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Design Optimal Health Insurance Policies from Multiple Perspectives

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DESIGN OPTIMAL HEALTH INSURANCE POLICIES FROM MULTIPLE PERSPECTIVES

by

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ABSTRACT

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by

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The majority of the literature about moral hazard focuses only on qualitative studies. If a health insurance plan imposes little copayment on the insured, the insured may be motivated to have more than necessary medical services, which would raise the insurer's share of cost. This is referred to as moral hazard. Furthermore, the involvement of a third party–healthcare providers adds more complications on moral hazard. Healthcare providers and patients might choose to collaborate to benefit more from insurance reimbursement, which consequently result in unnecessary loss of the insurer. In this dissertation, we attempt to solve these issues and focus on develop optimal insurance coverage from both insurer's and insured's perspectives.

Many companies promote high-deductible health plans (HDHP) to mitigate moral hazard. This dissertation aims to establish models to analyze the decision-making behaviors from the perspectives of both insurers and insureds. In Chapter 3, we analyze the incentive that people switch from low-deductible health plans (LDHPs) to HDHPs with different deductible and premium to maximize the insured's profit. In Section 3.4, We apply the idea of prospect theory, and compare the value functions under LDHP and HDHP to help the policyholders make the decision. This research is anticipated to make two folds of contributions. First, it would introduce a theoretical approach to quantify the moral hazard and study the insured's behavior. Second, it would reveal the optimal insurance design through a two-stage optimization scheme. The optimal strategy would take the interests of both parties into consideration and make the insurance a collaborative game. This would efficiently mitigate the moral hazard issue and further enhance the welfare of the entire society.

To

my parents,

my husband Gangfeng,

my sons Zach and Luka

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Chapter 1

Introduction

1.1 Motivation

High-deductible health plans (HDHPs) are health insurance policies with higher deductibles than traditional insurance plans. Individuals with HDHPs pay lower monthly insurance premiums but pay more out of pocket for medical expenses until their deductible is met. An HDHP may be used with or without a health savings account (HSA). An HSA allows pretax income to be saved to help pay for the higher costs associated with an HDHP

HDHPs have become increasingly popular since the Medicare Modernization Act of 2003 authorized portable, tax-advantaged health savings accounts (HSAs) that were designed to be coupled with these plans. According to America's Health Insurance Plans research, enrollment in HDHPs grew by more than 40 percent in 2006, and 34 percent in 2007, and 31 percent in 2008, from 3.2 million enrollees in January 2006 to more than 8 million enrollees in January 2009 (America's Health Insurance Plans 2009). As of January 2015, these plans accounted for 19.7 million HSA/HDHP enrollees, which was up from 17.4 million in 2014. (America's Health Insurance Plans 2015)

Why do HDHPs so prevail? What was the employer's motivation to incentivize the HDHPs? The employer's motivation is to save on the premium when employees switch from an LDHP to an HDHP. Let us illustrate this fact with a simple example:

Table 1.1: Comparison between an LDHP and an HDHP

	LDHP	HDHP
Monthly Premium	\$90	\$35
Deductible	\$250	\$1500
Coinsurance	10%	10%
OOPL(out of pocket limit)	\$1250	\$2500

Additionally, the employer offers an incentive to employees who switch from an LDHP to an HDHP. In this example, the incentive is \$600. For employees, the reduction in the insurance coverage caused by a switch from an LDHP to an HDHP in the worst scenario could be completely compensated by the incentive provided by the employer plus the savings from the premium reduction. For an employer, the contribution to the premium would be reduced by an amount greater than the amount of the incentive it provides. It turns out that both the employer and the employee benefit from the switch from the LDHP to the HDHP, while the health insurance provider loses. Specifically, when switching from the LDHP to the HDHP, the decrease in premium is steeper than the decrease in the insurance coverage. In other words, the HDHP is underpriced or the LDHP is overpriced, and thus leaves an "arbitrage" opportunity. Initially, we illustrated that the quasi-arbitrage condition seemingly implies an arbitrage opportunity, when we consider two insurance coverage plans with the same copay rate and the deductible OOPL(out of pocket limit) spread u, but a different deductible level d. However, after careful examination, we find that the arbitrage opportunity is based on the assumption that the underlying risk dose not change when the policyholder switches from the LDHP to the HDHP. However, this outcome does not occur in reality, because the underlying risks being priced are actually different, due to the alternation in insured's behavior under different plans. Under the HDHP, the insureds are incentivized to reduce the risk. After switching from the LDHP to the HDHP, the number of the doctor visits will be reduced. With the HDHP, the policyholder has to pay for doctor visits out of the pocket before meeting the high deductible. This requirement prevents nonessential visits. At this point, we realized that the seemingly inconsistent pricing between the LDHP and the HDHP did not create an arbitrage. Rather, it is a reasonable design to mitigate moral hazard by incentivizing the insureds to reduce excessive utilization of medical services for their own benefits.

Therefore, we call the quasi-arbitrage condition the “quasi” - arbitrage condition. We want to determine the conditions that motivate insureds to select the HDHP and then make efforts to reduce unnecessary medical expenses and thus mitigate the moral hazard.

To maximize the number of policyholders who switch from an LDHP to an HDHP, we use optimization models to first determine the level to set the incentive to reach an optimization. It takes different amounts of incentives to trigger the switch from an LDHP to an HDHP for different individuals. In other words, not all existing policyholders are willing to switch their health plans, even with a positive incentive amount. These amounts, if properly identified, could serve as proxies to quantify the moral hazard of different individuals. With a higher incentive, more policyholders are willing to switch. However, a high incentive decreases the benefit to the insurance company. Therefore, to maximize the expected profit under the insurer's perspective, we look at both existing policyholders who stay with the LDHP and also the policyholders who are willing to switch to the HDHP. For simplicity, the behaviors of policyholders are not considered in the optimal models in the initial model setting. In reality, the behaviors are effective, for example, under the same incentive, a single young healthy policyholder is more willing to switch from an LDHP to an HDHP than a married young healthy policyholder, especially, if her or his family members are covered under her or his health plan. Thus, in the following, we want to group existing policyholders with different behaviors by using prospect theory.

1.2 Literature Review

It is commonly acknowledged that high copayments for insureds help reduce moral hazard(the incentive of people to seek more care when they are insured), as evidenced in the seminal work of Arrow (1978) and Pauly (1968). Following their work, Zeckhauser (1970) discusses the tradeoff between risk-spreading and incentives, and arrives at the conclusion that insurance payments should be determined by the degree of illness (though difficult to verify) in addition to the actual medical cost. Pauly (1971) refers to this type of insurance as “indemnity” insurance. Later on, Harris and Raviv (1978) studied moral hazard under the general theory of agency. The results can be applied to the case of health insurance to show under what conditions moral hazard causes inefficiencies and how appropriate contractual arrangements such as indemnity insurance can resolve this problem. They manage to recover and extend the results of Zechhauser (1970) and others. Cutler and Zeckhauser (2000) outline several models to study moral hazard in insurance and conduct some qualitative analyses. Empirical evidence shows that moral hazard is quantitatively important. Recently, Einav et al. (2013) investigated the impact of individual selection on the mitigation of moral hazard through an empirical approach. Five years later, Einav and Finkelstein (2018) used the moral hazard to illustrate the value of and important complementarities between different empirical approaches.

Notably, the majority of the literature focuses only on qualitative or empirical studies only. The proposed thesis aims to establish some theoretical models for the quantitative analysis of moral hazard and attempts to answer some fundamental questions of interest to practitioners. Such questions include the following: what levels of the deductible and copay rate should be set to so as to prevent the moral hazard? By what amount should the premium should be reduced to incentivize insureds to select the high deductible plan without jeopardizing the insurer's profitability? To fully answer these questions, the project solves a two-stage optimal insurance design problem: first, identify the insured's response to given deductible levels and copay rates as well as the premium principle based on some optimization criterion from the insured's perspective; second, incorporate the insured's response functions and design an optimal insurance coverage form so as to maximize the insurer's profitability or minimize the capital requirement. We first find that the "quasi" - arbitrage conditions mitigate the moral hazard. Then, we build optimization insurance models to benefit both the insured and the insurer. Finally, we want to study the insured's behaviors with prospect theory. The selection behaviors of individuals implicitly reflect the anticipation of the demand for medical services.

In 1979, Daniel Kahneman and Amos Tversky introduced the model of risk attitudes called "prospect theory" to the standard risk measure expected utility theory. A version of prospect theory that employs cumulative rather than separable decision weights was also developed by Tversky and Kahneman (1992). This version is called cumulative prospect theory. Comparing prospect theory with expected utility theory, Camerer (2004) notes that some anomalies can be explained by the former but not the latter. However, why are there relatively few well-known applications of prospect theory so many years after the publication of the 1979 paper? Barberis (2013) suggests that it is challenging to know exactly how to apply it in economics after it was be published in 1979. It has become popular over the past decade, and researchers in the field of behavioral economics have invested much thought into how prospect theory should be applied in economic settings. Later, Kairies-Schwarz et al. (2017) conducted a laboratory experiment to show that the behaviors of the majority of participants are explained by cumulative prospect theory rather than expected utility theory when facing insurance choices. Prospect theory has recently been applied in different areas; for instance, Zou and Petrick (2019) apply it to finance, Ebrahimigharehbaghi et al. (2022) to energy, and Wu (2022) to health care.

In this dissertation, the decision-making process is studied in two steps. First, the optimization analysis is conducted from the insurer's perspective based on the models. Then, we study the behaviors of the insured with prospect theory. This thesis enhances an individual's understanding of his or her demand for health insurance, which in turn leads to a better individual decisions. Through this two-step optimization procedure, a robust optimal scheme is anticipated to be reached. More importantly, a study of the insured's behavior would promote the development of analytical tools to quantify and thus efficiently mitigate moral hazard, finally improving social welfare.

1.3 Plan of the Dissertation

The remainder of the dissertation is arranged in the following manner. In Chapter 2, we explain that the "Quasi" - Arbitrage condition is a reasonable design to mitigate moral hazard by incentivizing insureds to reduce the nonessential doctor's visits. Furthermore, we find the sufficient conditions to make the "Quasi" - Arbitrage Condition hold with different distributions.

