11-1-2019

Two Essays on Distribution, Fulfillment and Pricing Decisions for Retailers with E-Commerce Channel

Khosro Pichka
University of Wisconsin-Milwaukee

TWO ESSAYS ON DISTRIBUTION, FULFILLMENT AND PRICING DECISIONS FOR RETAILERS WITH E-COMMERCE CHANNEL

by

Khosro Pichka

A Dissertation Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

in Management Science

at

The University of Wisconsin-Milwaukee

December 2019

ABSTRACT

TWO ESSAYS ON DISTRIBUTION, FULFILLMENT AND PRICING DECISIONS FOR RETAILERS WITH E-COMMERCE CHANNEL

by

Khosro Pichka

The University of Wisconsin-Milwaukee, 2019

Under the Supervision of Professor Layth Alwan and Professor Xiaohang Yue

E-commerce has grown rapidly in the past decade. In 2015, e-commerce was accounted for 7.2 percent of all retail sales in the U.S., which is massively higher than 0.2 percent in 1998 (U.S. Department of Commerce 2017). Worldwide e-commerce sales also show the same trend and reached $2.356 trillion in 2018 and are expected to grow steadily (Statista 2017). This trend has impacted major areas of operations management including supply chain management and revenue management. Today, e-commerce companies cannot satisfy their customers' demand using traditional distribution systems. Therefore, retailers with e-commerce channels (e-tailers), cooperate with third party logistic service providers to perform or improve their logistic services and last mile deliveries. In a report on logistics trends, same-day delivery was considered "the next evolutionary step in parcel logistics" (McKinsey 2014). Third party providers are either individual drivers working through the e-tailer's mobile platform such as Amazon Flex (Amazon.com), or start-ups that provide on-demand urban delivery and aggregate demand via their own mobile platforms, such as DoorDash.com. In the first essay, we propose three mixed-integer mathematical models and an efficient heuristic algorithm to solve routing and location decisions for these distribution systems, which are different from traditional delivery services. We model this problem as a two echelon location-routing problem with open routes (2E-OLRP) since the third parties have their own fleet of vehicles. Besides a fast and low-cost delivery, retailers are trying to improve the whole shopping experience for

customers every day. While in-store sales are relatively low, traditional retailers are increasingly offering products through both e-commerce and brick-and-mortar channels and many online retailers are also opening physical stores. This integration, referred to as omni-channel retailing, can help retailers to have a more responsive demand fulfillment process and provide a satisfactory shopping experience for their customers. In the second essay, we model the consumers' behavior with discrete choice models and examine that how the probability of purchasing from each of these channels can change by different price and delivery options. We also propose two optimization models that consider pricing, fulfillment and inventory decisions. We show that e-tailers can increase their profit if they make these decisions simultaneously.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENTS

Firstly, I would like to express my sincere gratitude to my advisors Professor Layth Alwan and Professor Xiaohang Yue for their continuous support of my Ph.D. study and for their patience, motivation, immense knowledge, and many insightful suggestions. I would also like to thank Professor Ehsan Soofi for his priceless advice for both my research and career paths. He has been a true mentor and helpful in providing advice many times during my graduate studies.

Besides my advisors, I would like to thank the rest of my committee members: Professor Samar Mukhopadhyay and Professor Xiang Fang for their insightful comments and encouragement, but also for their thought provoking questions which pushed me to widen my research from various perspectives. I am also indebted to the University of Wisconsin-Milwaukee for their generous funding, which removed financial concerns from my decision to embark on this journey.

Lastly but most importantly, I would like to express my deepest gratitude to my family for their love and support. My beautiful wife, Maryam Behrouzinia, who has always believed in me and supported me all through my life. My lovely mother, Shahrbanoo Pichka, and my generous father, Ali Pichka, for all their sacrifices. My brothers: Akbar, Mohammad and Rahmat, who always inspired me with hope and faith, and my grandfather, Ebrahim Pichka, one of the most influential persons in my life. I am also grateful to my other family members and friends who have supported me along the way.

ABSTRACT

ESSAY 1: LOCATION AND ROUTING DECISIONS FOR RETAILERS WITH E-COMMERCE CHANNEL

by

Khosro Pichka

The University of Wisconsin-Milwaukee, 2019

Under the Supervision of Professor Layth Alwan and Professor Xiaohang Yue

In recent years, retailers with e-commerce channel tend to use existing extra capacity on the roads and traffic flow to deliver their products. Moreover, the number of shipments from suppliers to individual customers, many of whom are located in densely populated urban areas, has increased. Therefore, crowdsourced delivery in a multi echelon environments has become more common. Contrary to classic location-routing problems (LRPs), where the drivers had closed routes and had to return to the depot after delivery, this problem can be addressed as two echelon location-routing problem with open routes (2E-OLRP). In this problem, we seek to find a minimum-cost set of delivery routes that do not return to the main warehouse (i.e., depot) in the first echelon and do not return to intermediary distribution centers (i.e., satellites) in the second echelon due to the presence of individual drivers and third party logistics (3PL) providers. In spite of the large amount of research on LRPs, 2E-OLRP has received very little attention. We propose three mathematical formulations and a hybrid heuristic algorithm to deal with medium- and large-sized problem instances.

1.1. Introduction

With the recent growth of e-commerce transactions, retailers with e-commerce channel cannot satisfy their customers' needs by traditional delivery systems and they are replacing that with crowdsourced delivery. In a crowdsourced delivery system, retailers benefit from individual drivers such as commuters to deliver their packages to customers. Besides faster delivery times, this can also provide financial and environmental benefits for both retailers and the government. Walmart revealed in 2013 that they were considering offering their in-store customers to deliver the products to their online customers when they leave the store (Morphy 2013). Indeed, hiring of independent contractors and drivers to deliver packages in urban areas is becoming more widespread. Amazon Flex (Amazon.com), which allows individual drivers to deliver packages from Amazon distribution centers to delivery points, has been launched in more than fifty cities across the United States. Uber Rush (uber.com) is another example that shows the growing trend of outsourcing in urban transportation services.

1.1.1. Problem description

The two echelon open LRP (2E-OLRP) is an increasingly important real-world problem. Many retailers have switched to crowdsourced delivery to manage their last mile deliveries and are using their physical stores as small distribution centers. Customers are usually located in very populated urban areas and it's profitable for the retailers to distribute their products via a two-echelon approach. In the first echelon, product is transported from the very large central warehouses (i.e., depot) to urban distribution centers (i.e., satellite), which could be physical stores as well. In the second echelon, product is transported from the urban distribution centers to customers.

2E-OLRP is a variant of the 2E-LRP in which each route in the first (second) echelon is a sequence of satellites (customers), that starts at a main depot (satellite) and finishes at one of the satellites (customers) to whom goods are delivered by the available fleet. In contrast, in the classical 2E-LRP, all first (second) echelon vehicles return to the main depot (a satellite) after serving satellites (customers). In practice, the 2E-OLRP can arise when a supplier or producer does not have its own vehicle fleet or its fleet's capacity is not enough to serve all of its customers. Such a company may prefer to employ a third party logistics (3PL) provider to transfer goods between the depot, satellites and customers. Indeed, from a supply chain management perspective, it may be more economical for such companies to outsource the distribution of their products. Thus, in the 2E-OLRP, the contractee does not need to have the fleet at its own depot after serving all the satellites or customers in a single planning horizon.

An example of the 2E-OLRP, in which there is one main depot (triangle), four satellites (squares), and ten customers (circles) is illustrated in Figure 1 (Pichka et al. 2018). The dashed arrows show the routes of two vehicles in the first echelon while the solid arrows show the routes of five vehicles that transport goods from opened satellites to customers in the second echelon. As Figure 1.1 shows, in the first echelon, a vehicle starts its route from the main depot and serves one or more satellites and finishes its route at a satellite. The demand of an opened satellite equals the total demand of customers which we decide to assign to that satellite. If a satellite is not opened, no customer is assigned to that satellite. On the other hand, a second echelon route starts from an opened satellite and ends at a customer after serving one or more customers. One should note that all the customer demands should be satisfied. The demands satisfied by a first (second) echelon route cannot exceed the capacity of a first (second) echelon vehicle. We assume that the demand of a satellite in the first echelon cannot be satisfied by more than one vehicle. In other words, split deliveries are not accepted. Similarly, we assume that the demand of a customer cannot be satisfied by more than one second-echelon vehicle. Accordingly, a customer is assigned to a satellite and cannot be served by two vehicles from the same or different satellites. Moreover, the vehicles in the first and second echelon have different capacities and they can only serve in the echelon they are assigned to. There are an unlimited number of vehicles available at the main depot and each opened satellite. However, the number of vehicles used in the first and second echelon should be minimized to reduce costs.

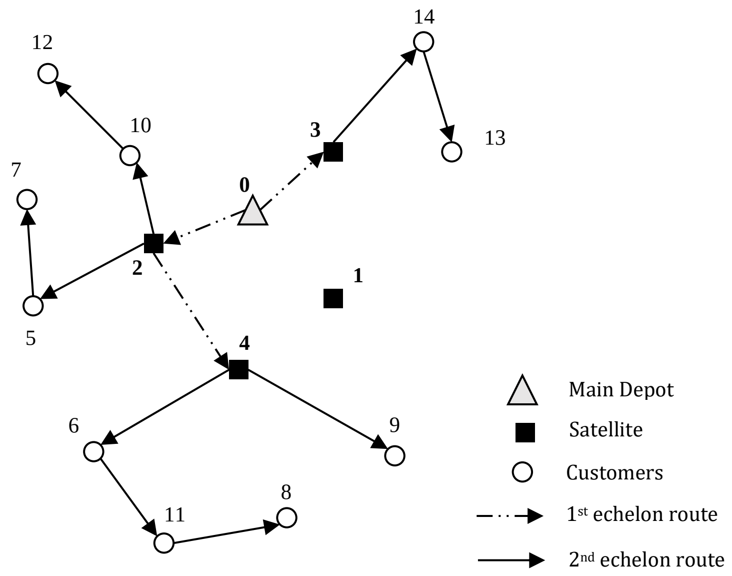


Figure 1.1. Example of the 2E-OLRP.

1.1.2. Summary of contributions

- Three new mixed-integer mathematical models are developed to model retailers' distribution system considering third party logistic service providers. We propose new decision variables to handle the open aspect of the routing problem.
- An efficient hybrid heuristic algorithm as well as a new solution encoding scheme are proposed to solve large-size instances more efficiently. The proposed hybrid heuristic outperforms mathematical models both in solution quality and solution time across the vast majority of instances.
- The proposed heuristic is compared to other existing algorithms in the literature for the classic closed-loop 2E-LRP. The results show that the proposed algorithm obtains

competitive results in terms of solution quality and computation time compared to those in the literature.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature on the 2E-LRP and open VRP (OVRP). Three mathematical programming formulations of the 2E-OLRP are developed in Section 3. Section 4 introduces a hybrid heuristic algorithm that we developed for solving the 2E-OLRP. Computational results on modified benchmark instances are reported in Section 5. Finally, conclusions and future work are discussed in Section 6.

1.2. Literature review

Cuda et al. (2015) published a survey on two echelon routing problems that included location routing, vehicle routing, and truck and trailer problems. Similarly, Prodhon and Prins (2014) and Drexl and Schneider (2015) published two recent surveys of the LRP and its variants and identified future directions for this area of research.

The literature on vehicle routing problems (VRPs) can be classified according to at least three aspects: (1) the number of echelons in the transportation network, (2) whether a VRP or LRP is considered; and (3) whether routes are open or closed. Below we first discuss papers that consider multiple echelon LRPs with closed routes. Then we review the studies that have considered open routes and crowdsourced delivery.

The location and routing decisions are interrelated and the benefit of considering both decisions in designing distribution systems has been shown in the literature (Salhi and Rand 1989). Contrary to the classical LRP, the 2E-LRP has only been studied by a few researchers. Jacobson and Madsen (1980) and Madsen (1983) are the classical papers that first considered the existence of multiple echelons in a location routing problem. They proposed heuristics and compared their performance for designing a newspaper distribution network. Lin and Lei (2009) considered three-echelon distribution systems consisting of distribution centers (DCs), big clients, and small retailers. They proposed a mathematical model and a hybrid genetic algorithm embedded with a routing heuristic to find near optimal solutions in terms of the location and number of DCs and routing in each echelon. They tested the performance of their heuristic method by comparing their results with exact solutions of small problem instances which were solved optimally. Finally, they designed a finished goods distribution system for a Taiwan label-stock manufacturer. Through the case study, they concluded that the inclusion of big clients in the first-level routing in the analysis leads to a better network design in terms of total logistics costs.

Boccia et al. (2010) proposed a tabu search (TS) heuristic which efficiently combines the following sub-problems: the location and the number of facilities in each echelon, the size of two different vehicle fleets, and the related routes in each echelon. They reported results on small, medium, and large problem instances. Crainic et al. (2011) proposed three mixed integer programming formulations for the 2E-LRP; three-index, two-index, and single-index formulations. They evaluated these mathematical models on a large set of examples derived from two-tiered city logistics system settings with various numbers and distributions of potential locations for the two types of facilities.

Nguyen et al. (2012b) proposed four constructive heuristics and a hybrid metaheuristic called the greedy randomized adaptive search procedure (GRASP) combined with a learning process (LP) and path relinking (PR). Three greedy randomized heuristics were used to generate trial solutions for the GRASP and learning process, and two variable neighborhood descent (VND) procedures were implemented to improve them. They showed that applying LP and PR improves the performance of their metaheuristic on the classical LRP and 2E-LRP instances. Nguyen et al. (2012a) proposed a multi-start iterated local search (MS-ILS) for the 2E-LRP. For generating initial solutions, they used three greedy randomized heuristics based on (a) the Clarke and Wright algorithm, (b) the nearest neighbor heuristic for the TSP, and (c) an insertion heuristic that constructs second-level routes one by one. The ILS run changes between two solution spaces: (i) 2E-LRP solutions and (ii) traveling salesman (TSP) tours covering the main depot and the customers. When a known solution (stored in a tabu list) is revisited, the number of iterations in each run is reduced. Also, they strengthened the MS-ILS algorithm by a path-relinking procedure (PR) which was used internally for intensification and/or post-optimization. On two sets of 2E-LRP instances, they showed that the MS-ILS, on average, outperforms two GRASP algorithms. Also, on capacitated location routing problem (CLRP) instances, their algorithm is more efficient than all algorithms in the literature except the LRGTS algorithm by Prins et al. (2007).

Schwengerer et al. (2012) presented a variable neighborhood search (VNS) algorithm for the 2E-LRP which is an extension of a previous efficient VNS for the LRP. Their algorithm uses seven different basic neighborhood structures parameterized with different perturbation sizes which leads to a total of 21 specific neighborhood structures. They also incorporated the idea of two consecutive local search methods that consider only recently changed solution parts. Their algorithm is efficient in terms of time and quality of solutions compared to the existing results. In Contardo et al. (2012) two algorithms are proposed to deal with the 2E-CLRP. The first one uses a branch-and-cut method based on a new two-index vehicle flow formulation which is strengthened with several families of valid inequalities. An adaptive large-neighborhood search (ALNS) meta-heuristic is also proposed to quickly find solutions. They show that ALNS outperforms existing heuristics on sets of instances from the literature. Moreover, their branch-and-cut method provides tight lower bounds and solves small- and medium-sized instances to optimality within a reasonable amount of time.

In a recent work, Winkenbach et al. (2015) presented a large-scale static and deterministic mixed-integer linear programming (MILP) model with modal choice to develop a profitable urban logistics service (ULS) by providing strategic decision making to postal operators. By considering operating data from La Poste, they identified the main elements of an optimal infrastructure and fleet design for the centralized consolidation of urban freight flows under a global service time constraint. Furthermore, they conducted a sensitivity analysis of the optimal design based on the input data. They proposed a routing cost estimation formula and an optimization-based heuristic to solve the large-scale MILP within a reasonable time and to near-optimality.

Vidovic et al. (2016) proposed a mathematical formulation for a 2E-LRP that considers non-hazardous recyclables collection with a profit and a distance dependent collection rate. The proposed model simultaneously decides the location of collection points, the location of intermediate consolidation points, and the routing of collection vehicles. They proposed a two-phase heuristic to solve the problem. In the first phase, sets of opened collection points and their locations, end user allocation to collection points, and collection quantity of recyclables are determined by applying a greedy heuristic. In the second phase, the proposed algorithm determines optimal routes of collection vehicles that visit city blocks and transfer stations considering the solution from the previous step.

Besides the 2E-LRP, the open vehicle routing problem (OVRP) is another variant of the VRP which has attracted a lot of attention by researchers in recent years. In the literature on the OVRP, several heuristics and meta-heuristics are proposed. Repoussis et al. (2007) developed a mathematical model for an OVRP with time windows and solved the problem using a greedy look-ahead route construction heuristic algorithm. Letchford et al. (2007) presented the first exact algorithm for the OVRP. The algorithm is based on branch-and-cut. They modified the classical integer programming formulation, introduced cutting planes for the OVRP, and compared the difficulty of open and closed versions of the problem.

Russell et al. (2008) employed a tabu search (TS) methodology to integrate the production and distribution of newspapers from plants to delivery locations to maximize overall productivity and profitability. Chiang et al. (2009) considered a similar case in a stochastic environment in which stochastic phenomena include the delivery start time, production rate, loading and unloading times, and travel times. They employed a two-phase meta-heuristic method to find an initial solution and an effective way to improve it.

Norouzi et al. (2012) presented a new multi-objective mathematical model for an OVRP with homogeneous vehicles and competitive time windows in order to maximize sales. They also proposed a multi-objective PSO algorithm. Fleszar et al. (2009) introduced a VNS method based on reversing segments of routes and exchanging segments between routes and compared their results to the best performing heuristics.

Some research studies have developed hybrid algorithms to solve OVRPs. Repoussis et al. (2010) proposed a population-based hybrid meta-heuristic algorithm that utilizes the basic solution framework of an evolutionary algorithm (EA) combined with a memory-based trajectory local search algorithm. Several other studies – including those by Salari et al. (2010), Liu and Jiang (2012), and Liu et al. (2014) have investigated OVRPs and presented heuristics/meta-heuristics to solve them.

In some recent studies, Vincent and Lin (2015) proposed a mathematical model and a simulated annealing-based heuristic for the open LRP (OLRP). They tested their mathematical program and heuristic on modified instances that have been adapted from CLRP benchmark instances. Compared to the linear relaxation of the mathematical model, exact solutions, and solutions produced by other heuristics in the literature, they showed that their proposed heuristic solves the OLRP very efficiently. Also, Vincent et al. (2016) proposed a mixed-integer linear program and a simulated annealing (SA) algorithm for the open VRP with cross-docking (OVRPCD) that minimizes the vehicle hiring cost and transportation cost. They first tested the SA algorithm by comparing the results with benchmark instances for the vehicle routing problem with cross-docking. They then tested the SA algorithm on three sets of

OVRPCD benchmark instances and the results were compared with those obtained by the mathematical model. They showed that both the mathematical model and SA can obtain optimal solutions to all small- and medium-sized instances. However, the computational time for SA is less than CPLEX. Furthermore, for large scale problems, SA outperforms CPLEX in both solution quality and computational time.

Arslan et al. 2018 considered the crowdsourced problem as dynamic pick-up and delivery problem. They studied a service platform that matches deliveries and ad hoc drivers. Their platform also has a dedicated fleet that can be used for the deliveries that cannot be outsourced. They proposed an exact approach to solve the matching problem as they update their input information. By numerical experiments, they showed that by using individual drivers they can benefit from a savings up to 37% compared to a traditional delivery system with dedicated vehicles.

Overall, despite the existence of dozens of outstanding articles on various kinds of 2E-LRPs, OVRPs, and LRPs, it appears that the problem considered in this study—the 2E-OLRP—remains unexplored. The 2E-OLRP arises in urban settings, especially in large cities where distances between satellites and customers are high and there is high demand density. In such contexts, two-echelon systems—in which intermediate distribution centers (satellites) are located on the outskirts of the city—may reduce total shipping costs. Furthermore, open routes are a possibility when companies do not have their own fleet, or their fleet is not large enough to serve all customers. These observations motivate the current study.

1.3. Flow-based mathematical models for the 2E-OLRP

We now introduce three mathematical formulations, namely P_1, P_2 and P_3 for the 2E-OLRP. The input parameters and decision variables used in the models are summarized in Table 1.1.

Table 1.1 Input parameters and decision variables in the mathematical models

Input parameters			
N_s	Number of satellites	Q_1	Capacity of vehicles in the 1st level
N_c	Number of customers	Q_2	Capacity of vehicles in the 2nd level
V_0	Depot	Q_i^s	Capacity of satellite i
V_s	Set of potential satellites	D_i	Demand required by customer i
V_c	Set of customers	C_{ij}	Cost for a vehicle to travel along (i, j)
K_1	Set of 1st-level vehicles (all of which are identical)	F_1	Cost of activating one vehicle in the 1st level
K_2	Set of 2nd-level vehicles (all of which are identical)	F_2	Cost of activating one vehicle in the 2nd level
O_i	Cost of opening satellite i		

Decision variables for two index mathematical model	
X_{ij}	1, if a first-level vehicle travels directly from node i to j 0, otherwise (binary) $(i, j \in V_0 \cup V_s,\ i \neq j)$
Y_{ij}	1, if a second-level vehicle travels directly from node i to j 0, otherwise (binary) $(i, j \in V_s \cup V_c,\ i \neq j)$
Z_i	1, if satelite i is opened 0, otherwise (binary) $(i \in V_s)$
V_{in}	1, if satelite i serves customer n 0, otherwise (binary) $(i \in V_s,\ n \in V_c)$
U_{ij}	Load carried by the vehicle during its trip from node i to j in the first echelon (real, ≥ 0) $(i, j \in V_0 \cup V_s,\ i \neq j)$
L_{ij}	Load carried by the vehicle during its trip from node i to j in the second echelon (real, ≥ 0) $(i, j \in V_s \cup V_c,\ i \neq j)$
δ_i	1, if node i is the last satellite that is visited by a first-level vehicle 0, otherwise (binary) $(i \in V_s)$
λ_i	1, if node i is the last customer that is visited by a second-level vehicle 0, otherwise (binary) $(i \in V_c)$

Decision variables for three index mathematical models	
X_{ijk}	1, if first-level vehicle k travels directly from node i to j 0, otherwise (binary) $(i, j \in V_0 \cup V_s,\ i \neq j;\ k \in K_1)$
Y_{ijk}	1, if second-level vehicle k travels directly from node i to j 0, otherwise (binary) $(i, j \in V_s \cup V_c,\ i \neq j;\ k \in K_2)$
Z_i	1, if satelite i is opened 0, otherwise (binary) $(i \in V_s)$
V_{in}	1, if satelite i serves customer n 0, otherwise (binary) $(i \in V_s,\ n \in V_c)$
U_{ijk}	Load carried by vehicle k during its trip from node i to j in the first echelon (real, ≥ 0) $(i, j \in V_0 \cup V_s,\ i \neq j;\ k \in K_1)$
L_{ijk}	Load carried by vehicle k during its trip from node i to j in the second echelon (real, ≥ 0) $(i, j \in V_s \cup V_c,\ i \neq j;\ k \in K_2)$
δ_{ik}	1, if node i is the last satellite that is visited by first-level vehicle k 0, otherwise (binary) $(i \in V_s;\ k \in K_1)$
λ_{ik}	1, if node i is the last customer that is visited by second-level vehicle k 0, otherwise (binary) $(i \in V_c;\ k \in K_2)$

Each customer *i* has demand D_i . In the first echelon, freight should be delivered from the depot v_0 to satellite set $V_s = \left\{ v_{s_1}, v_{s_2}, ..., v_{s_{N_s}} \right\}$. In the second echelon, the freight that has accumulated at satellite set $V_s = \left\{ v_{s_1}, v_{s_2}, ..., v_{s_{N_s}} \right\}$ should be shipped to the customer set $V_c = \left\{ v_{c_1}, v_{c_2}, ..., v_{c_{N_c}} \right\}$. A large number, K_1, of first-echelon vehicles are available at the depot, each with capacity Q_1 and fixed cost of F_1. Similarly, a large number, K_2, of second-echelon vehicles, each with capacity Q_2 and fixed cost F_2, are available. Each satellite *i* has capacity Q_i^s and opening cost O_i. C_{ij} is the travel cost between node *i* and *j*. This parameter gives travel costs in the first echelon if $i, j \in V_0 \cup V_s$ and second echelon if $i, j \in V_s \cup V_c$ and *i* and *j* are not both in V_s.

The three mathematical models differ as follows. P_1 and P_2 forbid direct travel from any satellite to the depot in the first echelon and from any customer to a satellite in the second echelon. This has been done by defining two sets of variables δ_i and λ_i for P_1 and, δ_{ik} and λ_{ik} for P_2 that are necessary for flow balance constraints for the last satellite or customer visited in a first or second echelon route respectively. These sets of variables are specifically defined to allow for open routes in the first and second echelon and improve the performance of the mathematical models, which will be discussed in detail in Section 5. Moreover, note that P_1 uses two index decision variables and P_3 uses three index decision variables to model 2E-OLRP.

In contrast to P_1 and P_2, in P_3, the problem is considered as a closed 2E-LRP except the distances (C_{ij}) from satellites to the depot in the first echelon and from customers to satellites in the second echelon equal zero. This is the classical approach that has also been used in several studies on the OVRP (Letchford et al. 2007, Fung et. al 2013, Liu et al. 2014, Lalla-Ruiz et al. 2016).

One should note that all the existing models in the literature for the classic 2E-LRP (Crainic et al. 2011, Nguyen et al. 2012a, Nguyen et al. 2012b, Contardo et al. 2012) use classical constraints regarding subtour elimination. However, in proposed mathematical models in this study, subtour elimination constraints are based on flow-based modeling techniques, which are adaptations of those introduced for the two echelon VRP (Perboli et. al 2011).

The following assumptions are considered in the proposed mathematical models:

1. Each customer's demand should be satisfied by a second-echelon vehicle that starts its route at an opened satellite and ends its route at a customer.
2. The total demand of the customers in the route of a second echelon vehicle must be less than the vehicle capacity.
3. Two or more second-echelon vehicles may begin their routes at the same satellite as long as the total demand of the customers they serve does not exceed the satellite's capacity.
4. A customer is assigned to at most one satellite and cannot be served by two second-echelon vehicles originating at the same or different satellites.
5. Each first-echelon vehicle starts from the main depot and serves one or more satellites and finishes its route at a satellite.
6. If a satellite is not opened, no customer may be served by that satellite. A satellite cannot be visited by more than one first-echelon vehicle (i.e. split deliveries are not allowed).
7. Time windows and satellite synchronization constraints are not considered.
8. A one-day planning horizon is considered. Locations are chosen and first echelon transportation occurs at night. Second echelon transportation takes place during the day.
9. A single, homogeneous product is being delivered.

Eight sets of decision variables have been defined in order to model the 2E-OLRP. These variables can be categorized into four main groups. The first group consists of the arc usage variables. X_{ij} , which is used in two index mathematical model P_1, is a binary variable that is equal to 1 if a vehicle travels directly from node *i* to *j* in the first echelon. Similarly, X_{ijk} that is used in three index models P_2 and P_3, is a binary variable that is equal to 1 if vehicle *k* travels directly from node *i* to *j* in the first echelon. Y_{ij} is again a binary variable that is used in P_1 and is equal to 1 if a second-echelon vehicle travels directly from node *i* to *j* while Y_{ijk} is a three index variable used in models P_2 and P_3 that is equal to 1 if vehicle *k* travels directly from node *i* to *j* in the second echelon. The second group of variables assign customers to satellites and activate the satellites. They are used in all three mathematical models. Z_i is a binary variable which is equal to 1 if satellite *i* is opened. V_{in} is a binary variable that is equal to 1 if customer *n* is assigned to satellite *i*. The third group of variables are related to the freight flows in the first and second echelon. U_{ij} and L_{ij} are the load carried by the vehicle when traveling directly from node *i* to *j* in the first and the second echelon respectively in two index mathematical model P_1. Similarly, U_{ijk} and L_{ijk} are the load carried by vehicle *k* when traveling directly from node *i* to *j* in the first and the second echelon respectively in three index mathematical models P_2 and P_3. The fourth group of the variables are defined to handle the open aspect of the problem. They are only used in mathematical models P_1 and P_2, but not P_3. δ_i is a binary variable which is equal to 1 if satellite *i* is the last satellite that is visited by a first-echelon vehicle and λ_i is a binary variable which is equal to 1 if customer *i* is the last customer that is visited by a second-echelon vehicle, both in mathematical model P_1. Likewise, δ_{ik} and λ_{ik} are used in three index mathematical model P_2. δ_{ik} is a binary variable which is equal to 1 if satellite *i* is the last satellite that is visited by a first-echelon vehicle *k* and λ_{ik} is a binary variable which is equal to 1 if customer *i* is the last customer that is visited by a second-echelon vehicle *k*. We use all groups of variables in mathematical model P_1 and P_2 but we only use the first three groups of variables in mathematical model P_3.

1.3.1. Mathematical model #1 (P_1)

The objective of P_1 is to minimize overall cost including vehicle traveling costs, vehicle hiring costs, and satellite opening costs. The MIP model for P_1 is as follows:

$$\text{Min} \quad \left(\sum_{i \in V_0 \cup V_s} \sum_{j \in V_s} X_{ij} C_{ij} + \sum_{i \in V_s \cup V_c} \sum_{j \in V_c} Y_{ij} C_{ij} \right) + \left(\sum_{i \in V_s} \delta_i F_1 + \sum_{i \in V_c} \lambda_i F_2 \right) + \sum_{i \in V_s} Z_i O_i \tag{1}$$

s.t.

$$\sum_{\substack{i \in V_0 \cup V_s \\ i \neq j}} X_{ij} = Z_j, \qquad \forall j \in V_s, \tag{2}$$

$$\sum_{j \in V_s} \delta_j \geq \sum_{j \in V_s} X_{ij}, \qquad \forall i \in V_0, \tag{3}$$

$$\sum_{\substack{i \in V_0 \cup V_s \\ i \neq j}} X_{ij} \leq \sum_{\substack{i \in V_s \\ i \neq j}} X_{ji} + M\delta_j, \qquad \forall j \in V_s, \tag{4}$$

$$\sum_{\substack{i \in V_0 \cup V_s \\ i \neq j}} X_{ij} \geq \sum_{\substack{i \in V_s \\ i \neq j}} X_{ji} - M\delta_j, \qquad \forall j \in V_s, \tag{5}$$

$$\sum_{\substack{j \in V_0 \cup V_s \\ i \neq j}} X_{ij} \leq (1 - \delta_i), \qquad \forall i \in V_s, \tag{6}$$

$$\delta_i \leq Z_i, \qquad \forall i \in V_s, \tag{7}$$

$$\sum_{i \in V_s} V_{ij} = 1, \qquad \forall j \in V_c, \tag{8}$$

$$\sum_{j \in V_c} D_j V_{ij} \leq Z_i Q_i^s, \qquad \forall i \in V_s \tag{9}$$

$$Z_i \le \sum_{n \in V_c} V_{in}, \qquad \forall i \in V_s \tag{10}$$

$$U_{ij} \le Q_1 X_{ij}, \qquad \forall i \in V_0 \cup V_s,\ \forall j \in V_s,\ i \ne j, \tag{11}$$

$$\sum_{i \in V_s} U_{ij} = 0, \qquad \forall j \in V_0, \tag{12}$$

$$\sum_{j \in V_s} U_{ij} = \sum_{n \in V_c} D_n, \qquad \forall i \in V_0, \tag{13}$$

$$\sum_{\substack{j \in V_0 \cup V_s \\ j \ne i}} U_{ji} = \sum_{\substack{j \in V_s \\ j \ne i}} U_{ij} + \sum_{n \in V_c} V_{in} D_n, \qquad \forall i \in V_s, \tag{14}$$

$$\sum_{\substack{i \in V_s \cup V_c \\ i \ne j}} Y_{ij} = 1, \qquad \forall j \in V_c, \tag{15}$$

$$\sum_{j \in V_c} \lambda_j \ge \sum_{j \in V_c} Y_{ij}, \qquad \forall i \in V_s, \tag{16}$$

$$\sum_{\substack{i \in V_s \cup V_c \\ i \ne j}} Y_{ij} \le \sum_{\substack{i \in V_c \\ i \ne j}} Y_{ji} + M\lambda_j, \qquad \forall j \in V_c, \tag{17}$$

$$\sum_{\substack{i \in V_s \cup V_c \\ i \ne j}} Y_{ij} \ge \sum_{\substack{i \in V_c \\ i \ne j}} Y_{ji} - M\lambda_j, \qquad \forall j \in V_c, \tag{18}$$

$$\sum_{\substack{j \in V_s \cup V_c \\ i \ne j}} Y_{ij} \le (1 - \lambda_i), \qquad \forall i \in V_c, \tag{19}$$

$$Y_{ij} = 0, \qquad \forall i \in V_s,\ j \in V_s,\ i \ne j, \tag{20}$$

$$L_{ij} \le Q_2 Y_{ij}, \qquad \forall i \in V_s \cup V_c,\ \forall j \in V_c,\ i \ne j, \tag{21}$$

$$\sum_{i \in V_c} L_{ij} = 0, \qquad \forall j \in V_s, \tag{22}$$

$$\sum_{j \in V_c} L_{ij} = \sum_{n \in V_c} V_{in} \times D_n, \qquad \forall i \in V_s, \tag{23}$$

$$\sum_{\substack{j \in V_s \cup V_c \\ i \ne j}} L_{ji} = \sum_{\substack{j \in V_c \\ i \ne j}} L_{ij} + D_i, \qquad \forall i \in V_c, \tag{24}$$

$$L_{ij} \le (Q_2 - D_i) Y_{ij}, \qquad \forall i \in V_c,\ \forall j \in V_c,\ i \ne j, \tag{25}$$

$$L_{ij} \ge D_j Y_{ij}, \qquad \forall i \in V_s \cup V_c,\ \forall j \in V_c,\ i \ne j, \tag{26}$$

Constraints (2) to (14) relate to the first echelon. Constraint (2) ensures that each satellite is visited exactly once if it's opened. Constraint (3) ensures that if a first-echelon vehicle is hired, then it must finish its route at a satellite. Constraints (4) and (5) are flow balance constraints. They ensure that the number of times the vehicle arrives at the satellite equals the number of times the vehicle departs from the satellite unless the satellite is the last satellite that is met on a route. If a satellite is the last satellite that is visited on a first-echelon route, then constraint (6) ensures that the vehicle does not travel to any other node after serving that satellite. Constraint (7) ensures that a satellite that is not opened cannot be considered as a final satellite. Constraint (8) ensures that each customer is assigned to exactly one satellite to be served. Constraint (9) ensures that the maximum customer demand that is assigned to a satellite is less than its capacity, if that satellite is opened. It also ensures that no customer is served by that satellite if it is not opened. Constraint (10) ensures that every opened satellite must serve at least one customer. Constraint (11) limits the maximum load carried by a vehicle while it travels from node *i* to *j*. It must be less than the vehicle capacity in the first level, Q_1. The load is also equal to zero if no vehicle travels from node *i* to *j*. Constraint (12) ensures that the load carried by any first-echelon vehicle from any satellite to the depot should be equal to zero. Constraint (13) ensures that the total load of all first-echelon vehicles that are starting their routes from the depot should be equal to the total demand of all customers. Constraint (14) ensures that the proper load is delivered to each satellite in the first echelon. Constraints (11) to (14) together eliminate first-echelon subtours.

