENCES AND WORKSHOP ATTENDED

- INFORMS 2019, Presented
- Prognostics and Health Management 2019, Presented
- MERC 2019, Attended
- INFORMS 2018, Attended
- Microsoft Azure IoT Workshop
- INFORMS 2017, Presented
- IEEE Energy Conversion Congress & Exposition (ECCE), Attended, September 2016.
- The Food-Water-Energy-Nexus, Attended, January 2016.
- INFORMS 2016, Attended.
- ICRERA 2015, Presented.
- ICRERA 2014, Attended.
- PEMWA 2014, Attended.
- ANSYS Workshop, November 2016.
- Minitab Workshop, May 2016.
- MATLAB Workshop, Jan 2013.
- ISO-9000 Workshop, Feb 2012.