In Chapter 3, an optimization analysis is conducted from the insurer's perspective based on the model assumptions presented in Chapter 2. The primary objective of this chapter is to thoroughly model and identify under what conditions, if any, the insurer maximizes profit. We start with the simple case in which the cost of each doctor's visit is a deterministic constant, and we determine at what level to set the incentive to reach an optimization. We assume that the probability of switching from LDHP to HDHP is a function that changes with the incentive amount. Under the insurance, different policyholders have their own utility functions; thus, we next analyze the maximum expected utility of the profit. For simplicity, we assume the utility function is a memoryless exponential utility function. Finally, the maximum expected utility of the insurer's profit can be found in different intervals of inventive. In reality, the cost of a doctor's visit is not constant. Thus, we discuss the insurer's maximum expected utility of the profit using an aggregate loss model. At the end of this chapter, we revisit this switching rate. In particular, we aim to find the conditions that qualify the switching rate as increasing and concave in terms of both probability and expectation decision criteria. Furthermore, the interactive decision-making process (between the insured and the insurer) would stimulate a natural categorization of the insureds in the sense that insureds with similar conditions would be motivated to select similar coverage plans. This natural categorization would serve as a proxy of a health status based categorization and thus facilitate the creation of a more accurate pricing system in the health insurance market. More importantly, a study of the insured's behavior should promote the development of analytical tools to quantify and thus efficiently mitigate moral hazard.

In Chapter 4, we summarize the results of this dissertation and briefly discuss the future research plans.

Chapter 2

Quasi - Arbitrage Condition

2.1 Introduction and motivation

It has been recognized in the insurance literature that medical insurance may increase usage by lowing the marginal cost of care to the individual, which is a characteristic that has been termed "moral hazard" by Pauly (1968). A review of the relevant academic literature shows that a great number of scholars realize the negative impact of moral hazard. For example, the moral hazard is defined by Boyd et al. (1998) as "the intangible loss - producing propensities of the individual assured" or as "comprehends all of the nonphysical hazards of risk". Moral hazard represents a deviation from correct human behavior that may pose a problem for an insurer. If a health insurance plan imposes little copayment on the insured, the insured may be motivated to seek more medical services than necessary, which would raise the insurer's share of cost. The extreme case is full insurance, and policyholders will not worry about the money out of the pocket expenses during illness. Furthermore, the involvement of a third-party health care providers adds more complications to moral hazard. Health care providers and patients might choose to collaborate to benefit more from insurance reimbursement, which consequently results in unnecessary loss for the insurer. This thesis aims to motivate insureds to switch from a low deductible health plan to a high deductible health plan and then make efforts to reduce unnecessary medical expenses and thus mitigate the moral hazard.

Let N be the random counts of natural illness of an insured person during a one-year period. Not all these illnesses result in doctor visits. How the insured person decides whether to visit a doctor depends on his or her specific health insurance plan. Individuals on a low deductible health plan are more willing to see a doctor than those on a high deductible health plan. In the special case, when $d = 0$, which means the insured is covered by the full insurance. The insured is not interested in making efforts to reduce the risk exposure. The insured is more willing to see the doctor. Therefore, most health insurance policies impose a deductible and a positive copay rate in practice. The other special case is when $d = \infty$, which means that this person is not covered by insurances. In this case, this person can still see the doctor. For this moment, we investigate only the effect of deductible level d. In this case, the annual counts of doctor visits are denoted by $N(d)$.

We assume the cost during each doctor's visit is a deterministic constant. For notational convenience, we assume the cost is the base unit 1, and other quantities, such as the deductible and OOPL, are expressed in terms of this unit. For example, if, in reality, the fixed cost of a doctor's visit is \$200, and the deductible is \$1000, then the deductible level in the model would be set to $d = 5$.

Consider a health insurance plan with copay rate α, deductible level d, and a deductible - OOPL spread of u (i.e., OOPL $= (1-\alpha)u + d$). We assume d and u are both positive. Under such a plan, the loss random variable is $L(d) = 1 \times N(d) = N(d)$, and the payment random variable for the insurance company is

$$P(L(d), d) = \alpha(L(d) - d)_+ + (1-\alpha)(L(d) - d - u)_+ \tag{2.1}$$

The cost (or pure premium) of this insurance coverage is thus

$$\begin{aligned} \pi(d) &= E[P(L(d), d)] \\ &= \alpha E[(L(d) - d)_+] + (1-\alpha)E[(L(d) - d - u)_+] \\ &= \alpha E[(N(d) - d)_+] + (1-\alpha)E[(N(d) - d - u)_+] \end{aligned} \tag{2.2}$$

In the following, we shall analyze the properties of the pure premium $\pi(d)$ as a function of d. Ideally, we want to establish the following quasi-arbitrage condition:

$$\pi(d_1) - \pi(d_2) \geq d_2 - d_1 \tag{2.3}$$

for some $0 \leq d_1 \leq d_2$

Intuitively, the quasi-arbitrage condition seemingly implies an arbitrage opportunity. Specifically, consider two insurance coverage plans with the same copay rate α and deductible

OOPL spread u, but different deductible levels $d_1$and d_2 with $d_1 \leq d_2$. The maximum amount of difference between the coverages of these two plans is $d_2 - d_1$. The quasi-arbitrage condition (2.3) implies that the savings in premium by raising the deductible level from d_1 to d_2 exceed the maximum possible loss in coverage. In this sense, the insurance plan with deductible d_1 is overpriced or the insurance plan with deductible d_2 is underpriced. Either way, insureds are motivated to pursue the insurance product with deductible d_2. In a hypothetical market where insurance products are allowed to be freely transacted, this situation seems to create an arbitrage opportunity due to the pricing inconsistency. However, after a careful examination, it is clear that this situation is not a true arbitrage opportunity, because the underlying risks being priced, namely $L(d_1)$ and $L(d_2)$, are actually different due to the alternation in the insured's behavior under different plans. An arbitrage opportunity is based on the assumption that the underlying risk $L(d_1) = L(d_2)$. However, this situation is not the case because under the HDHP, insureds are incentivized to reduce the risk. Therefore, we call condition (2.3) the "quasi"-arbitrage condition.

However, the seemingly inconsistent pricing between the LDHP and the HDHP dose not create an arbitrage. However, the "quasi"-arbitrage condition is the foundation for an insurance company to design a highly deductible health plan, so as to motivate insureds to select this plan and then make efforts to reduce unnecessary medical expenses and thus mitigate the moral hazard.

2.2 Constant cost per doctor's visit

To achieve the "quasi"-arbitrage condition, we start with the simple case. The simplest model to characterize the relationship between the variables is

$$N(d) = \sum_{i=1}^{N} I_i \tag{2.4}$$

where $\{I_i$, i =1,2,...$\}$are i.i.d. Bernoulli random variables with $P\{I_i = 1\} = p(d)$. Intuitively, $N(d)$ is a thinning version of N.

The function $p(d)$ is assumed to satisfy the following properties:

- $p(0) = 1$ and p(∞)=p_∞ for some $p_\infty > 0$.
- $p(d)$ is strictly decreasing and convex.

For further analysis, we shall use the form: $p(d) = p_\infty + (1 - p_\infty)e^{-d}$

The convexity of $p(d)$ is assumed to reflect the marginal diminishing effect. That is, $p(d)$ is decreasing in d, but the speed of this decrease slows down as d increases. The above two assumptions immediately imply that $p'(d) < 0$ for any d.

The random sum

$$N(d) = I_1 + I_2 + ... + I_N$$

where N has a counting distribution, then $N(d)$ has the following distribution function:

$$\begin{aligned} F_{N(d)}(x) &= Pr(N(d) \leqslant x) \\ &= \sum_{n=0}^{\infty} p_n Pr(N(d) \leqslant x | N = n) \\ &= \sum_{n=0}^{\infty} p_n F_I^{*n}(x) \end{aligned}$$

where $F_I(x) = Pr(I \leqslant x)$ is the common distribution function of the $I_j s$ and $p_n = Pr(N = n)$. The distribution of $N(d)$ is called a compound distribution. And $F_I^{*n}(x)$ is the "$n-fold$ convolution" of the cdf of I. It can be obtained as

$$F_I^{*0}(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geqslant 0 \end{cases}$$

and if I has a discrete counting distribution, with probabilities at 0, 1, 2, ... ,

$$F_I^{*k}(x) = \sum_{y=0}^{x} F_I^{*(k-1)}(x-y) f_I(y)$$

for $x = 0, 1, ..., k = 2, 3, ...$

The corresponding probability function is

$$f_I^{*k}(x) = \sum_{y=0}^{x} f_I^{*(k-1)}(x-y) f_I(y)$$

for $x = 0, 1, ..., k = 2, 3, ...$

Then, in this case, $N(d)$ has a discrete distribution with probability function,

$$f_{N(d)}(x) = Pr(N(d) = x) = \sum_{n=0}^{\infty} p_n f_I^{*n}(x)$$

where $x = 0, 1, ...$

The probability generating function (pgf) of $N(d)$ is

$$\begin{aligned}
P_{N(d)}(z) &= E\left[z^{N(d)}\right] \\
&= E\left[z^0\right] Pr(N=0) + \sum_{n=1}^{\infty} E\left[z^{I_1+I_2+...+I_n}|N=n\right] Pr(N=n) \\
&= Pr(N=0) + \sum_{n=1}^{\infty} E\left[\prod_{j=1}^{n} z^{I_j}\right] Pr(N=n) \\
&= \sum_{n=0}^{\infty} Pr(N=n)\left[P_I(z)\right]^n \\
&= E\left[P_I(z)^N\right] \\
&= P_N\left[P_I(z)\right]
\end{aligned}$$

Due to the independence of I_1, I_2, ... , I_n for fixed n, the probability generating functions are a useful tool for dealing with discrete random variables taking values 0, 1, 2, ... The pgf transforms a sum into a product and enables it to be handled much more easily. The following examples illustrate how it works.