Constraints (15) to (26) are related to the second echelon. Constraints (15) to (19) are similar to constraints (2) to (6) respectively. Constraint (15) ensures that each customer is visited exactly once. Constraint (16) ensures that if a second-echelon vehicle is hired, then it must finish its route at a customer. Constraints (17) and (18) are flow balance constraints. Constraints (17) and (18) ensure that the number of arriving routes to a customer should be equal to the number of departing routes from that customer, if a given customer is not the last customer visited by a given second-echelon vehicle. Constraint (19) ensures that a second-echelon vehicle does not travel to any other node after serving its last customer.

Constraint (20) forbids any routes traveling directly from a satellite to another satellite. Constraint (21) ensures that the load carried by a second-echelon vehicle is less than the vehicle capacity. The load is zero If no vehicle travels along a specific link. Constraint (22) ensures that the carried load from any customer to any satellite is zero. Constraint (23) ensures that the total load of all second-echelon vehicles that depart satellite *i* is equal to total demand of customers served by that satellite. Constraint (24) ensures that the combined load of all vehicles that depart customer *i* is equal to the combined load of all vehicles entering that node minus the demand of that customer. Like constraints (11) to (14), constraints (21) to (24) eliminate second-echelon subtours that do not contain a satellite. Constraint (25) ensures that the load carried by a second-echelon vehicle when leaving customer *i* never exceeds $Q_2 - D_i$. Constraint (26) ensures that the load carried by a second echelon vehicle just prior to its arrival at customer *j* is at least D_j. Decision variable domains are shown in Table 1.1.

1.3.2. Mathematical model #2 (P_2)

Similar to P_1, the objective of P_2 is to minimize overall cost which consists of three parts: vehicle traveling costs, vehicle hiring costs, and satellite opening costs. Our MIP model for P_2 is as follows:

$$\text{Min} \quad \left(\sum_{k\in K_1} \sum_{i\in V_0\cup V_s} \sum_{j\in V_s} X_{ijk} \times C_{ij} + \sum_{k\in K_2} \sum_{i\in V_s\cup V_c} \sum_{j\in V_c} Y_{ijk} \times C_{ij} \right) + \left(\sum_{k\in K_1} \sum_{i\in V_s} \delta_{ik} \times F_1 + \sum_{k\in K_2} \sum_{i\in V_c} \lambda_{ik} \times F_2 \right) + \sum_{i\in V_s} Z_i \times O_i \quad (27)$$

s.t.

$$\sum_{k\in K_1} \sum_{\substack{i\in V_0\cup V_s \\ i\neq j}} X_{ijk} = Z_j, \qquad \forall j\in V_s, \quad (28)$$

$$\sum_{j\in V_s} X_{ijk} \le 1, \qquad \forall i\in V_0\cup V_s,\ k\in K_1, \quad (29)$$

$$\sum_{i\in V_s} \delta_{ik} \le 1, \qquad \forall k\in K_1, \quad (30)$$

$$\sum_{k\in K_1} \delta_{ik} \le 1, \qquad \forall i\in V_s, \quad (31)$$

$$\sum_{j\in V_s} \delta_{jk} \ge \sum_{j\in V_s} X_{ijk}, \qquad \forall i\in V_0,\ k\in K_1, \quad (32)$$

$$\sum_{\substack{i\in V_0\cup V_s \\ i\neq j}} X_{ijk} \le \sum_{\substack{i\in V_s \\ i\neq j}} X_{jik} + M\delta_{jk}, \qquad \forall j\in V_s,\ k\in K_1, \quad (33)$$

$$\sum_{\substack{i\in V_0\cup V_s \\ i\neq j}} X_{ijk} \ge \sum_{\substack{i\in V_s \\ i\neq j}} X_{jik} - M\delta_{jk}, \qquad \forall j\in V_s,\ k\in K_1, \quad (34)$$

$$\sum_{\substack{j\in V_0\cup V_s \\ i\neq j}} X_{ijk} \le (1-\delta_{ik}), \qquad \forall i\in V_s,\ k\in K_1, \quad (35)$$

$$\delta_{ik} \le Z_i, \qquad \forall i\in V_s,\ k\in K_1, \quad (36)$$

$$\sum_{i\in V_s} V_{ij} = 1, \qquad \forall j\in V_c, \quad (37)$$

$$\sum_{j\in V_c} D_j \times V_{ij} \le Z_i \times Q_i^s, \qquad \forall i\in V_s \quad (38)$$

$$Z_i \le \sum_{n\in V_c} V_{in}, \qquad \forall i \in V_s \tag{39}$$

$$U_{ijk} \le Q_1 X_{ijk}, \qquad \forall i \in V_0 \cup V_s,\ \forall j \in V_s,\ i \ne j,\ k \in K_1, \tag{40}$$

$$\sum_{i\in V_s} U_{ijk} = 0, \qquad \forall j \in V_0,\ k \in K_1, \tag{41}$$

$$\sum_{k\in K_1} \sum_{j\in V_s} U_{ijk} = \sum_{n\in V_c} D_n, \qquad \forall i \in V_0, \tag{42}$$

$$\sum_{k\in K_1} \sum_{\substack{j\in V_0\cup V_s \\ j\ne i}} U_{jik} = \sum_{k\in K_1} \sum_{\substack{j\in V_s \\ j\ne i}} U_{ijk} + \sum_{n\in V_c} V_{in} D_n, \qquad \forall i \in V_s, \tag{43}$$

$$\sum_{k\in K_2} \sum_{\substack{i\in V_s\cup V_c \\ i\ne j}} Y_{ijk} = 1, \qquad \forall j \in V_c, \tag{44}$$

$$\sum_{j\in V_c} Y_{ijk} \le 1, \qquad \forall i \in V_s \cup V_c,\ k \in K_2, \tag{45}$$

$$\sum_{i\in V_c} \lambda_{ik} \le 1, \qquad \forall k \in K_2, \tag{46}$$

$$\sum_{k\in K_2} \lambda_{ik} \le 1, \qquad \forall i \in V_c, \tag{47}$$

$$\sum_{j\in V_c} \lambda_{jk} \ge \sum_{j\in V_c} Y_{ijk}, \qquad \forall i \in V_s,\ k \in K_2, \tag{48}$$

$$\sum_{\substack{i\in V_s\cup V_c \\ i\ne j}} Y_{ijk} \le \sum_{\substack{i\in V_c \\ i\ne j}} Y_{jik} + M\lambda_{jk}, \qquad \forall j \in V_c,\ k \in K_2, \tag{49}$$

$$\sum_{\substack{i\in V_s\cup V_c \\ i\ne j}} Y_{ijk} \ge \sum_{\substack{i\in V_c \\ i\ne j}} Y_{jik} - M\lambda_{jk}, \qquad \forall j \in V_c,\ k \in K_2, \tag{50}$$

$$\sum_{\substack{j\in V_s\cup V_c \\ i\ne j}} Y_{ijk} \le (1-\lambda_{ik}), \qquad \forall i \in V_c,\ k \in K_2, \tag{51}$$

$$Y_{ijk} = 0, \qquad \forall i \in V_s,\ j \in V_s,\ i \ne j, k \in K_2, \tag{52}$$

$$\sum_{j\in V_c} Y_{ijk} + \sum_{j\in V_s\cup V_c} Y_{jnk} \le 1 + V_{in}, \qquad \forall i \in V_s, \forall n \in V_c, \forall k \in K_2, \tag{53}$$

$$L_{ijk} \le Q_2 \times Y_{ijk}, \qquad \forall i \in V_s \cup V_c,\ \forall j \in V_c,\ i \ne j,\ k \in K_2, \tag{54}$$

$$\sum_{i\in V_c} L_{ijk} = 0, \qquad \forall j \in V_s,\ k \in K_2, \tag{55}$$

$$\sum_{k \in K_2} \sum_{j \in V_c} L_{ijk} = \sum_{n \in V_c} V_{in} \times D_n, \qquad \forall i \in V_s, \tag{56}$$

$$\sum_{k \in K_2} \sum_{\substack{j \in V_s \cup V_c \\ i \neq j}} L_{jik} = \sum_{k \in K_2} \sum_{\substack{j \in V_c \\ i \neq j}} L_{ijk} + D_i, \qquad \forall i \in V_c, \tag{57}$$

$$L_{ijk} \leq (Q_2 - D_i) \times Y_{ijk}, \qquad \forall i \in V_c,\ \forall j \in V_c,\ i \neq j,\ k \in K_2, \tag{58}$$

$$L_{ijk} \geq D_j \times Y_{ijk}, \qquad \forall i \in V_s \cup V_c,\ \forall j \in V_c,\ i \neq j,\ k \in K_2, \tag{59}$$

Constraints (28) to (43) are related to the first echelon. Constraint (28) ensures that exactly one (zero) first-echelon vehicle arrives at each opened (closed) satellite. Constraint (29) ensures that each first-echelon vehicle travels to at most one location from any given location. Constraint (30) ensures that each first-echelon vehicle serves at most one satellite as its final satellite. Constraint (31) ensures that each satellite can be the final satellite in at most one vehicle's route. Constraint (32) ensures that if a first-echelon vehicle is hired, then it must finish its route at a satellite. Constraints (33) and (34) are flow balance constraints. If a given satellite is not the last satellite visited by a given first-echelon vehicle, then constraints (33) and (34) ensure that the number of times the vehicle arrives at the satellite equals the number of times the vehicle departs from the satellite. Constraint (35) ensures that each first-echelon vehicle does not travel to any other node after serving its last satellite. Constraint (36) ensures that a satellite is not considered as a final satellite if it is not opened. Constraint (37) ensures that each customer is served by exactly one satellite.

Constraint (38) is the capacity constraint for opened satellites. It states that if a satellite is opened, the maximum customer demand that can be served from that satellite is less than the satellite's capacity. It also ensures that if a satellite is not opened, no customer is served by that satellite. Constraint (39) ensures that every opened satellite serves at least one customer. Constraint (40) limits the maximum load carried by a vehicle while it travels from node i to j. It must be less than the vehicle capacity in the first level, Q_1. Also, if no vehicle travels along a specific link, the load carried on that link is zero. Constraint (41) ensures that the load carried by any first-echelon vehicle from any satellite to the depot should be equal to zero. Constraint (42) ensures that the combined load of all first-echelon vehicles when starting their routes from the depot should be equal to the total demand of all customers. Constraint (43) ensures that the proper load is delivered to each satellite in the first echelon. Constraints (40) to (43) together eliminate first-echelon subtours not containing the depot.

Constraints (44) to (59) are related to the second echelon. Constraint (44) ensures that exactly one second-echelon vehicle arrives at each customer. Constraint (45) ensures that each second-echelon vehicle travels to at most one location from any given location. Constraint (46) ensures that each second-echelon vehicle serves at most one customer as its final customer. Constraint (47) ensures that each customer is the final customer in at most one vehicle's route. Constraint (48) ensures that if a second-echelon vehicle is hired, then it must finish its route at a customer. Constraints (49) and (50) are flow balance constraints. If a given customer is not the last customer visited by a given second-echelon vehicle, then constraints (49) and (50) ensure that the number of times the vehicle arrives at the customer equals the number of times the vehicle departs from the customer. Constraint (51) ensures that each second-echelon vehicle does not travel to any other node after serving its last customer.

Constraint (52) forbids second-echelon vehicles from traveling directly from a satellite to another satellite. Constraint (53) ensures that if customer n is not served by satellite i, there is no route connecting satellite i to customer n. Constraints (54) to (57) are similar to constraints (40) to (43) respectively. Constraint (54) ensures that the load carried by each second-echelon vehicle is less than the vehicle capacity. Also, if no vehicle travels along a specific link, the load carried on that link is zero. Constraint (55) ensures that the load carried by any second-echelon vehicle from any customer to any satellite is zero. Constraint (56) ensures that the combined load of all second-echelon vehicles that depart satellite i is equal to total demand of customers served by that satellite. Constraint (57) ensures that the proper amount of freight is delivered to each customer. In other words, the combined load of all vehicles that depart customer *i* is equal to the combined load of all vehicles arriving at that customer minus the demand of that customer. Constraints (54) to (57) eliminate second-echelon subtours that do not contain a satellite. Constraint (58) ensures that the load carried by a second-echelon vehicle when departing customer *i* never exceeds $Q_2 - D_i$. Constraint (59) ensures that the load carried by a second echelon vehicle just prior to its arrival at customer *j* is at least D_j.

1.3.3. Mathematical model #3 (P_3)

As mentioned before, we consider a closed 2E-LRP for P_3 except the distances (C_{ij}) from satellites to the depot (in the first echelon) and from customers to satellites (in the second echelon) equal zero. The MIP model P_3 is as follows:

$$\text{Min} \left(\sum_{k \in K_1} \sum_{i \in V_0 \cup V_s} \sum_{j \in V_0 \cup V_s} X_{ijk} \times C_{ij} + \sum_{k \in K_2} \sum_{i \in V_s \cup V_c} \sum_{j \in V_s \cup V_c} Y_{ijk} \times C_{ij} \right) + \left(\sum_{k \in K_1} \sum_{i \in V_0} \sum_{j \in V_s} X_{ijk} \times F_1 + \sum_{k \in K_2} \sum_{i \in V_s} \sum_{j \in V_c} Y_{ijk} \times F_2 \right) + \sum_{i \in V_s} Z_i \times O_i \tag{60}$$

s.t.

(28)-(29), (37)-(45), (52)-(59)

$$\sum_{\substack{i \in V_0 \cup V_s \\ i \neq j}} X_{ijk} = \sum_{\substack{i \in V_0 \cup V_s \\ i \neq j}} X_{jik}, \qquad \forall j \in V_0 \cup V_s,\ k \in K_1, \tag{61}$$

$$\sum_{\substack{i \in V_s \cup V_c \\ i \neq j}} Y_{ijk} = \sum_{\substack{i \in V_s \cup V_c \\ i \neq j}} Y_{jik}, \qquad \forall j \in V_s \cup V_c,\ k \in K_2, \tag{62}$$

The objective function (60) consists of three parts: vehicle traveling costs, vehicle hiring costs, and satellite opening costs. Constraint (61) ensures that each first-echelon vehicle that enters a node also departs from that node; this flow balance constraint ensures that all first-echelon vehicle routes are cyclic. Constraint (62) ensures that each second-echelon vehicle that enters a node also departs from that node; this flow balance constraint ensures all second echelon routes are cyclic.

1.4. Hybrid heuristic for the 2E-OLRP

Since the 2E-OLRP is an NP-hard problem, the proposed mathematical models are not able to find optimal solutions for some medium- and large-sized problems. Thus, heuristic methods are needed to find good solutions in a reasonable amount of time. A two-phase heuristic algorithm is proposed in this study. In the first phase, called the "*Satellite location and first echelon routing*" phase, two major decisions are made: (1) the set of opened satellites and the assignments of customers to those opened satellites is decided and (2) the first-echelon vehicle routes are constructed. A simulated annealing (SA) algorithm is used as a local search engine in this phase to improve solutions. In the second phase of the algorithm, called the "*second echelon routing*" phase, the vehicle routes in the second echelon are decided by another SA algorithm based on the solution from the first phase. However, before the SA in the second echelon starts, a good initial solution for the second echelon routes is made using a modified Clarke and Wright algorithm (CWA). In every iteration of the first phase SA, the second phase SA runs for many iterations to find the best possible routing solution for the second echelon based on the solution from the first phase of the algorithm.

Based on Nagy and Salhi (2007), the proposed algorithm in this paper is in the class of hierarchal heuristics. Karaoglan et al. (2012) and Wu et al. (2002) are two studies in the LRP literature that use this type of heuristic. SA is a stochastic search method, which is adopted from the annealing process of materials in physics (Kirkpatrick et al. 1983). SA is a widely-used method for finding good solutions to NP-hard combinatorial problems (Vincent and Lin 2015, Vincent et al. 2016). However, as we describe in the following subsections, the proposed heuristic in this study differs from the previous ones in both the problem that the heuristic is applied to and the type of neighborhood structures used.

Section 1.4.1 presents the solution representation used in our heuristic method. Section 1.4.2 presents our procedure for finding an initial feasible solution. Section 1.4.3 describes the first phase of the algorithm, called the satellite location and first echelon routing phase. Section 1.4.4 describes the second phase, called the second echelon routing phase. The steps of the entire heuristic are discussed in detail in Section 1.4.5.

1.4.1. Solution representation

In our heuristic method, feasible solutions are represented using (a) a matrix defining the first-echelon routes, (b) a customer-to-satellite assignment matrix and (c) several matrices defining the second-echelon routes. Figure 1.2 displays the solution representation for the example presented in Figure 1. Figure 2b shows the customer-to-satellite assignment matrix. Here, the first column in each row represents the satellites and the numbers to the right of each satellite in each row represent the customers assigned to that satellite. Based on the presented example, customers 7, 5, 10, 12 are assigned to satellite 2; customers 11, 8, 6 and 9 are assigned to satellite 4; and customers 14 and 13 are assigned to satellite 3. There is no customer assigned to satellite 1, which is not opened. Furthermore, Figure 2a indicates the routes, from the main depot, which is shown by (0), to the satellites in the first echelon. For instance, the first route in the first echelon is (0), (2), (4) and the second route is (0), (3). The routes start at the depot and end at a satellite because in the 2E-OLRP the vehicles are not required to return to the depot. Also, there is a matrix dedicated to each opened satellite in Figure 2c, which shows the second echelon vehicle routes which start at satellites and end at customers. This representation is very easy to implement (Ghaffari-Nasab et al. 2013).

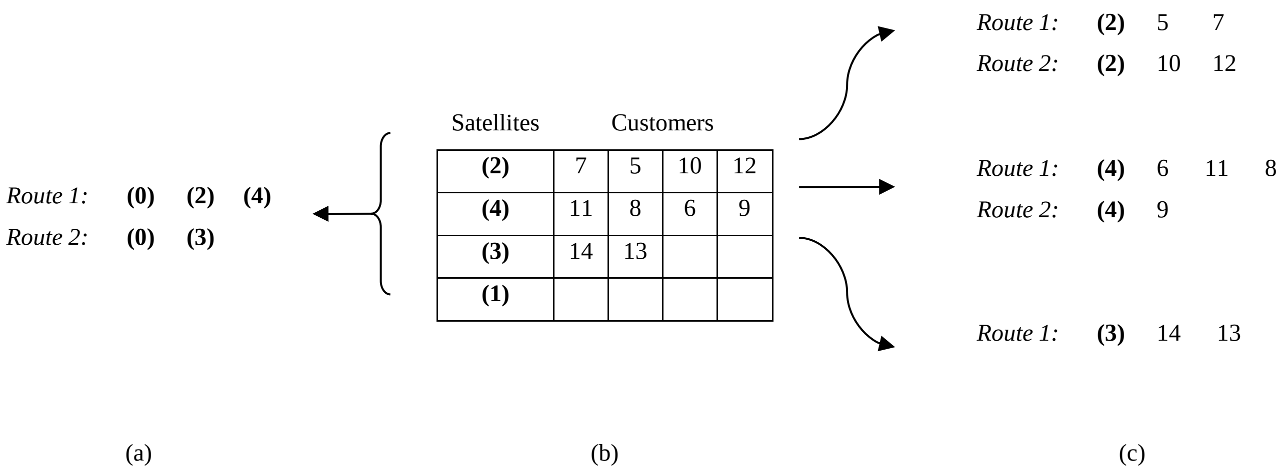


Figure 1.2. Solution representation used in heuristic method. (a) first-echelon vehicle routes, (b) customer-to-satellite assignment, (c) second-echelon vehicle routes.

1.4.2. Initial solution

Starting heuristic algorithms with high quality initial solutions can save computational time. In each phase of our algorithm we do not begin with a randomly generated initial solution. Rather, we use an intelligent heuristic approach to generate initial solutions. Our method for creating an initial feasible solution consists of the same two phases described at the beginning of Section 1.4. In the first phase we use a math-based heuristic based on a facility location problem and in the second phase we use a modified version of the Clarke and Wright algorithm. These approaches are briefly described below.

Math-based Heuristic (MH): The first phase of the proposed heuristic assigns customers to satellites based on the capacity of the satellites and the direct distance between customers and satellites and then generates routes for the first echelon based on this assignment. Note that if we only consider the assignment of customers to satellites and ignore routing issues, the 2E-OLRP reduces to a capacitated facility location problem (FLP). After assigning customers based on the MH, we create the first echelon routes randomly. The objective of the FLP is to minimize the satellite opening costs and the total direct distance from satellites to customers while respecting satellite capacities. Based on the notation and decision variables in Table 1.1, the formulation for problem MH is shown below:

$$\text{Min} \quad \sum_{i \in V_s} \sum_{j \in V_c} V_{ij} \times C_{ij} + \sum_{i \in V_s} Z_i \times O_i \tag{63}$$

s.t.

$$\sum_{i \in V_s} V_{ij} = 1, \qquad \forall j \in V_c \tag{64}$$

$$\sum_{j \in V_c} D_j \times V_{ij} \le Z_i \times Q_i^s, \qquad \forall i \in V_s \tag{65}$$

$$Z_i \in \{0,1\}, \qquad \forall i \in V_s, \tag{66}$$

$$V_{ij} \in \{0,1\}, \qquad \forall i \in V_s,\ \forall j \in V_c, \tag{67}$$

In the above formulation, the objective function (63) minimizes the total cost which is the sum of traveling or customer assignment costs and satellite opening costs. Constraint (64) ensures that each customer is assigned to exactly one satellite. Constraint (65) guarantees that the total demand of the customers assigned to a satellite is less than the satellite's capacity and it ensures that customers are only assigned to opened satellites. Constraints (66) and (67) are binary constraints on the decision variables. Problem MH —a facility location problem— is NP-hard. However, in preliminary experiments we were able to optimally solve instances with up to 200 customers quickly using IBM ILOG CPLEX 12.5. Therefore, in phase one of our algorithm, we use CPLEX to solve problem MH to optimality and then we generate the first echelon routes randomly after that.

Modified Clarke and Wright Heuristic (MCWH): In order to generate an initial solution for the second phase of the algorithm, which is the routing decision for the second echelon, the well-known Clarke and Wright heuristic (Clarke and Wright, 1964) is used. This heuristic begins with customer-to-satellite assignments that are determined in the first phase of the algorithm. The MCWH then gives the number of vehicles needed for each satellite and the assignment of customers to the second-echelon vehicles that depart each satellite. Note that, in the 2E-OLRP, the routes are open and vehicles are not required to return to satellites after their last delivery. Thus, the classical Clarke and Wright heuristic is modified based on the open route assumption. In particular, in our modified Clarke and Wright heuristic, the customers in each route are visited in order of increasing distance (C_{ij}) from the satellite that serves them. Thus, the closest customer to the satellite is served first in that route and the farthest customer is served last.

1.4.3. Satellite location and first echelon routing phase

In this phase, six neighborhood structures are implemented to search a large part of the solution space through a SA algorithm. Five of these six structures are related to the satellite location and customer assignment decisions, and one is related to the first echelon routing. These structures are described below:

- *Closing an opened satellite and opening a closed one*: An opened satellite is chosen based on a probability. The more customers assigned to a satellite, the greater probability it is chosen to be closed. A closed satellite is also chosen randomly, and all the customers from the selected opened satellite are transferred to the selected closed satellite. This neighborhood structure investigates different combinations of opened satellites.
- *Closing an opened satellite*: The opened satellite with the minimum number of assigned customers is selected and all of its customers are transferred to another opened satellite after checking the satellite capacity. The opened satellites are sorted according to increasing opening cost and checked one by one to find an opened satellite with enough capacity. This neighborhood structure decreases satellite opening costs if possible.

- *Opening a closed satellite*: An opened satellite is chosen based on a probability that is proportional to the number of customers assigned to it and a random number of its customers are assigned to a randomly selected closed satellite after checking the satellite capacity constraint.
- *Changing all customers of two opened satellites*: Two opened satellites are chosen randomly and their customers are swapped if satellite capacity allows.
- *Changing some customers of two opened satellites*: Two opened satellites are chosen randomly and a random number of their customers are swapped if the satellite capacity constraint allows. Each satellite accepts the same number of new customers from the other satellite.
- *Swapping two opened satellites in the first echelon routes*: Two opened satellites are chosen randomly from the same or different first-echelon routes and are swapped between their respective positions in their respective routes. The customer assignment is not changed. This neighborhood structure searches for different first-echelon routing options.

1.4.4. Second echelon routing phase

In the second phase of the heuristic, we use six major neighborhood structures which are well-known in the VRP and LRP literature (Karaoglan et al. 2012). These are listed below. Note that several of these neighborhoods - including swap, insert, 2-Opt, move, and merge – may change the customer-to-satellite assignments that are initially decided during the first phase of the heuristic. As described below, there are three varieties of each of the first three neighborhoods and one variety of each of the last three neighborhoods.

- *Swap*: Two customers from (a) the same route, or from different routes which originate (b) from the same or (c) different satellites, are swapped between their respective routes.

- *Insert*: One customer is randomly selected and is inserted into a new random position in the (a) same route or in another route which is connected to (b) the same satellite or (c) another satellite.

- *2-Opt*: Two customers are selected randomly and the path connecting these two customers is reversed if (a) the customers are in the same route. If they are from different routes originating from (b) the same or (c) different satellites, the second parts of the routes containing these two customers are swapped. That is, the second part of the route containing the first customer is linked to the first part of the route containing the second customer, and the second part of the route containing the second customer is linked to the first part of the route containing the first customer. The inter-tour *2-opt* neighborhood is commonly known as the *2-opt-star* (*2-opt**).

- *3-Opt*: Three ordered customers in the same route are selected randomly, and the sequence from the second to the third customer is moved to the position that immediately follows the first selected customer.

- *Move*: A route is randomly selected and its origin is changed to the satellite that is closest to the first customer of that route. If the closest satellite to the first customer has no capacity, the second closest satellite is chosen, and so on. This will continue until a new feasible solution is found (if any).

- *Merge*: Two routes originating at the same satellite or different satellites are randomly selected and they are merged together; the first route is added to the end of the second route or vice versa depending on which option has the minimum cost.

1.4.5. General structure of the proposed heuristic

The complete logic of our proposed heuristic for the 2E-OLRP is shown in Table 1.2. The process begins by constructing an initial feasible solution ($S_{Current}$) using the MH and MCWH. During each iteration of the main SA algorithm, one of the first phase neighborhood structures is chosen randomly with the same probability (1/6) to generate a new customer-to-satellite assignment and/or new first echelon vehicle routes. Then the MCWH is used to generate initial second-echelon vehicle routes. This new solution becomes the initial solution ($S^2_{Current}$) for the second phase of the algorithm. In the second phase, a SA subroutine uses the six second-phase neighborhood structures to generate second-phase neighbors (S^2_{Next}) and thereby improve $S^2_{Current}$. In particular, during each iteration of the SA subroutine, twelve neighbors are generated by applying all twelve varieties of the six, second-phase neighborhood structures, and the neighbor with the best (minimum) objective function is chosen as the candidate solution (S^2_{Next}) which is compared to $S^2_{Current}$.

The best solution (S^2_{Best}) from the second phase of the algorithm becomes the neighboring solution (S_{Next}) in the main SA algorithm. This neighbor is then compared to the current solution ($S_{Current}$) in terms of the objective function. If it is better, it replaces $S_{Current}$. If not, it replaces $S_{Current}$ with probability $\exp(-\frac{Obj_{Next} - Obj_{Current}}{T})$. After each iteration of the main SA, the temperature, T is multiplied by a cooling rate called α ($0 < \alpha < 1$). The SA algorithm starts with an initial temperature ($T_{Initial}$) and stops when the temperature is less than T_{Final}.

Table 1.2. Pseudocode of proposed heuristic

Capture input parameter values for 2E-OLRP instance from text file;

$T \leftarrow T_{Initial}$;

Generate an initial solution ($S_{Current}$) and compute initial objective value ($Obj_{Current}$) using MH and MCWH;

$Obj_{Best} \leftarrow Obj_{Current}$, $S_{Best} \leftarrow S_{Current}$;

While ($T > T_{Final}$) **do**

  Set $Iter = 0$;

  While ($Iter < NumIter$) **do**

    Generate a random number from one to six ($RandNum_1$);

    Switch ($RandNum_1$)

    Case 1: Generate next solution by *Closing an opened satellite and opening a closed one*

    Case 2: Generate next solution by *Closing an opened satellite*

    Case 3: Generate next solution by *Opening a closed satellite*

    Case 4: Generate next solution by *Changing all customers of two opened satellites*

    Case 5: Generate next solution by *Changing some customers of two opened satellites*

    Case 6: Generate next solution by *Swapping two opened satellites*

    Generate a full initial solution ($S^2_{Current}$) for second echelon routing using MCWH and compute its objective value ($Obj^2_{Current}$);

    $T^2 \leftarrow T^2_{Initial}$;

    $Obj^2_{Best} \leftarrow Obj^2_{Current}$, $S^2_{Best} \leftarrow S^2_{Current}$;

    While ($T^2 > T^2_{Final}$) **do**

      $Iter^2 \leftarrow 0$;

      While ($Iter^2 < NumIter^2$) **do**

        Create a pool of twelve neighboring solutions as follows:

        Three neighbors are generated by three types of *Swap* operator;

        Three neighbors are generated by three types of *Insert* operator;

        Three neighbors are generated by three types of *2-Opt* operator;

        One neighbor is generated by *3-Opt* operator;

        One neighbor is generated by *Move* operator;

        One neighbor is generated by *Merge* operator;

        Select the best solution (S^2_{Next}) in the pool and compute its objective value (Obj^2_{Next});

        If ($Obj^2_{Next} < Obj^2_{Current}$) **Then**

          $S^2_{Current} \leftarrow S^2_{Next}$, $Obj^2_{Current} \leftarrow Obj^2_{Next}$

        Else

          Generate a random number between 0 and 1 ($RandNum_2$);

          If ($RandNum_2 < \exp(-(Obj^2_{Next} - Obj^2_{Current})/T^2)$) **Then**

            $S^2_{Current} \leftarrow S^2_{Next}$, $Obj^2_{Current} \leftarrow Obj^2_{Next}$;

        If ($Obj^2_{Current} < Obj^2_{Best}$) **Then**

          $S^2_{Best} \leftarrow S^2_{Current}$, $Obj^2_{Best} \leftarrow Obj^2_{Current}$;

        $Iter^2 \leftarrow Iter^2 + 1$;

      $T^2 \leftarrow T^2 * \alpha$;

    $S_{Next} \leftarrow S^2_{Best}$, $Obj_{Next} \leftarrow Obj^2_{Best}$;

    If ($Obj_{Next} < Obj_{Current}$) **Then**

      $Obj_{Current} \leftarrow Obj_{Next}$, $S_{Current} \leftarrow S_{Next}$;

    Else

      Generate a random number between 0 and 1 ($RandNum_3$);

      If ($RandNum_3 < \exp(-(Obj_{Next} - Obj_{Current})/T)$) **Then**

        $Obj_{Current} \leftarrow Obj_{Next}$, $S_{Current} \leftarrow S_{Next}$;

    If ($Obj_{Current} < Obj_{Best}$) **Then**

      $Obj_{Best} \leftarrow Obj_{Current}$, $S_{Best} \leftarrow S_{Current}$;

    $Iter \leftarrow Iter + 1$;

  $T \leftarrow T * \alpha$;

Return S_{Best} and Obj_{Best}

1.5. Numerical experiments

The presented mathematical formulations and heuristic method were coded into Microsoft Visual C++ 2010 Professional. IBM ILOG Concert Technology was used to define the model within C++ and call the mixed integer linear programming solver IBM ILOG CPLEX 12.5 to solve instances within the Windows 7 environment on a Dell desktop computer with an Intel Core i7, 2.6 GHz processor and 16 GB of RAM. Text files defining all problem instances used in this paper are available from the authors upon request.