Example 2.2.1. We assume that the natural illness count N follows a geometric distribution with mean β, then $N(d)\sim$ *Geo*$(\beta p(d))$.

Proof. Since $N \sim$ Geo(β), then the pmf is,

$$P(k) = \left(\frac{\beta}{\beta+1}\right)^k \left(\frac{1}{\beta+1}\right)$$

the pgf is,

$$P_N(s) = \frac{1}{1+\beta(1-s)}$$

from (2.4),

$$N(d) = \sum_{i=1}^{N} I_i$$

where, $\{I_i$, i =1,2,...$\}$are i.i.d. Bernoulli random variables with $P\{I_i = 1\} = p(d)$, and the pgf is,

$$P_I(z) = 1 - p(d) + p(d)z$$

Thus, the pgf of $N(d)$ is,

$$\begin{aligned}
P_{N(d)}(z) &= P_N\left[P_I(z)\right] \\
&= \frac{1}{1 + \beta\left(1 - P_I(z)\right)} \\
&= \frac{1}{1 + \beta\left(1 - (1 - p(d)) - p(d)z\right)} \\
&= \frac{1}{1 + \beta p(d)(1 - z)}
\end{aligned}$$

which shows, $N(d)\sim$ $Geo\left(\beta p(d)\right)$ □

Example 2.2.2. We assume the natural illness count N follows a Poisson distribution with mean λ, then $N(d)\sim$ $Poi(\lambda p(d))$.

Proof. Since $N \sim Poi(\lambda)$, then the pgf is,

$$P_N(s) = e^{\lambda(s-1)}$$

thus, the pgf of $N(d)$ is,

$$\begin{aligned}
P_{N(d)}(z) &= P_N[P_I(z)] \\
&= e^{\lambda(P_I(z)-1)} \\
&= e^{\lambda(1-p(d)+p(d)z-1)} \\
&= e^{\lambda(-p(d)+p(d)z)} \\
&= e^{\lambda p(d)(z-1)}
\end{aligned}$$

which shows, $N(d)\sim$ $Poi(\lambda p(d))$ □

Example 2.2.3. We assume that the natural illness count N follows a binomial distribution with mean nq, then $N(d) \sim Bin(n, p(d)q)$.

Proof. Since $N \sim Bin(n, q)$, then the pgf is,

$$P_N(s) = (1 - q + qs)^n$$

Thus, the pgf of $N(d)$ is,

$$\begin{aligned}
P_{N(d)}(z) &= P_N\left[P_I(z)\right] \\
&= (1 - q + qP_I(z))^n \\
&= (1 - q + q(1 - p(d) + p(d)z)^n \\
&= (1 - qp(d) + qp(d)z)^n
\end{aligned}$$

which shows, $N(d) \sim Bin(n, p(d)q)$. □

In Model (2.4), the insured person is assumed to select his or her illnesses for a doctor visit according to a constant rate, which may not reflect the reality. In reality, as the insured's total medical cost exceeds or even approaches the deductible level, the insured person may be more motivated to visit a doctor more often and thus meet the deductible and enjoy the benefit of insurance. In this sense, a more accurate way to model the relationship between N and $N(d)$ is through a Markov process, for example, a non-homogeneous Poisson process, which will be the subject of future research.

In the following propositions and examples, first, we determine the conditions for the quasi-arbitrage condition that hold if N follows a Poisson distribution or binomial distribution; then, we consider the geometric distribution.

Proposition 2.2.4. *We assume the natural illness count N follows a Poisson distribution with mean λ, the probability the insured decides to visit the doctor is, $p(d) = p_\infty + (1 - p_\infty)e^{-d}$; for simplicity, we assume the health insurance plan has copay rate $\alpha = 1$. Then the quasi - arbitrage condition holds if this inequality holds for all d:*

$$-1 - \lambda(1 - p_\infty) \leqslant -\frac{1}{P\left(N(\lambda p_\infty) > d\right)} \tag{2.5}$$

Proof. From (2.2) and the assumption $\alpha = 1$, we obtain $\pi(d) = E[(N(d) - d)_+]$. Since $N \sim Poi(\lambda)$, then $N(d) \sim Poi(\lambda(d))$, where $\lambda(d) = \lambda p(d)$.

Let: $\varphi(\lambda, y) = E[(N(\lambda) - y)_+]$. Thus, we have,

$$\pi'(d) = \frac{\partial}{\partial \lambda}\varphi(\lambda, y)|_{(\lambda,y)=(\lambda(d),d)} \times \frac{\partial \lambda(d)}{\partial d} + \frac{\partial}{\partial y}\varphi(\lambda, y)|_{(\lambda,y)=(\lambda(d),d)}$$

The second term is direct,

$$\frac{\partial}{\partial y}\varphi(\lambda, y)|_{(\lambda,y)=(\lambda(d),d)} = -P(N(\lambda(d)) > d)$$

In the first term,

$$\lambda'(d) = \lambda p'(d)$$

we focus on the other part:

$$\begin{aligned} \frac{\partial}{\partial \lambda}\varphi(\lambda, y) &= \frac{\partial}{\partial \lambda} E\left[(N(\lambda) - y)_+\right] \\ &\triangleq \frac{\partial}{\partial \lambda} E\left[u(N(\lambda))\right] \end{aligned}$$

where, $u(z) = (z-y)_+$, for Poisson distribution, $p_n(\lambda) = \frac{\lambda^n}{n!}e^{-\lambda}$, thus,

$$\begin{aligned}
\frac{\partial}{\partial\lambda}\varphi(\lambda, y) &= \frac{\partial}{\partial\lambda}\sum_{n=0}^{\infty} p_n(\lambda)u(n) \\
&= \sum_{n=0}^{\infty} \frac{\partial}{\partial\lambda} p_n(\lambda)u(n) \\
&= \sum_{n=0}^{\infty} \frac{\partial}{\partial\lambda} \frac{\lambda^n}{n!} e^{-\lambda}u(n) \\
&= \sum_{n=0}^{\infty} \frac{n\lambda^{n-1}}{n!} e^{-\lambda}u(n) + \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda}(-1)u(n) \\
&= \sum_{n=0}^{\infty} \frac{n\lambda^{n-1}}{n!} e^{-\lambda}u(n) - \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda}u(n) \\
&= \sum_{n=0}^{\infty} \frac{\lambda^{n-1}}{(n-1)!} e^{-\lambda}u(n) - \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda}u(n) \\
&= \sum_{n=1}^{\infty} \frac{\lambda^{n-1}}{(n-1)!} e^{-\lambda}u(n) - \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda}u(n) \\
&= \sum_{n=0}^{\infty} \frac{\lambda^{n'}}{(n')!} e^{-\lambda}u(n'+1) - \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda}u(n) \\
&= E\left[u(N(\lambda)+1)\right] - E\left[u(N(\lambda))\right]
\end{aligned}$$

substitute $N(\lambda)+1$ and $N(\lambda)$ for z,

$$\begin{aligned}
\frac{\partial}{\partial\lambda}\varphi(\lambda, y) &= E\left[(N(\lambda)+1-y)_+\right] - E\left[(N(\lambda)-y)_+\right] \\
&= E\left[(N(\lambda)-(y-1))\mathbb{I}\{N(\lambda) > y-1\}\right] - E\left[(N(\lambda)-y)\mathbb{I}\{N(\lambda) > y\}\right] \\
&= E[(N(\lambda)-(y-1))\mathbb{I}\{N(\lambda) \in (y-1, y]\}] + E[\mathbb{I}\{N(\lambda) > y\}] \\
&= E[(N(\lambda)-(y-1))\mathbb{I}\{N(\lambda) = [y]\}] + E[\mathbb{I}\{N(\lambda) > y\}] \\
&= ([y]-(y-1))P(N(\lambda) \in (y-1, y]) + P(N(\lambda) > y)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\pi'(d) &= \lambda'(d)\left[([d]-(d-1))\,P\left(N(\lambda(d))\in(d-1,d]\right)+P(N(\lambda(d))>d)\right]-P(N(\lambda(d))>d)\\
&\leqslant \lambda'(d)P(N(\lambda(d))>d)-P(N(\lambda(d))>d)\\
&=(\lambda p'(d)-1)P(N(\lambda(d))>d)
\end{aligned}$$

We want to show $\pi'(d)\leqslant -1$; Thus $(\lambda p'(d)-1)\leqslant -\frac{1}{P(N(\lambda(d))>d)}$ is the sufficient condition for the quasi - arbitrage condition holds. We can simply the inequality by amplification and minimization.

$$LHS=-1+\lambda p'(d)\leqslant -1+\lambda p'(0)=-1-\lambda(1-p_\infty)$$

$$RHS=-\frac{1}{P(N(\lambda(d))>d)}\geqslant -\frac{1}{P(N(\lambda(\infty))>d)}=-\frac{1}{P(N(\lambda p_\infty)>d)}$$

Thus,

$$-1-\lambda(1-p_\infty)\leqslant -\frac{1}{P(N(\lambda p_\infty)>d)}$$

□

Corollary 2.2.5. *There exists λ_0, s.t. (2.5) holds for all $\lambda>\lambda_0$.*

Proof. If $\lambda\to\infty$, then $\lambda p_\infty\to\infty$. Thus, $P(N(\lambda p_\infty)>d)\to 1$, and $-\lambda(1-p_\infty)\to-\infty$; therefore, (2.5) holds for all $\lambda>\lambda_0$. □

Example 2.2.6. Assuming that the natural illness count N follows a Poisson distribution with mean λ, following Proposition 2.2.4, the quasi - arbitrage condition holds if the following inequality holds when $d_1=0$, and $d_2\to 0$,

$$-1-\lambda(1-p_\infty)\leqslant -\frac{1}{1-e^{-\lambda}} \tag{2.6}$$

Furthermore, there exists λ_1 *s.t.* (2.6) holds for all $\lambda>\lambda_1$ when $d=0$.

Proof. We want to find the sufficient condition for $\pi'(0)\leqslant -1$. Since $N\sim Poi(\lambda)$, then $N(\lambda)\sim Poi(\lambda p(d))$; therefore, $P(N(\lambda)>d)=1-\frac{(\lambda p(d))^d e^{-\lambda p(d)}}{d!}$. If $d=0$, $P(N(\lambda)>0)=1-e^{\lambda}$, the from (2.6), we obtain $-1-\lambda(1-p_\infty)\leqslant -\frac{1}{1-e^{-\lambda}}$.

The following are some examples of λ_1 when $d=0$;

p_∞	0.1	0.2	0.3
λ_1	2.341	2.6	2.937

$\square$

Proposition 2.2.7. *We assume the natural illness count N follows a binomial distribution with mean nq. Then the probability the insured decides to visit the doctor is, $p(d) = p_\infty + (1-p_\infty)e^{-d}$; for simplicity, we assume the health insurance plan has copay rate $\alpha = 1$. Then, the quasi - arbitrage condition holds if this inequality holds for all d:*

$$(np'(d)q - 1)P(N'(q') > d) \leqslant -1, \tag{2.7}$$

where $N'(q) \sim bin(n-1, p(d)q)$, and $q' = p(d)q$

Proof. Since $N \sim Bin(n, q)$, then $N(d) \sim Bin(n, q')$, where $q' = p(d)q$. Let $\varphi(q', d) = E[(N(q') - d)_+]$; therefore,

$$\pi'(d) = \frac{\partial}{\partial q'}\varphi(q', y)|_{(q',y)=(p(d)q,d)} \times \frac{\partial p(d)}{\partial d} + \frac{\partial}{\partial y}\varphi(q', y)|_{(q',y)=(p(d)q,d)}$$

In the first term,

$$
\begin{aligned}
\frac{\partial}{\partial q'}\varphi(q',y) &= \frac{\partial}{\partial q'}E[(N(q')-y)_+] \\
&\triangleq \frac{\partial}{\partial q'}E[u(N(q'))], u(z)=(z-y)_+ \\
&= \frac{\partial}{\partial q'}\sum_{k=0}^{n} p_n(q')u(n) \\
&= \frac{\partial}{\partial q'}\sum_{k=0}^{n} \binom{n}{k}(q')^k(1-q')^{n-k}u(n) \\
&= \sum_{k=0}^{n}\binom{n}{k}k(q')^{k-1}(1-q')^{n-k}u(n) - \sum_{k=0}^{n}\binom{n}{k}(q')^k(n-k)(1-q')^{n-k-1}u(n) \\
&= n\sum_{k=1}^{n}\binom{n-1}{k-1}(q')^{k-1}(1-q')^{n-k}u(n) - n\sum_{k=0}^{n}\binom{n-1}{k}(q')^k(1-q')^{n-k-1}u(n) \\
&= n\sum_{k=0}^{n+1}\binom{n}{k}(q')^k(1-q')^{n-k-1}u(n+1) - n\sum_{k=0}^{n}\binom{n-1}{k}(q')^k(1-q')^{n-k-1}u(n) \\
&= n\cdot E[u(N'+1)] - n\cdot E[u(N')], N' \sim Bin(n-1,q') \\
&= n\cdot(E[u(N'+1)] - E[u(N')]) \\
&= n\cdot(E[(N'+1-y)_+] - E[(N'-y)_+]) \\
&= n\cdot[([y]-(y-1))P(N'\in(y-1,y]) + P(N'>y)]
\end{aligned}
$$

Here, because $0 < [y]-(y-1) < 1$, thus, $0 < ([y]-(y-1))P(N'\in(y-1,y]) < 1$, and $p'(d)([y]-(y-1))P(N'\in(y-1,y]) < 0$. Therefore,

$$
\pi'(d) \leqslant np'(d)q\cdot P(N'(p(d)q) > d) - P(N(p(d)q) > d)
$$

We want to show $\pi'(d) \leqslant -1$. Therefore, we need $np'(d)q\cdot P(N'(q') > d) - P(N(q') > d) \leqslant -1$. Since $N(q') \sim Bin(n,q')$, and $N'(q') \sim Bin(n-1,q')$, then $P(N'(q') > d) \leqslant P(N(q') > d)$, thus, $(np'(d)q - 1)P(N'(q') > d) \leqslant -1$ is the sufficient condition for the quasi-arbitrage condition to hold. □

Corollary 2.2.8. *There exists n_0, s.t. (2.7) holds for all $n > n_0$.*

Proof. If $n \to \infty$, then $np'(d)q - 1 \to -\infty$. Therefore, (2.7) holds for all $n > n_0$. □

Corollary 2.2.9. *We assume that the natural illness count N follows a binomial distribution with mean nq. Then following Proposition 2.2.7, the quasi - arbitrage condition holds if the following inequality holds when $d_1 = 0$ and $d_2 \to 0$:*

$$-n(1-p_\infty)q - 1 \leqslant -\frac{1}{1-(1-q)^{n-1}} \tag{2.8}$$

Proof. Since $P'(0) = -(1-p_\infty)$, and $N'(q') \sim Bin(n-1, p(d)q)$, then $P(N'(q') > 0) = 1 - P(N'(q') = 0) = 1 - (1-p(d)q)^{n-1}$. Therefore, from (2.7), $(-n(1-p_\infty)q - 1)(1-(1-q)^{n-1}) \leqslant -1$ is the condition that ensures the quasi - arbitrage condition holds. □

Proposition 2.2.10. *We assume that the natural illness count N follows a geometric distribution with mean β. The pure premium $\pi(d)$ admits the following expression:*

$$\pi(d) = \beta(d) \times q_\beta(d)^d(\alpha + (1-\alpha)q_\beta(d)^u) \tag{2.9}$$

with $\beta(d) = \beta p(d)$ and $q_\beta(d) = \frac{\beta(d)}{1+\beta(d)}$.

Proof. Since $N \sim$ Geo(β), then $N(d) \sim$ Geo(β(d)). Recalling the memoryless property of the geometric distribution, i.e., E[$N(d)$-d | $N(d) \geq d$] = E[$N(d)$], we have

$$E[(N(d)-d)_+] = E[N(d) - d \mid N(d) \geq d] \times P\{N(d) \geq d\} = \beta(d) \times q_\beta(d)^d.$$

Similarly,

$$E[(N(d)-d-u)_+] = \beta(d) \times q_\beta(d)^{d+u}.$$

Therefore,

$$\begin{aligned}\pi(d) &= \alpha E[(N(d)-d)_+] + (1-\alpha)E[(N(d)-d-u)_+] \\ &= \beta(d) \times q_\beta(d)^d(\alpha + (1-\alpha)q_\beta(d)^u).\end{aligned}$$

□

Proposition 2.2.11. *We assume that the natural illness count N follows a geometric distribution with mean β. If β is sufficiently large, then $\pi'(d) \leq$ -1 for all d such that $p'(d) \leq$ - $\frac{1}{\beta}$.*

Proof. Following (2.5), we have

$$\ln \pi(d) = \ln \beta(d) + d \ln q_\beta(d) + \ln(\alpha + (1-\alpha) q(d)^u),$$

and thus,

$$\frac{\pi'(d)}{\pi(d)} = \frac{\beta'(d)}{\beta(d)} + \ln q_\beta(d) + d \times \frac{q_\beta'(d)}{q_\beta(d)} + u \times \frac{(1-\alpha) q_\beta'(d) q_\beta(d)^{u-1}}{\alpha + (1-\alpha) q_\beta(d)^u}.$$

Note that $\beta'(d) = \beta p'(d) \leq 0$, $q_\beta(d) \leq 1$, and $q_\beta'(d) = \frac{\beta p'(d)}{(1+\beta p(d))^2} \leq 0$. Therefore,

$$\pi'(d) \leq \pi(d) \frac{\beta'(d)}{\beta(d)} = \beta p'(d) \times q_\beta(d)^d (\alpha + (1-\alpha) q_\beta(d)^u)$$

Since β is sufficiently large, and $p(d) \geq c_0 > 0$ for all d, thus $q_\beta(d) \approx 1$ and $\pi'(d) \leq \beta p'(d)$. If $p'(d) \leq -\frac{1}{\beta}$, then $\pi'(d) \leqslant -1$, which implies that the quasi - arbitrage condition holds. □

Proposition 2.2.12. *We assume that the natural illness N follows a geometric distribution with mean β, the probability the insured decides to visit the doctor is $p(d) = p_\infty + (1-p_\infty)e^{-d}$, and the health insurance plan has copay rate $\alpha = 1$. Then the quasi - arbitrage condition holds if $d_1 = 0$, and $d_2 \to 0$ such that there exist β_0, s.t. $\beta \geqslant \beta_0$.*

Proof. Following (2.6), we have

$$\pi(d) = \beta(d) \times \left(\frac{\beta(d)}{1+\beta(d)} \right)^d$$

Thus,

$$\begin{aligned}
\pi'(d) &= \beta'(d) \left(\frac{\beta(d)}{1+\beta(d)} \right)^d + \beta(d) \left(\frac{\beta(d)}{1+\beta(d)} \right)^d \left[\frac{d \cdot \beta'(d)}{(1+\beta(d))^2} \cdot \frac{1+\beta(d)}{\beta(d)} + \ln\left(\frac{\beta(d)}{1+\beta(d)} \right) \right] \\
&= \beta'(d) \left(\frac{\beta(d)}{1+\beta(d)} \right)^d + \beta(d) \left(\frac{\beta(d)}{1+\beta(d)} \right)^d \left[\frac{d \cdot \beta'(d)}{\beta(d)(1+\beta(d))} + \ln\left(\frac{\beta(d)}{1+\beta(d)} \right) \right] \\
&= \beta'(d) \left(\frac{\beta(d)}{1+\beta(d)} \right)^d + \frac{d \cdot \beta'(d)}{1+\beta(d)} \cdot \left(\frac{\beta(d)}{1+\beta(d)} \right)^d + \beta(d) \ln\left(\frac{\beta(d)}{1+\beta(d)} \right) \left(\frac{\beta(d)}{1+\beta(d)} \right)^d
\end{aligned}$$

We want to show $\pi'(0) \leqslant -1$. From Proposition 2.2.10, we have $\beta(d) = \beta \cdot p(d)$, thus,