1.5.1. Test instances[*]

The above math model and heuristic method were tested on four data sets (i.e. sets of problem instances) from the literature. The first data set, from Nguyen et al. (2012a) includes 24 2E-LRP instances which can also be used for our problem, the 2E-OLRP, which has open routes. In these instances, there are 5 to 10 satellites and 25, 50, 100, or 200 customers. More details regarding the generation of these problem instances are provided in Nguyen et al. (2012a).

The second data set is also from Nguyen et al. (2012a). These instances were originally created by Prodhon (2006) for the CLRP and were transformed into 2E-LRP instances by considering a central depot at coordinates (0,0) and assuming the depots in the original instance are satellites in the 2E-LRP. Similarly, they can also be used for the 2E-OLRP. There are 30 such instances with 20-200 customers and 5-10 satellites.

The third data set contains 13 2E-OLRP instances, that we adopt from the CLRP instances of Barreto (2004). This is done by considering the depots in the original instances as satellites; creating a central depot at coordinate (0,0); and assuming that distances in the first

[*] All the instances were downloaded from http://prodhonc.free.fr

echelon from the central depot to satellites are twice the Euclidian distances, rounded up to the nearest integer. Also, the vehicle capacity in the first echelon is 1.5 times the maximum satellite capacity. Furthermore, the fixed vehicle cost in the first echelon is five times the vehicle cost in the second echelon. The number of satellites ranges from 2 to 14, and the number of customers ranges from 8 to 318. All costs including vehicle costs and satellite opening costs are considered integer for the sake of simplicity and for comparison with future studies. In case any costs are not integral in the original instance, they are rounded up.

1.5.2. Parameter settings

Parameter calibration is an important issue that can affect the results. Based on a set of preliminary experiments, we decided to assign parameter values based on three problem sizes: small (less than 50 nodes), medium (50-100 nodes), and large (more than 100 nodes). We consider two possible values for each parameter that appears in Table 1.2. In particular, we consider the values 1 and 5 for $NumIter$ and $NumIter^2$; 300 and 400 for $T_{Initial}$; 200 and 250 for $T^2_{Initial}$; 0.98 and 0.99 for α; and 0.4 and 0.5 for T_{Final} and T^2_{Final}. Based on a set of preliminary experiments, T_{Final} and T^2_{Final} were both fixed to 0.5 since they did not have any effect on the quality of the solutions. To find the best parameter values for each problem size, we ran the algorithm five times on one benchmark instance for each problem size using each of the 32 possible combinations of parameter values. The combination of parameter values with the minimum objective function was selected as the best setting for each problem size. Table 1.3 shows the parameter settings for the three problem sizes.

Table 1.3. Parameter settings for heuristic

	$NumIter$	$NumIter^2$	$T_{Initial}$	$T^2_{Initial}$	α	T_{Final}	T^2_{Final}
Small-size	1	1	400	200	0.98	0.5	0.5
Medium-size	1	5	400	200	0.99	0.5	0.5
Large-size	5	5	400	250	0.99	0.5	0.5

1.5.3. Computational results

The efficiency of the proposed heuristic is tested in this section using four different predesigned datasets from the literature. Also, the proposed mathematical programs are solved using CPLEX with a two-hour time limit. Due to the complexity of the problem, in most cases CPLEX is not able to find an optimal solution. Therefore, in order to evaluate the quality of solutions obtained by the heuristic approach, the LP relaxations of the 2E-OLRP instances are also solved using CPLEX.

Since the 2E-OLRP is a new problem, there are no results from the literature to which we can compare the heuristic's performance. Thus, the proposed heuristic is also tested on the 2E-LRP instances in the literature by changing the heuristic algorithm based the assumptions in the 2E-LRP.

Table 1.4 shows the results when the heuristic and CPLEX are tested on the instances in Nguyen's dataset using formulations P_1, P_2 and P_3. In this table, the first column shows the name of the problem instance. Columns 2-5 show the results from CPLEX for P_1. Column "LB" shows the lower bound on the optimal objective value found by CPLEX at termination. Column "UB" shows the objective value of the best feasible solution found by CPLEX within the time limit. Column 4 shows the CPU runtime in seconds used by CPLEX. The fifth column "Gap (%)" shows the gap, $\left(\frac{UB-LB}{UB}\right)$, that exists after the time limit is reached. Columns 6-9 and 10-13 are the same as columns 2-5 but show the results for formulation P_2 and P_3, respectively. Results for the proposed heuristic are shown in the last three columns. Column

“Cost” shows the average of the objective values found in five runs of the heuristic. Column “Time” shows the average CPU runtime used in five runs of the heuristic. The last column “Gap (%)” shows the gap between the results of the hybrid SA heuristic and the highest lower bound obtained by CPLEX using P_1 or P_2 according to the following formula: $\left(\frac{HybridSA_{Ave} - LB_{best}}{HybridSA_{Ave}}\right)$. The first term in the numerator is taken from the “Ave.” column.

As shown in Table 1.4, only four of 24 instances (shown in bold) are optimally solved by CPLEX using P_2 or P_3 but six are optimally solved using P_1. For other sixteen instances CPLEX can find a feasible but not provably optimal solution within the predefined time limit using P_1, P_2 or P_3. Overall, formulation P_1 performs significantly better than P_2 and P_3 considering the average “Gap (%)” and final lower bound. P_1 also finds optimal solutions for two medium-sized problems, 50-5N and 50-5MNb, where P_2 and P_3 are not able to find such solutions. The results also show that the heuristic finds the optimal solution for small-sized instances and good solutions for the medium- and large-sized instances in a reasonable amount of time. Figure 1.3 provides a visualization of the best solution found by the heuristic for a relatively large problem, 50-5MN, with 13.12% gap. As shown in the figure, the heuristic finds a good solution for the problem in a reasonable amount of time. Moreover, the average amount of “Gap (%)” for this dataset is 7.23% which shows the strength of the proposed heuristic.

Table 1.5 has the same structure as Table 4 for Prodhon’s dataset. Here, once again, formulation P_1 performs significantly better than P_2 and P_3 in obtaining better feasible solutions, tighter lower bounds and a smaller “Gap%”. CPLEX and the heuristic generally find the optimal solution for the small instances using P_1, P_2 or P_3. One should note that the P_1 solved two medium instances optimally where P_2 and P_3 did not. However, the proposed heuristic is much faster than CPLEX. For the medium- and large-sized instances, CPLEX is not able to find any feasible solution but the heuristic approach finds good quality solutions compared to the lower bound obtained by CPLEX. The average "Gap (%)" of 5.56% shows that the proposed heuristic has good performance in a reasonable amount of time (about a minute). Note that for some large-sized instances, even the LP relaxation cannot be solved to optimality within the two-hour time limit using CPLEX.

Table 1.6 shows the results for the mathematical models and heuristic for Barreto's dataset. Here, CPLEX is not able to find feasible solutions for most of the large-sized instances. However, both CPLEX and the heuristic find optimal solutions for the small- and medium-sized instances. The time used by heuristic and 2.82% average "Gap (%)" demonstrate the efficiency of the proposed heuristic. Similar to the results from Tables 1.4-1.5, formulation P_1 performs significantly better than P_2 and P_3 by finding better lower bounds and feasible solutions. P_1 solves one medium instance optimally and find better feasible solutions than P_2 and P_3 for two other medium instances.

Overall, based on the results from these data sets, we can conclude that P_1 and P_2, which are new proposed methods to model open vehicle routing problems, performs significantly better than P_3, which is the most common technique in the literature for modeling open problems. One should note that P_1, which is a two index mathematical model, also outperforms P_2 which is a three index mathematical model. Contrary to the 2E-LRP, the 2E-OLRP is an asymmetric routing problem. Considering this fact, two new sets of decision variables, δ_i and λ_i and, δ_{ik} and λ_{ik}, are proposed in P_1 and P_2, respectively, that are appropriate for handling the asymmetric characteristic of the problem. As mentioned in Section 3, these variables are used to add new constraints to the 2E-OLRP. These constraints can help mathematical model P_1 and P_2 obtain better results than P_3.

To have a better evaluation of the proposed heuristic, we now compare its performance in solving 2E-LRP instances to two of the best algorithms in the 2E-LRP literature. In order to do this comparison, the proposed heuristic is changed based on the 2E-LRP objective function and constraints. Since routes are closed in the classical 2E-LRP, our MCWH is changed to the classical Clarke and Wright heuristic to find an appropriate initial solution for the second phase of the heuristic algorithm. Tables 1.7 and 1.8 show the results of this comparison. The results for the proposed heuristic are shown under the heading “Hybrid SA” which includes the average and best results out of five runs. Columns 2-5 show the results from two previous studies on the 2E-LRP: Nguyen et al. (2012a) and Contardo et al. (2012). The last column, “Gap (%)” shows the difference between hybrid SA and best solution found by these previous studies; it is computed using the following formula: $\left(\frac{Hybrid\ SA_{best} - Best\ Solution}{Best\ Solution}\right)$.

In Table 1.7 (for Nguyen’s dataset), the proposed heuristic finds the same best solutions as the other algorithms for the small-sized instances. For the medium-sized instances, the best solutions found by the proposed algorithm are similar to those in the literature in most cases. The proposed heuristic competes well against the other algorithms and obtains results very close to those in the literature on large-sized instances too. The overall gap of 1.65% shows the strength of the proposed hybrid SA. For small-sized instances, all three algorithms solve the problems optimally in less than 5 seconds. For medium-sized instances with 50 customers, although the proposed hybrid SA is faster than Nguyen’s algorithm in some cases, Contardo’s algorithm generally obtains better solutions in less time. For large-sized instances with 100 and 200 customers, there is no pattern and each algorithm is the fastest on some instances and none of them can be considered as the fastest algorithm. Based on average objective value and average runtime, our hybrid SA performs very close to the other algorithms in the literature.

In Table 1.8, our heuristic is compared to the same algorithms by Nguyen et al. (2012a) and Contardo et al. (2012) but this time on Prodhon’s dataset. Our heuristic finds the same best solutions as the other algorithms for the small-sized instances with 20 customers. Our heuristic also finds the same best solutions as the other algorithms for most of the medium-sized instances with 50 customers. For most large-sized instances with 100 and 200 customers, the quality of solutions obtained by proposed hybrid SA is very close to the best solution found by the other two heuristics. Note that the hybrid SA finds a better solution for instance (ppw-100×10-1a). To our knowledge, this is a new best solution for this instance. The overall 1.17% gap shows the good performance of our proposed heuristic. Regarding runtime, the other two algorithms are generally faster than our hybrid SA on the small- and medium-sized instances; however, the difference in speed between the algorithms is less obvious for the large-sized instances.

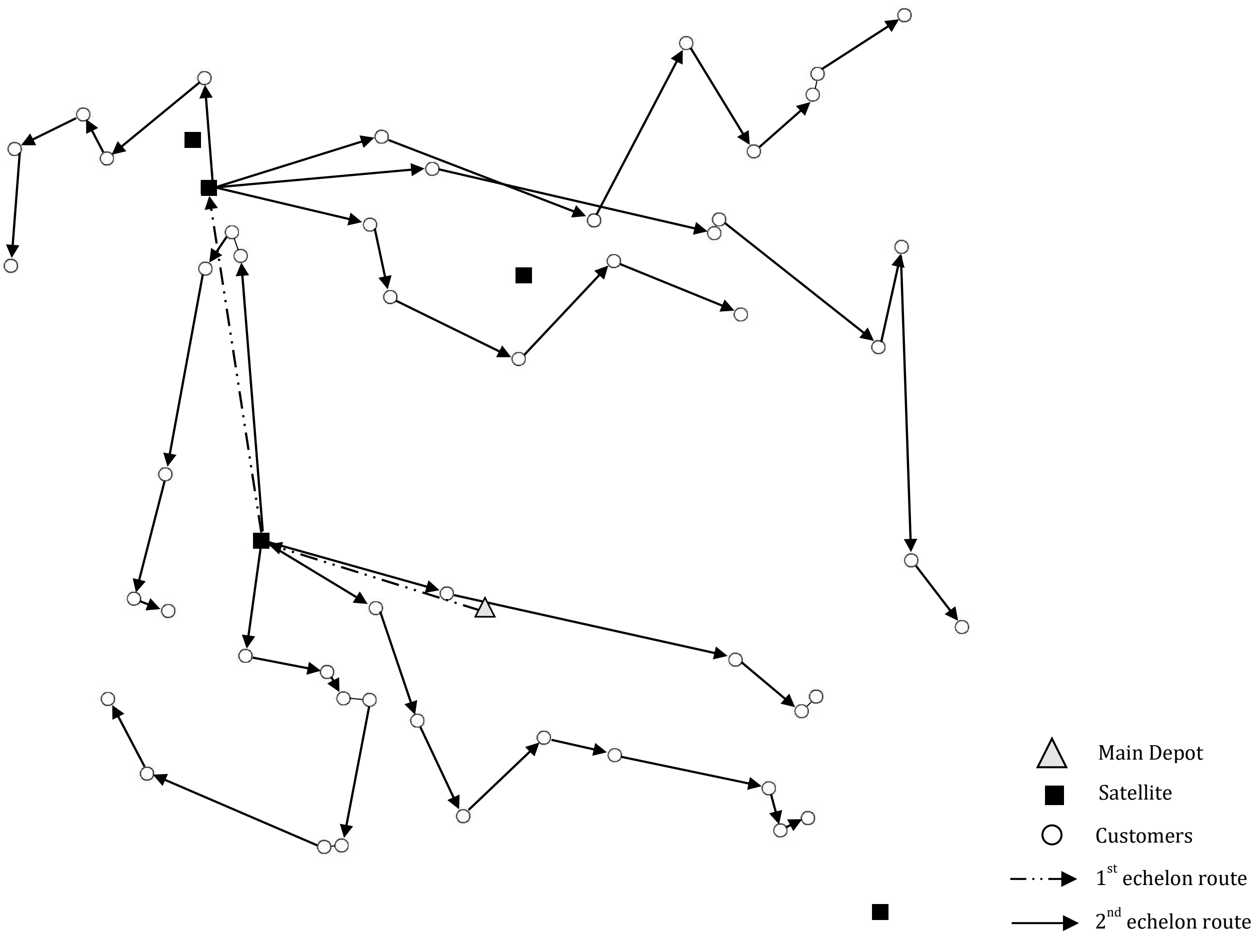


Figure 1.3. Best solution found by the proposed heuristic for instance 50-5MN.

Table 1.4. Comparison of proposed heuristic and CPLEX for Nguyen's dataset.

Instance	CPLEX P_1 Cost LB	UB	Time	Gap (%)	P_2 Cost LB	UB	Time	Gap (%)	P_3 Cost LB	UB	Time	Gap (%)	Heuristic Cost Ave.	Time Ave.	Gap (%)
25-5N	57,448.00	**57,448**	22	0.00	57,448.00	**57,448**	149	0.00	57,448.00	**57,448**	234	0.00	**57,448.00**	2.80	0.00
25-5Nb	48,605.00	**48,605**	8	0.00	48,605.00	**48,605**	31	0.00	48,605.00	**48,605**	36	0.00	**48,605.00**	2.60	0.00
25-5MN	54,079.00	**54,079**	21	0.00	54,079.00	**54,079**	1012	0.00	54,079.00	**54,079**	1165	0.00	**54,079.00**	3.00	0.00
25-5MNb	47,109.00	**47,109**	8	0.00	47,109.00	**47,109**	108	0.00	47,109.00	**47,109**	85	0.00	**47,109.00**	2.60	0.00
50-5N	97,575.00	**97575**	2030	0.00	88,987.60	105,852	7200	15.93	91,467.40	NA	7200	NA	99,459.60	42.40	1.89
50-5Nb	74,874.80	78,131	7200	4.17	72,626.70	105,915	7200	31.43	72,645.23	103,859	7200	30.05	79,014.20	36.60	5.24
50-5MN	84,151.90	88,373	7200	4.78	78,205.40	124,862	7200	37.37	81,096.58	108,503	7200	25.26	93,344.20	41.20	9.85
50-5MNb	82185.00	**82,185**	4403	0.00	79,141.10	92,090	7200	14.06	78,317.30	91,197	7200	14.12	82,973.20	33.80	0.95
50-10N	84,087.50	85,541	7200	1.70	77,640.60	115,190	7200	32.60	77,142.66	128,351	7200	39.90	88,411.20	42.80	4.89
50-10Nb	69,399.00	69,590	7200	0.27	67,028.20	71,834	7200	6.69	67,198.38	92,778	7200	27.57	71,412.60	35.20	2.82
50-10MN	100,054.00	107,129	7200	6.60	88,536.03	NA	7200	NA	88,436.04	NA	7200	NA	103,307.40	46.20	3.15
50-10MNb	84,659.50	85,209	7200	0.64	75,165.00	100,363	7200	25.11	76,299.76	105,344	7200	27.57	86,851.60	27.80	2.52
100-5N	127,828.00	321,050	7200	60.18	119,521.26	NA	7200	NA	116,242.84	NA	7200	NA	145,141.20	92.60	11.93
100-5Nb	113,176.00	488,082	7200	76.81	108,607.19	NA	7200	NA	103,359.80	NA	7200	NA	127,503.80	78.20	11.24
100-5MN	130,785.00	212,032	7200	38.32	121,187.83	NA	7200	NA	119,273.65	NA	7200	NA	154,019.80	87.20	15.09
100-5MNb	111,940.00	284,336	7200	60.63	108,107.47	NA	7200	NA	102,646.71	NA	7200	NA	131,711.20	71.80	15.01
100-10N	137,856.00	NA	7200	NA	NA	NA	7200	NA	131,459.79	NA	7200	NA	168,829.80	95.80	18.35
100-10Nb	117,659.00	NA	7200	NA	114,284.91	NA	7200	NA	110,151.19	NA	7200	NA	132,636.40	79.80	11.29
100-10MN	133,546.00	NA	7200	NA	122,501.00	NA	7200	NA	122,490.80	NA	7200	NA	160,999.20	96.60	17.05
100-10MNb	115,032.00	NA	7200	NA	112,217.00	NA	7200	NA	104,544.22	NA	7200	NA	132,755.40	76.80	13.35
200-10N	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	282,161.40	491.20	NA
200-10Nb	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	221,897.40	433.40	NA
200-10MN	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	256,618.20	550.60	NA
200-10MNb	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	239,992.80	420.20	NA
Average															7.23

Bold values are optimal.

All times in seconds.

Table 1.5. Comparison of proposed heuristic and CPLEX for Prodhon's dataset.

Instance	CPLEX P_1 Cost LB	UB	Time	Gap (%)	P_2 Cost LB	UB	Time	Gap (%)	P_3 Cost LB	UB	Time	Gap (%)	Heuristic Cost Ave.	Time Ave.	Gap (%)
ppw-20×5-1a	66,263.00	**66,263**	37	0.00	65,609.18	**66,263**	7200	0.99	65,384.67	**66,263**	7200	1.33	**66,263**	2.20	0.00
ppw-20×5-1b	48,013.00	**48,013**	7	0.00	48,013.00	**48,013**	65	0.00	48,013.00	**48,013**	40	0.00	**48,013**	1.60	0.00
ppw-20×5-2a	64,049.00	**64,049**	49	0.00	63,346.97	**64,049**	7200	1.10	64,049.00	**64,049**	4941	0.00	**64,049**	2.00	0.00
ppw-20×5-2b	46,986.00	**46,986**	9	0.00	46,986.00	**46,986**	45	0.00	46,986.00	**46,986**	38	0.00	**46,986**	2.20	0.00
ppw-50×5-1a	89,065.70	93,346	7200	4.59	82,827.48	NA	7200	NA	82,844.24	NA	7200	NA	95,697	48.60	6.93
ppw-50×5-1b	74,970.60	77,554	7200	3.33	64,993.70	NA	7200	NA	73,148.40	81,823	7200	10.60	77,575	33.00	3.36
ppw-50×5-2a	96,635.10	99,292	7200	2.68	84,173.74	NA	7200	NA	85,201.49	NA	7200	NA	99,322	48.20	2.71
ppw-50×5-2b	86,261.00	**86,261**	2612	0.00	85,516.84	98,081	7200	12.81	85,130.10	109,609	7200	22.33	87,703	35.00	1.64
ppw-50×5-2BIS	81,552.80	85,572	7200	4.70	75,601.03	NA	7200	NA	75,447.30	NA	7200	NA	84,943	49.40	3.99
ppw-50×5-2bBIS	75,202.10	77,600	7200	3.09	63,467.10	88,958	7200	28.65	64,233.04	81,917	7200	21.59	77,219	36.20	2.61
ppw-50×5-3a	87,370.70	88,669	7200	1.46	84,817.70	NA	7200	NA	84,536.97	NA	7200	NA	89,139	32.60	1.98
ppw-50×5-3b	76,932.00	**76,932**	2324	0.00	74,804.30	89,787	7200	16.69	74,549.49	93,810	7200	20.53	77,366	33.00	0.56
ppw-100×5-1a	206,730.00	270,926	7200	23.70	NA	NA	7200	NA	NA	NA	7200	NA	252,505	110.20	18.13
ppw-100×5-1b	209,224.00	479,798	7200	56.39	164,719.00	NA	7200	NA	158,055.91	NA	7200	NA	219,945	77.20	4.87
ppw-100×5-2a	186,146.00	192,330	7200	3.22	169,871.00	NA	7200	NA	NA	NA	7200	NA	194,818	111.40	4.45
ppw-100×5-2b	166,473.00	174,331	7200	4.51	155,516.00	NA	7200	NA	148,883.88	NA	7200	NA	172,303	76.40	3.38
ppw-100×5-3a	183,321.00	194,929	7200	5.95	162,191.00	NA	7200	NA	NA	NA	7200	NA	193,370	72.60	5.18
ppw-100×5-3b	154,944.00	166,313	7200	6.84	152,155.00	NA	7200	NA	142,457.93	NA	7200	NA	164,925	75.20	6.05
ppw-100×10-1a	256,582.00	1,019,710	7200	74.84	NA	NA	7200	NA	NA	NA	7200	NA	296,497	112.40	13.46
ppw-100×10-1b	231,498.00	NA	7200	NA	229,167.00	NA	7200	NA	226,749.71	NA	7200	NA	258,309	83.20	10.38
ppw-100×10-2a	228,702.00	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	255,000	110.80	10.31
ppw-100×10-2b	208,857.00	NA	7200	NA	204,790.00	NA	7200	NA	202,452.62	NA	7200	NA	230,859	82.00	9.53
ppw-100×10-3a	228,188.00	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	262,076	73.40	12.93
ppw-100×10-3b	206,581.00	NA	7200	NA	202,138.00	NA	7200	NA	199,224.18	NA	7200	NA	232,034	80.00	10.97
ppw-200×10-1a	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	453,948	517.40	NA
ppw-200×10-1b	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	398,867	419.40	NA
ppw-200×10-2a	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	433,233	581.60	NA
ppw-200×10-2b	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	386,033	416.00	NA
ppw-200×10-3a	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	431,722	370.00	NA
ppw-200×10-3b	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	350,890	392.80	NA
Average															5.56

Table 1.6. Comparison of proposed heuristic and CPLEX for Barreto's dataset.

Instance	CPLEX												Heuristic		
	*P*₁				*P*₂				*P*₃						
	Cost		Time	Gap (%)	Cost		Time	Gap (%)	Cost		Time	Gap (%)	Cost	Time	Gap (%)
	LB	UB			LB	UB			LB	UB			Ave.	Ave.	
Christofides69-50x5	538.00	**538**	733	0.00	516.90	548	7200	5.68	515.2051	548	7200	5.98	564.8	42.60	4.75
Christofides69-75x10	743.23	764	7200	2.72	708.06	NA	7200	NA	633.0747	NA	7200	NA	813	70.20	8.58
Christofides69-100x10	809.13	858	7200	5.70	778.45	NA	7200	NA	722.6704	NA	7200	NA	872.8	84.60	7.30
Daskin95-88x8	654.53	NA	7200	NA	486.44	NA	7200	NA	486.2554	NA	7200	NA	697	62.60	6.09
Daskin95-150x10	NA	NA	7200	NA	NA	NA	7200	NA	NA	NA	7200	NA	65,868.2	112.60	NA
Gaskell67-21x5	855.00	**855**	5	0.00	855.00	**855**	36	0.00	855	**855**	141	0.00	**855**	3.20	0.00
Gaskell67-22x5	1,157.00	**1,157**	5	0.00	1,157.00	**1,157**	88	0.00	1,157.00	**1,157**	7	0.00	**1,157.00**	3.60	0.00
Gaskell67-29x5	1,219.00	**1,219**	22	0.00	1,219.00	**1,219**	1803	0.00	1,219.00	**1,219**	1050	0.00	**1,219.00**	3.20	0.00
Gaskell67-32x5	1,437.00	**1,437**	41	0.00	1,422.07	1,437	7200	1.04	1,421.92	1,437	7200	1.05	1,437.00	3.20	0.00
Gaskell67-32x5b	1,415.00	**1,415**	20	0.00	1,415.00	**1,415**	6641	0.00	1,415.00	**1,415**	397	0.00	**1,415.00**	3.00	0.00
Gaskell67-36x5	499.00	**499**	8	0.00	499.00	**499**	239	0.00	499	**499**	69	0.00	**499**	4.20	0.00
Min92-27x5	10,449.00	**10,449**	15	0.00	10,449.00	**10,449**	1072	0.00	10,449.00	**10,449**	61	0.00	**10,449.00**	3.40	0.00
Min92-134x8	21,154.10	NA	7200	NA	18,459.70	NA	NA	NA	NA	NA	7200	NA	22,784.60	134.00	7.16
Average															2.82

Table 1.7. Comparison of proposed heuristic and other heuristics on Nguyen's 2E-LRP instances

Instance	Nguyen et al. 2012a		Contardo et al. 2012		Hybrid SA				
	Cost	Time	Cost	Time	Cost		Time		Gap (%)
					Ave.	Min.	Ave.	Min.	
25-5N	**80,370**	3.1	**80,370**	0.1	**80,370**	**80,370**	2.8	2	0.00%
25-5Nb	**64,562**	2.6	**64,562**	0	**64,562**	**64,562**	2.2	2	0.00%
25-5MN	**78,947**	3.2	**78,947**	0.5	**78,947**	**78,947**	2.4	2	0.00%
25-5MNb	**64,438**	4.1	**64,438**	0	**64,438**	**64,438**	2.6	2	0.00%
50-5N	138,126	13.7	**137,815**	33.9	138,468	138,444	44.2	43	0.46%
50-5Nb	111,062	11.7	**110,094**	32.3	112,298	111,840	32.6	31	1.59%
50-5MN	**123,484**	9.1	**123,484**	13.1	124,014	123,854	42	41	0.30%
50-5MNb	**105,401**	13.6	**105,401**	23.1	106,416	106,313	32.8	31	0.87%
50-10N	116,132	46.6	**115,725**	21.3	116,700	116,132	42.8	42	0.35%
50-10Nb	**87,315**	22.4	**87,315**	20.7	90,085	89,744	32.6	32	2.78%
50-10MN	135,748	37.5	**135,519**	39	136,095	135,568	42.8	42	0.04%
50-10MNb	**110,613**	42.4	**110,613**	19.1	110,907	110,703	34.6	33	0.08%
100-5N	196,910	13.1	**193,228**	154.3	199,487	198,444	83	81	2.70%
100-5Nb	159,086	33.1	**158,927**	133	163,272	162,813	61.4	60	2.45%
100-5MN	207,119	25.5	**204,682**	135.7	207,672	205,742	79.6	78	0.52%
100-5MNb	166,115	41.3	**165,744**	112.9	170,789	169,151	62.6	60	2.06%
100-10N	215,792	132.5	**212,847**	70.8	219,053	217,056	82.4	79	1.98%
100-10Nb	156,401	76.9	**155,489**	70.3	160,211	158,269	60	58	1.79%
100-10MN	205,964	156.1	**201,275**	104.7	213,790	209,270	98.2	81	3.97%
100-10MNb	170,706	192.4	**170,625**	114.2	175,593	173,158	64.8	60	1.48%
200-10N	353,685	240.8	**347,395**	237.2	358,961	356,391	490.2	483	2.59%
200-10Nb	262,072	358.8	**256,171**	340.8	271,189	269,577	439.2	430	5.23%
200-10MN	332,345	523.1	**326,454**	354.7	344,443	339,479	499	486	3.99%
200-10MNb	292,523	690	**289,742**	481.7	309,742	302,502	422.2	416	4.40%
Average	163,954.8	112.2	162,369.2	104.7	167,395.8	165,948.6	114.8	111.4	1.65%

Bold values indicate best performing algorithm for each instance.

Table 1.8. Comparison of proposed heuristic and other heuristics on Prodhon's 2E-LRP instances

Instance	Nguyen et al. 2012a		Contardo et al. 2012		Hybrid SA				
	Cost	Time	Cost	Time	Cost		Time		Gap (%)
					Ave.	Min.	Ave.	Min.	
ppw-20×5-1a	**89,075**	2.4	**89,075**	2.8	**89,075**	**89,075**	2.8	2	0.00%
ppw-20×5-1b	**61,863**	2.6	**61,863**	0.2	**61,863**	**61,863**	3.4	2	0.00%
ppw-20×5-2a	85,290	1.6	**84,478**	2.8	**84,478**	**84,478**	2.8	2	0.00%
ppw-20×5-2b	**60,838**	1.4	**60,838**	0	**60,838**	**60,838**	3.6	3	0.00%
ppw-50×5-1a	134,855	9.2	**130,843**	8.2	135,592	135,008	48.6	47	3.18%
ppw-50×5-1b	**101,530**	15.8	**101,530**	8.7	102,211	101,879	34	32	0.34%
ppw-50×5-2a	132,159	12.9	**131,825**	37.2	132,515	132,131	48.6	48	0.23%
ppw-50×5-2b	110,547	18.6	**110,332**	21.1	110,684	110,395	34.4	34	0.06%
ppw-50×5-2BIS	122,654	27.6	**122,599**	43.2	122,965	122,809	49	48	0.17%
ppw-50×5-2bBIS	105,776	30	**105,696**	57.4	106,077	105,835	35	34	0.13%
ppw-50×5-3a	**128,379**	16.2	**128,379**	17.8	128,630	**128,379**	50	49	0.00%
ppw-50×5-3b	**104,006**	17	**104,006**	14.2	104,088	**104,006**	33.4	33	0.00%
ppw-100×5-1a	320,130	37.3	**319,137**	445	324,359	323,693	158.4	158	1.43%
ppw-100×5-1b	258,205	32	**257,349**	607.9	263,623	260,754	109.6	109	1.32%
ppw-100×5-2a	234,179	29.6	**231,305**	43.7	236,313	235,752	158.4	157	1.92%
ppw-100×5-2b	195,426	24.9	**194,729**	602.4	195,809	195,450	108.8	107	0.37%
ppw-100×5-3a	245,944	68	**244,194**	167.9	254,910	250,992	157	156	2.78%
ppw-100×5-3b	195,254	54.3	**194,110**	210.9	201,451	198,343	110.2	108	2.18%
ppw-100×10-1a	358,939	336.4	358,068	65.6	359,389	**356,669**	161.4	159	-0.39%
ppw-100×10-1b	302,584	333.5	**297,167**	118.7	304,761	301,509	116.2	114	1.46%
ppw-100×10-2a	306,303	362	**305,402**	163	309,946	308,712	158	157	1.08%
ppw-100×10-2b	**264,389**	294.1	265,138	225.9	268,084	266,692	119.2	116	0.87%
ppw-100×10-3a	**313,249**	370.8	313,517	106	322,141	322,096	161.4	160	2.82%
ppw-100×10-3b	266,383	340.4	**264,096**	181.6	276,589	275,951	116.4	114	4.49%
ppw-200×10-1a	554,598	700.9	**552,816**	648.7	563,172	561,835	641.6	613	1.63%
ppw-200×10-1b	452,286	723.7	**448,236**	927.6	461,742	460,476	465	448	2.73%
ppw-200×10-2a	502,173	220.2	**498,199**	339.3	506,715	506,124	625.4	623	1.59%
ppw-200×10-2b	425,311	267.3	**423,048**	1161.5	433,382	429,033	443	424	1.41%
ppw-200×10-3a	**533,732**	676.3	534,569	285.2	543,974	543,312	618.6	605	1.79%
ppw-200×10-3b	418,800	323.6	**404,284**	439.1	411,150	409,961	419.6	413	1.40%
Average	246,161.9	178.4	244,560.9	231.8	249,217.6	248,135.0	173.1	169.2	1.17%

1.6. Conclusion

In this paper, we studied crowed sourced delivery problem in a multi echelon environment. This problem can be modeled as a two-echelon open location routing problem (2E-OLRP). This problem is a new variant of the classical location routing problem (LRP) which considers location and routing decisions in two echelon supply chains in which third party logistics providers are used. Three new mathematical models and a hybrid simulated annealing (SA) heuristic are developed to solve the 2E-OLRP. Subtour elimination constraints are also developed in the mathematical models to eliminate routes that are not connected to the depot (a satellite) in the first (second) echelon. A new solution encoding scheme is used which is easy to implement and simplifies the search for obtaining better solutions. Our methods are tested on two sets of problem instances from the 2E-LRP literature and one set of well-known LRP benchmark instances that we transform into 2E-OLRP instances. IBM ILOG CPLEX is used to search for feasible and optimal solutions and obtain a lower bound on the optimal value for these 2E-OLRP instances after a two-hour time limit. The proposed hybrid SA heuristic outperforms CPLEX both in solution quality and solution time across the vast majority of instances. We also note that CPLEX fails to find feasible solutions to many medium- and large-sized 2E-OLRP instances, whereas the proposed SA heuristic finds feasible solutions to all of them in a reasonable amount of time.