$\beta'(d) = \beta \cdot p'(d)$. Furthermore, $\beta'(0) = \beta \cdot p'(0)$, and $p'(0) = -(1 - p_\infty)$. Therefore,

$$\beta p'(0) + \beta p(0) \cdot \ln\left(\frac{\beta p(0)}{1 + \beta p(0)}\right) \leqslant -1$$

$$-\beta(1 - p_\infty) + \beta \cdot \ln\left(\frac{\beta}{1 + \beta}\right) \leqslant -1$$

$$\ln\left(\frac{\beta}{1 + \beta}\right) \leqslant \frac{-1}{\beta} + (1 - p_\infty)$$

$$\frac{\beta}{1 + \beta} \leqslant e^{-\frac{1}{\beta}} \cdot e^{1 - p_\infty}$$

$$\frac{\beta e^{\frac{1}{\beta}}}{1 + \beta} \leqslant e^{1 - p_\infty}$$

Since $0 \leqslant 1 - p_\infty \leqslant 1$, then $1 \leqslant e^{1-p_\infty} \leqslant e$. Therefore,

$$\frac{\beta e^{\frac{1}{\beta}}}{1 + \beta} \leqslant e$$

We want to show the monotonicity at the LHS:

$$\begin{aligned} \frac{\partial}{\partial \beta}\left(\frac{\beta e^{\frac{1}{\beta}}}{1 + \beta}\right) &= \frac{(e^{\frac{1}{\beta}} + \beta e^{\frac{1}{\beta}}(-1)\beta^{-2})(1 + \beta) - \beta e^{\frac{1}{\beta}}}{(1 + \beta)^2} \\ &= \frac{e^{\frac{1}{\beta}}[(1 - \frac{1}{\beta})(1 + \beta) - \beta]}{(1 + \beta)^2} \\ &= -\frac{e^{\frac{1}{\beta}}\frac{1}{\beta}}{(1 + \beta)^2} \end{aligned}$$

Since $(1 + \beta)^2 > 0$, then $\frac{\partial}{\partial \beta}\left(\frac{\beta e^{\frac{1}{\beta}}}{1+\beta}\right) < 0$ only if $\beta > 0$. Therefore, $\frac{\beta e^{\frac{1}{\beta}}}{1+\beta}$ decreases with β if $\beta > 0$, which means that there is exactly one solution for the inequality $\frac{\beta e^{\frac{1}{\beta}}}{1+\beta} \leqslant e$. By using a graphing software, we find $\beta \geqslant 0.465941$. $\square$

2.3 Random cost per doctor's visit

As in the previous sections, we assume that the natural illness count N has a Poisson, binomial or geometric distribution. The medical expenses, unlike in the previous section, are now assumed to be random. Specifically, let $\{X_1, X_2, ...\}$ denote the medical expenses for different visits. When a deductible is imposed, the insured would start to screen their visits to doctors. There are two ways to model this screening procedure. One way is to screen based on the medical expense. Following this way, the total medical expenses under the a policy with deductible d are

$$L(d) = \sum_{i=1}^{N} X_i \times \mathbb{I}\{X_i > l(d)\}, \tag{2.10}$$

where $l(d)$ is a function to reflect the insured's attitude toward deductible level d. We assume $l(d)$ is increasing concave with $l(0) = 0$ and $\lim_{d\to\infty} l(d) = l_0$ for some $l_0 \subseteq (0, d)$.

The main drawback of this model is the screening procedure. Namely, it assumes the insured knows in advance the medical cost of each illness and then makes a decision whether to visit or not. This situation does not reflect practical situations.

Another way to model the screening is to add a screening indictor that is independent of medical expenses. Specifically,

$$L(d) = \sum_{i=1}^{N} X_i \times I_i, \tag{2.11}$$

where $\{I_1, I_2, ...\}$ are independent of $\{X_1, X_2, ...\}$ and $\{I_1, I_2, ...\} \overset{i.i.d.}{\sim} I$ with $P\{I = 1\} = p(d)$ and $P\{I = 0\} = 1 - p(d)$. Note that this screening process is consistent with that in Section 2.1. With such a screening, Model (2.11) is equivalent to the following

$$L(d) = \sum_{i=1}^{N(d)} X_i, \tag{2.12}$$

As in the previous section, we learn that the conditions ensuring the quasi- arbitrage condition holds if N follows a Poisson distribution or binomial distribution; then, we examine the geometric distribution.

Proposition 2.3.1. *We assume that the natural illness count N follows a Poisson distribution with mean λ, and the probability the insured decides to visit the doctor is, $p(d) =$*

$p_\infty + (1-p_\infty)e^{-d}$. *During each visit, the medical expenses* X_i *follows an independent and identical distribution. For simplicity, we assume that the health insurance plan has copay rate* $\alpha = 1$*; then, the quasi - arbitrage condition holds if this inequality holds for all d,*

$$E[X]\lambda p'(d) - 1 \leqslant -\frac{1}{P\left(\sum_{i=1}^{N(\lambda)} X_i > d\right)} \tag{2.13}$$

Proof. From (2.2) and the assumption $\alpha = 1$, we obtain $\pi(d) = E[(N(d)-d)_+]$, where $N(d) = \sum_{i=1}^{N(\lambda(d))} X_i$. Since $N \sim Poi(\lambda)$, then $N(d) \sim Poi(\lambda(d))$, where $\lambda(d) = \lambda p(d)$. For simplicity, let $N(\lambda(d)) \triangleq N(\lambda)$.

Let $\varphi(\lambda, y) = E[(N(\lambda) - y)_+]$. Then, we have

$$\pi'(d) = \frac{\partial}{\partial \lambda}\varphi(\lambda, y)|_{(\lambda,y)=(\lambda(d),d)} \times \frac{\partial \lambda(d)}{\partial d} + \frac{\partial}{\partial y}\varphi(\lambda, y)|_{(\lambda,y)=(\lambda(d),d)}$$

The second term is direct,

$$\frac{\partial}{\partial y}\varphi(\lambda, y)|_{(\lambda,y)=(\lambda(d),d)} = -P(N(\lambda) > d) = -P(\sum_{i=1}^{N(\lambda(d))} X_i > d)$$

In the first term,

$$\lambda'(d) = \lambda p'(d)$$

We focus on the other part:

$$\begin{aligned}
\varphi(\lambda, y) &= E\left[(\sum_{i=1}^{N(\lambda(d))} X_i - y)_+\right] \\
&= E\left[E\left[(\sum_{i=1}^{N(\lambda(d))} X_i - y)_+ | N(\lambda)\right]\right] \\
&\triangleq E[u(N(\lambda))]
\end{aligned}$$

where, $u(n) = E\left[(\sum_{i=1}^{n} X_i - y)_+\right]$, for a Poisson distribution, $p_n(\lambda) = \frac{\lambda^n}{n!}e^{-\lambda}$, thus,

$$\begin{aligned}
\frac{\partial}{\partial \lambda}\varphi(\lambda, y) &= \frac{\partial}{\partial \lambda}\sum_{n=0}^{\infty} p_n(\lambda)u(n) \\
&= \sum_{n=0}^{\infty} \frac{\partial}{\partial \lambda} p_n(\lambda)u(n) \\
&= \sum_{n=0}^{\infty} \frac{\partial}{\partial \lambda}\frac{\lambda^n}{n!}e^{-\lambda}u(n) \\
&= \sum_{n=0}^{\infty} \frac{n\lambda^{n-1}}{n!}e^{-\lambda}u(n) + \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}(-1)u(n) \\
&= \sum_{n=0}^{\infty} \frac{n\lambda^{n-1}}{n!}e^{-\lambda}u(n) - \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}u(n) \\
&= \sum_{n=0}^{\infty} \frac{\lambda^{n-1}}{(n-1)!}e^{-\lambda}u(n) - \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}u(n) \\
&= \sum_{n=1}^{\infty} \frac{\lambda^{n-1}}{(n-1)!}e^{-\lambda}u(n) - \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}u(n) \\
&= \sum_{n=0}^{\infty} \frac{\lambda^{n'}}{(n')!}e^{-\lambda}u(n'+1) - \sum_{n=0}^{\infty}\frac{\lambda^n}{n!}e^{-\lambda}u(n) \\
&= E\left[u(N(\lambda)+1)\right] - E\left[u(N(\lambda))\right]
\end{aligned}$$

Here, $u(N(\lambda)) = E\left[(\sum_{i=1}^{N(\lambda)} X_i - y)_+ | N(\lambda)\right] = E\left[(\sum_{i=1}^{N(\lambda)} X_i - y)\cdot \mathbb{I}\{\sum_{i=1}^{N(\lambda)} X_i > y\}\right]$.

$$
\begin{aligned}
u(N(\lambda)+1) &= E\left[(\sum_{i=1}^{N(\lambda)+1} X_i - y)_+ | N(\lambda)\right] \\
&= E\left[(\sum_{i=1}^{N(\lambda)} X_i + X_{N(\lambda)+1} - y)_+\right] \\
&= E\left[(\sum_{i=1}^{N(\lambda)} X_i + X_{N(\lambda)+1} - y) \cdot \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i > y - X_{N(\lambda)+1}\right\}\right] \\
&= E\left[(\sum_{i=1}^{N(\lambda)} X_i + X_{N(\lambda)+1} - y) \cdot \left(\mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i > y\right\} + \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i \in (y - X_{N(\lambda)+1}, y]\right\}\right)\right] \\
&= E\left[(\sum_{i=1}^{N(\lambda)} X_i + X_{N(\lambda)+1} - y) \cdot \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i > y\right\}\right] + \\
&\quad E\left[(\sum_{i=1}^{N(\lambda)} X_i + X_{N(\lambda)+1} - y) \cdot \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i \in (y - X_{N(\lambda)+1}, y]\right\}\right]
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
E[u(N(\lambda)+1)] - E[u(N(\lambda))] &= E\left[E[(X_{N(\lambda)+1})\mathbb{I}\{\sum_{i=1}^{N(\lambda)} X_i > y\}\Big| N(\lambda)]\right] + \\
&\quad E\left[E\left[\left(\sum_{i=1}^{N(\lambda)} X_i - (y - X_{N(\lambda)+1})\right) \cdot \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i \in (y - X_{N(\lambda)+1}, y]\right\} \Big| N(\lambda)\right]\right] \\
&= E\left[X_{N(\lambda)+1}\mathbb{I}\{\sum_{i=1}^{N(\lambda)} X_i > y\}\right] + \\
&\quad E\left[\left(\sum_{i=1}^{N(\lambda)} X_i - (y - X_{N(\lambda)+1})\right) \cdot \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i \in (y - X_{N(\lambda)+1}, y]\right\}\right] \\
&= E[X] \cdot P(\sum_{i=1}^{N(\lambda)} X_i > y) + \\
&\quad E\left[\left(\sum_{i=1}^{N(\lambda)} X_i - (y - X_{N(\lambda)+1})\right) \cdot \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i \in (y - X_{N(\lambda)+1}, y]\right\}\right]
\end{aligned}
$$

Substituting d for y, we obtain,

$$\pi'(d) = \left(E[X] \cdot P(\sum_{i=1}^{N(\lambda)} X_i > d) + E\left[\left(\sum_{i=1}^{N(\lambda)} X_i - (d - X_{N(\lambda)+1})\right) \cdot \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i \in (d - X_{N(\lambda)+1}, d]\right\}\right]\right) \cdot$$

$$\lambda p'(d) - P(\sum_{i=1}^{N(\lambda)} X_i > d)$$

$$= (E[X]\lambda p'(d) - 1) \cdot P(\sum_{i=1}^{N(\lambda)} X_i > d) +$$

$$E\left[\left(\sum_{i=1}^{N(\lambda)} X_i - (d - X_{N(\lambda)+1})\right) \cdot \mathbb{I}\left\{\sum_{i=1}^{N(\lambda)} X_i \in (d - X_{N(\lambda)+1}, d]\right\}\right] \lambda p'(d)$$

$$\leqslant (E[X]\lambda p'(d) - 1) \cdot P(\sum_{i=1}^{N(\lambda)} X_i > d)$$

We want to show $\pi'(d) \leqslant -1$; therefore, $E[X]\lambda p'(d) - 1 \leqslant -\frac{1}{P(\sum_{i=1}^{N(\lambda)} X_i > d)}$ is the sufficient condition for the quasi - arbitrage condition to hold. □

Corollary 2.3.2. *There exists λ_0, s.t. (2.13) holds for all $\lambda > \lambda_0$.*

Proof. If $\lambda \to \infty$, then $P(\sum_{i=1}^{N(\lambda)} X_i > d) \to 1$, and $E[X]\lambda p'(d) - 1 \to -\infty$. Therefore, (2.13) holds for all $\lambda > \lambda_0$. □

Corollary 2.3.3. *If $d = 0$, the quasi - arbitrage condition holds if the following inequality holds:*

$$-E[X]\lambda(1 - p_\infty) - 1 \leqslant -\frac{1}{1 - e^{-\lambda}}$$

Proof. If $d = 0$, then $p'(0) = -(1 - p_\infty)$ and $P(\sum_{i=0}^{N(\lambda)} X_i > 0) = 1 - e^{-\lambda}$. Therefore, $\pi'(0) \leqslant -1$ holds, if $-E[X]\lambda(1 - p_\infty) - 1 \leqslant -\frac{1}{1-e^{-\lambda}}$. If the medical expenses X_i follow an exponential distribution with mean θ, or in other words, if $E[X] = \theta$, with fixed θ and p_∞, then we can find the corresponding λ, similar to Example 2.2.6. □

Proposition 2.3.4. *Assuming that the natural illness count N follows a binomial distribution with mean nq, the probability the insured decides to visit the doctor is, $p(d) = p_\infty + (1 - p_\infty)e^{-d}$. During each visit, the medical expenses X_i follow an independent and identical distribution. For simplicity, we assume the health insurance plan has copay rate*

$\alpha = 1$, then the quasi - arbitrage condition holds if this inequality holds for all d:

$$E[X]nq \cdot p'(d) - 1 \leqslant -\frac{1}{P(\sum_{i=1}^{N'} X_i > d)} \tag{2.14}$$

where, $N' \sim Bin(n-1, q \cdot p(d))$

Proof. Since $N \sim Bin(n,q)$, then $N(d) \sim Bin(n,q')$, where $q' = p(d)q$, here, $\pi(d) = E[(\sum_{i=1}^{N(d)} X_i - d)_+]$. Let $\varphi(q',y) = E[(\sum_{i=1}^{N(d)} X_i - y)_+] = E\left[E[(\sum_{i=1}^{N(d)} X_i - y)_+ \middle| N(d)]\right] = E[u(n)]$, where, $u(n) = E[(\sum_{i=1}^{n} X_i - y)_+]$. Therefore,

$$\pi'(d) = \frac{\partial}{\partial q'}\varphi(q',y)|_{(q',y)=(p(d)q,d)} \times \frac{\partial p(d)}{\partial d} + \frac{\partial}{\partial y}\varphi(q',y)|_{(q',y)=(p(d)q,d)}$$

In the first term,

$$\begin{aligned}\frac{\partial}{\partial q'}E[u(n)] &= \frac{\partial}{\partial q'}P_n(q') \cdot u(n)\\ &= n \cdot (E[u(N'+1)] - E[u(N')])\end{aligned}$$

by the Proposition 2.2.7, where $N' \sim Bin(n-1, p(d)q)$

$$u(N') = E\left[(\sum_{i=1}^{N'} X_i - y)_+ \middle| N'\right] = E\left[(\sum_{i=1}^{N'} X_i - y) \cdot \mathbb{I}\{\sum_{i=1}^{N'} X_i > y\}\right]$$

$$\begin{aligned}u(N'+1) &= E\left[(\sum_{i=1}^{N'+1} X_i - y)_+ \middle| N'\right]\\ &= E\left[(\sum_{i=1}^{N'} X_i + X_{N'+1} - y) \cdot \left(\mathbb{I}\{\sum_{i=1}^{N'} X_i > y\} + \mathbb{I}\{\sum_{i=1}^{N'} X_i \in (y - X_{N'+1}, y]\}\right)\right]\\ &= E\left[(\sum_{i=1}^{N'} X_i + X_{N'+1} - y) \cdot \mathbb{I}\{\sum_{i=1}^{N'} X_i > y\}\right] +\\ &\quad E\left[(\sum_{i=1}^{N'} X_i + X_{N'+1} - y) \cdot \mathbb{I}\{\sum_{i=1}^{N'} X_i \in (y - X_{N'+1}, y]\}\right]\end{aligned}$$

By the law of total expectation,

$$E[u(N'+1)] - E[u(N')] = E\left[X_{N'+1} \cdot \mathbb{I}\{\sum_{i=1}^{N'} X_i > y\}\right] +$$

$$E\left[(\sum_{i=1}^{N'} X_i + X_{N'+1} - y) \cdot \mathbb{I}\{\sum_{i=1}^{N'} X_i \in (y - X_{N'+1}, y]\}\right]$$

Therefore,

$$\pi'(d) \leqslant np'(d)q \cdot E[X] \cdot P(\sum_{i=1}^{N'} X_i > d) - P(\sum_{i=1}^{N} X_i > d)$$

We want to show $\pi'(d) \leqslant -1$, in other words, we need $np'(d)q \cdot E[X] \cdot P(\sum_{i=1}^{N'} X_i > d) - P(\sum_{i=1}^{N} X_i > d) \leqslant -1$. Since $N(q') \sim Bin(n, q')$, and $N'(q') \sim Bin(n-1, q')$, then $P(N'(q') > d) \leqslant P(N(q') > d)$. Thus, $(np'(d)q \cdot E[X] - 1)P(\sum_{i=1}^{N'} X_i > d) \leqslant -1$ is the sufficient condition for the quasi-arbitrage condition holds. □

Corollary 2.3.5. *There exists n_0, s.t. (2.14) holds for all $n > n_0$.*

Proof. If $n \to \infty$, then $np'(d)q \cdot E[X] - 1 \to -\infty$. Therefore, (2.14) holds for all $n > n_0$. □

Corollary 2.3.6. *We assume that the natural illness count N follows a binomial distribution with mean nq, Then, following Proposition 2.3.4, the quasi - arbitrage condition holds if the following inequality holds when $d = 0$:*

$$-n(1 - p_\infty)q \cdot E[X] - 1 \leqslant -\frac{1}{1 - (1-q)^{n-1}}$$

Proof. Since $P'(0) = -(1 - p_\infty)$, and $N'(q') \sim Bin(n-1, p(d)q)$, then $P(N'(q') > 0) = 1 - P(N'(q') = 0) = 1 - (1 - p(d)q)^{n-1}$. Therefore, from (2.14), $-n(1-p_\infty)q \cdot E[X] - 1 \leqslant -\frac{1}{1-(1-q)^{n-1}}$ is the condition for the quasi - arbitrage condition to hold. □

Proposition 2.3.7. *We assume that the natural illness count N has a geometric distribution with mean β. The medical expenses X_i follow an exponential distribution with mean θ, $\{X_i, i = 1, 2, ...\} \overset{i.i.d.}{\sim} Exp(\theta)$. The pure premium $\pi(d)$ admits the following expression:*

$$\pi(d) = \beta(d) \times q_\beta(d)^d (\alpha + (1-\alpha) q_\beta(d)^u) \tag{2.15}$$

where $\beta(d) = \theta(1 + \beta p(d))$ and $q_\beta(d) = e^{-\frac{1}{\beta(d)}}$

Proof. Since $N \sim Geo(\beta)$, $X_i \sim Exp(\theta)$, then $N(d) \sim Geo(\beta p(d))$. According to Klugman et al. (2012), the distribution of the aggregate loss model $L(d)$ when the frequency distribution is geometric with mean β, and the severity distribution that is an exponential distribution with mean θ is a mixture distribution of zero and geometric over exponential model, whose distribution has already been established. Since we only consider the losses $L(d) > d$, thus $L(d) \sim Exp(\theta(1 + \beta p(d)))$. Thus, we have

$$
\begin{aligned}
E[(L(d) - d)_+] &= E[L(d) - d \mid L(d) > d] \times P(L(d) > d) \\
&= \theta(1 + \beta p(d)) \times p(L(d) > d) \\
&= \theta(1 + \beta p(d))(e^{-\frac{d}{(1+\beta p(d))\theta}})
\end{aligned}
$$