Future research on this topic might proceed in several directions. More realistic constraints such as vehicle synchronization at satellites, or simultaneous pickup and delivery might be considered. Moreover, uncertainties in customer demand, travel time, and/or service time might be incorporated into the modeling framework. Another main area of future research may be developing new exact or heuristic methods for solving the 2E-OLRP that exploit the problem characteristics. Finally, as more companies outsource their transportation affairs to third parties, it may be beneficial for future studies to spend more effort analyzing the financial savings achieved by companies that have made outsourcing decisions.

References

Arslan, A. M., Agatz, N., Kroon, L., & Zuidwijk, R. (2018). Crowdsourced delivery—A dynamic pickup and delivery problem with ad hoc drivers. Transportation Science.

Barreto, S. S. "Análise e Modelização de Problemas de localização-distribuição (Analysis and modelization of location-routing problems)." University of Aveiro, campus universitário de Santiago (in Portuguese) (2004).

Boccia, M., Crainic, T., Sforza, A., & Sterle, C. (2010). A metaheuristic for a two echelon location-routing problem. In P. Festa (Ed.), Symposium on experimental algorithms (SEA 2010). Lecture Notes in Computer Science, 6049, 288–301.

Chiang, W. C., Russell, R., Xu, X., & Zepeda, D. (2009). A simulation/metaheuristic approach to newspaper production and distribution supply chain problems. International Journal of Production Economics, 121(2), 752-767.

Clarke, G. U., & Wright, J. W. (1964). Scheduling of vehicles from a central depot to a number of delivery points. Operations Research, 12(4), 568-581.

Crainic, T. G., Sforza, A., Sterle, C. (2011). Location-routing models for two-echelon freight distribution system design. Technical report CIRRELT-2011-40.

Contardo, C., Hemmelmayr, V., & Crainic, T. G. (2012). Lower and upper bounds for the two-echelon capacitated location-routing problem. Computers & Operations Research, 39(12), 3185–3199.

Cuda, R., Guastaroba, G., & Speranza, M. G. (2015). A survey on two-echelon routing problems. Computers & Operations Research, 55, 185-199.

Drexl, M., & Schneider, M. (2015). A survey of variants and extensions of the location-routing problem. European Journal of Operational Research, 241(2), 283-308.

Fleszar, K., Osman, I. H., & Hindi, K. S. (2009). A variable neighbourhood search algorithm for the open vehicle routing problem. European Journal of Operational Research, 195(3), 803-809.

Fung, R. Y., Liu, R., & Jiang, Z. (2013). A memetic algorithm for the open capacitated arc routing problem. Transportation Research Part E: Logistics and Transportation Review, 50, 53-67.

Ghaffari-Nasab, N., Jabalameli, M. S., Aryanezhad, M. B., & Makui, A. (2013). Modeling and solving the bi-objective capacitated location-routing problem with probabilistic travel times. International Journal of Advanced Manufacturing Technology, 67, 2007-2019.

Jacobsen, S., & Madsen, O. (1980). A comparative study of heuristics for a two-level routing-location problem. European Journal of Operational Research, 6, 378–387.

Karaoglan, I., Altiparmak, F., Kara, I., & Dengiz, B. (2012). The location-routing problem with simultaneous pickup and delivery: Formulations and a heuristic approach. Omega, 40(4), 465-477.

Kirkpatrick, S., Gelatt, C. D., Vecchi, M. P. (1983). Optimization by simulated annealing. Science, 20, 671-680.

Lalla-Ruiz, E., Expósito-Izquierdo, C., Taheripour, S., & Voß, S. (2016). An improved formulation for the multi-depot open vehicle routing problem. OR Spectrum, 38(1), 175-187.

Letchford, A. N., Lysgaard, J., & Eglese, R. W. (2007). A branch-and-cut algorithm for the capacitated open vehicle routing problem. Journal of the Operational Research Society, 58(12), 1642-1651.

Lin, J.-R., & Lei, H.-C. (2009). Distribution systems design with two-level routing considerations. Annals of Operations Research, 172, 329–347.

Liu, R., & Jiang, Z. (2012). The close–open mixed vehicle routing problem. European Journal of Operational Research, 220(2), 349-360.

Liu, R., Jiang, Z., & Geng, N. (2014). A hybrid genetic algorithm for the multi-depot open vehicle routing problem. OR Spectrum, 36(2), 401-421.

Madsen, O. (1983). Methods for solving combined two level location-routing problems of realistic dimensions. European Journal of Operational Research, 12, 295–301.

McKinsey (2014), Same-day delivery: The next evolutionary step in parcel logistics, https://www.mckinsey.com/industries/travel-transport-and-logistics/our-insights/same-day-delivery-the-next-evolutionary-step-in-parcel-logistics

Morphy, E. (2013). About Walmart's idea to crowdsource its same-day delivery service.

U.S. Department of Commerce. (2017). New Insights on Retail E-Commerce (ESA Issue Brief # 04-17). https://www.esa.gov/reports/new-insights-retail-e-commerce.

Nagy, G., & Salhi, S. (2007). Location-routing: Issues, models and methods. European Journal of Operational Research, 177(2), 649–672.

Nguyen, V.-P., Prins, C., & Prodhon, C. (2012a). A multi-start iterated local search with tabu list and path relinking for the two-echelon location-routing problem. Engineering Applications of Artificial Intelligence, 25(1), 56–71.

Nguyen, V. P., Prins, C., & Prodhon, C. (2012b). Solving the two-echelon location routing problem by a GRASP reinforced by a learning process and path relinking. European Journal of Operational Research, 216(1), 113–126.

Norouzi, N., Tavakkoli-Moghaddam, R., Ghazanfari, M., Alinaghian, M., & Salamatbakhsh, A. (2012). A new multi-objective competitive open vehicle routing problem solved by particle swarm optimization. Networks and Spatial Economics, 12(4), 609-633.

Perboli, G., Tadei, R., & Vigo, D. (2011). The two-echelon capacitated vehicle routing problem: Models and math-based heuristics. Transportation Science, 45(3), 364-380.

Pichka, K., Bajgiran, A. H., Petering, M. E., Jang, J., & Yue, X. (2018). The two echelon open location routing problem: Mathematical model and hybrid heuristic. Computers & Industrial Engineering, 121, 97-112.

Prodhon, C., (2006). The Location-routing Problem. Ph.D. Thesis, Troyes University of Technology, Troyes, France.

Prodhon, C., & Prins, C. (2014). A survey of recent research on location-routing problems. European Journal of Operational Research, 238(1), 1-17.

Prins, C., Prodhon, C., Soriano, P., Ruiz, A., Wolfler Calvo, R. (2007). Solving the capacitated location routing problem by a cooperative Lagrangean relaxation granular tabu search heuristic. Transportation Science, 41, 470–483.

Repoussis, P. P., Tarantilis, C. D., Bräysy, O., & Ioannou, G. (2010). A hybrid evolution strategy for the open vehicle routing problem. Computers & Operations Research, 37(3), 443-455.

Repoussis, P. P., Tarantilis, C. D., & Ioannou, G. (2007). The open vehicle routing problem with time windows. Journal of the Operational Research Society, 58, 355-367.

Russell, R., Chiang, W. C., & Zepeda, D. (2008). Integrating multi-product production and distribution in newspaper logistics. Computers & Operations Research, 35(5), 1576-1588.

Salari, M., Toth, P., & Tramontani, A. (2010). An ILP improvement procedure for the open vehicle routing problem. Computers & Operations Research, 37(12), 2106-2120.

Salhi, S., & Rand, G. K. (1989). The effect of ignoring routes when locating depots. European Journal of Operational Research, 39(2), 150-156.

Statista (2017). B2C e-commerce sales worldwide from 2012 to 2018". https://www.statista.com/statistics/261245/b2c-e-commerce-sales-worldwide.

Tuzun, D., & Burke, L. I. (1999). A two-phase tabu search approach to the location routing problem. European Journal of Operational Research, 116(1), 87-99.

Schwengerer, M., Pirkwieser, S., & Raidl, G. R. (2012). A variable neighborhood search approach for the two-echelon location-routing problem. In J.-K. Hao &M. Middendorf (Eds.), Evolutionary computation in combinatorial optimization. Lecture Notes in Computer Science, 7245, 13–24.

Vidović, M., Ratković, B., Bjelić, N., & Popović, D. (2016). A two-echelon location-routing model for designing recycling logistics networks with profit: MILP and heuristic approach. Expert Systems with Applications, 51, 34-48.

Vincent, F. Y., & Lin, S. Y. (2015). A simulated annealing heuristic for the open location-routing problem. Computers & Operations Research, 62, 184-196.

Vincent, F. Y., Jewpanya, P., & Redi, A. P. (2016). Open vehicle routing problem with cross-docking. Computers & Industrial Engineering, 94, 6-17.

Winkenbach, M., Kleindorfer, P. R., & Spinler, S. (2015). Enabling urban logistics services at La Poste through multi-echelon location-routing. Transportation Science, 50(2), 520-540.

Wu, T. H., Low, C., & Bai, J. W. (2002). Heuristic solutions to multi-depot location-routing problems. Computers & Operations Research, 29(10), 1393-1415.

ABSTRACT

ESSAY 2: JOINT FULFILMENT AND PRICING DECISIONS FOR OMNI-CHANNEL RETAILERS

by

Khosro Pichka

The University of Wisconsin-Milwaukee, 2019

Under the Supervision of Professor Layth Alwan and Professor Xiaohang Yue

The rapid growth of online sales has encouraged many traditional brick-and-mortar retailers to fulfill their customers' demand through multiple channels, which is known as omni-channel retailing. Omni-channel retailers sell their products through e-commerce channels and brick-and-mortar stores in different geographical locations. They have specific fulfillment centers (e-fulfillment centers) in different geographical locations to fulfill their online orders; however, they may also fulfill their online orders using their in-store inventory. Moreover, it has recently been more common to offer the in-store customers the option to ship their orders if the store is out of inventory. On the other hand, retailers can benefit from pricing decisions to control their customers' channel preference. Therefore, the omni-channel retailer faces an optimization problem to maximize the total profit by considering both revenue and fulfillment costs. We model the customers' demand by a Multinomial Logit (MNL) choice model. Given this choice model, in our first proposed optimization model, we assume that in-store demands can also be satisfied with e-fulfillment inventory from the same geographical zone. In the second optimization model, we also assume multiple shipping options and inventory decisions. By numerical experiments, we show that an omni-channel retailer can increase its profit by having control over initial inventory assignment and fulfilling the demand of in-store customers by e-fulfillment inventory.

2.1. Introduction

Omni-channel retailing is the integration of demand fulfillment process across retailers' multiple channels. The main purpose of this integration is to give consumers a better shopping experience whether it's an online customer picking up the product in the store or a customer at a physical store asking for shipping the product to her home. Omni-channel retailing is growing fast. Customers expect to have several options for shipping their packages, including store pick-up when they buy their products online. They also expect to be able to check the retailers in-store inventory on their mobile app (e.g., Walmart mobile app) and locate the exact aisle for the product. Based on a research study, almost seventy percent of the consumers expect to view in-store inventory online and almost forty percent are unlikely to visit the store if they cannot see the store inventory online (Forrester Research 2014).

If the retailers want to compete in today's market, they should focus on convenience of their customers. Omni-channel retailing helps the retailers to achieve this goal. This can help customer to compare the online price with the in-store price and to have a package shipped to her home if the store is out of inventory or to pick up a package at store without paying the shipping cost. Omni-channel have several benefits for retailers as well. They can increase their market share by covering both the online sales and sales through traditional channel. They can also decrease the lost sales that happens in pure e-commerce companies, such as Amazon, by utilizing the in-store inventories for online orders. By utilizing such inventory, retailers can improve the performance and the responsiveness of their distribution systems.

2.1.1. Omni-channel retailing vs. multi-channel retailing

Although there are several similarities, omni-channel retailing is different from multi-channel retailing. Both multi-channel and omni-channel retailing represent selling products through multiple channels including brick-and-mortar and e-commerce channels, but the key difference is how the demand fulfillment process is combined across different channels. A traditional multi-channel retailer may have sales from its website and physical stores, but these two channels are performing separately and have very little interaction with each other. In fact, e-commerce and brick-and mortar channels are two distinct businesses; for example, they have their own inventory and never share when they are out of stock. But in omni-channel retailing, these two channels are integrated. For example, in omni-channel retailing, a customer who bought the product online can pick it up in-store or when the store is out of inventory, the retailer can ship the product to the customer from another warehouse dedicated to online orders.

2.1.2. Problem description

In this paper, we study both pricing and fulfillment decisions for omni-channel retailers. The retailer has specific fulfillment centers (e-fulfillment centers) in different geographical zones to primarily satisfy online orders in the same geographical zone. They can fulfill an online order in a given zone by the e-fulfillment inventory in the same zone or from an e-fulfillment center in another zone. They also have physical stores in all the geographical zones (i.e., brick-and-mortar channels). The physical store inventory can be used to fulfill the online demand in any zone. But the in-store demand can only be fulfilled from the in-store inventory or from the e-fulfillment inventory in the same geographical zone, if the store is out of inventory. We also assume that, the in-store and online prices are decision variables and the retailer can use these decisions to change the demand pattern for each channel. In fact, as it has been often assumed in revenue management literature, the retailer can use the price to control the demand in each channel. retailers can offer promotions for slow-moving in-store products to change their demand patterns for brick-and-mortar channels. One should be noted that the retailer is allowed to decide not to satisfy an order when it has enough inventory, but it will lose revenue. However, some portion of the lost revenue can be recovered by selling the leftover inventory at a lower price. Indeed, we study cross-channel pricing and fulfillment problem in this paper. For two zones, Figure 2.1 illustrates the different ways that an online order or a store demand can be fulfilled using available inventory.

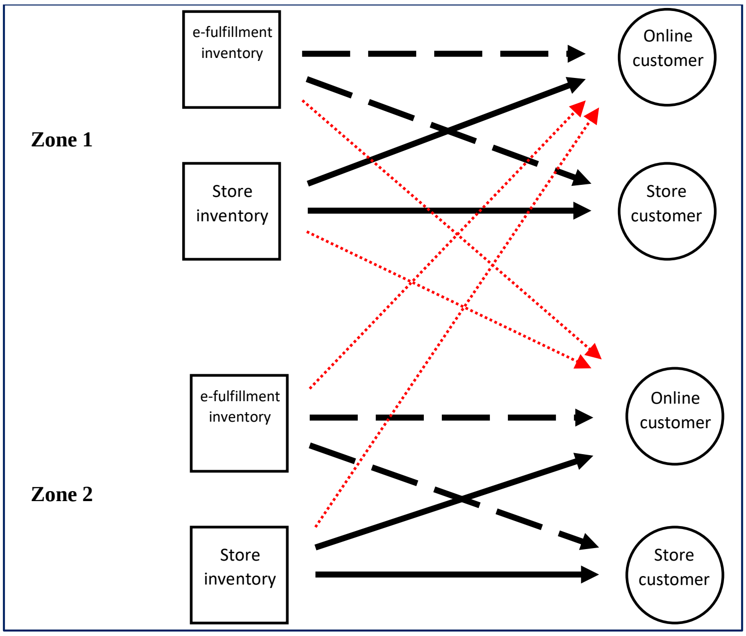


Figure 2.1 Omni-channel fulfillment options (two zones)

Although there are several benefits for both retailers and customers in implementing omni-channel retailing, new operational difficulties have arisen since it was introduced, but not all of them has addressed yet. One challenge is how to balance the inventory for online sales and in-store demands and how to use the store and e-fulfillment inventory across channels. The other challenge relates to making fulfillment decisions for online orders. Sometimes retailers decide to ship the order from the farther location to keep the inventory level in a closer fulfillment center prepared for future demand. Finally, pricing decisions and its impact on each channel is another challenge. These decisions were traditionally made separately but pricing controls can affect the fulfillment decisions as well. In fact, retailers can move customers between channels and change the customers' shipping preferences by offering prices for different channels while simultaneously making fulfillment decisions to maximize profit. For example, if the e-fulfillment center in one zone is out of inventory for online orders, the retailer has to ship those orders either form another e-fulfilment center or physical store and incurring extra shipping cost. However, if retailers make the pricing decisions with these fulfillment decisions simultaneously, then they can offer a proportion of this extra fulfillment cost as a promotion on in-store price and increase the probability of customers buying from physical stores or increase the probability of customers buying online to then pick up the orders from physical stores. For example, if the e-fulfillment center is out of inventory, the retailer might pay $10 extra cost to ship an online order from the in-store inventory which is not designed primarily for fulfilling online orders. But if the pricing decisions are made simultaneously, then the retailer can offer a part of this extra cost, like $3, as a discount to the in-store price to increase the probability that customers pick up the product from the store.

2.1.3. Summary of contributions

- The customer demand is modeled by multinomial logit (MNL) choice model and pricing control is used to manage the demand through channels.
- We propose two new optimization models for omni-channel retailing considering proposed pricing controls, and fulfillment decisions simultaneously, whereas the second model also includes delivery time and inventory decisions. We also consider the assumption that in-store demand can be fulfilled from e-fulfillment centers.
- Both optimization models are non-linear in constraints and objective function. Therefore, both models are linearized using standard techniques that are introduced in the literature and the models are solved by CPLEX solver.

- We design extensive numerical experiments that provide several insights for omni-channel retailing. The numerical results provide several key managerial insights.

The rest of this paper is organized as follows. Section 2.2 reviews the relevant literature on the pricing and fulfillment decisions considering e-commerce channel. Problem description and mathematical programming formulations of the joint pricing and fulfillment decisions are developed in Section 2.3. Computational results are reported in Section 2.4. Several managerial insights are provided in Section 2.5 based on computational studies. Finally, conclusions and future work are discussed in Section 2.6.

2.2. Literature review

The first stream of literature that is related to this study is revenue management and pricing literature, specifically the studies that consider consumers' choice. The idea of using choice models in revenue management was formally introduced by Talluri and Van Ryzin (2004). They modeled a reserve management problem using a Multinomial Logit (MNL) choice model. Based on this choice model, they calculated the probability of purchase for each product as a function of the set of products offered. Their control problem was to decide which subset of products to offer at each point in time. They also proposed an estimation procedure for this setting based on the expectation-maximization (EM) method that jointly estimates arrival rates and choice model parameters when no-purchase outcomes are unobservable. Talluri and Van Ryzin (2006), Bodea and Ferguson (2014), and Strauss et al. (2018) provide comprehensive reviews for pricing and application of choice models in revenue management.

Akçay et al. (2010) considered a dynamic pricing problem for substitutable and perishable products where the product demands are based on consumer choice model. They modeled this problem as a stochastic dynamic program and characterized its optimal prices.

They introduced a linear random utility framework that captures the cases of vertical and horizontal product differentiation. Kunnumkal and Topaloglu (2010) proposed a new dynamic programming decomposition method considering customer choice behavior to allocate the revenue associated with an itinerary having different flight legs and to solve a single-leg revenue management problem for each flight leg in the airline network. Their approach finds the revenue allocations by solving an auxiliary optimization problem considering the probabilistic nature of the customer choices. They compare their approach with two standard benchmark methods, one is a deterministic linear programming formulation and the other one is a dynamic programming decomposition. By computational experiments, they show that their approach outperforms the benchmark methods.

Vulcano et al. (2010) studied the application of a choice-based revenue management system using real data from a U.S. airline company. They first estimated the parameters of the choice model then used these estimates in a simulation study to assess the revenue performance of the expected marginal seat revenue capacity control policies. Their simulation results show one to five percent average revenue improvements using choice models. In another study, Vulcano et al. (2012) proposed a method for estimating substitute and lost demand when only sales and product availability data are observable, and the seller knows the aggregate market share. They combined a MNL choice model with a nonhomogeneous Poisson model of arrivals over multiple periods. They applied the expectation-maximization (EM) method to this model and treated the observed demand as an incomplete observation of primary demand and were able to estimate the parameters efficiently. They showed the effectiveness of their procedure on simulated data and with two industry data sets.

Rusmevichientong and Topaloglu (2012) studied the assortment optimization problem under the MNL choice model where the parameters of the logit model are unknown and uncertain. They maximized the worst case expected revenue over all parameter values in the uncertainty set. They did not consider inventory constraints in static case but allowed for limited initial inventory to be allocated over time in the dynamic setting. They showed the robustness of their method by numerical experiments and it's benefit when there is significant uncertainty in the parameter values. They also compared their method with other methods in the literature and showed that there can be up to ten percent improvement in the worst-case performance. Rusmevichientong et al (2014) also studied the assortment optimization problem under the MNL choice model where the parameters of the choice model are random, which is also called the mixture-of-logits model. This is because there are multiple customer segments, and each segment could have different preferences for the products. Their objective was to maximize the expected revenue per customer for all customer segments. They showed that the problem is NP-complete and then focused on assortments consisting of products with the highest revenues. By numerical experiments, they showed that revenue-ordered assortments perform remarkably well in terms of profit. Newman et al. (2014) also developed a parameter estimation method for MNL discrete choice models where one of the alternatives is never chosen in the sample data set. Their method is based on decomposing the log-likelihood function into marginal and conditional components easily enabling efficient incorporation of the price and other product attributes efficiently. Their method is computationally efficient and provides consistent parameter estimates. They showed the computational performance of their method using simulations and with industry data.

Paul et al. (2018) considered both assortment and pricing problems when customers' behavior follows a nonparametric choice model. In their problem, each customer arrives with a preference list and will purchase the highest-ranking offered product in her preference list. It is assumed that the set of customer classes is derived from paths in a tree and the order of nodes visited along each path gives the corresponding preference list. They first proposed a dynamic programming solution for assortment problem to maximize expected revenue by finding which products to offer. Then they studied the joint assortment and pricing problem, where they decide both prices and set of offered products simultaneously. They provide optimal solutions when customers have some universal ranking of the products, and the tree takes the form of a single path. They showed that the tree choice model captures customer purchasing behavior more precisely than the MNL choice model in the majority of test cases by running computational experiments on both synthetic data and real hotel purchase data.

The other streams of literature that are related to our study are papers on fulfillment decisions with the presence of e-commerce channel and the few papers that consider both pricing and e-fulfillment decisions. Agatz et al. (2008) provides a comprehensive review of papers in different areas of supply chain optimization considering multi-channel fulfillment.

Xu et al. (2009) studies the benefit of reevaluating the delivery decisions for shipment of online orders. Online retailers assign orders to one or multiple warehouses to minimize their operational costs. However, they consider this assignment a myopic decision because it cannot account for any future customer orders or future inventory replenishment. They examined the benefits of periodically reevaluating these real-time assignments and proposed efficient heuristics for the reassignment for a large set of customer orders to minimize the total number of shipments. They showed significant improvement by testing their heuristics on real data from a major online retailer. In a similar study, Acimovic and Graves (2014) studied the distribution decisions for online retailers and developed a heuristic that makes fulfillment decisions by minimizing the immediate outbound shipping cost plus an estimate, which are from the dual values of a transportation linear program, of future expected outbound shipping costs. They take orders with multiple items into account for opportunities in reducing the number of shipments. In fact, when they consider a specific order that includes a particular item, they first choose the fulfillment centers that also has on-hand inventory of the other items in the customer's order and try to fulfill this customer's order from the fulfillment center that has both the particular SKU and the other items. They showed the efficiency of their algorithm by running an experiment on real-data and were able to reduce outbound shipping costs by about 1% while keeping the same service level for customers.

A recent study by Lei et al. (2018) considered joint pricing and fulfillment problem for e-tailers. They first modeled the exact control problem to maximize the total expected profits for the online retailer as the total expected revenues minus total expected shipping costs. Since the optimal solution for this problem is difficult to obtain, they proposed two heuristic algorithms. The first heuristic algorithm directly uses the solution of a deterministic approximation of joint pricing and fulfillment problem. The second heuristic algorithm improves the first algorithm by adjusting the original control parameters according to the observed demand. They showed that the second heuristic algorithm significantly outperforms the first heuristic control and is very close to a benchmark that jointly reoptimizes the full deterministic problem at the beginning of every period. As a second study, Harsha et al. (2016) considered joint pricing and fulfillment problem for a retailer that sells products through both e-commerce and brick-and-mortar channels. They model customer channel preference by MNL choice models. They proposed an optimization model for joint pricing controls and fulfillment and then solved the model optimally. They also proposed solutions and examples where the e-commerce warehouse is capacitated. By their experiments, they showed that compared to the retailer's actual sales data, the omni-channel model results in an average of seven percent increase in sales. However, compared to our study, they do not consider inventory decisions and different shipping preferences. Also, they do not allow the e-fulfilment center inventory in one zone to be used for physical store demand in that zone or online orders in other zones.

2.3. Problem formulation

We first provide the problem statement and justification. We then introduce the retailer's demand model and two mixed-integer nonlinear mathematical models that consider joint pricing controls and fulfillment decisions, where the second mixed-integer model takes more assumptions and constraints into account. We also introduce some additional decision variables and apply linearization techniques to transform the nonlinear version of the models to mixed-integer linear models to be able to solve the models much more efficiently.

2.3.1. Problem justification

Although many retailers have started to integrate their activities across different channels, there are still many retailers that make the pricing decisions separately for each channel. However, a few studies have recently shown that this is not the optimal approach and the retailers can increase their profit when they solve the price optimization problem for both channels simultaneously (Cao et. al 2016, Harsha et al. 2019). In fact, the price in one channel can affect the demand on the other channels. This assumption has been broadly used in revenue management literature where researchers use pricing to control demand for different class of products (i.e., price controls). For example, a very low in-store price can motivate a large portion of customers to purchase form the brick-and-mortar channel instead of the e-commerce channel. This implies that the offered prices for each channel can significantly change the probability of customers purchasing from each of those channels. However, there are still customers that prefer online shopping due to different reasons such as distance from the store or lack of free time.

One of the main advantages of omni-channel retailing compared to pure e-commerce or pure brick-and-mortar environments is sharing inventory between multiple channels and zones. In our proposed optimization models, online orders can be fulfilled from multiple zones, either from inventory dedicated to e-commerce orders (e-fulfillment centers) or in-store inventory. The omni-channel retailer has physical stores and e-fulfillment centers in different geographical zones, therefore, they can fulfill online orders from different zones if the closest e-fulfillment center is out of stock. We also allow the retailer to fulfill its in-store demand from the nearby e-fulfillment center. Unlike from existing literature, the new approach in our proposed models is that we adopt both pricing decisions (price controls) and fulfillment decisions across channels simultaneously. Indeed, omni-channel retailers can increase their profit if they make their fulfillment decisions (fulfillment costs) while considering pricing decisions. For example, the retailer may offer promotions to increase their online sales, but this may also cause a significant increase in their distribution costs. However, it could be more profitable if they deplete the in-store inventory by offering in-store promotions. This could increase the in-store demand and shift customers to the stores to avoid high shipping costs for online orders. Therefore, retailers can benefit from considering fulfillment decisions when they make pricing decisions.

2.3.2. Retailer demand model

The main purpose of omni-channel is to provide a seamless experience for customers. Customers should be able to see prices for different channels and different delivery times. Then they can choose the alternative that maximizes their utility. Thus, an omni-channel model should take this choice behavior into account. The model should consider the effect of an e-commerce price on brick-and-mortar demand and the effect of an in-store price on the e-commerce demand. In order to model this behavior, we consider Z geographical zones in the retailer network and assume that the customer choice of a channel in one zone is independent of brick-and-mortar prices in another zone. We also assume that the customer choice is independent of competitors' prices. Therefore, considering only the price, the channel demand in one zone is only dependent on the online and in-store prices in the same zone. However, in the second optimization model, we also take the delivery times into account and assume that this can change the customer's choice as well.

We first present an overview of discrete-choice models in general and then discuss the parametric choice model that is applied in this study.

2.3.2.1 General choice models

Discrete choice models have been widely used in revenue management. Strauss et al. (2018) provides a comprehensive review on choice models and their applications in revenue management. In this paper, the channel choice for customers is modeled by a discrete choice model. In general, customers' choice behavior can be modeled by the fact that the customers will maximize their utility by making a choice. In other words, a decision maker is modeled to select the alternative with highest utility among available ones when the choice is made. Choice models are commonly composed of utility functions with observable features and unknown parameters that can be estimated from a sample of observed choices made by decision makers. It is impossible to estimate a choice model that will always predict the correct choice for all individuals. Therefore, the decision maker utility is considered to be a random function. The concept of random utility was first used in Thurstone (1927). The utilities of the alternatives are random variables and the probability that an alternative is chosen is defined as the probability of that alternative having the greatest utility among the available alternatives (Ben-Akiva and Lerman 1985). Assume that the set of choices offered to customers equals

$C = c \cup \{0\}$, where c is the set of alternatives and 0 is the case when the customer does not choose any of the offered alternatives. Then the customer utility equals:

$$U_i = u_i + \xi_i$$

where u_i is the mean utility of customer choosing alternative $i \in C$ and ξ_i is the random component. As mentioned above, the mean utility is often modeled as a linear combination of observed attributes:

$$u_i = \boldsymbol{\beta}^{\mathsf{T}} \mathbf{x}_i$$

where $\boldsymbol{\beta}$ is an unknown vector of parameters that can be estimated and $\mathbf{x}_i$ is a vector of attributes for alternative *i*. Then the probability that a decision maker chooses alternative *i* is:

$$P_i = P\left(U_i \geq U_j,\ \forall j \in C\right)$$

2.3.2.2 The Multinomial Logit (MNL) choice model

The MNL model is one of the most widely used choice models in revenue management (Talluri and Van Ryzin 2004, Vulcano et al. 2010, Vulcano et al. 2012) and also in travel demand forecasting, economics, and marketing. It is derived by assuming that random components ξ_i are independent and identically distributed (i.i.d.) with Gumbel, also called double-exponential distribution, which has the following cumulative distribution (Ben-Akiva and Lerman 1985):

$$F(x) = P(\xi_i \leq x) = e^{-e^{-\mu(x-\eta)}}$$

Where *x* corresponds to error random variable and η is the location parameter (mode) and the μ is a positive scale parameter. The mean and the variance of ξ_i are:

$$E[\xi_i] = \eta + \gamma / \mu$$
$$Var[\xi_i] = \frac{\pi^2}{6\mu^2}$$

where $\pi \approx 3.14$ and $\gamma \approx 0.57$, which is Euler constant. Because the utility is an ordinal measure, the assumption of zero mean and a scale parameter of one are without loss of generality (see Ben-Akiva and Lerman 1985 for derivations).

In an omni-channel environment, the customer can easily see the online price and compare it with brick-and-mortar price. Therefore, observing a lower price for the same product may change the customer decision. This can actually affect the customer channel preferences. Assume that there are total of *Z* geographical zones (e.g., the U.S. states) in omni-channel network and customer who purchases from brick-and-mortar channel in zone *z* has utility equal to U_{bz} and by purchasing from online channel, her utility is U_{ez}. Therefore, the customer utility by purchasing from each channel in zone *z* equals:

$$U_{bz} = u_{bz} + \xi_{bz}$$

$$U_{ez} = u_{ez} + \xi_{ez}$$

where $u_{iz}, i \in \{b, e\}$ is mean utility of choice *i* and $\xi_{iz}, i \in \{b, e\}$ is an i.i.d. Gumble random variable with mean zero and scale parameter one for all *i*. The customer can also decide not to buy from either of these channels and receive a mean utility equal to zero. As mentioned before, because the utility is ordinal, we can assume its value is zero without loss of generality. Under this utility model, it is well-known (Ben-Akiva and Lerman 1985) that the choice probabilities equal:

$$P_{iz} = \frac{e^{u_{iz}}}{\sum_{j \in \{e,b\}} e^{u_{jz}} + e^{u_0}} \qquad \forall i \in \{e, b\}, \forall z \in Z$$

where P_{iz} is the probability that a customer purchases from channel *i* in zone *z* and u_0 is the mean utility of the non-purchase choice. It is also assumed that the customer in one geographical zone *z* only compares the e-commerce price with the brick-and-mortar price in the same zone.

As noted before, the mean utility of a choice of a customer can be modeled by a linear function of attributes such as price, channel-specific attributes including shipping times, and holiday effects. However, in our first mathematical model, we assume that the mean utility of choice *i*, u_{iz}, $i \in \{b,e\}$, is only a linear function of the channel prices (Harsha et al. 2019).

$$u_{bz} = \beta_{0bz} + \beta_{1bz} p_{bz} \tag{1}$$

$$u_{ez} = \beta_{0ez} + \beta_{1ez} p_{ez} \tag{2}$$

where $\beta_{0iz}, i \in \{b,e\}$ is a constant and $\beta_{1iz}, i \in \{b,e\}$ is the coefficient of price in online or brick-and-mortar channels, $p_{iz}, i \in \{b,e\}$ is the price offered to customers in e-commerce and brick-and-mortar channels in zone *z*. (Please note that the P_{iz} is used for channel probabilities). Now, assume that we have *T* periods during the planning horizon and the market size at time period *t* in zone *z* equals n_z^t, then the e-commerce and brick-and-mortar channel demand functions at time period *t* in zone *z* follow the form below, which is a function of the price vector $\mathbf{P}_z^t$:

$$\begin{aligned} D_{bz}^t\left(\mathbf{P}_z^t\right) &= n_z^t \times P_{bz} \\ &= n_z^t \times \frac{e^{\beta_{0bz}+\beta_{1bz}p_{bz}^t}}{e^{\beta_{0bz}+\beta_{1bz}p_{bz}^t} + e^{\beta_{0ez}+\beta_{1ez}p_{ez}^t} + 1} \end{aligned} \tag{3}$$

$$D_{ez}^{t}\left(\mathbf{P}_{z}^{t}\right)=n_{z}^{t}\times P_{ez} \tag{4}$$
$$=n_{z}^{t}\times\frac{e^{\beta_{0ez}+\beta_{1ez}p_{ez}^{t}}}{e^{\beta_{0bz}+\beta_{1bz}p_{bz}^{t}}+e^{\beta_{0ez}+\beta_{1ez}p_{ez}^{t}}+1}$$

where P_{bz} and P_{ez} are probabilities that customers purchase from brick-and-mortar and e-commerce, respectively. $\mathbf{P}_{z}^{t}=\left(p_{bz}^{t},p_{ez}^{t}\right)$ is the price vector including online and brick-and-mortar prices offered at time *t*, in zone *z*.

2.3.3. Mathematical models for joint pricing and fulfillment problem

We propose two mathematical models for joint pricing and fulfillment problem in this section. The second model has more assumptions than the first model in that it considers initial inventory allocation, inventory holding cost, and different types of shipments. Both proposed models are nonlinear models, but we show that they can be transformed to linear versions to be solved efficiently.

2.3.3.1 Optimization model #1 (M_1)

Consider an omni-channel retailer that sells its products in *Z* geographical zones. The customer can buy the product from a brick-and-mortar channel or order the product through online channel. For both physical stores and fulfillment centers, there is limited amount of inventory available. The selling horizon is finite and divided into *T* periods. At the beginning of period *t*, the retailer reveals the price vector $\mathbf{P}_{z}^{t}=\left(p_{bz}^{t},p_{ez}^{t}\right)$ in each zone $z\in Z$. The resulting demand will be estimated in each zone in brick-and-mortar and online channels using choice probabilities and market size, $D_{bz}^{t}\left(\mathbf{P}_{z}^{t}\right)$ and $D_{ez}^{t}\left(\mathbf{P}_{z}^{t}\right)$, respectively. The retailer then fulfills the demand using the available inventory in its network. Replenishment is not considered in this study since the pricing and fulfillment decisions taken together are sufficiently complicated operational decisions. This is also common in the literature (Xu et al. 2009, Acimovic and Graves 2014, and Lei et al. 2018) because the planning horizon could be considered the time between two replenishments. We assume that there is no holding cost in mathematical model M_1 because of short planning horizon. However, inventory costs could be included in the products price set Ω. As seen later, we include the holding cost and initial inventory assignment in mathematical model M_2. We also assume that there is no shortage cost because of the short planning period. Retailer will not pay any penalty if the demand is higher that its fixed initial inventory. But if the retailer has enough inventory and decides not to satisfy the order, it will only lose revenue.

We assume that the retailer has one physical store and one e-fulfillment center in each zone. We also assume that the store demand in zone *z* can be fulfilled either by the in-store inventory or the e-fulfillment center inventory in the same zone, *z*. However, the online demand in zone *z* can be fulfilled by in-store inventory or the e-fulfillment center inventory from the same or any other geographical zone. When the e-fulfillment center in zone *z* is out of inventory for an order, then the retailer should decide form which zone, and from which type of inventory to fulfill that demand. Shipping an online order from another zone could cause increased logistical costs for retailers but by having the pricing controls, the retailers could encourage a customer to order that product online and pick it up from a physical store in the same zone. This is an example that how the omni-channel retailers could benefit from joint pricing controls and fulfillment decisions to maximize their revenue.

We have used three types of variables to model this problem. The first type includes integer variables that are used to show the fulfillment quantities. Specifically S_{bwz}^{t} and S_{ewz}^{t} are decision variables that are used for the fulfilled brick-and-mortar demand in zone *z* at time *t* that are satisfied from zone *w* with in-store and e-fulfillment inventory, respectively. In addition, O_{bwz}^{t} and O_{ewz}^{t} are decision variables that are used for the fulfilled e-commerce demand in zone *z* at time *t* that are satisfied from zone *w* with in-store and e-fulfillment inventory, respectively. The second type of the variables are for leftover inventory. We use L_{bz} and L_{ez} for the leftover inventory at the end of the planning horizon in zone *z* for in-store and e-fulfillment inventory, respectively. The last type of variables are pricing variables. P_{bz}^{t} and P_{ez}^{t} are used for pricing decisions in zone *z* at time *t* for brick-and-mortar and online price offered to customer, respectively. The notation and variables are listed in Table 2.1.

Table 2.1. Input parameters and decisions variables in mathematical model M_1

Input parameters	
Z	Number of geographical zones
T	Number of time periods
Ω	Set of all feasible prices in each channel in each zone
x_{bz}	Initial in-store inventory in zone z
x_{ez}	Initial e-fulfillment inventory in zone z
c^o_{ewz}	Fulfillment cost of e-commerce demand in zone *z* from an e-fulfillment center in zone *w*
c^o_{bwz}	Fulfillment cost of e-commerce demand in zone *z* from an in-store inventory in zone *w*
c^s_{ewz}	Fulfillment cost of store demand in zone *z* from an e-fulfillment center in zone *w*
c^s_{bwz}	Fulfillment cost of store demand in zone *z* from an in-store inventory in zone *w*
n^t_z	Market size in zone *z* at period *t*
v	Salvage value for leftover inventory after the planning horizon
Decision Variables	
P^t_{bz}	Brick-and-mortar price in zone *z* at period *t*
P^t_{ez}	E-commerce price in zone *z* at period *t*
S^t_{bwz}	Fulfilled brick-and-mortar demand in zone *z* by in-store inventory in zone *w* at period *t*
S^t_{ewz}	Fulfilled brick-and-mortar demand in zone *z* by e-fulfillment inventory in zone *w* at period *t*
O^t_{bwz}	Fulfilled e-commerce demand in zone *z* by in-store inventory in zone *w* at period *t*
O^t_{ewz}	Fulfilled e-commerce demand in zone *z* by e-fulfillment inventory in zone *w* at period *t*
L_{bz}	Leftover e-fulfillment inventory in zone *z* after the planning horizon
L_{ez}	Leftover in-store inventory in zone *z* after the planning horizon

The optimization model M_1 is as follows:

$$Max \sum_{t\in T}\sum_{w\in Z}\sum_{z\in Z}\left[P_{bz}^{t}\left(S_{bwz}^{t}+S_{ewz}^{t}\right)+P_{ez}^{t}\left(O_{bwz}^{t}+O_{ewz}^{t}\right)\right]+v\left[\sum_{z\in Z}\left(L_{bz}+L_{ez}\right)\right]$$
$$-\sum_{t\in T}\sum_{w\in Z}\sum_{z\in Z}\left(c_{bwz}^{s}S_{bwz}^{t}+c_{ewz}^{s}S_{ewz}^{t}+c_{bwz}^{o}O_{bwz}^{t}+c_{ewz}^{o}O_{ewz}^{t}\right) \tag{5}$$

s.t.

$$\sum_{w\in Z}\left(S_{bwz}^{t}+S_{ewz}^{t}\right)\le D_{bz}^{t}\left(\mathbf{P_z^t}\right) \qquad \forall z\in Z, \forall t\in T, \tag{6}$$

$$\sum_{w\in Z}\left(O_{bwz}^{t}+O_{ewz}^{t}\right)\le D_{ez}^{t}\left(\mathbf{P_z^t}\right) \qquad \forall z\in Z, \forall t\in T, \tag{7}$$

$$L_{bz}=x_{bz}-\sum_{t\in T}\sum_{w\in Z}\left(S_{bzw}^{t}+O_{bzw}^{t}\right) \qquad \forall z\in Z \tag{8}$$

$$L_{ez}=x_{ez}-\sum_{t\in T}\sum_{w\in Z}\left(S_{ezw}^{t}+O_{ezw}^{t}\right) \qquad \forall z\in Z \tag{9}$$

$$\mathbf{P_z^t}=\left(P_{bz}^{t},P_{ez}^{t}\right)_{t\in T, z\in Z}\in\Omega \tag{10}$$

$$S_{bwz}^{t},S_{ewz}^{t},O_{bwz}^{t},O_{ewz}^{t}\ge 0 \qquad \forall z\in Z, \forall w\in Z, \forall t\in T \tag{11}$$

$$L_{ez},L_{bz}\ge 0 \qquad \forall z\in Z \tag{12}$$

The objective function (5) includes three primary terms. The first term considers the revenue from the fulfilled demand from brick-and-mortar and e-commerce channel, respectively. This is nonlinear since fulfillment decision variables are multiplied against pricing variables. The second term considers the revenue from the leftover inventory, both in-store and e-fulfillment centers. The third term considers the fulfillment cost of online and brick-and-mortar demand which are fulfilled by either the in-store inventory or e-fulfillment inventory in the same or other geographical zones. As a cost, this part is subtracted from the revenue of the first two parts since we are trying to maximize the profit.

Constraint (6) ensures that the brick-and-mortar sales in zone *z* at period *t* is less than the brick-and-mortar demand in the same zone and time period. Constraint (7) guarantees that the e-commerce sales in zone *z*, period *t* is less than the e-commerce demand in zone *z* and time period *t*. Constraint (8) forces the leftover in-store inventory in zone *z* to be equal to the initial in-store inventory in zone *z* minus the used inventory for the in-store demand in the same zone and e-commerce demand for the same and all the other zones. Constraint (9) ensures that the leftover e-fulfillment inventory in zone *z* to be equal to initial inventory in that zone minus the used inventory for the in-store demand in the same zone and e-commerce demand in the same and all the other zones. Constraint (10) ensures that the prices are from the price set. Constraint (11) and (12) are non-negativity constraints.

The mathematical model M_1 has multiple nonlinear terms, which presents computational difficulties. As noted earlier, the first term of the objective function is nonlinear. Also, in constraint (6) and (7), the demand function is a nonlinear function including exponential terms as seen in (3) and (4). Therefore, we introduce additional variables and transform the model to a mixed-integer linear mathematical model that can be solved much more efficiently (Harsha et al. 2016, Sherali and Adams 1998).

We first introduce Y_{ezi}^t, which is a binary variable. It will be equal to 1 if the *i*-th price in the discrete online price set at zone *z*, at time *t*; $\Omega_{ez}^t = \left\{ p_{ezi}^t \right\}_{i \in I_{ez}^t}$ is chosen, where I_{ez}^t is the number of discrete prices that are offered in that set. The second variable is Y_{bzi}^t, which is also a binary variable. It will be equal to 1 if the *i*-th price in the discrete brick-and-mortar price set at zone z, at time t; $\Omega_{bz}^{t}=\left\{p_{bzi}^{t}\right\}_{i\in I_{bz}^{t}}$ is chosen, where I_{bz}^{t} is the number of discrete prices that are available in that set. We also define the following transformations:

$$r_{bzi}^{t}=e^{\beta_{0bz}+\beta_{1bz}p_{bzi}^{t}} \tag{13}$$

$$r_{ezi}^{t}=e^{\beta_{0bz}+\beta_{1bz}p_{ezi}^{t}} \tag{14}$$

$$R_{z}^{t}=\frac{1}{\sum_{i\in I_{bz}^{t}}Y_{bzi}^{t}r_{bzi}^{t}+\sum_{i\in I_{ez}^{t}}Y_{ezi}^{t}r_{ezi}^{t}+1} \tag{15}$$

$$U_{bzi}^{t}=R_{z}^{t}Y_{bzi}^{t} \tag{16}$$

$$U_{ezi}^{t}=R_{z}^{t}Y_{ezi}^{t} \tag{17}$$

$$V_{bzi}^{t}=\sum_{w\in Z}\left(S_{ewz}^{t}+S_{bwz}^{t}\right)Y_{bzi}^{t} \tag{18}$$

$$V_{ezi}^{t}=\sum_{w\in Z}\left(O_{ewz}^{t}+O_{bwz}^{t}\right)Y_{ezi}^{t} \tag{19}$$

where r_{bzi}^{t} and r_{ezi}^{t} are constants. Now we can reformulate the optimization model M_1, as a mixed-integer linear program that can be solved efficiently using integer programming software packages. The linear version of the M_1 is as follows:

$$Max \sum_{t\in T}\sum_{z\in Z}\left[\left(\sum_{i\in I_{bz}^t} P_{bzi}^t V_{bzi}^t\right)+\left(\sum_{i\in I_{ez}^t} P_{ezi}^t V_{ezi}^t\right)\right]+v\left[\sum_{z\in Z}\left(L_{bz}+L_{ez}\right)\right]$$
$$-\sum_{t\in T}\sum_{w\in Z}\sum_{z\in Z}\left(c_{bwz}^s S_{bwz}^t + c_{ewz}^s S_{ewz}^t + c_{bwz}^o O_{bwz}^t + c_{ewz}^o O_{ewz}^t\right) \quad (20)$$

$$\sum_{w\in Z}\left(S_{bwz}^t + S_{ewz}^t\right) \le n_z^t \sum_{i\in I_{bz}^t} r_{bzi}^t U_{bzi}^t \qquad \forall z\in Z, \forall t\in T, \quad (21)$$

$$\sum_{w\in Z}\left(O_{bwz}^t + O_{ewz}^t\right) \le n_z^t \sum_{i\in I_{ez}^t} r_{ezi}^t U_{ezi}^t \qquad \forall z\in Z, \forall t\in T, \quad (22)$$

$$L_{bz} = x_{bz} - \sum_{t\in T}\sum_{w\in Z}\left(S_{bzw}^t + O_{bzw}^t\right) \qquad \forall z\in Z \quad (23)$$

$$L_{ez} = x_{ez} - \sum_{t\in T}\sum_{w\in Z}\left(S_{ezw}^t + O_{ezw}^t\right) \qquad \forall z\in Z \quad (24)$$

$$\sum_{i\in I_{bz}^t} Y_{bzi}^t = 1 \qquad \forall t\in T, \forall z\in Z, \quad (25)$$

$$\sum_{i\in I_{ez}^t} Y_{ezi}^t = 1 \qquad \forall t\in T, \forall z\in Z, \quad (26)$$

$$\sum_{i\in I_{bz}^t} V_{bzi}^t = \sum_{w\in Z}\left(S_{bwz}^t + S_{ewz}^t\right) \qquad \forall z\in Z, \forall t\in T, \quad (27)$$

$$\sum_{i\in I_{ez}^t} V_{ezi}^t = \sum_{w\in Z}\left(O_{bwz}^t + O_{ewz}^t\right) \qquad \forall z\in Z, \forall t\in T, \quad (28)$$

$$V_{bzi}^t \le \sum_{w\in Z}\left(S_{bwz}^t + S_{ewz}^t\right) \qquad \forall i\in I_{bz}^t, \forall z\in Z, \forall t\in T, \quad (29)$$

$$V_{ezi}^t \le \sum_{w\in Z}\left(O_{bwz}^t + O_{ewz}^t\right) \qquad \forall i\in I_{ez}^t, \forall z\in Z, \forall t\in T, \quad (30)$$

$$V_{bzi}^t \le n_z^t r_{bzi}^t U_{bzi}^t \qquad \forall i\in I_{bz}^t, \forall z\in Z, \forall t\in T, \quad (31)$$

$$V_{ezi}^t \le n_z^t r_{ezi}^t U_{ezi}^t \qquad \forall i\in I_{ez}^t, \forall z\in Z, \forall t\in T, \quad (32)$$

$$\sum_{i \in I_{bz}^t} U_{bzi}^t = R_z^t \qquad \forall z \in Z, \forall t \in T, \tag{33}$$

$$\sum_{i \in I_{ez}^t} U_{ezi}^t = R_z^t \qquad \forall z \in Z, \forall t \in T, \tag{34}$$

$$U_{bzi}^t \leq R_z^t \qquad \forall i \in I_{bz}^t, \forall z \in Z, \forall t \in T, \tag{35}$$

$$U_{ezi}^t \leq R_z^t \qquad \forall i \in I_{ez}^t, \forall z \in Z, \forall t \in T, \tag{36}$$

$$U_{bzi}^t \leq Y_{bzi}^t \qquad \forall i \in I_{bz}^t, \forall z \in Z, \forall t \in T, \tag{37}$$

$$U_{ezi}^t \leq Y_{ezi}^t \qquad \forall i \in I_{ez}^t, \forall z \in Z, \forall t \in T, \tag{38}$$

$$R_z^t + \sum_{i \in I_{bzi}^t} r_{bzi}^t U_{bzi}^t + \sum_{i \in I_{ezi}^t} r_{ezi}^t U_{ezi}^t = 1 \qquad \forall z \in Z, \forall t \in T, \tag{39}$$

$$Y_{bzi}^t \in \{0,1\} \qquad \forall z \in Z, \forall w \in Z, \forall t \in T, \forall i \in I_{bz}^t \tag{40}$$

$$Y_{ezi}^t \in \{0,1\} \qquad \forall z \in Z, \forall w \in Z, \forall t \in T, \forall i \in I_{ez}^t, \tag{41}$$

$$S_{bwz}^t, S_{ewz}^t, O_{bwz}^t, O_{ewz}^t \geq 0 \qquad \forall z \in Z, \forall w \in Z, \forall t \in T \tag{42}$$

$$L_{ez}, L_{bz} \geq 0 \qquad \forall z \in Z \tag{43}$$

$$R_z^t \geq 0, \qquad \forall z \in Z, \forall t \in T, \tag{44}$$

$$U_{bzi}^t, \mathrm{V}_{bzi}^t \geq 0, \qquad \forall z \in Z, \forall t \in T, \forall i \in I_{bz}^t, \tag{45}$$

$$U_{ezi}^t, \mathrm{V}_{ezi}^t \geq 0, \qquad \forall z \in Z, \forall t \in T, \forall i \in I_{ez}^t, \tag{46}$$

Objective function (20) includes the same terms as objective function (5), namely revenue from selling in both channels and leftover inventory subtracted by fulfillment costs, but without any nonlinear terms. Constraints (21) and (22) are ensuring that the sales are less than demand in both channels. Constraints (23) and (24) are for leftover inventory balance.

Constraints (25) and (26) ensure that exactly one price is chosen in brick-and-mortar and e-commerce channel in each zone at each time period, respectively. Constraints (27) and (28) ensure that variables V_{bzi}^{t} and V_{ezi}^{t} are equal to fulfilled demand in brick-and-mortar and online channels. Constraints (29) and (30) are tighter bounds on variables V_{bzi}^{t} and V_{ezi}^{t}. Constraints (31) and (32) ensure that variables V_{bzi}^{t} and V_{ezi}^{t} are less than demand in each channel. Since variable U_{bzi}^{t} equals $R_{z}^{t}Y_{bzi}^{t}$ and Y_{bzi}^{t} is a binary decision variable, then constraint (33) ensures that the sum over index *i* leads to equality of variables U_{bzi}^{t} and R_{z}^{t}. Similar to constraint (33), constraint (34) ensures the equality of U_{ezi}^{t} and R_{z}^{t}. Constraints (35) and (36) are tighter bound on constraints (33) and (34). Constraints (37) and (38) also ensure that the variables U_{bzi}^{t} and U_{ezi}^{t} are less than one. Constraint (39) ensures the transformation introduced in (15). Constraints (40)-(46) introduce the non-negativity and binary constraints.

2.3.3.2 Optimization model #2 (M_2)

Price is not the only factor that can change the customer's choice. As customer can also consider her convenience when purchasing online. Mathematical model M_2 considers multiple shipping options and their delivery times along with their prices. Based on a research study on the omni-channel retailing (Forrester Research 2014), seventy five percent of the customers mentioned that a fast and free shipping option can change their decision when they buy online. We also consider inventory decisions in mathematical model M_2.

Again, consider an omni-channel retailer that sells its products in *Z* geographical zones. The customer can buy the product from a brick-and-mortar channel or buy the product through an online channel, with two shipping options: fast (expedited shipping) and very fast (next day delivery). They could also be considered as regular shipping versus fast shipping. Without loss of generality, we assume that the customer's mean utility for store-pick-up is the same as her mean utility for buying the product from the brick-and-mortar channel and we do not include both of them in the model to avoid model complexity and computational inefficiencies. However, more shipping options could be added as a choice to customers. Therefore, we assume three purchasing options in total and define the following customer's utility function:

$$u_{bz} = \beta_{0bz} + \beta_{1bz} p_{bz} + \beta_{2bz} t_{bz} + \beta_{3bz} t_{bz} p_{bz} \tag{47}$$

$$u_{fz} = \beta_{0fz} + \beta_{1fz} p_{fz} + \beta_{2fz} t_{fz} + \beta_{3fz} t_{fz} p_{fz} \tag{48}$$

$$u_{sz} = \beta_{0sz} + \beta_{1sz} p_{sz} + \beta_{2sz} t_{sz} + \beta_{3sz} t_{sz} p_{sz} \tag{49}$$

where $p_{iz}, i \in \{b, f, s\}$ and $t_{iz}, i \in \{b, f, s\}$ are the prices and delivery times offered to customers for brick-and-mortar channel, expedited shipping and next day delivery in zone *z*. $\beta_{0iz}, i \in \{b, f, s\}$ is a constant and $\beta_{1iz}, i \in \{b, f, s\}$ is the price coefficient for brick-and-mortar channel, expedited shipping and next day delivery, respectively. $\beta_{2iz}, i \in \{b, f, s\}$ is the delivery time coefficient and $\beta_{3iz}, i \in \{b, f, s\}$ is the coefficient for interaction of price and delivery time. There are *T* periods and the market size at time period *t* in zone *z* equals n_z^t, then the e-commerce and brick-and-mortar channel demand functions with different shipping options at time period *t* in zone *z* follow the form below, which is a function of the price vector $\mathbf{P}_z^t$:

$$D_{bz}^t\left(\mathbf{P}_z^t\right) = n_z^t \times \frac{e^{u_{bz}}}{e^{u_{bz}} + e^{u_{fz}} + e^{u_{sz}} + 1} \tag{50}$$

$$D_{fz}^t\left(\mathbf{P}_z^t\right) = n_z^t \times \frac{e^{u_{fz}}}{e^{u_{bz}} + e^{u_{fz}} + e^{u_{sz}} + 1} \tag{51}$$

$$D_{sz}^t\left(\mathbf{P}_z^t\right) = n_z^t \times \frac{e^{u_{sz}}}{e^{u_{bz}} + e^{u_{fz}} + e^{u_{sz}} + 1} \tag{52}$$

where the $\mathbf{P}_z^t = \left(p_{bz}^t, p_{fz}^t, p_{sz}^t \right)$ is the price vector including online and brick-and-mortar prices offered at time *t*, in zone *z*.

We also consider inventory decisions along with pricing control and fulfillment decisions in mathematical model M_2. The initial inventory levels for physical stores and e-fulfillment centers are not fixed and they are decided at the beginning of the planning horizon. Since it's common for retailers to follow a system wide periodic review policy, more specifically a base-stock policy (Acimovic and Graves 2017), the retailers have to decide how to allocate their initial inventory throughout their network. At the beginning of period *t*, the retailer sets the price vector $\mathbf{P}_z^t = \left(p_{bz}^t, p_{fz}^t, p_{sz}^t \right)$ in each zone $z \in Z$. After the retailer decides the price, the demand will be estimated in each zone in brick-and-mortar and online channels for each shipping option, $D_{bz}^t\left(\mathbf{P}_z^t\right)$, $D_{fz}^t\left(\mathbf{P}_z^t\right)$ and $D_{sz}^t\left(\mathbf{P}_z^t\right)$. The retailer then fulfills the demand using the available inventory in its network. Here, we assume that the retailer has one physical store and one e-fulfillment center in each zone. Similar to model M_1, we assume that there is no shortage cost because of the short planning horizon. Retailer will not pay any penalty if the demand is higher that its total initial inventory (i.e., total system-wide initial inventory is fixed). But if the retailer has enough inventory and decides not to satisfy the order, it will lose revenue. As noted earlier, in-store inventory can be used to help retailers to fulfill online customers with expedited and next day shipping options.

We have used three type of variables in model M_2. The first type are integer variables that are used to show the fulfillment quantities. S_{bwz}^t and S_{ewz}^t are decision variables that are used for the fulfilled brick-and-mortar demand in zone *z* at time *t* that are satisfied from zone *w* with in-store and e-fulfillment inventory, respectively. O_{bwz}^{ft} and O_{ewz}^{ft} are decision variables that are used for the fulfilled e-commerce demand with expedited shipping in zone *z* at time *t*, that are satisfied from zone *w* with in-store and e-fulfillment inventory, respectively. O_{bwz}^{st} and O_{ewz}^{st} are decision variables that are used for online demand with next day delivery in zone *z* at time *t* that are satisfied from zone *w* with in-store and e-fulfillment inventory, respectively. The second type of the variables are pricing variables P_{bz}^{t}, P_{fz}^{t} and P_{sz}^{t}. We also have one more set of variables, X_{ez}^{t} and X_{bz}^{t}, which are integer variables determining the inventory for physical store and e-fulfillment center at time *t*, in zone *z*. The notation and variables are listed in Table 2.2.

Table 2.2. Input parameters and decisions variables in mathematical model M_2

Input parameters	
Z	Number of geographical zones
T	Number of time periods
Ω	Set of all feasible prices in each channel in each zone
x	Total initial inventory before the planning horizon
c_{ewz}^{f}	Fulfillment cost of expedited e-commerce demand in zone *z* from an e-fulfillment center in zone *w*
c_{bwz}^{f}	Fulfillment cost of expedited e-commerce demand in zone *z* from an in-store inventory in zone *w*
c_{ewz}^{s}	Fulfillment cost of next day e-commerce demand in zone *z* from an e-fulfillment center in zone *w*
c_{bwz}^{s}	Fulfillment cost of next day e-commerce demand in zone *z* from an in-store inventory in zone *w*
c_{ewz}^{s}	Fulfillment cost of store demand in zone *z* from an e-fulfillment center in zone *w*
c_{bwz}^{s}	Fulfillment cost of store demand in zone *z* from an in-store inventory in zone *w*
n_{z}^{t}	Market size in zone *z* at period *t*
v	Salvage value for leftover inventory after the planning horizon
h	Inventory holding cost
Decision Variables	
P_{bz}^{t}	Brick-and-mortar price in zone *z* at period *t*
P_{fz}^{t}	E-commerce price with fast shipping in zone *z* at period *t*
P_{sz}^{t}	E-commerce price with very fast shipping in zone *z* at period *t*
S_{bwz}^{t}	Fulfilled brick-and-mortar demand in zone *z* by brick-and-mortar inventory in zone *w* at period *t*
S_{ewz}^{t}	Fulfilled brick-and-mortar demand in zone *z* by e-fulfillment inventory in zone *w* inventory at period *t*
O_{bwz}^{ft}	Fulfilled expedited e-commerce demand in zone *z* by brick-and-mortar inventory in zone *w* at period *t*
O_{ewz}^{ft}	Fulfilled expedited e-commerce demand in zone *z* by e-fulfillment inventory in zone *w* at period *t*
O_{bwz}^{st}	Fulfilled next day e-commerce demand in zone *z* by brick-and-mortar inventory in zone *w* at period *t*
O_{ewz}^{st}	Fulfilled next day e-commerce demand in zone *z* from e-fulfillment inventory in zone *w* at period *t*
X_{bz}^{t}	In-store inventory in zone *z* at the end of period *t*
X_{ez}^{t}	E-fulfillment inventory in zone *z* at the end of period *t*

The optimization model M_2 is as follows:

$$Max \sum_{t\in T}\sum_{w\in Z}\sum_{z\in Z}\left[P_{bz}^{t}\left(S_{bwz}^{t}+S_{ewz}^{t}\right)+P_{fz}^{t}\left(O_{bwz}^{ft}+O_{ewz}^{ft}\right)+P_{ez}^{t}\left(O_{bwz}^{st}+O_{ewz}^{st}\right)\right]+v\left[\sum_{z\in Z}\left(X_{bz}^{T}+X_{ez}^{T}\right)\right]$$
$$-h\left[\sum_{t\in T}\sum_{z\in Z}\left(X_{bz}^{t}+X_{ez}^{t}\right)\right]-\sum_{t\in T}\sum_{w\in Z}\sum_{z\in Z}\left(c_{bwz}^{s}S_{bwz}^{t}+c_{ewz}^{s}S_{ewz}^{t}+c_{bwz}^{f}O_{bwz}^{ft}+c_{ewz}^{f}O_{ewz}^{ft}+c_{bwz}^{s}O_{bwz}^{st}+c_{ewz}^{s}O_{ewz}^{st}\right) \quad (53)$$

s.t.

$$\sum_{w\in Z}\left(S_{bwz}^{t}+S_{ewz}^{t}\right)\le D_{bz}^{t}\left(\mathbf{P_z^t}\right) \qquad \forall z\in Z,\forall t\in T, \quad (54)$$

$$\sum_{w\in Z}\left(O_{bwz}^{ft}+O_{ewz}^{ft}\right)\le D_{fz}^{t}\left(\mathbf{P_z^t}\right) \qquad \forall z\in Z,\forall t\in T, \quad (55)$$

$$\sum_{w\in Z}\left(O_{bwz}^{st}+O_{ewz}^{st}\right)\le D_{sz}^{t}\left(\mathbf{P_z^t}\right) \qquad \forall z\in Z,\forall t\in T, \quad (56)$$

$$X_{bz}^{t}=X_{bz}^{t-1}-\sum_{w\in Z}\left(S_{bzw}^{t}+O_{bzw}^{ft}+O_{bzw}^{st}\right) \qquad \forall z\in Z,\forall t\in T, \quad (57)$$

$$X_{ez}^{t}=X_{ez}^{t-1}-\sum_{w\in Z}\left(S_{ezw}^{t}+O_{ezw}^{ft}+O_{ezw}^{st}\right) \qquad \forall z\in Z,\forall t\in T, \quad (58)$$

$$\sum_{z\in Z}X_{ez}^{0}+\sum_{z\in Z}X_{bz}^{0}=x \quad (59)$$

$$\mathbf{P_z^t}=\left(P_{bz}^{t},P_{fz}^{t},P_{sz}^{t}\right)_{t\in T,z\in Z}\in\Omega \quad (60)$$

$$S_{bwz}^{t},S_{ewz}^{t},O_{bwz}^{ft},O_{ewz}^{ft},O_{bwz}^{st},O_{ewz}^{st}\ge 0 \qquad \forall z\in Z,\forall w\in Z,\forall t\in T \quad (61)$$

$$X_{ez}^{t},X_{bz}^{t}\ge 0 \qquad \forall z\in Z,\forall t\in T \quad (62)$$

Objective function (53) includes revenue from three components, online sales with expedited shipping, online sales with next day delivery and sales from brick-and mortar channel (or buy online and pick up in store). The second part is for revenue from leftover inventory. The third and the fourth parts consider the inventory and fulfillment and costs, respectively. Constraints (54)-(56) ensure that the sales in each channel, under each shipping option are less than the demand in those channels. Constraints (57) and (58) ensure inventory balance for in-store and e-fulfillment inventory. Constraint (59) ensures that the total inventory distributed throughout the system is equal to initial inventory. Constraint (60) ensures that prices should be chosen from a price set. Constraints (61) to (62) are non-negativity constraints.

Similar to M_1, mathematical model M_2 also has multiple nonlinear terms. Therefore, we use the previous technique and introduce variables to transform the model to a mixed-integer linear mathematical model that can be solved more efficiently. The first defined variable is Y_{fzi}^{t}. It is equal to 1 if the *i*-th price in the discrete online price set for fast shipping at zone *z*, at time *t*; $\Omega_{fz}^{t} = \left\{ p_{fzi}^{t} \right\}_{i \in I_{fz}^{t}}$ is chosen, where I_{fz}^{t}, is the number of discrete prices that are available in that set. We similarly define Y_{szi}^{t} and Y_{bzi}^{t}, which are also binary variables.