Similarly,

$$
E[(L(d) - d - u)_+] = \theta(1 + \beta p(d))e^{-\frac{d+u}{\theta(1+\beta p(d))}}
$$

Therefore,

$$
\begin{aligned}
\pi(d) &= \alpha E[(L(d) - d)_+] + (1 - \alpha)E[(L(d) - d - u)_+] \\
&= \theta(1 + \beta p(d))e^{-\frac{d}{\theta(1+\beta p(d))}}(\alpha + (1 - \alpha)e^{-\frac{u}{\theta(1+\beta p(d))}}) \\
&= \beta(d) \times q_\beta(d)^d(\alpha + (1 - \alpha)q_\beta(d)^u)
\end{aligned}
$$

where $\beta(d) = \theta(1 + \beta p(d))$ and $q_\beta(d) = e^{-\frac{1}{\beta(d)}}$ □

Proposition 2.3.8. *We assume that the natural illness count N follows a geometric distribution with mean β. The medical expenses X_i follow an exponential distribution with mean θ, $\{X_i, i = 1, 2, ...\} \overset{i.i.d.}{\sim}$ Exp(θ). If β is sufficiently large, then $\pi'(d) \leqslant -1$ for all d such that $p'(d) \leqslant -\frac{1}{\beta}$.*

Proof. From Proposition 2.3.7. we know that the pure premium for the aggregate model has the same equation as the nonrandom model if the natural illness count N follows a geometric distribution. Therefore, this proof is the same as Proposition 2.2.10. □

Proposition 2.3.9. *We assume that the natural illness N follows a geometric distribution with mean β, The medical expenses X_i follow an exponential distribution with mean θ, $\{X_i, i = 1, 2, ...\} \overset{i.i.d.}{\sim}$ Exp(θ).The probability that the insured decides to visit the doctor is $p(d) = p_\infty + (1 - p_\infty)e^{-d}$. For simplicity, we assume the health insurance plan has copay rate $\alpha = 1$. Then the quasi - arbitrage condition holds for all β when $d = 0$.*

Proof. According to (2.15), we have $\pi(d) = \beta(d) \times q_\beta(d)^d$. Therefore,

$$
\pi'(d) = \beta'(d) \times q_\beta(d)^d + \beta(d) \times q_\beta(d)^d(\ln q(d) + \frac{d}{q(d)} \cdot q'(d))
$$

substituting 0 for d,

$$\pi'(0) = \theta\beta p'(0) - 1$$

where $p'(0) = -(1 - p_\infty)$. We want to show $\pi'(0) \leqslant -1$, therefore, $-\theta\beta(1 - p_\infty) \leqslant 0$, which is true for all β. $\square$

Chapter 3

Optimal design of health insurance policy

In Chapter 2, we found that the quasi-arbitrage condition helps to mitigate the moral hazard from the insured's perspective, and established some conditions for the quasi-arbitrage condition holds when N, the random counts of the natural illness of an insured person during a one-year period follows the $(a, b, 0)$ class distributions. In this chapter, we investigate into the expected profit from the insurer's perspective. In Section 3.1, we establish the insurer's profit formula, and maximize the expected profit. Then, in Section 3.2, we analyze the maximum expected utility, because each policyholder has their own utility. Finally, similarly to Chapter 2, we also consider the random cost per doctor's visit in Section 3.3. Initially, we set the probability that policyholders switch from the LDHP to the HDHP as a fixed function of the incentive. In Section 3.4, we analyze this probability by using prospect theory.

Suppose the insurer charges $\pi(d_1)$ for LDHP and $G(d_2)$ for HDHP; here, $G(d_2)$ is not necessarily equal to $\pi(d_2) = E[(d_2)]$, and the pure premium for risk is $L(d_2)$. If we set the price of the HDHP, $G(d_2)$ equals to the net premium $\pi(d_2)$, and there is no profit for the insurance company. If we set $G(d_2)$ too high, fewer policyholders want to switch to the HDHP. Rather, $G(d_2)$ is up to the insurer's choice to determine; it has to be assumed that $G(d_2) \geq \pi(d_2)$. For simplicity, we assume the insurer charges a pure premium for risk $L(d_1)$ for LDHP. We aim to find a reasonable incentive that could maximize the insurer's profit after policyholders transfer from the LDHP to the HDHP.

If a policyholder switches from the LDHP to the HDHP, the maximum loss in coverage is:

$d_2 - d_1$. The savings in the premium is $\pi(d_1) - G(d_2)$. Therefore, $I = \pi(d_1) - G(d_2) - (d_2 - d_1)$ can be regarded as the amount of incentive. I needs to satisfy this inequality, $0 \leq I \leq \pi(d_1) - \pi(d_2) - (d_2 - d_1) \triangleq I_m$. On the one hand, I has to be positive to incentivize policyholders to switch; otherwise, the policyholders would rather stay with the LDHP instead of switching. On the other hand, the insurer can provide the maximum incentive when the insurance company earns the least profit; in other words, $G(d_2) = \pi(d_2)$. Denote the maximum incentive as I_m. The positive value of I_m is guaranteed by the quasi-arbitrage condition.

The insurer needs to determine what level to set I to reach optimization. Notably, even with a positive incentive, not all the existing policyholders are willing to switch from the LDHP to the HDHP. We assume the probability of switching is $\lambda(I)$. As the amount of the incentive increases, the probability of switching increases, but the increasing speed level slows down because of marginal diminishing returns. To set up some basic properties of $\lambda(I)$, consider two special cases: if the incentive amount is zero, no one wants to switch. If the incentive amount is provided at its maximum possible level I_m, some policyholders may want to stay on the LDHP. Thus, it is reasonable to assume that:

- $\lambda(I)$ is increasing and concave in I.
- $\lambda(0) = 0$, $\lambda_{max} \triangleq \lambda(I_m) < 1$.

For convenience, $\lambda(I) = 1 - (1 - \frac{I}{A})^{\alpha}$. This special form will be used in this chapter, where $\alpha > 1$ and A is a constant such that $I < A$ for all I. With the probability of switching from the LDHP to the HDHP, $\lambda(I)$, we can maximize expected profit for two groups of policyholders: one group chooses to stay on the LDHP with probability $1 - \lambda(I)$, and the other group switches to the HDHP with probability $\lambda(I)$.

3.1 Maximize expected profit

Maximizing the profit is the most important consideration from the insurer's perspective, and we can start addressing profit maximization with the profit function.

Suppose there are n policyholders originally covered under the LDHP. They make independent decisions There are M policyholders who decide to switch from the LDHP to the HDHP; therefore, after the launch of the HDHP, there are only $n - M$ LDHP policyholders, where $M \sim Bin(n, \lambda(I))$ is a binomial random variable, and $N \sim Bin(n, 1-\lambda(I))$. The total profit B of this insurance portfolio can be added from two parts: the first part generates the profit of policyholders staying on the LDHP, and the second part generates the profit of policyholders switching from the LDHP to the HDHP.

$$B = \sum_{i=1}^{n-M} (\pi(d_1) - L_i(d_1)) + \sum_{j=1}^{M} (\pi(d_1) + d_1 - d_2 - I - L_j(d_2)) \tag{3.1}$$

where $\{L_i(d_1), i = 1, 2, ...\}$ represents the individual losses from LDHP policyholders, and $\{L_j(d_2), j = 1, 2, ...\}$ represents the individual losses from HDHP policyholders. The expected profit becomes:

$$E[B] = n(1 - \lambda(I))(\pi(d_1) - E[L(d_1)]) + n\lambda(I)(\pi(d_1) + d_1 - d_2 - I - E[L(d_2)])$$

The insurer's expected profit for each LDHP policyholder is

$$\pi(d_1) - E[L(d_1)] = 0$$

The expected gain from launching each HDHP is:

$$\lambda(I)(G(d_2) - E[L(d_2)]) = \lambda(I)(I_m - I) \triangleq g_1(I)$$

where $G(d_2) = \pi(d_1) + d_1 - d_2 - I$, and

$g_1(I)$ is defined as the expected profit under the assumption that the cost per doctor's visit is constant. Our goal is to maximize $g_1(I)$ over $[0, I_m]$.

Lemma 3.1.1. *$g_1(I) = \lambda(I)(I_m - I)$ has the following properties:*

(i) $g_1(0) = 0, g_1(I_m) = 0$

(ii) $g_1(I)$ is concave in I, and obtains its maximum at $I^*(d_2)$, in other words, $g_1'(I^*(d_2)) = 0$.