We also define the following transformations:

$$r_{bzi}^{t} = \exp\left(\beta_{0bz} + \beta_{1bz} p_{bzi}^{t} + \beta_{2bz} t_{bz} + \beta_{3bz} t_{bz} p_{bzi}^{t} \right) \quad (64)$$

$$r_{fzi}^{t} = \exp\left(\beta_{0fz} + \beta_{1fz} p_{fzi}^{t} + \beta_{2fz} t_{fz} + \beta_{3fz} t_{fz} p_{fzi}^{t} \right) \quad (65)$$

$$r_{szi}^{t} = \exp\left(\beta_{0sz} + \beta_{1fz} p_{szi}^{t} + \beta_{2sz} t_{sz} + \beta_{3sz} t_{sz} p_{szi}^{t} \right) \quad (66)$$

$$R_{z}^{t} = \frac{1}{\sum_{i \in I_{bz}^{t}} Y_{bzi}^{t} b_{bzi}^{t} + \sum_{i \in I_{ez}^{t}} Y_{fzi}^{t} b_{fzi}^{t} + \sum_{i \in I_{ez}^{t}} Y_{szi}^{t} b_{szi}^{t} + 1} \quad (67)$$

$$U_{bzi}^{t} = R_{z}^{t} Y_{bzi}^{t} \quad (68)$$

$$U_{fzi}^{t} = R_{z}^{t} Y_{fzi}^{t} \quad (69)$$

$$U_{szi}^{t} = R_{z}^{t} Y_{szi}^{t} \tag{70}$$

$$V_{bzi}^{t} = \sum_{w \in Z} \left(S_{ewz}^{t} + S_{bwz}^{t} \right) Y_{bzi}^{t} \tag{71}$$

$$V_{fzi}^{t} = \sum_{w \in Z} \left(O_{ewz}^{ft} + O_{bwz}^{ft} \right) Y_{fzi}^{t} \tag{72}$$

$$V_{szi}^{t} = \sum_{w \in Z} \left(O_{ewz}^{st} + O_{bwz}^{st} \right) Y_{szi}^{t} \tag{73}$$

where r_{bzi}^{t} , r_{fzi}^{t} and r_{szi}^{t} are constants. Reformulated M2 is as follows:

$$\begin{aligned} Max \ & \sum_{t \in T} \sum_{z \in Z} \left[\left(\sum_{i \in I_{bz}^{t}} P_{bzi}^{t} V_{bzi}^{t} \right) + \left(\sum_{i \in I_{fz}^{t}} P_{fzi}^{t} V_{fzi}^{t} \right) + \left(\sum_{i \in I_{sz}^{t}} P_{szi}^{t} V_{szi}^{t} \right) \right] + v \left[\sum_{z \in Z} \left(X_{bz}^{T} + X_{ez}^{T} \right) \right] \\ & - h \left[\sum_{t \in T} \sum_{z \in Z} \left(X_{bz}^{t} + X_{ez}^{t} \right) \right] - \sum_{t \in T} \sum_{w \in Z} \sum_{z \in Z} \left(c_{bwz}^{s} S_{bwz}^{t} + c_{ewz}^{s} S_{ewz}^{t} + c_{bwz}^{f} O_{bwz}^{ft} + c_{ewz}^{f} O_{ewz}^{ft} + c_{bwz}^{s} O_{bwz}^{st} + c_{ewz}^{s} O_{ewz}^{st} \right) \end{aligned} \tag{74}$$

$$\sum_{w \in Z} \left(S_{bwz}^{t} + S_{ewz}^{t} \right) \le n_{z}^{t} \sum_{i \in I_{bz}^{t}} r_{bzi}^{t} U_{bzi}^{t} \qquad \forall z \in Z, \forall t \in T, \tag{75}$$

$$\sum_{w \in Z} \left(O_{bwz}^{ft} + O_{ewz}^{ft} \right) \le n_{z}^{t} \sum_{i \in I_{fz}^{t}} r_{fzi}^{t} U_{fzi}^{t} \qquad \forall z \in Z, \forall t \in T, \tag{76}$$

$$\sum_{w \in Z} \left(O_{bwz}^{st} + O_{ewz}^{st} \right) \le n_{z}^{t} \sum_{i \in I_{sz}^{t}} r_{szi}^{t} U_{szi}^{t} \qquad \forall z \in Z, \forall t \in T, \tag{77}$$

$$X_{bz}^{t} = X_{bz}^{t-1} - \sum_{w \in Z} \left(S_{bzw}^{t} + O_{bzw}^{ft} + O_{bzw}^{st} \right) \qquad \forall z \in Z, \forall t \in T, \tag{78}$$

$$X_{ez}^{t} = X_{ez}^{t-1} - \sum_{w \in Z} \left(S_{ezw}^{t} + O_{ezw}^{ft} + O_{ezw}^{st} \right) \qquad \forall z \in Z, \forall t \in T, \tag{79}$$

$$\sum_{z \in Z} X_{ez}^{0} + \sum_{z \in Z} X_{bz}^{0} = x \tag{80}$$

$$\sum_{i \in I_{bz}^t} Y_{bzi}^t = 1 \qquad \forall t \in T, \forall z \in Z, \tag{81}$$

$$\sum_{i \in I_{fz}^t} Y_{fzi}^t = 1 \qquad \forall t \in T, \forall z \in Z, \tag{82}$$

$$\sum_{i \in I_{sz}^t} Y_{szi}^t = 1 \qquad \forall t \in T, \forall z \in Z, \tag{83}$$

$$\sum_{i \in I_{bz}^t} V_{bzi}^t = \sum_{w \in Z} \left(S_{bwz}^t + S_{ewz}^t \right) \qquad \forall z \in Z, \forall t \in T, \tag{84}$$

$$\sum_{i \in I_{fz}^t} V_{fzi}^t = \sum_{w \in Z} \left(O_{bwz}^{ft} + O_{ewz}^{ft} \right) \qquad \forall z \in Z, \forall t \in T, \tag{85}$$

$$\sum_{i \in I_{sz}^t} V_{szi}^t = \sum_{w \in Z} \left(O_{bwz}^{st} + O_{ewz}^{st} \right) \qquad \forall z \in Z, \forall t \in T, \tag{86}$$

$$V_{bzi}^t \le \sum_{w \in Z} \left(S_{bwz}^t + S_{ewz}^t \right) \qquad \forall i \in I_{bz}^t, \forall z \in Z, \forall t \in T, \tag{87}$$

$$V_{fzi}^t \le \sum_{w \in Z} \left(O_{bwz}^{ft} + O_{ewz}^{ft} \right) \qquad \forall i \in I_{fz}^t, \forall z \in Z, \forall t \in T, \tag{88}$$

$$V_{szi}^t \le \sum_{w \in Z} \left(O_{bwz}^{st} + O_{ewz}^{st} \right) \qquad \forall i \in I_{sz}^t, \forall z \in Z, \forall t \in T, \tag{89}$$

$$V_{bzi}^t \le n_z^t r_{bzi}^t U_{bzi}^t \qquad \forall i \in I_{bz}^t, \forall z \in Z, \forall t \in T, \tag{90}$$

$$V_{fzi}^t \le n_z^t r_{fzi}^t U_{fzi}^t \qquad \forall i \in I_{fz}^t, \forall z \in Z, \forall t \in T, \tag{91}$$

$$V_{szi}^t \le n_z^t r_{szi}^t U_{szi}^t \qquad \forall i \in I_{sz}^t, \forall z \in Z, \forall t \in T, \tag{92}$$

$$\sum_{i \in I_{bz}^t} U_{bzi}^t = R_z^t \qquad \forall z \in Z, \forall t \in T, \tag{93}$$

$$\sum_{i \in I_{fz}^t} U_{fzi}^t = R_z^t \qquad \forall z \in Z, \forall t \in T, \tag{94}$$

$$\sum_{i \in I_{sz}^t} U_{szi}^t = R_z^t \qquad \forall z \in Z, \forall t \in T, \tag{95}$$

$$U_{bzi}^{t} \le R_{z}^{t} \qquad \forall i \in I_{bz}^{t}, \forall z \in Z, \forall t \in T, \tag{96}$$

$$U_{fzi}^{t} \le R_{z}^{t} \qquad \forall i \in I_{fz}^{t}, \forall z \in Z, \forall t \in T, \tag{97}$$

$$U_{szi}^{t} \le R_{z}^{t} \qquad \forall i \in I_{sz}^{t}, \forall z \in Z, \forall t \in T, \tag{98}$$

$$U_{bzi}^{t} \le Y_{bzi}^{t} \qquad \forall i \in I_{bz}^{t}, \forall z \in Z, \forall t \in T, \tag{99}$$

$$U_{fzi}^{t} \le Y_{fzi}^{t} \qquad \forall i \in I_{fz}^{t}, \forall z \in Z, \forall t \in T, \tag{100}$$

$$U_{szi}^{t} \le Y_{szi}^{t} \qquad \forall i \in I_{sz}^{t}, \forall z \in Z, \forall t \in T, \tag{101}$$

$$R_{z}^{t} + \sum_{i \in I_{bzi}^{t}} r_{bzi}^{t} U_{bzi}^{t} + \sum_{i \in I_{fzi}^{t}} r_{fzi}^{t} U_{fzi}^{t} + \sum_{i \in I_{szi}^{t}} r_{szi}^{t} U_{szi}^{t} = 1 \qquad \forall z \in Z, \forall t \in T, \tag{102}$$

$$S_{bwz}^{t}, S_{ewz}^{t}, O_{bwz}^{ft}, O_{ewz}^{ft}, O_{bwz}^{st}, O_{ewz}^{st} \ge 0 \qquad \forall z \in Z, \forall w \in Z, \forall t \in T \tag{103}$$

$$X_{ez}^{t}, X_{bz}^{t} \ge 0 \qquad \forall z \in Z \tag{104}$$

$$Y_{bzi}^{t} \in \{0,1\} \qquad \forall z \in Z, \forall w \in Z, \forall t \in T, \forall i \in I_{bz}^{t} \tag{105}$$

$$Y_{fzi}^{t} \in \{0,1\} \qquad \forall z \in Z, \forall w \in Z, \forall t \in T, \forall i \in I_{fz}^{t}, \tag{106}$$

$$Y_{szi}^{t} \in \{0,1\} \qquad \forall z \in Z, \forall w \in Z, \forall t \in T, \forall i \in I_{sz}^{t}, \tag{107}$$

$$R_{z}^{t} \ge 0, \qquad \forall z \in Z, \forall t \in T, \tag{108}$$

$$U_{bzi}^{t}, \mathrm{V}_{bzi}^{t} \ge 0, \qquad \forall z \in Z, \forall t \in T, \forall i \in I_{bz}^{t}, \tag{109}$$

$$U_{fzi}^{t}, \mathrm{V}_{fzi}^{t} \ge 0, \qquad \forall z \in Z, \forall t \in T, \forall i \in I_{fz}^{t}, \tag{110}$$

$$U_{szi}^{t}, \mathrm{V}_{szi}^{t} \ge 0, \qquad \forall z \in Z, \forall t \in T, \forall i \in I_{sz}^{t}, \tag{111}$$

Objective function (74) includes the same parts as objective function (53), revenue from selling on both channels and leftover inventory subtracted by the fulfillment and inventory costs, but all linear terms. Constraints (75) to (77) are ensuring that the sales are less than demand in both channels. Constraints (78) to (80) enforce inventory balance. Constraints (81) to (83) ensure that exactly one price is offered in brick-and-mortar and e-commerce channel in each zone at each time period under each shipping option. Constraints (84) to (86) ensure that variables V_{bzi}^{t}, V_{fzi}^{t} and V_{szi}^{t} are equal to fulfilled demand in brick-and-mortar and online channels using fast and very fast shipping. Constraints (87) to (89) are tighter bounds on V_{bzi}^{t}, V_{fzi}^{t} and V_{szi}^{t}. Constraints (90) to (92) ensure that variables V_{bzi}^{t}, V_{fzi}^{t} and V_{szi}^{t} are less than the estimated demand in each channel for each shipping method. Since variable U_{bzi}^{t} equals $R_{z}^{t}Y_{bzi}^{t}$ and Y_{bzi}^{t} is a binary decision variable, then constraint (93) ensures the equality of variables U_{bzi}^{t} and R_{z}^{t} for all prices. Constraints (94) and (95) ensures the equality of U_{fzi}^{t} and R_{z}^{t}, U_{szi}^{t} and R_{z}^{t} for all prices. Constraints (96) to (98) are tighter bound on constraints (94) to (95). Constraint (99) to (101) ensure that the variables U_{bzi}^{t}, U_{fzi}^{t} and U_{szi}^{t} are less than one. Constraint (102) ensures the transformation introduced in (67). Constraints (103)-(111) enforce the non-negativity and binary constraints.

2.4. Numerical experiments

Numerical experiments will be conducted in this section to show the performance of the proposed mathematical models M_1 and M_2 and their dynamics. Two sets of instances are used for each mathematical model, one with 15 geographical zones and one with 40 zones. Each dataset has 50 instances, including 25 different initial inventories and 2 different market sizes. First, several experiments are performed for sensitivity analysis. Then, all of the instances are solved optimally to provide robust results for different operational scenarios.

2.4.1. Computational results for mathematical model #1 (M_1)

In order to illustrate how the optimal solution changes based on different parameter settings, we consider an omni-channel retailer with two sets of networks, one with 15 geographical zones (physical store and e-fulfillment centers in each zone) and one with 40 zones. First, we consider an omni-channel retailer with 15 geographical zones (Z=15) as 15 U.S. states and a two-week planning horizon. Figure 2.2 shows the 15 states that are considered in this setting. The states with a circle inside are the states that the omni-channel retailer has facilities in. The retailer has one physical store and one e-fulfillment center in each zone. The U.S. states are assumed to be on a Cartesian plane between (0,0) and (110,110) points. Then the coordinates of the physical stores in each state are estimated compared to the (0,0). For example, the store location in Maine is assumed to be at (100,100), and California is (10,50). The e-fulfillment centers are located close to the physical stores. Then the location of the customers in one zone are assumed to be between the physical store and the e-fulfillment center in that zone.

Figure 2.2. Omni-channel retailer distribution network with 15 zones

We assume discrete price sets with three prices for each channel at each time period, at each geographical zone. Specifically, we assume one product and possible prices of \$95, \$100 and \$105 for in-store price and prices of \$105, \$110 and \$115 for online price. Based on the notation in the mathematical models, we assume the brick-and-mortar price set at zone 1, at period 1 equals $\Omega^{1}_{b1} = \{\$95,\ \$100,\ \$105\}$. Then the decision variable Y^{t}_{bzi} ensures that one of the above prices in the price set is chosen for brick-and-mortar channel. Two different market sizes equal to 20 and 30 products are also assumed for all zones at every period.

For each setting (i.e., a network with 15 or 40 zones), 25 instances are generated with different initial inventories. Since the initial inventories can be lower or higher than the market share, they are simulated from a wide range to illustrate the robustness of the mathematical model and also the effect of the initial inventory on revenue, cost, and profit under different scenarios. Specifically, for a network with 15 geographical zones, initial inventories are generated uniformly that range from 5 to 25 products for each in-store inventory and 5 to 20 products for each e-fulfillment center's inventory. The lower number in the interval (e.g., 5 in interval of 5 to 20) is increased gradually in each simulation scenario to increase the whole level of inventory in the omni-channel network. In fact, the total inventory in the retailer's network ranges from 294 products to 629 products.

The salvage price for leftover inventory is also assumed to be $20. The fulfillment costs are calculated based on Euclidian distance. The cost of shipping an online order from the in-store inventory is assumed to be twice as high as the fulfilling that order from the e-fulfillment center. Also, the brick-and-mortar demand that is fulfilled by e-fulfillment inventory in the same zone will cost twice as much as fulfilling that from in-store inventory.

In order to estimate the MNL choice model parameters in mathematical model M_1, we simulate different set of e-commerce and in-store prices in range of $40 to $140. We then simulate the choice of customers when they observe two distinct e-commerce and brick-and-mortar price. We assume that the in-store prices are cheaper since the retailer should pay for shipping costs for online orders. Customers are generally assumed to purchase cheaper products with higher probability; however, we have some observations that the customer chooses the higher price. This is reasonable assumption since the online shopping is more convenient for many customers.

The parameters of the MNL choice model are estimated using Python package, *PyLogit*. In order to reduce the complexity, one set of coefficients are used for modeling the customers' choice behavior in different zones. The estimated coefficients are 0.506 for intercept and -0.006 for price coefficient for mathematical model M_1. The proposed mathematical formulations were coded into Microsoft Visual C++ 2010 Professional. ILOG Concert Technology was used to define the model within C++ and call the mixed integer linear programming solver IBM ILOG CPLEX 12.5 to solve instances within the Windows 10 environment on a Dell desktop computer with an Intel Core i7, 2.6 GHz processor and 16 GB of RAM. Text files defining all problem instances used in this paper are available from the author upon request.

The retailer's revenue includes revenue from sale and revenue from leftover inventory in optimization model M_1. The sum of these two revenues is considered total revenue. Figure 2.3 shows how the total revenue, revenue from sales and fulfillment cost are changing with different levels of initial inventories. As mentioned before, we consider 15 zones, two planning periods, price sets range from \$95 to \$115, salvage price of 20 dollars and 25 instances with different simulated initial inventories. The left vertical axis (primary) shows the revenue, and the right vertical axis (secondary) shows the fulfillment cost. The horizontal axis shows the sum of initial inventories in the whole retailer's network. As it can be seen from the graph, the sale's revenue (dashed grey line) increases gradually as the total initial inventory increases. However, it remains constant at around \$40,000 when the inventory level passes 400 units of products. This clearly is the point that the total initial inventory is higher than the retailer's market share. Also, in all instances, the retailer chooses \$105 and \$115 for in-store and online price, respectively.

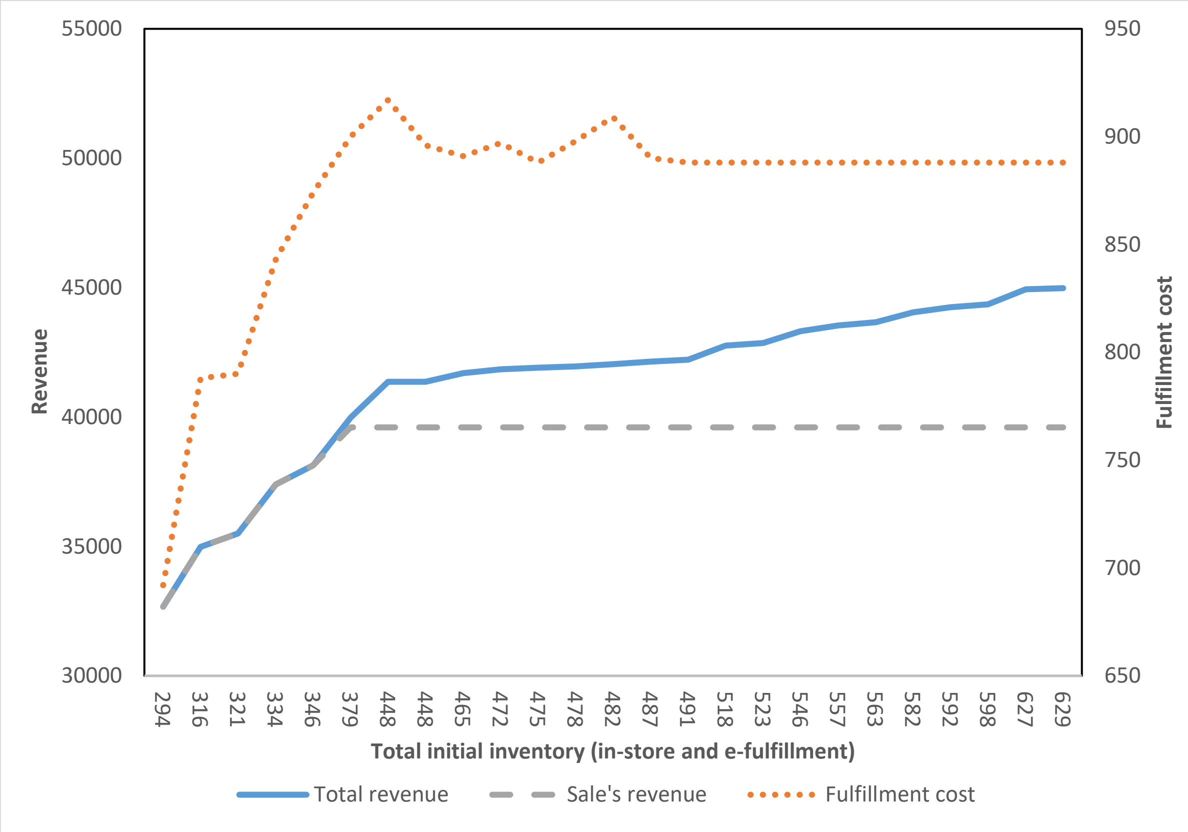


Figure 2.3. Effect of initial inventory on revenue and fulfillment cost

(Total revenue and Sale's revenue should be read on the left vertical axis and Fulfillment cost should be read on the right vertical axis)

One should note that there is a probability that the customers do not purchase from any channel based on the choice model. Therefore, the total demand for the retailer is less than the total available market share. But the total revenue, solid blue line increases as the total inventory increases since a part of the revenue is coming from the leftover inventory. Although the revenue from sale remains fixed as the total initial inventory increases, the revenue from salvaged items increases. Finally, similar to sale's revenue, the fulfillment cost (dotted red line) increases as the inventory level increases, but it becomes stable at 400 since the sales amount does not increase. Some variations can also be seen between 400 to 480 inventories that are due to different fulfillment decisions considering the fact that the initial inventories are simulated randomly.

We can also see the same trend for revenue and fulfillment cost with a different salvage price. Figure 2.4 shows the revenue and fulfillment cost with a salvage price equal to $40, which is as twice as large of the previous salvage price. All the other parameters are the same as previous simulation. Again, 25 instances with different initial inventories ranging from 294 to 629 are solved. Similar to Figure 2.3, the revenue from sale and fulfillment costs are increased as the initial inventory increases, but they stay stable after the inventory level reaches the captured market share. Total revenue increases as inventory increases but it reaches a higher point, around 50,000, since the revenue from leftover inventory is higher due to higher salvage price. Also, under new salvage price, the retailer again chooses $105 and $115 for its brick-and-mortar and e-commerce channel prices for all instances, respectively.

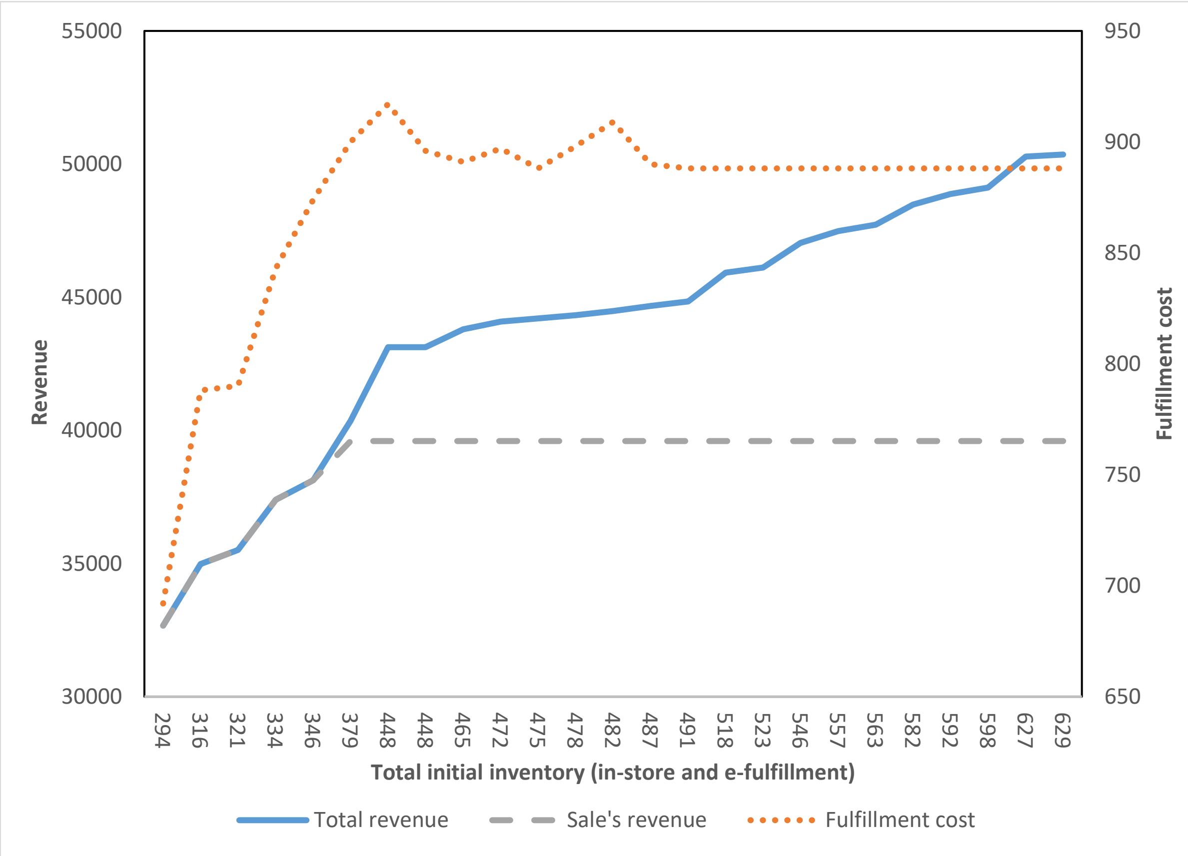


Figure 2.4. Effect of salvage price on revenue and fulfillment cost

Figure 2.5 illustrates the effect of different fulfillment policies on the revenue and fulfillment cost of the retailer. In this graph, we have the same parameters as in Figure 2.3, but the retailer is not allowed to ship the in-store orders. Overall, we have the same trends for total and sale's revenue and fulfilment cost, but it takes longer for retailer to achieve the maximum market share. In fact, the maximum sale's revenue appears to happen at around 490 initial inventory, but this number was lower and around 400 when we have the possibility of shipping the in-store demand as well. This is due to the retailers' flexibility to fulfill their customer demand. Also, similar to sale's revenue, fulfillment cost is also increasing for inventory less than 490. One should note that the optimal retailer's prices are $105 and $115 for in-store and online price in all cases.

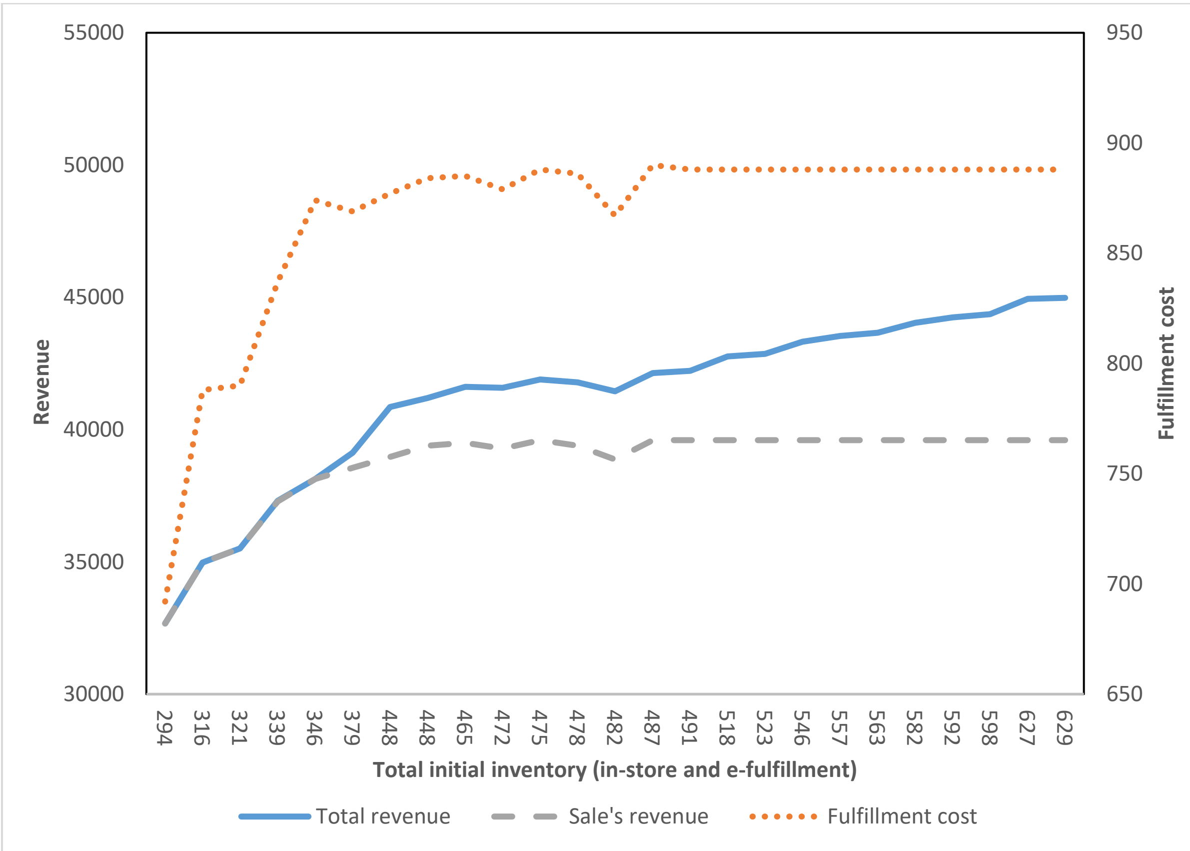


Figure 2.5. Effect of "shipping the in-store demand" option on revenue and fulfillment cost

To show the effect of price variation on the model, we assume that the retailer offers in-store promotions and chooses the in-store price from a set of \$40, \$45 and \$50, instead of \$95, \$100 and \$105. All the other parameters are the same as previous examples. As a result of the retailer's demand model, we expect this decision to increase the customers' purchase probabilities from brick-and-mortar channel, and to decrease the leftover inventories at the physical stores. As noted earlier, we obtain the optimal solution for 25 instances with 25 different initial inventories, and the total leftover inventories are compared in Figure 2.6.

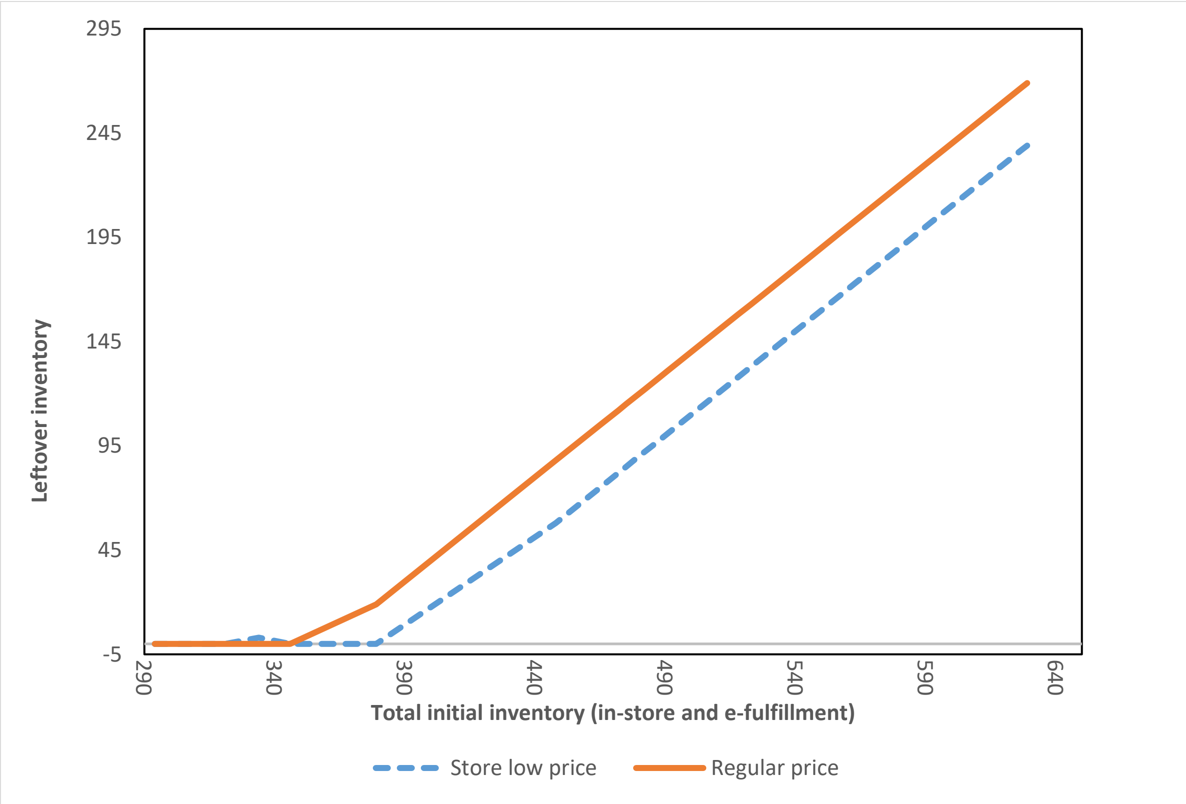


Figure 2.6. Effect of in-store promotions on total leftover inventories

As illustrated in Figure 2.6, the total leftover inventory is lower when the retailer offers low prices for brick-and-mortar channel. In fact, the retailer can use price controls to increase the probability of customers purchasing from physical stores to deplete the in-store inventory. Using price controls to move customers to physical stores also help retailers to reduce their fulfillment cost. However, this can eventually increase the fulfillment cost because we assume that the stores will need to ship their customers' demand if they are out of inventory. This can be seen in Figure 2.7, where the total fulfillment costs are higher when the retailer offer lower prices for the brick-and-mortar channel. However, offering promotions can in general increase the retailers market share and they can sell more. Thus, they have to fulfill higher portion of the total market size and have higher fulfillment costs. This is the case in Figure 2.7 when the cost under each of these prices becomes constant (for instances with higher than 500 initial inventory). However, the total revenue could be higher under the regular prices since the major source of revenue is from sale's revenue. It should be noted that the optimal in-store price for retailer is $40 for the instance with 294 initial inventories, which is not the highest price is the price set. This shows that the retailers can have lower revenue by offering a lower price but having lower fulfillment costs as well, and therefore a higher profit in total.

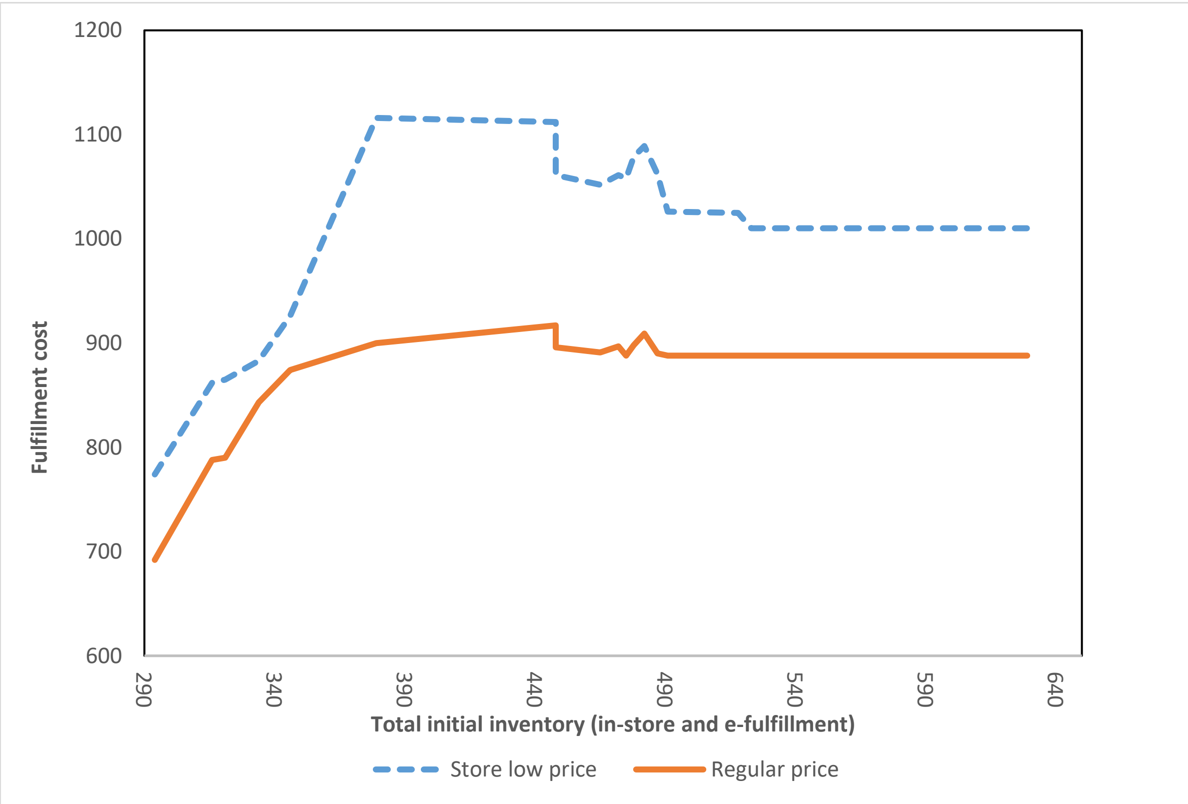


Figure 2.7. Effect of in-store promotions on fulfillment costs

Although the in-store inventory moves faster, and we have lower inventory in total under promotions, the fulfillment cost may increase for the retailer. This increase is because we assume the retailer can fulfill the in-store demand by e-fulfillment inventory in the same zone if the store is out of inventory. However, if we remove this assumption from the model, and assume that the in-store demand can only be fulfilled by in-store inventory, we could have a different pattern. This has been shown in Figure 2.8.

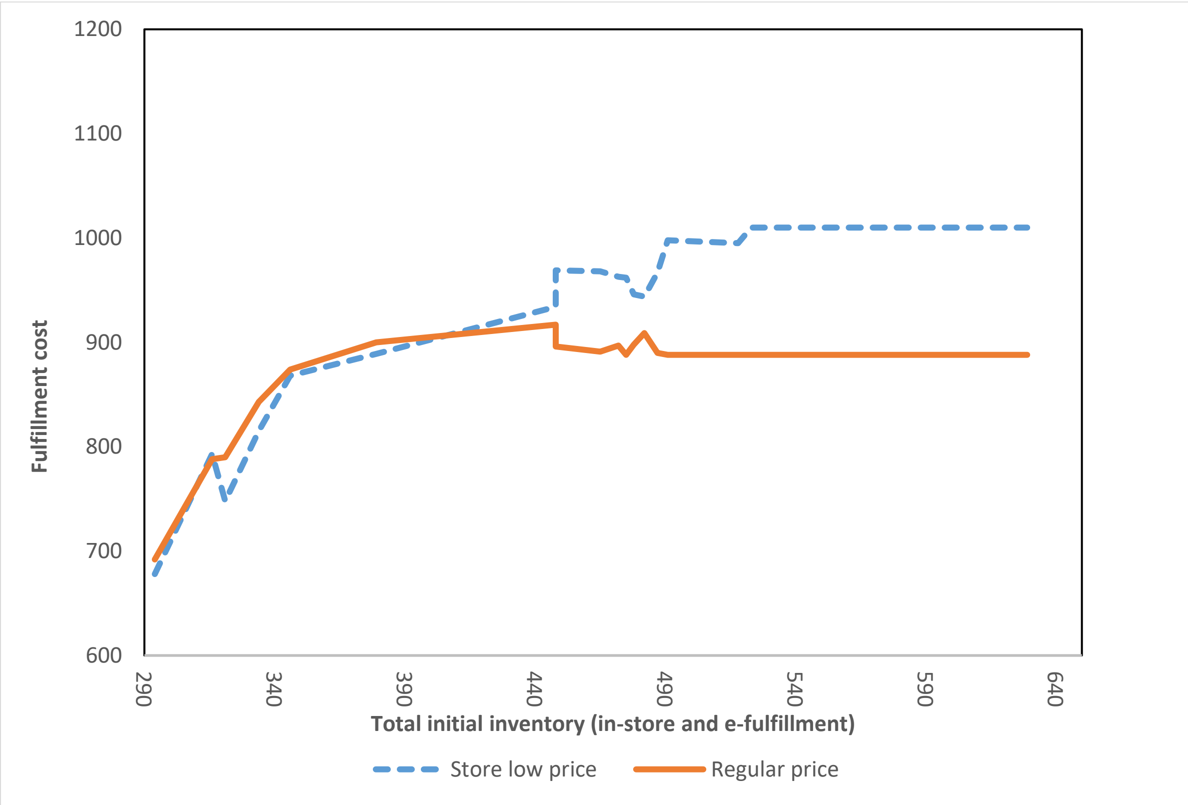


Figure 2.8. Effect of in-store promotions on fulfillment costs when the in-store demand fulfilled only by in-store inventory

Similar to Figure 2.7, Figure 2.8 compares the fulfillment cost under in-store promotion and the fulfillment cost with regular prices, but with the assumption that shipment of in-store demand is not allowed. It can be seen that when the retailer doesn't ship in-store demand, the fulfillment cost is much lower compared to a situation that allows for shipping of in-store demand (Figure 2.7). It is reasonable not to ship the in-store demand when the retailer offers promotions because retailers usually offer promotions to deplete their inventory in one of their channels, for example, brick-and-mortar channel. In fact, if they offer in-store promotions, they don't want to fulfill that demand with inventory in e-commerce channel.

One of the major assumptions in mathematical model M_1 is that the retailers are able to ship the in-store demand to the customers when the store is out of inventory. In such cases, the store can benefit from sharing inventory with e-fulfillment centers. To illustrate how profitable this assumption is and how the retailers can increase their profit under this scenario, Tables 2.3 and 2.4 show the revenue, fulfillment cost, and profit under different scenarios for mathematical model M_1.

We consider a network with 15 geographical zones for an omni-channel retailer in Table 2.3 and a retailer with 40 zones (see Figure 2.9) in Table 2.4. The online price set that the retailer can choose from is \$105, \$110, and \$115. The in-store price is chosen from the set of \$95, \$100, and \$105 in both tables. The salvage price is \$20, and the 25 instances are solved with different initial inventories in both tables. The initial inventories in Table 2.3 are generated uniformly from 5 to 25 products, but the lower point in simulation is increased gradually to have instances with a wide range of initial inventories. But in Table 2.4, the initial inventories are generated from a range of 25 to 50 products for e-fulfillment centers and between 50 to 100 for store inventories. Again, the minimum inventories are increased throughout the instances to have a wider range of initial inventories. We consider market sizes equal to 20 and 30 for Table 2.3 and market sizes equal to 90 and 100 for Table 2.4.

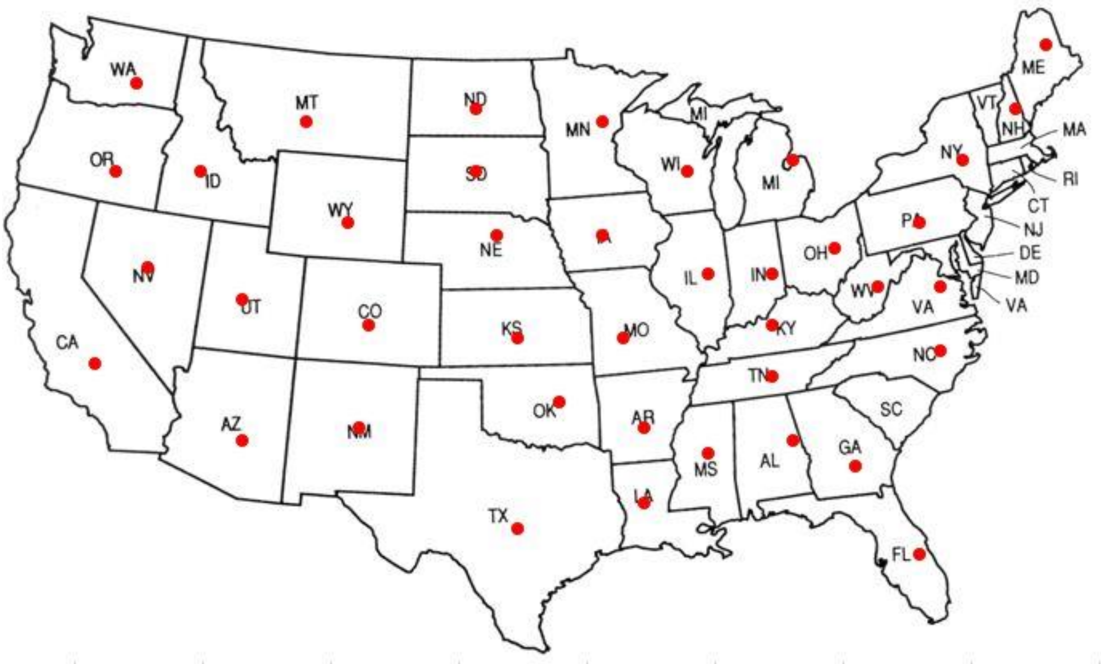


Figure 2.9. Omni-channel retailer distribution network with 40 zones

Table 2.3. Analysis of mathematical model M_1 under different operational scenarios with 15 zones

Total Initial Inv.	$n_z^t = 20$						$n_z^t = 30$					
	$S_{ewz}^t \geq 0$			$S_{ewz}^t = 0$			$S_{ewz}^t \geq 0$			$S_{ewz}^t = 0$		
	Total Rev.	Fulfill. Cost	Profit	Total Rev.	Fulfill. Cost	Profit	Total Rev.	Fulfill. Cost	Profit	Total Rev.	Fulfill. Cost	Profit
294	32,670	692	31,978	32,670	692	31,978	33,570	604	32,966	33,570	604	32,966
316	35,505	790	34,715	35,505	790	34,715	36,405	677	35,728	36,405	677	35,728
321	34,980	788	34,192	34,980	788	34,192	35,880	672	35,208	35,880	672	35,208
334	38,130	874	37,256	38,130	874	37,256	39,030	760	38,270	39,030	760	38,270
346	37,395	843	36,552	37,310	837	36,473	38,295	737	37,558	38,295	737	37,558
379	39,980	944	39,036	39,130	869	38,261	42,495	852	41,643	42,495	852	41,643
448	41,360	917	40,443	40,850	877	39,973	49,740	1,074	48,666	49,740	1,074	48,666
448	42,040	909	41,131	41,445	867	40,578	53,310	1,174	52,136	53,310	1,174	52,136
465	41,360	896	40,464	41,190	884	40,306	49,740	1,064	48,676	49,740	1,064	48,676
472	41,700	891	40,809	41,615	885	40,730	51,525	1,107	50,418	51,525	1,107	50,418
475	41,840	897	40,943	41,585	879	40,706	52,260	1,128	51,132	52,260	1,128	51,132
478	41,960	898	41,062	41,790	886	40,904	52,890	1,158	51,732	52,890	1,158	51,732
482	42,140	890	41,250	42,140	890	41,250	53,835	1,179	52,656	53,835	1,179	52,656
487	41,900	888	41,012	41,900	888	41,012	52,575	1,148	51,427	52,575	1,148	51,427
491	42,220	888	41,332	42,220	888	41,332	54,255	1,201	53,054	54,255	1,201	53,054
518	42,760	888	41,872	42,760	888	41,872	57,090	1,301	55,789	57,090	1,301	55,789
523	42,860	888	41,972	42,860	888	41,972	57,615	1,306	56,309	57,615	1,306	56,309
546	43,320	888	42,432	43,320	888	42,432	59,520	1,361	58,159	59,265	1,339	57,926
557	43,540	888	42,652	43,540	888	42,652	59,740	1,348	58,392	59,740	1,348	58,392
563	44,040	888	43,152	44,040	888	43,152	60,240	1,335	58,905	60,240	1,335	58,905
582	43,660	888	42,772	43,660	888	42,772	59,860	1,338	58,522	59,860	1,338	58,522
592	44,240	888	43,352	44,240	888	43,352	60,440	1,334	59,106	60,440	1,334	59,106
598	44,360	888	43,472	44,360	888	43,472	60,560	1,332	59,228	60,560	1,332	59,228
627	44,940	888	44,052	44,940	888	44,052	61,140	1,332	59,808	61,140	1,332	59,808
629	44,980	888	44,092	44,980	888	44,092	61,180	1,332	59,848	61,180	1,332	59,848
Average	41,355	875	40,480	41,246	867	40,379	51,728	1,114	50,613	51,717	1,113	50,604

Table 2.4. Analysis of mathematical model M_1 under different operational scenarios with 40 zones

	$n_z^t = 90$						$n_z^t = 100$					
	$S_{ewz}^t \geq 0$			$S_{ewz}^t = 0$			$S_{ewz}^t \geq 0$			$S_{ewz}^t = 0$		
Total Initial Inv.	Total Rev.	Fulfill. Cost	Profit	Total Rev.	Fulfill. Cost	Profit	Total Rev.	Fulfill. Cost	Profit	Total Rev.	Fulfill. Cost	Profit
4,499	489,915	10,483	479,432	489,490	10,435	479,055	491,890	10,064	481,826	492,060	10,079	481,981
4,656	495,520	10,482	485,038	493,055	10,293	482,762	508,545	10,492	498,053	508,715	10,508	498,207
4,421	481,810	10,050	471,760	481,800	10,062	471,738	483,785	9,784	474,001	483,785	9,789	473,996
4,507	490,755	10,400	480,355	488,525	10,251	478,274	492,730	10,007	482,723	492,815	10,020	482,795
4,614	494,585	10,449	484,136	493,405	10,354	483,051	504,050	10,356	493,694	503,880	10,363	493,517
4,254	464,190	9,539	454,651	464,360	9,558	454,802	466,505	9,289	457,216	466,335	9,294	457,041
4,646	495,320	10,402	484,918	493,450	10,265	483,185	507,495	10,395	497,100	507,240	10,399	496,841
4,634	494,985	10,471	484,514	492,435	10,262	482,173	506,150	10,369	495,781	506,065	10,374	495,691
4,795	498,300	10,396	487,904	496,760	10,279	486,481	522,885	10,819	512,066	523,225	10,845	512,380
4,670	495,800	10,447	485,353	492,645	10,193	482,452	509,930	10,466	499,464	510,015	10,482	499,533
4,706	496,520	10,495	486,025	493,960	10,283	483,677	513,540	10,562	502,978	513,710	10,576	503,134
4,738	497,160	10,402	486,758	495,885	10,298	485,587	516,900	10,633	506,267	517,155	10,653	506,502
4,618	494,760	10,478	484,282	490,765	10,167	480,598	504,555	10,280	494,275	504,470	10,287	494,183
4,843	499,165	10,269	488,896	498,835	10,242	488,593	528,265	10,920	517,345	527,915	10,919	516,996
4,730	497,000	10,354	486,646	494,025	10,132	483,893	516,230	10,577	505,653	516,315	10,588	505,727
4,960	501,410	10,287	491,123	500,325	10,189	490,136	540,295	11,326	528,969	540,465	11,350	529,115
4,739	497,180	10,326	486,854	494,545	10,123	484,422	517,175	10,599	506,576	517,175	10,607	506,568
4,819	498,780	10,328	488,452	496,230	10,118	486,112	525,575	10,787	514,788	525,660	10,800	514,860
4,930	500,905	10,205	490,700	500,395	10,164	490,231	537,230	11,124	526,106	537,315	11,139	526,176
4,888	500,160	10,236	489,924	498,885	10,137	488,748	532,735	10,980	521,755	532,990	10,992	521,998
4,831	499,020	10,265	488,755	496,130	10,046	486,084	527,005	10,822	516,183	526,835	10,819	516,016
4,857	499,540	10,233	489,307	497,235	10,071	487,164	529,565	10,899	518,666	529,650	10,912	518,738
4,958	501,560	10,207	491,353	500,285	10,103	490,182	540,085	11,220	528,865	540,000	11,216	528,784
5,040	503,200	10,175	493,025	502,095	10,087	492,008	546,400	11,610	534,790	541,640	11,216	530,424
5,190	506,200	10,164	496,036	504,840	10,061	494,779	549,400	11,481	537,919	545,660	11,184	534,476
Average	495,750	10,302	485,448	494,014	10,167	483,847	516,757	10,634	506,122	516,444	10,616	505,827

In Table 2.3, the columns two to seven are the results with market size equal to 20 products at each zone, and columns eight to thirteen show the results when the market size equals 30 products. The first column in Table 2.3, “Total Initial Inv.” shows the total initial inventory for each instance. The next three columns of Table 2.3 show the total revenue, fulfillment cost, and profit for omni-channel retailers that consider the shipment of in-store demand to customers. But the next three columns, five to seven, show the results without this assumption. As it can be seen above, considering the assumption of fulfilling the demand of the in-store customers with e-fulfillment centers will increase the average profit of the retailer under both 20 and 30 market size. This assumption may also increase the average fulfillment costs, but this is due to the higher number of shipments of in-store demands. Also, it can be seen that increasing the market size can increase the retailers profit with the same initial inventory. Furthermore, from instance with lower inventories to instance with higher inventories, the retailer’s profit is increasing. In all instances, the retailer chooses the highest price for e-commerce ($115) and highest price for brick-and-mortar channel ($105). Table 2.4 has exactly the same columns as Table 2.3, which show the results for a retailer with 40 zones and higher initial inventories and larger market share, 90 and 100 units of products. Similar to Table 2.3, in Table 2.4, the average fulfillment cost increases when we allow for shipment of in-store demands. However, retailer covers more demand and has a higher average revenue and a higher average profit under both market shares. Again, in all instances, the retailer’s optimal prices are $115 and $105 for online and in-store products.

2.4.2. Computational results for mathematical model #2 (M_2)

We performed similar experiments for mathematical model M_2. We first simulate price and shipping time for different products in range of $90 to $160. We then simulate the choice of customers when they observe different prices and shipping options for e-commerce and brick-and-mortar channel. The delivery times for very fast, fast, and store pick-up (i.e., in-store purchase) are taken to e 1, 3 and 7 days. It is assumed that the customers do not always choose the cheapest price or the fastest shipping method. Then the Python *PyLogit* package is used to estimate the model's coefficients. The estimated intercept, price, time and interaction coefficients are -0.01031, 0.00018, -0.07217, and 0.00129, respectively.

We used the same instances we used before for mathematical model M_1. In fact, we used two problem sets, one with a network of 15 geographical zones and one with a network of 40 zones and a two-week planning horizon. Each zone has one physical store and one e-fulfillment center. The initial inventories, price sets, and the salvage value are the same as for model M_1. The only difference is that we are considering initial inventory assignments and inventory holding cost. We assume that the holding cost is also $20 per item per period. Before providing tables of our results, we perform some sensitivity analysis.

Figure 2.10 shows the effect of holding cost on the optimal solution on instances with 15 geographical zones and market size equal to 20 products, at each time at each zone. As it can be seen, the retailer has higher profit with lower holding cost. However, the profit increases as the initial inventory increases up to 450 units of product, but it stays non-increasing or even drops for instances with inventory higher than 450. This is because that the retailers cannot increase the revenue due to market share limitation, but still need to pay higher holding cost for larger initial inventory.

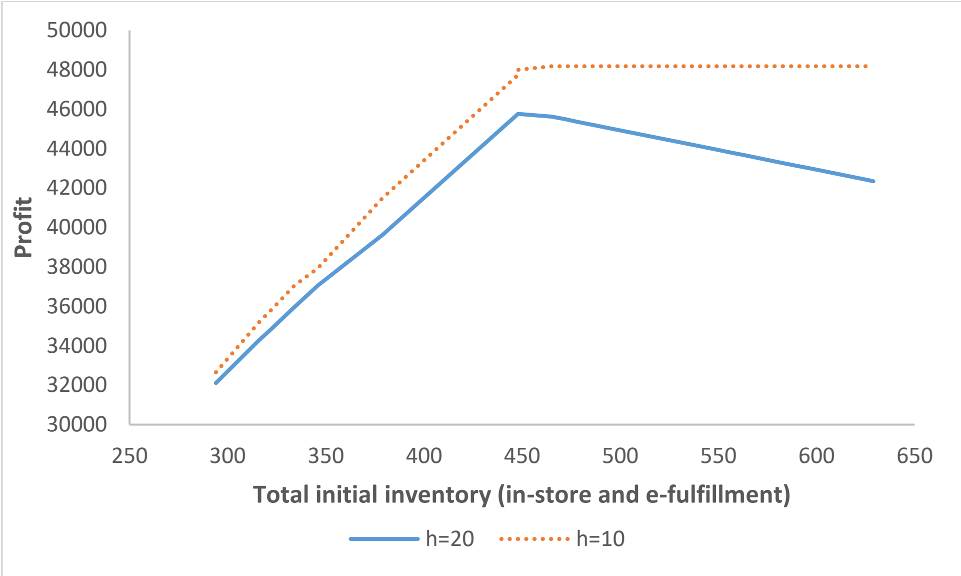


Figure 2.10. Effect of holding cost on retailer’s profit (market share=20)

We have a similar graph when the market share is equal to 30 units of products. Figure 2.11 similarly shows the retailers obtain a higher profit when they have lower holding cost. However, we don’t see any kink on the graph like Figure 2.10. This can be because the retailer has the ability to increase the revenue since there is a higher market size in this setting and the revenue increases as the initial inventory increases.

Figure 2.11. Effect of holding cost on retailer’s profit with a different market share (30)

In order to show how retailers can benefit from considering initial inventory considerations in mathematical model M_2, we compare the results with scenarios that the initial inventories are assumed to be fixed. Tables 2.5 and 2.6 are using the same instances as Tables 2.3 and 2.4, respectively. We assume the same online and brick-and-mortar price sets and the same salvage price. The only difference is that Tables 2.5 and 2.6 are also considering holding costs and initial inventory assignments.

Again, a network with 15 geographical zones is considered in Table 2.5 and a retailer with 40 zones in Table 2.6. The online price set for fast delivery is \$105, \$110, and \$115, the online price set for very fast delivery is \$115, \$120, and \$125, and the in-store price is chosen among \$95, \$100, and \$105 in both tables. The salvage price and holding cost are both \$20. The initial inventories are the same as Table 2.3 and Table 2.4. Market sizes are equal to 20 and 30 for Table 2.5 and market sizes equal to 90 and 100 for Table 2.6.

The first column in Table 2.5 shows the sum of total initial inventory (in-store and e-fulfillment). Columns two to nine show the results with market share equal to 20 products at each zone at each zone, and columns nine to sixteen show the results when the market share equals 30 products. The columns two to five, show the total revenue, holding cost, fulfillment cost and profit for omni-channel retailers that have the ability to assign the in-store and e-fulfillment inventory at the beginning of the planning period and fulfills the in-store demand by shipments. But the next four columns are for fixed initial inventory and no shipping option for in-store customers. However, the sum of total in-store and e-fulfillment centers' inventory are equal in both cases. As it can be seen from the last row of the Table 2.5, the average holding cost and average fulfillment cost both increase when the retailer has fixed initial inventories and does not allow for shipment of in-store demand. The revenue also decreases with fixed inventory. This is because the retailer has less flexibility in inventory and fulfillment decisions.

However, the average profit is higher when the retailer has the ability to assign the initial inventories and ship the in-store demand under both market sizes. Also, contrary to Tables 2.3 and 2.4, optimal prices for retailer are not the highest prices available in each channel's price set in Table 2.5. For example, in the instance with initial inventory equal to 379, under variable initial inventories, the optimal price for fast delivery is \$105 at period two and zone three but it's \$115 at period two and zone two. This shows that the retailers can benefit from joint pricing and fulfillment decisions to maximize their profit. Although the retailer can use the maximum price in each price set to increase the revenue, they choose prices that maximize their profit considering their fulfillment costs. This is the case for some other instances as well.

Tables 2.6 shows the robustness of the obtained results in Table 2.5. Similar to Table 2.5, Table 2.6 shows the same pattern for retailer's holding cost and fulfillment costs, but under different initial inventories and market shares. Again, the retailer's average profit increases when initial inventories are decision variables and the in-store orders can be shipped. Also, it can be seen that increasing the market size can increase the retailer's profit with the same initial inventory. Moreover, in some instances like the one with 4,499 fixed initial inventory, the optimal prices in each channel are not the highest prices and the retailer offers lower prices to attract more demand in each channel and have lower fulfillment costs. Furthermore, from instance with lower inventories to instance with higher inventories, the retailer's profit is increasing. However, we have some instances that both result in a same profit.

Table 2.5. Analysis of mathematical model M_2 under different operational scenarios with 15 zones

	$n_z^t = 20$								$n_z^t = 30$							
	Variable X_{bz}^t $S_{ewz}^t \geq 0$				*Fixed* X_{bz}^t $S_{ewz}^t = 0$				*Variable* X_{bz}^t $S_{ewz}^t \geq 0$				*Fixed* X_{bz}^t $S_{ewz}^t = 0$			
Total Initial Inv.	Total Rev.	Holding Cost	Fulfill. Cost	Profit	Total Rev.	Holding Cost	Fulfill. Cost	Profit	Total Rev.	Holding Cost	Fulfill. Cost	Profit	Total Rev.	Holding Cost	Fulfill. Cost	Profit
294	33,990	1,380	501	32,109	33,930	1,460	514	31,956	33,720	0	489	33,231	33,840	120	497	33,223
316	36,640	1,820	542	34,278	36,630	1,820	560	34,250	36,030	0	511	35,519	36,450	420	580	35,450
321	37,215	1,920	552	34,743	37,185	1,940	565	34,680	36,555	0	516	36,039	36,825	280	532	36,013
334	38,710	2,180	578	35,952	38,740	2,240	588	35,912	38,000	80	533	37,387	38,670	820	570	37,280
346	40,090	2,420	602	37,068	39,995	2,440	614	36,941	39,500	320	557	38,623	39,920	740	597	38,583
379	43,355	3,080	651	39,624	43,415	3,120	679	39,616	43,625	980	623	42,022	43,945	1,300	671	41,974
448	50,940	4,460	718	45,762	50,580	4,540	774	45,266	51,970	2,360	761	48,849	51,905	2,440	837	48,628
448	50,940	4,460	718	45,762	50,690	4,500	754	45,436	51,750	2,360	760	48,630	51,805	2,380	849	48,576
465	51,450	5,100	720	45,630	51,345	5,120	735	45,490	53,925	2,700	795	50,430	53,925	2,700	812	50,413
472	51,590	5,380	720	45,490	51,335	5,440	730	45,165	54,730	2,840	809	51,081	54,730	2,840	822	51,068
475	51,650	5,500	720	45,430	51,650	5,500	746	45,404	55,075	2,900	815	51,360	55,075	2,900	889	51,286
478	51,710	5,620	720	45,370	51,540	5,660	719	45,161	55,420	2,960	821	51,639	55,230	3,000	817	51,413
482	51,790	5,780	720	45,290	51,135	5,920	737	44,478	55,880	3,040	829	52,011	55,795	3,080	901	51,814
487	51,890	5,980	720	45,190	51,860	5,980	741	45,139	56,455	3,140	855	52,460	56,415	3,140	892	52,383
491	51,970	6,140	720	45,110	51,970	6,140	731	45,099	56,915	3,220	847	52,848	56,790	3,240	874	52,676
518	52,510	7,220	720	44,570	52,510	7,220	720	44,570	60,020	3,760	901	55,359	59,840	3,820	962	55,058
523	52,610	7,420	720	44,470	52,610	7,420	720	44,470	60,595	3,860	911	55,824	60,310	3,920	964	55,426
546	53,070	8,340	720	44,010	53,070	8,340	722	44,008	63,030	4,320	936	57,774	62,935	4,340	950	57,645
557	53,290	8,780	720	43,790	53,290	8,780	720	43,790	63,885	4,580	945	58,360	63,920	4,600	994	58,326
563	53,410	9,020	720	43,670	53,410	9,020	720	43,670	64,585	4,660	953	58,972	64,575	4,700	961	58,914
582	53,790	9,780	720	43,290	53,790	9,780	720	43,290	66,765	5,040	974	60,751	66,755	5,040	981	60,734
592	53,990	10,180	720	43,090	53,990	10,180	720	43,090	67,820	5,240	982	61,598	67,530	5,300	1,064	61,166
598	54,110	10,420	720	42,970	54,110	10,420	720	42,970	68,490	5,360	988	62,142	68,270	5,400	1,058	61,812
627	54,690	11,580	720	42,390	54,690	11,580	720	42,390	71,535	5,940	1,017	64,578	71,500	5,940	1,035	64,525
629	54,730	11,660	720	42,350	54,730	11,660	720	42,350	71,570	6,000	1,027	64,543	71,500	6,020	1,026	64,454
Average	49,205	6,225	684	42,296	49,128	6,249	696	42,184	55,114	3,026	806	51,281	55,138	3,139	845	51,154

Table 2.6. Analysis of mathematical model M_2 under different operational scenarios with 40 zones

	$n_z^t = 90$								$n_z^t = 100$							
	Variable X_{bz}^t $S_{ewz}^t \geq 0$				*Fixed* X_{bz}^t $S_{ewz}^t = 0$				*Variable* X_{bz}^t $S_{ewz}^t \geq 0$				*Fixed* X_{bz}^t $S_{ewz}^t = 0$			
Total Initial Inv.	Total Rev.	Holding Cost	Fulfill. Cost	Profit	Total Rev.	Holding Cost	Fulfill. Cost	Profit	Total Rev.	Holding Cost	Fulfill. Cost	Profit	Total Rev.	Holding Cost	Fulfill. Cost	Profit
4,499	491,495	28,780	8,435	454,280	490,970	28,920	8,793	453,257	494,320	22,720	8,912	462,688	493,180	23,000	9,087	461,093
4,656	510,740	32,000	8,728	470,012	509,700	32,360	9,080	468,260	513,635	26,060	9,080	478,495	510,910	26,720	9,332	474,858
4,421	518,575	33,940	8,846	475,789	518,860	33,880	9,350	475,630	519,755	28,260	9,095	482,400	519,380	27,960	9,598	481,822
4,507	522,820	33,400	9,304	480,116	519,685	34,060	8,866	476,759	523,140	27,800	9,146	486,194	521,985	28,180	9,487	484,318
4,614	534,600	35,560	9,568	489,472	532,660	36,000	9,128	487,532	535,760	29,920	9,268	496,572	535,925	30,000	9,684	496,241
4,254	532,985	36,020	9,193	487,772	532,200	36,220	9,469	486,511	536,180	30,000	9,246	496,934	536,105	30,060	9,624	496,421
4,646	535,440	36,240	9,212	489,988	534,400	36,480	9,556	488,364	538,035	30,340	9,280	498,415	536,585	30,660	9,656	496,269
4,634	535,585	36,760	9,210	489,615	534,850	36,880	9,478	488,492	539,385	30,620	9,273	499,492	539,510	30,600	9,636	499,274
4,795	538,570	36,480	9,653	492,437	537,830	36,600	9,290	491,940	540,520	30,800	9,296	500,424	539,235	31,100	9,680	498,455
4,670	538,470	36,900	9,657	491,913	536,565	36,900	9,329	490,336	541,120	31,300	9,733	500,087	536,565	36,900	9,329	490,336
4,706	542,840	37,680	9,428	495,732	541,560	38,000	9,788	493,772	543,390	32,480	9,780	501,130	542,840	37,680	9,428	495,732
4,738	545,785	38,060	9,739	497,986	545,325	38,180	9,497	497,648	546,860	32,680	9,774	504,406	545,325	38,180	9,497	497,648
4,618	547,575	37,980	9,912	499,683	546,010	38,360	9,518	498,132	548,595	32,740	9,362	506,493	548,045	32,680	10,115	505,250
4,843	547,485	38,100	9,780	499,605	546,105	38,380	9,521	498,204	550,185	32,420	9,388	508,377	547,250	32,800	9,781	504,669
4,730	552,940	39,320	9,783	503,837	551,850	39,520	10,002	502,328	555,485	33,880	9,955	511,650	554,590	34,000	9,454	511,136
4,960	554,695	39,940	9,767	504,988	553,035	40,340	9,964	502,731	558,455	34,260	9,972	514,223	554,695	39,940	9,767	504,988
4,739	556,565	40,060	9,821	506,684	554,130	40,620	9,893	503,617	560,485	34,340	9,564	516,581	558,865	34,660	9,890	514,315
4,819	557,725	40,340	10,046	507,339	556,780	40,280	9,870	506,630	561,890	34,560	9,590	517,740	559,565	34,820	9,925	514,820
4,930	558,940	40,680	9,884	508,376	558,090	40,880	9,995	507,215	560,740	35,220	9,956	515,564	558,940	40,680	9,884	508,376
4,888	562,085	41,180	9,995	510,910	560,750	41,640	10,095	509,015	565,845	35,700	9,661	520,484	565,060	35,920	9,997	519,143
4,831	565,645	42,020	10,121	513,504	565,840	42,320	10,257	513,263	571,980	36,320	9,762	525,898	571,520	36,400	10,124	524,996
4,857	570,990	42,360	10,240	518,390	570,715	42,380	10,379	517,956	575,260	36,840	9,822	528,598	573,775	37,180	10,114	526,481
4,958	569,970	42,640	10,208	517,122	569,160	42,880	10,380	515,900	575,560	36,880	9,826	528,854	573,520	37,320	10,173	526,027
5,040	578,615	44,220	10,453	523,942	577,905	44,380	10,549	522,976	583,650	38,600	10,338	534,712	582,920	38,880	9,946	534,094
5,190	595,095	47,060	10,925	537,110	593,310	47,480	10,931	534,899	599,060	41,600	10,374	547,086	598,630	41,840	10,569	546,221
Average	546,649	38,309	9,676	498,664	545,531	38,558	9,719	497,255	549,572	32,654	9,578	507,340	548,197	33,926	9,751	504,519

2.5. Managerial implications

We gained the following insights from the computational experiments. First, retailers can use their transactional data to see how their customers behave under different pricing scenarios for future purchases. A retailer can use pricing controls to shift their customers between channels and move them toward a specific channel. It's also profitable for retailers to consider pricing controls when they make fulfillment decisions. Pricing decisions can change their market share and also the proportion of the demand in each channel. Retailers may get higher revenue if they increase their prices, but they may decrease their total market share.

Second, the fulfilled demand is limited by the amount of initial inventory and the retailer's market share, whichever has the minimum quantity. Thus, the retailer cannot increase its revenue from sales by increasing its initial inventory if its initial inventory is higher than market share. However, they can change their market share by pricing decisions. The retailers can use the results of its demand model, specifically the MNL model, and calculate their market share based on the probability of selling through any of their channels. In Figure 2.3, the market size is equal to 20 products for 15 geographical zones for a two-period planning horizon. Therefore, the retailer should be able to sell 600 (i.e., 2 periods × 15 zones × 20 products) units of products in total if it has the sufficient initial inventory, however, it cannot increase its sale's revenue when its initial inventory passes 400 units of product. Because there is a probability that the retailer does not sell the product through any of the channels. However, the retailer can change this probability by different pricing decisions. For example, based on the results for Figure 2.3 and also Table 2.3, the probability of selling through both channels equals one subtracted by the probability of not selling through any of the channels, which will be 0.64 using the final in-store price equal to \$105 and online price equal to \$115. This can be calculated using estimated coefficients from choice model:

$$1-\frac{1}{\exp\left(0.506-0.006\times\$105\right)+\exp\left(0.506-0.006\times\$115\right)+1}$$

Third, retailers can use pricing decisions to change the demand of each channel and move customers between their channels and deplete channel-specific inventories. As was seen in Figure 2.6, offering lower prices and promotions for in-store products, increases the probability of customers purchasing form brick-and-mortar channel and lower the total leftover inventory. This can be the beneficial when retailers have slow-moving in-store inventories and using pricing decisions can help these retailers in such scenarios. One should note that this strategy can also decrease the fulfillment cost because more customers are purchasing in-store and the retailers save on shipments. However, in this study, we assumed that the retailer can ship the in-store demand if the store is out of inventory. Thus, the lower in-store prices may increase the brick-and-mortar demand more than store inventory leading the stores to ship high proportion of their in-store demand, which could cause a higher fulfillment cost compared to offering regular prices. In summary, lower prices may help the retailers to increase their inventory usage rate, but it may cause higher fulfillment costs if they need to ship the extra demand flowing to stores.

Fourth, the shipment of in-store demand can increase the retailer's market share and fulfillment costs, but on average, it increases the retailer's profit. On the other hand, since most online retailers follow a system-wide periodic review policy, specifically a base-stock policy (Acimovic and Graves 2017), they can also increase their profit if they can decide how to allocate their initial inventory throughout their network.

Fifth, higher holding costs can decrease the retailers profit in general. But, with higher holding cost, retailers should be more sensitive about their level of initial inventory because it may change their profit trend. With lower holding cost, they may have a stable profit with initial inventory higher than their market share, but with higher holding cost their profit decreases more sharply since revenue cannot overcome the cost which include holding and fulfillment costs (Figure 2.10).

2.6. Conclusion

We proposed two optimization models for omni-channel retailing in this study. Both models include pricing and fulfillment decisions simultaneously, whereas the second model also includes inventory decisions as well. In both models, the omni-channel retailer can benefit from sharing inventory for online and in-store demands. One new assumption that is considered in these models is that the in-store demand can be shipped to customers from the inventory that is dedicated for the e-commerce channel. In the second model, the retailer offers different shipping options and it can control the initial inventory allocation for each channel. The customer demand is modeled by an MNL choice model. As a result of using this choice model, the customers' choice probabilities and, consequently, the demand functions have nonlinear terms. Accordingly, both models are linearized using standard techniques that are introduced in the literature and the models are solved by IBM CPLEX solver. We designed several numerical experiments that showed the benefits the retailers can obtain from joint pricing controls and cross-channel fulfillment. These models can be implemented in practice for real world problem as well.

One of the areas that can be extended in this study is considering more realistic assumption in the optimization model. For example, we might consider multiple products in one shipment and the possibility of split deliveries. There are usually multiple products in one online order that can be fulfilled from different locations. The other assumption that could be included is the last mile delivery decisions. The retailer may hire a fleet of vehicles for delivery purposes or use the online shipping platforms as discussed in essay one. One of the other future research areas could be using modern data analytic techniques for customer choice prediction. For example, machine learning algorithms such as Neural Networks (NN) could be used to predict the customer behavior.

References

Acimovic, J., & Graves, S. C. (2017). Mitigating spillover in online retailing via replenishment. Manufacturing & Service Operations Management, 19(3), 419-436.

Acimovic, J., & Graves, S. C. (2014). Making better fulfillment decisions on the fly in an online retail environment. Manufacturing & Service Operations Management, 17(1), 34-51.

Agatz, N. A., Fleischmann, M., & Van Nunen, J. A. (2008). E-fulfillment and multi-channel distribution–A review. European Journal of Operational Research, 187(2), 339-356.

Akçay, Y., Natarajan, H. P., & Xu, S. H. (2010). Joint dynamic pricing of multiple perishable products under consumer choice. Management Science, 56(8), 1345-1361.

Ben-Akiva, M. E., & Lerman, S. R., (1985). Discrete choice analysis: theory and application to travel demand (Vol. 9). MIT press.

Bodea, T., & Ferguson, M. (2014). Segmentation, revenue management, and pricing analytics. Routledge.

Cao, J., So, K. C., & Yin, S. (2016). Impact of an "online-to-store" channel on demand allocation, pricing and profitability. European Journal of Operational Research, 248(1), 234-245.

Forrester Research (2015) Forrester research ecommerce forecast, 2014 to 2019 (US). Accessed April 21, 2016,

https://www.forrester.com/report/Forrester+Research+eCommerce+Forecast+2014+To+2019+US/../E-RES116713.

Harsha, P., Subramanian, S., & Uichanco, J. (2019). Dynamic Pricing of Omnichannel Inventories: Honorable Mention—2017 M&SOM Practice-Based Research Competition. Manufacturing & Service Operations Management, 21(1), 47-65

Harsha, P., Subramanian, S., & Uichanco, J. (2016). Omni-channel revenue management through integrated pricing and fulfillment planning. Ross School of Business, University of Michigan.

Kunnumkal, S., & Topaloglu, H. (2010). A new dynamic programming decomposition method for the network revenue management problem with customer choice behavior. Production and Operations Management, 19(5), 575-590.

Lei, Y., Jasin, S., & Sinha, A. (2018). Joint dynamic pricing and order fulfillment for e-commerce retailers. Manufacturing & Service Operations Management, 20(2), 269-284.

Newman, J. P., Ferguson, M. E., Garrow, L. A., & Jacobs, T. L. (2014). Estimation of choice-based models using sales data from a single firm. Manufacturing & Service Operations Management, 16(2), 184–197.

Paul, A., Feldman, J., & Davis, J. M. (2018). Assortment optimization and pricing under a nonparametric tree choice model. Manufacturing & Service Operations Management.

Rusmevichientong, P., & Topaloglu, H. (2012). Robust assortment optimization in revenue management under the multinomial logit choice model. Operations Research, 60(4), 865-882.

Rusmevichientong, P., Shmoys, D., Tong, C., & Topaloglu, H. (2014). Assortment optimization under the multinomial logit model with random choice parameters. Production and Operations Management, 23(11), 2023-2039.

Sherali, H. D., & Adams, W. P. (1998). Reformulation-linearization techniques for discrete optimization problems. In Handbook of combinatorial optimization (pp. 479-532). Springer, Boston, MA.

Strauss, A. K., Klein, R., & Steinhardt, C. (2018). A review of choice-based revenue management: Theory and methods. European Journal of Operational Research, 271(2), 375-387.

Talluri, K., & Van Ryzin, G. (2004). Revenue management under a general discrete choice model of consumer behavior. Management Science, 50 (1), 15–33.

Talluri, K. T., & Van Ryzin, G. J. (2006). The theory and practice of revenue management (Vol. 68). Springer Science & Business Media.

Thurstone, L. L. (1927). A law of comparative judgment. Psychological Review, 34(4), 273.

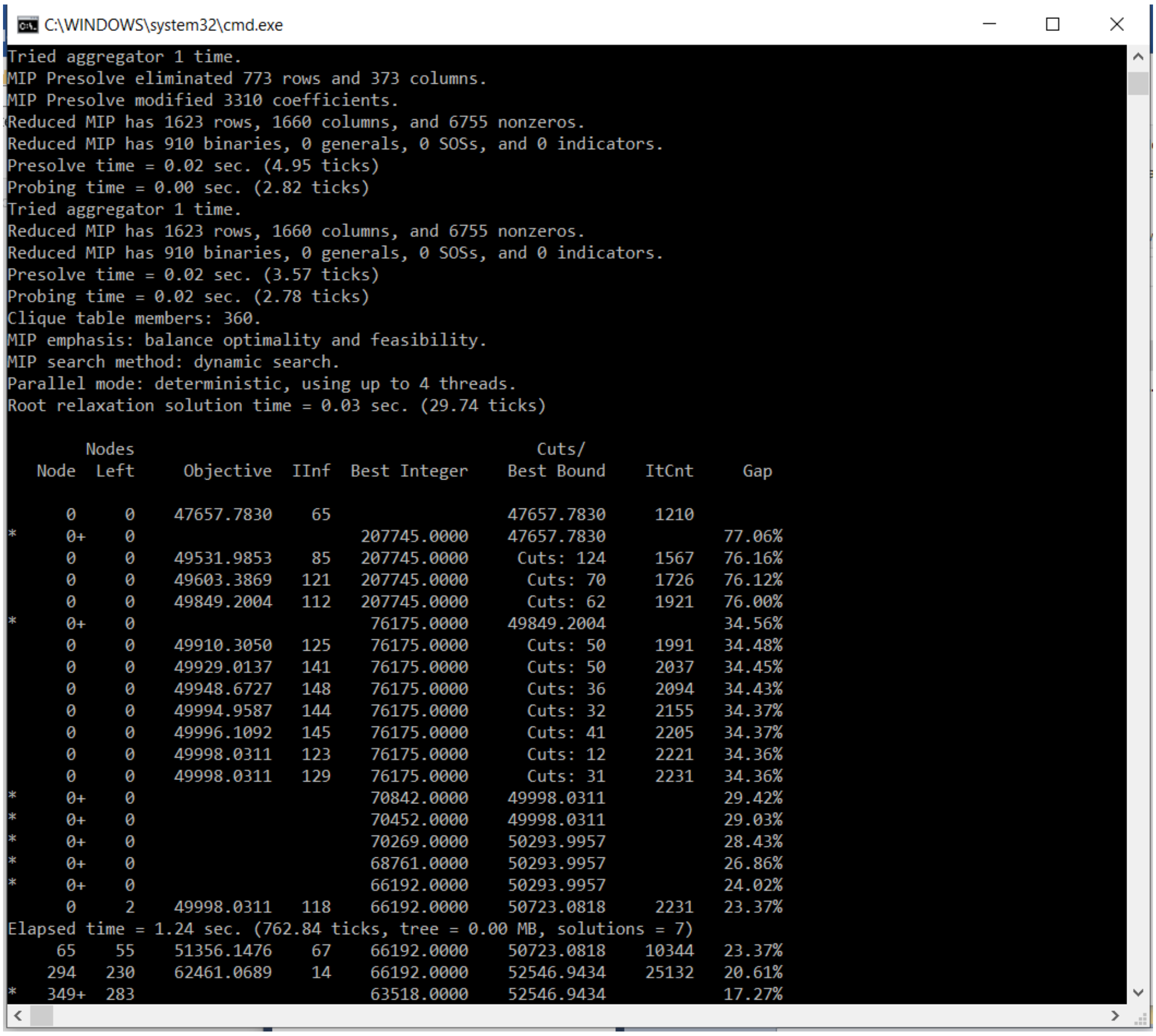
Vulcano, G., Van Ryzin, G., & Chaar, W. (2010). Choice-based revenue management: An empirical study of estimation and optimization. Manufacturing & Service Operations Management, 12 (3), 371–392.

Vulcano, G., Van Ryzin, G., & Ratliff, R. (2012). Estimating primary demand for substitutable products from sales transaction data. Operations Research, 60 (2), 313–334.

Xu, P. J., Allgor, R., & Graves, S. C. (2009). Benefits of reevaluating real-time order fulfillment decisions. Manufacturing & Service Operations Management, 11(2), 340-355.

Appendices

A. Screenshots of a solution of the C++/CPLEX code for optimization model P_1 in essay 1



```
C:\WINDOWS\system32\cmd.exe
Tried aggregator 1 time.
MIP Presolve eliminated 773 rows and 373 columns.
MIP Presolve modified 3310 coefficients.
Reduced MIP has 1623 rows, 1660 columns, and 6755 nonzeros.
Reduced MIP has 910 binaries, 0 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.02 sec. (4.95 ticks)
Probing time = 0.00 sec. (2.82 ticks)
Tried aggregator 1 time.
Reduced MIP has 1623 rows, 1660 columns, and 6755 nonzeros.
Reduced MIP has 910 binaries, 0 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.02 sec. (3.57 ticks)
Probing time = 0.02 sec. (2.78 ticks)
Clique table members: 360.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 4 threads.
Root relaxation solution time = 0.03 sec. (29.74 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt     Gap

      0     0    47657.7830    65                  47657.7830     1210
*     0+    0                        207745.0000    47657.7830            77.06%
      0     0    49531.9853    85   207745.0000     Cuts: 124     1567   76.16%
      0     0    49603.3869   121   207745.0000      Cuts: 70     1726   76.12%
      0     0    49849.2004   112   207745.0000      Cuts: 62     1921   76.00%
*     0+    0                         76175.0000    49849.2004            34.56%
      0     0    49910.3050   125    76175.0000      Cuts: 50     1991   34.48%
      0     0    49929.0137   141    76175.0000      Cuts: 50     2037   34.45%
      0     0    49948.6727   148    76175.0000      Cuts: 36     2094   34.43%
      0     0    49994.9587   144    76175.0000      Cuts: 32     2155   34.37%
      0     0    49996.1092   145    76175.0000      Cuts: 41     2205   34.37%
      0     0    49998.0311   123    76175.0000      Cuts: 12     2221   34.36%
      0     0    49998.0311   129    76175.0000      Cuts: 31     2231   34.36%
*     0+    0                         70842.0000    49998.0311            29.42%
*     0+    0                         70452.0000    49998.0311            29.03%
*     0+    0                         70269.0000    50293.9957            28.43%
*     0+    0                         68761.0000    50293.9957            26.86%
*     0+    0                         66192.0000    50293.9957            24.02%
      0     2    49998.0311   118    66192.0000    50723.0818     2231   23.37%
Elapsed time = 1.24 sec. (762.84 ticks, tree = 0.00 MB, solutions = 7)
     65    55    51356.1476    67    66192.0000    50723.0818    10344   23.37%
    294   230    62461.0689    14    66192.0000    52546.9434    25132   20.61%
*   349+  283                         63518.0000    52546.9434            17.27%
```

C:\WINDOWS\system32\cmd.exe

```
Optimal solution=

X[0 1 ] =1  7662
X[1 5 ] =1  4516

Y[1 10] =1 1589
Y[1 12] =1 1352
Y[1 28] =1 1372
Y[5 13] =1 543
Y[5 21] =1 1060
Y[6 14] =1 1193
Y[7 25] =1 540
Y[8 7] =1 1372
Y[9 17] =1 853
Y[10 30] =1 1426
Y[12 22] =1 1454
Y[13 9] =1 507
Y[14 24] =1 799
Y[16 26] =1 433
Y[17 16] =1 657
Y[19 15] =1 775
Y[20 29] =1 916
Y[21 6] =1 1960
Y[22 19] =1 533
Y[23 20] =1 1012
Y[25 27] =1 2425
Y[26 18] =1 1586
Y[28 23] =1 646
Y[29 11] =1 1621
Y[30 8] =1 469

Z[1] =1 SateliteOpeningCost=5527        Z[5] =1 SateliteOpeningCost=3650
V[1 7] =1
V[1 9] =1
V[1 11] =1
V[1 12] =1
V[1 13] =1
V[1 14] =1
V[1 16] =1
V[1 19] =1
V[1 20] =1
V[1 21] =1
V[1 26] =1
```

C:\WINDOWS\system32\cmd.exe

```
V[5 6] =1
V[5 8] =1
V[5 10] =1
V[5 15] =1
V[5 17] =1
V[5 18] =1
V[5 22] =1
V[5 23] =1
V[5 24] =1
V[5 25] =1
V[5 27] =1
V[5 28] =1

Delta[5] =1
Lambda[11] =1    Lambda[15] =1    Lambda[18] =1    Lambda[24] =1    Lambda[27] =1    U[01] =380
U[15] =147


L[1 10] =97
L[1 12] =57
L[1 28] =79
L[5 13] =74
L[5 21] =73
L[6 14] =34
L[7 25] =40
L[8 7] =60
L[9 17] =45
L[10 30] =83
L[12 22] =41
L[13 9] =60
L[14 24] =11
L[16 26] =21
L[17 16] =32
L[19 15] =11
L[20 29] =39
L[21 6] =53
L[22 19] =36
L[23 20] =60
L[25 27] =13
L[26 18] =7
L[28 23] =73
L[29 11] =23
L[30 8] =68
```

```
C:\WINDOWS\system32\cmd.exe
V[5 15] =1
V[5 17] =1
V[5 18] =1
V[5 22] =1
V[5 23] =1
V[5 24] =1
V[5 25] =1
V[5 27] =1
V[5 28] =1

Delta[5] =1
Lambda[11] =1   Lambda[15] =1   Lambda[18] =1   Lambda[24] =1   Lambda[27] =1   U[01] =380
U[15] =147


L[1 10] =97
L[1 12] =57
L[1 28] =79
L[5 13] =74
L[5 21] =73
L[6 14] =34
L[7 25] =40
L[8 7] =60
L[9 17] =45
L[10 30] =83
L[12 22] =41
L[13 9] =60
L[14 24] =11
L[16 26] =21
L[17 16] =32
L[19 15] =11
L[20 29] =39
L[21 6] =53
L[22 19] =36
L[23 20] =60
L[25 27] =13
L[26 18] =7
L[28 23] =73
L[29 11] =23
L[30 8] =68


Total seconds elapsed = 9
Press any key to continue . . .
```

B. Screenshots of a solution of the C++/CPLEX code for optimization model M_1 in essay 2

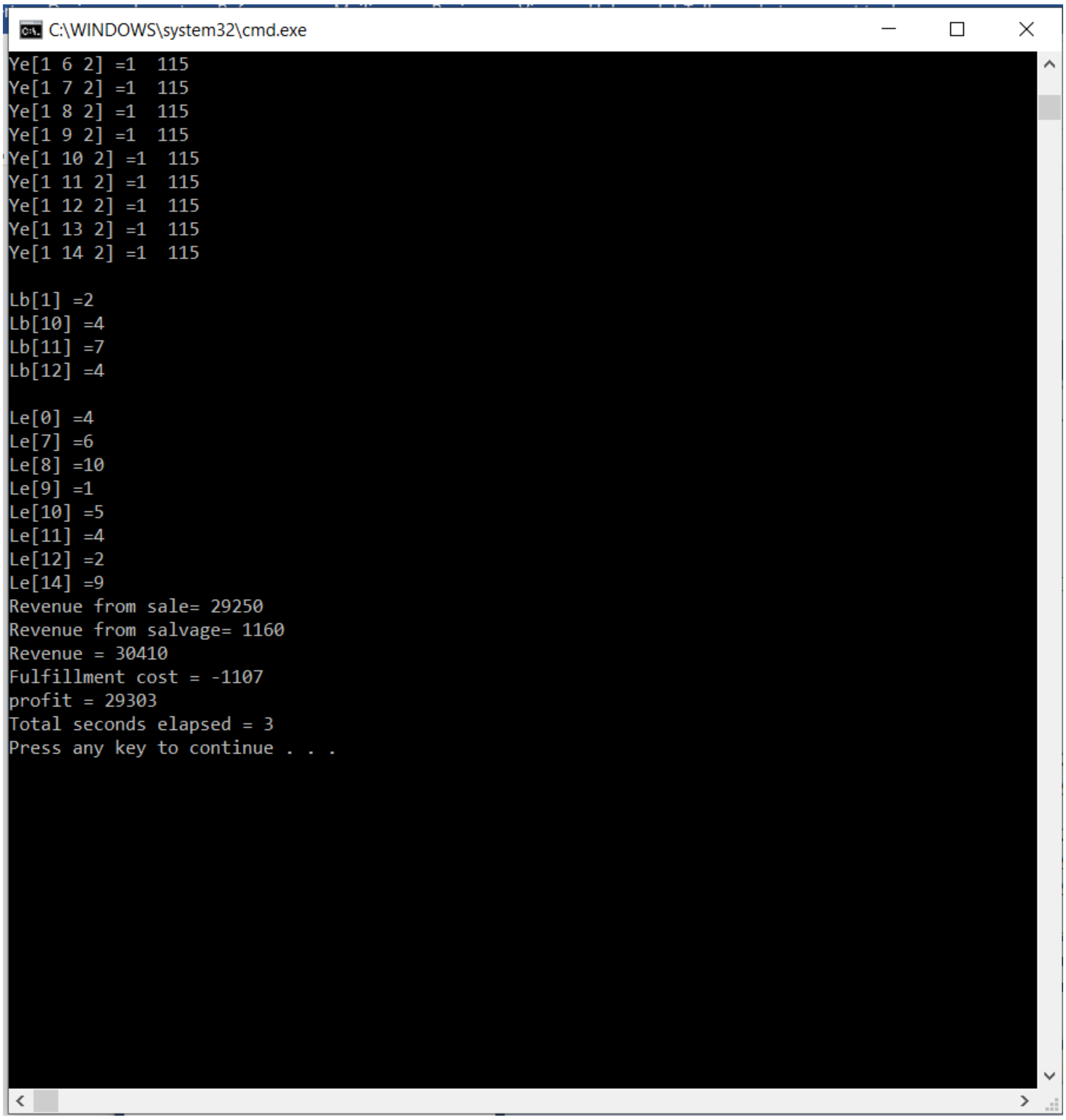
C:\WINDOWS\system32\cmd.exe

```
Our random number seed = 1560963184
26133
23602
RAND_MAX = 32767

Tried aggregator 2 times.
MIP Presolve eliminated 1200 rows and 841 columns.
MIP Presolve modified 300 coefficients.
Aggregator did 60 substitutions.
Reduced MIP has 600 rows, 1500 columns, and 4470 nonzeros.
Reduced MIP has 120 binaries, 990 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.02 sec. (6.01 ticks)
Found incumbent of value 8960.000000 after 0.02 sec. (8.17 ticks)
Probing fixed 0 vars, tightened 344 bounds.
Probing time = 0.00 sec. (0.37 ticks)
Tried aggregator 1 time.
MIP Presolve modified 120 coefficients.
Reduced MIP has 600 rows, 1500 columns, and 4470 nonzeros.
Reduced MIP has 120 binaries, 990 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.03 sec. (2.34 ticks)
Probing time = 0.00 sec. (0.32 ticks)
Clique table members: 60.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 4 threads.
Root relaxation solution time = 0.01 sec. (6.96 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt     Gap

*     0+    0                         8960.0000   202631.1851              ---
      0     0    30534.8502    88     8960.0000    30534.8502      450  240.79%
*     0+    0                        29298.0000    30534.8502             4.22%
      0     0    29750.3811   120    29298.0000      Cuts: 33      591    1.54%
*     0+    0                        29299.0000    29750.3811             1.54%
      0     0    29713.4675   139    29299.0000      Cuts: 89      644    1.41%
      0     0    29710.1872   155    29299.0000      Cuts: 59      706    1.40%
      0     0    29700.7005   172    29299.0000     Cuts: 135      790    1.37%
      0     0    29693.1036   172    29299.0000     Cuts: 168      873    1.35%
      0     0    29687.7105   170    29299.0000     Cuts: 106      919    1.33%
      0     0    29686.3941   161    29299.0000      Cuts: 48      945    1.32%
*     0+    0                        29301.0000    29686.3941             1.32%
      0     0    29684.0211   159    29301.0000      Cuts: 50      993    1.31%
      0     0    29681.6878   169    29301.0000      Cuts: 26     1011    1.30%
```

```
C:\WINDOWS\system32\cmd.exe
Objective value= 29303
Optimal solution=

Sb[0 0 0] =8  3
Sb[0 1 1] =8  3
Sb[0 2 2] =7  3
Sb[0 3 3] =5  3
Sb[0 4 4] =7  3
Sb[0 5 5] =6  3
Sb[0 6 6] =8  3
Sb[0 7 7] =8  3
Sb[0 8 8] =8  3
Sb[0 9 9] =8  3
Sb[0 10 10] =8  3
Sb[0 11 11] =8  3
Sb[0 12 12] =8  3
Sb[0 13 13] =8  3
Sb[0 14 14] =8  3
Sb[1 0 0] =8  3
Sb[1 1 1] =8  3
Sb[1 2 2] =8  3
Sb[1 3 3] =8  3
Sb[1 4 4] =8  3
Sb[1 6 6] =6  3
Sb[1 7 7] =7  3
Sb[1 8 8] =8  3
Sb[1 9 9] =4  3
Sb[1 10 10] =8  3
Sb[1 11 11] =8  3
Sb[1 12 12] =8  3
Sb[1 13 13] =4  3
Sb[1 14 14] =7  3

Se[0 2 2] =1  6
Se[0 3 3] =3  6
Se[0 4 4] =1  6
Se[0 5 5] =2  6
Se[1 5 5] =8  6
Se[1 6 6] =2  6
Se[1 7 7] =1  6
Se[1 9 9] =4  6
Se[1 13 13] =4  6
Se[1 14 14] =1  6
```

C:\WINDOWS\system32\cmd.exe

```
Oe[1 11 11] =5  2
Oe[1 11 14] =5  2
Oe[1 13 12] =5  2
Oe[1 13 13] =5  2

Yb[0 0 2] =1  50
Yb[0 1 2] =1  50
Yb[0 2 2] =1  50
Yb[0 3 2] =1  50
Yb[0 4 2] =1  50
Yb[0 5 2] =1  50
Yb[0 6 2] =1  50
Yb[0 7 2] =1  50
Yb[0 8 2] =1  50
Yb[0 9 2] =1  50
Yb[0 10 2] =1  50
Yb[0 11 2] =1  50
Yb[0 12 2] =1  50
Yb[0 13 2] =1  50
Yb[0 14 2] =1  50
Yb[1 0 2] =1  50
Yb[1 1 2] =1  50
Yb[1 2 2] =1  50
Yb[1 3 2] =1  50
Yb[1 4 2] =1  50
Yb[1 5 2] =1  50
Yb[1 6 2] =1  50
Yb[1 7 2] =1  50
Yb[1 8 2] =1  50
Yb[1 9 2] =1  50
Yb[1 10 2] =1  50
Yb[1 11 2] =1  50
Yb[1 12 2] =1  50
Yb[1 13 2] =1  50
Yb[1 14 2] =1  50

Ye[0 0 2] =1  115
Ye[0 1 2] =1  115
Ye[0 2 2] =1  115
Ye[0 3 2] =1  115
Ye[0 4 2] =1  115
Ye[0 5 2] =1  115
Ye[0 6 2] =1  115
Ye[0 7 2] =1  115
```


C:\WINDOWS\system32\cmd.exe

```
Ye[1 6 2] =1  115
Ye[1 7 2] =1  115
Ye[1 8 2] =1  115
Ye[1 9 2] =1  115
Ye[1 10 2] =1  115
Ye[1 11 2] =1  115
Ye[1 12 2] =1  115
Ye[1 13 2] =1  115
Ye[1 14 2] =1  115

Lb[1] =2
Lb[10] =4
Lb[11] =7
Lb[12] =4

Le[0] =4
Le[7] =6
Le[8] =10
Le[9] =1
Le[10] =5
Le[11] =4
Le[12] =2
Le[14] =9
Revenue from sale= 29250
Revenue from salvage= 1160
Revenue = 30410
Fulfillment cost = -1107
profit = 29303
Total seconds elapsed = 3
Press any key to continue . . .
```

CURRICULUM VITAE

Khosro Pichka

Education:

- **Sheldon B. Lubar School of Business** (2014-2019)
  University of Wisconsin-Milwaukee
  Ph.D. in Management Science-Operations and Supply Chain Management

- **University of Tafresh, Tafresh, Iran** (2009-2012)
  M.Sc. in Industrial Engineering

- **Isfahan University of Technology, Isfahan, Iran** (2004-2009)
  B.Sc. in Industrial Engineering

Publications:

Journal Papers:

- K. Pichka, A. Bajgiran, X. Yue, J. Jang, M. Petering, "The two echelon open location routing problem: Mathematical model and hybrid heuristic", 121 (2018): 97-112.

- A. Azadeh, A. Ziaeifar, K. Pichka, S. Asadzadeh, "An intelligent algorithm for optimum forecasting of manufacturing lead times in fuzzy and crisp environments", *International Journal of Logistics Systems and Management,* 16.2 (2013) 186-210.

- A. Ziaeifar, R. Tavakkoli-Moghaddam, K. Pichka, "Solving a new mathematical model for a hybrid flow shop scheduling problem with a processor assignment by a genetic algorithm", *International Journal of Advanced Manufacturing Technology*, 61 (2012) 339-349.

- A. Ziaeifar, K. Pichka, H. Rafiei, M. Rabbani, "A genetic algorithm approach towards scheduling flexible flow shop consisting of series M/M/n stages", *Bulletin of Calcutta. Mathematic Society,* 103.1 (2011) 47-58.

Conference Proceedings-Presentations:

- K. Pichka, A. Bajgiran, X. Yue, J. Jang, M. Petering, "Two Echelon Location Routing Problem in The Presence of Third Party Logistics", *INFORMS 2016, Nashville, TN.*

- K. Pichka, B. Ashjari, A. Ziaeifar, "Distribution network design: a new model for open vehicle routing problem with multiple depots", *10th International Industrial Engineering Conference (IIEC) 2014, Tehran, Iran.*

- A. Behrouznia, A. Azadeh, K. Pichka, Pazhoheshfar, M. Saberi, "Prediction of manufacturing lead-time based on Adaptive Neuro-Fuzzy Inference System (ANFIS)", *International Symposium on Innovations in Intelligent Systems and Applications (INISTA) 2011, Istanbul, Turkey*.

Academic Experience:

- Instructor (Fall 2018-Spring 2019)
  "Operations Planning and Control"
  Sheldon B. Lubar School of Business, University of Wisconsin-Milwaukee.

- Teaching Assistant (Fall 2016-Spring 2017)
  "Introduction to Management Statistics" course
  Sheldon B. Lubar School of Business, University of Wisconsin-Milwaukee.

- Project Assistant (Fall 2015- Spring 2016)
  Sheldon B. Lubar School of Business, University of Wisconsin-Milwaukee.

- Teaching Assistant (Spring 2015)
  "Engineering Drawing and Computer-Aided Design/Drafting" course
  College of Engineering and Applied Sciences, University of Wisconsin-Milwaukee.

Industry Experience:

- **Iranian Offshore Engineering and Construction Company**, Tehran, Iran
  Scheduling and planning engineer (July 2011-July 2013)

Academic Honors:

- UWM Distinguished Graduate Student Fellowship (DGSF) (Fall 2017-Spring 2018)
- UWM Sheldon B. and Marianne Lubar Scholarship (Summer 2017)
- UWM Chancellor's Graduate Student Award (Fall 2015-Present)

Selected Graduate Courses:

- Statistical Analysis
- Bayesian Data Analysis
- Probability Models for Operations Decisions
- Advanced Computational Methods in Operation Research

Technical Skills:

- Practical experience with softwares such as:
  C++ (Proficient), R, SAS, Win-Bugs, MINITAB, MATLAB, CPLEX, LINGO, MSP, PRIMAVERA.