Proof. (*i*) $g_1(0) = \lambda(0)(I_m) = 0$, $g_1(I_m) = \lambda(I_m)(I_m - I_m) = 0$

(*ii*) Since $\lambda(I)$ is an increasing concave function, $\lambda'(I) > 0$, $\lambda''(I) < 0$. Taking the first derivative of $g_1(I)$ respects with I, we obtain $g_1'(I) = \lambda'(I)(I_m - I) - \lambda(I)$, and it is difficult to determine whether $g_1(I)$ is increasing or decreasing in I. Then, taking the second derivate of $g_1(I)$, we obtain $g_1''(I) = \lambda''(I)(I_m - I) - 2\lambda'(I) < 0$; therefore, $g_1''(I)$ is concave in I, and there exists $I^*(d_2)$, such that $g_1'(I^*(d_2)) = 0$. □

Proposition 3.1.2. *If $I_m < I^*(d_2)$, then the maximum expected profit is obtained at $I = I_m$;*

therefore, max $E[B] = g_1(I_m)$*. If* $I_m \geq I^*(d_2)$*, then the maximum expected profit is obtained at* $I = I^*(d_2)$*; therefore, max* $E[B] = g_1(I^*(d_2))$*.*

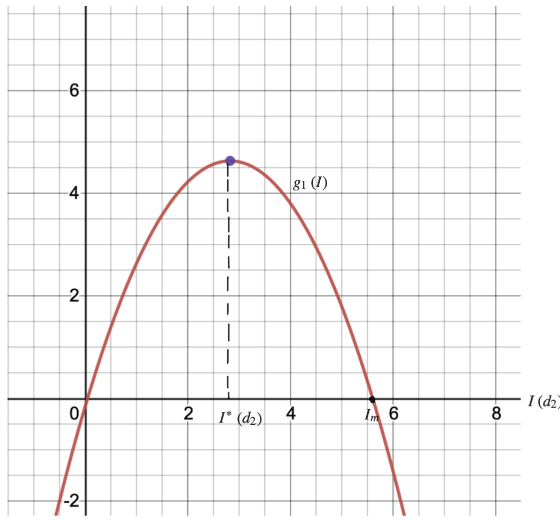

Figure 3.1: The figure shows $max\ E[B] = g_1(I^*(d_2))$

Proposition 3.1.3. *We assume that the natural illness count N follows a geometric distribution with mean β. Assuming the cost of each doctor's visit is a deterministic constant, the maximum expected profit is obtained at $I = I^*(d_2)$.*

Proof. Since $N \sim$ Geo(β), then $N(d)\sim$ Geo(β(d)), where $\beta(d) = \beta p(d)$ and $q_\beta(d) = \frac{\beta(d)}{1+\beta(d)}$.

The goal is to find the incentive obtains the maximum expected profit. We know,

$$g_1(I) = (1-(1-\frac{I}{A})^2)(I_m - I)$$

Then,

$$\begin{aligned} g_1'(I) &= -2(1-\frac{I}{A})(-\frac{1}{A})(I_m - I) + (1-(1-\frac{I}{A})^2)(-1) \\ &= \frac{2}{A}(1-\frac{I}{A})(I_m - I) - 1 + (1-\frac{I}{A})^2 \\ &= \frac{2}{A}(I_m) - \frac{2I_mI}{A^2} - \frac{2I}{A} + \frac{2I^2}{A^2} - \frac{2I}{A} + \frac{I^2}{A^2} \\ &= \frac{3I^2}{A^2} - (\frac{4}{A} + \frac{2I_m}{A^2})I + \frac{2I_m}{A} \end{aligned}$$

Thus,

$$\begin{aligned} g'(I_m) &= \frac{3I_m^2}{A^2} - (\frac{4}{A} + \frac{2I_m}{A^2})I_m + \frac{2I_m}{A} \\ &= \frac{3I_m^2}{A^2} - \frac{4I_m}{A} - \frac{2I_m^2}{A^2} + \frac{2I_m}{A} \\ &= \frac{I_m^2}{A^2} - \frac{2I_m}{A} \end{aligned}$$

Since A is chosen such that $I < A$ for all I, thus,

$$0 < \frac{I_m}{A} < 1, (\frac{I_m}{A})^2 - 2(\frac{I_m}{A}) < 0, g_1'(I_m) < 0.$$

From proposition 3.1.2, this expression is equivalent to $I_m \geq I^*(d_2)$; then, the maximum $E[B]$ is obtained at $I = I^*(d_2)$.

$I^*(d_2)$ can be solved by the equation $g_1'(I) = 0$,

$$\frac{3(I^*(d_2))^2}{A^2} - (\frac{4}{A} + \frac{2I_m}{A^2})I^*(d_2) + \frac{2I_m}{A} = 0$$

$$I^*(d_2) = \frac{2A}{3} + \frac{I_m}{3} \pm \frac{A}{3}\sqrt{(\frac{I_m}{A} - 1)^2 + 3}$$

here,

$$\frac{A}{3}\sqrt{(\frac{I_m}{A}-1)^2+3} > \frac{A}{3}$$

$$\frac{2A}{3}+\frac{A}{3}\sqrt{(\frac{I_m}{A}-1)^2+3} > A,$$

but, A is chosen such that $I < A$ for all I.

Thus,

$$I^*(d_2) = \frac{2A}{3}+\frac{I_m}{3}-\frac{A}{3}\sqrt{(\frac{I_m}{A}-1)^2+3}$$

$\square$

3.2 Maximize expected utility

Under the insurance, different policyholders have their own utility functions. Following the setup profit (3.1),

$$
\begin{aligned}
B &= \sum_{i=1}^{n-M} (\pi(d_1) - L_i(d_1)) + \sum_{j=1}^{M} (\pi(d_1) + d_1 - d_2 - I - L_j(d_2)) \\
&= n\pi(d_1) - [(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M} (L_j(d_2) - (d_1 - d_2 - I))]
\end{aligned}
$$

Assuming that the insurer has utility function u, the optimization problem can be formulated as maximizing the expected utility of the profit B, as follows:

$$
\max_{(I,d_2)\in D} E\left[u(n\pi(d_1) - \left[\left(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M} (L_j(d_2) - (d_1 - d_2 - I)) \right] \right) \right] \triangleq g_1(I, d_2) \quad (3.2)
$$

where

$$
D = \{(I, d_2) | I \leq E[L(d_1)] + d_1 - E[L(d_2)] - d_2 = I_m\}
$$

We want to maximize $g_1(I, d_2)$, which is the bivariate function. Let us solve this optimization problem I_m step by step. First, assume the incentive I is fixed to enable finding the deductible when the expected profit is maximized. Then, given the properties of I_m, optimize I to achieve the maximum expected profit. A special case is where u is the identical function. In the following, we assume the utility function u to be an exponential utility function to derive explicit solutions.

Proposition 3.2.1. *Assume u is an exponential utility function, $u(x) = 1 - e^{-\gamma x}$, and assume the cost during each doctor's visit is a deterministic constant. Then, maximum expected utility of profit B can be expressed as*

$$
\max E[M_{L(d_1)}(\gamma)^{-n}] \cdot E[z^M], z = \frac{M_{L(d_1)}(\gamma)}{M_{L(d_2)-(d_1-d_2-I)}(\gamma)}
$$

Proof.

$$
\max_{(I,d_2)\in D} E\left[u\left(n\pi(d_1) - \left[\left(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M} (L_j(d_2) - (d_1 - d_2 - I)) \right) \right] \right) \right]
$$

$$= \max_{(I,d_2)\in D} E\left[1 - e^{-\gamma\left(n\pi(d_1) - (\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M}(L_j(d_2) - (d_1 - d_2 - I)))\right)}\right]$$
$$\Leftrightarrow \min_{(I,d_2)\in D} E\left[e^{-\gamma n\pi(d_1) + \gamma\left(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M}(L_j(d_2) - (d_1 - d_2 - I))\right)}\right]$$
$$\Leftrightarrow \min_{(I,d_2)\in D} E\left[e^{\gamma\left(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M}(L_j(d_2) - (d_1 - d_2 - I))\right)}\right]$$
$$\Leftrightarrow \max_{(I,d_2)\in D} E\left[e^{-\gamma\left(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M}(L_j(d_2) - (d_1 - d_2 - I))\right)}\right]$$
$$\Leftrightarrow \min_{(I,d_2)\in D} \left(-E\left[e^{-\gamma\left(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M}(L_j(d_2) - (d_1 - d_2 - I))\right)}\right]\right)$$
$$\Leftrightarrow \min_{(I,d_2)\in D} \left(1 - E\left[e^{-\gamma\left(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M}(L_j(d_2) - (d_1 - d_2 - I))\right)}\right]\right)$$
$$\Leftrightarrow \min_{(I,d_2)\in D} E\left[u\left(\sum_{i=1}^{n-M} L_i(d_1) + \sum_{j=i}^{M}(L_j(d_2) - (d_1 - d_2 - I))\right)\right]$$

Let $X_i = L_i(d_1)$, $Y_j = L_j(d_2) - (d_1 - d_2 - I)$

We want to,

$$\min_{(I,d_2)\in D} E\left[u(\sum_{i=1}^{n-M} X_i + \sum_{j=1}^{M} Y_j)\right]$$

where $M \sim Bin(n, \lambda(I))$

$$= \min E\left[1 - e^{-\gamma(\sum_{i=1}^{n-M} X_i + \sum_{j=1}^{M} Y_j)}\right]$$
$$\Leftrightarrow \max E\left[e^{-\gamma(\sum_{i=1}^{n-M} X_i + \sum_{j=1}^{M} Y_j)}\right]$$
$$= \max E\left[E\left[e^{-\gamma(\sum_{i=1}^{n-M} X_i + \sum_{j=1}^{M} Y_j)}|M\right]\right]$$
$$= \max E\left[E\left[e^{-\gamma\sum_{i=1}^{n-M} X_i}|M\right] \cdot E\left[e^{-\gamma\sum_{j=1}^{M} Y_j}|M\right]\right]$$
$$= \max E\left[M_X(\gamma)^{-(n-m)} \cdot M_Y(\gamma)^{-M}\right]$$
$$= \max E\left[M_X(\gamma)^{-n} \cdot \left(\frac{M_X(\gamma)}{M_Y(\gamma)}\right)^M\right]$$
$$= \max E\left[M_X(\gamma)^{-n}\right] \cdot E[z^M], z = \frac{M_X(\gamma)}{M_Y(\gamma)}$$
$$= \max E\left[M_{L(d_1)}(\gamma)^{-n}\right] \cdot E\left[z^M\right], z = \frac{M_{L(d_1)}(\gamma)}{M_{L(d_2)-(d_1-d_2-I)}(\gamma)}$$

□

Corollary 3.2.2. *Assume the natural illness count N follows a geometric distribution with mean β, and assume the cost during each doctor's visit is a deterministic constant. Let u to be an exponential utility function, $u(x) = 1 - e^{-\gamma x}$. The expected utility is maximized as follows:*

$$\left(1 - (1 - \frac{I}{A})^{\alpha}\right)(1 + \beta(d_2) - \beta(d_2)e^{\gamma})\, e^{(d_1 - d_2 - I)\gamma} + (1 - \frac{I}{A})^{\alpha} \triangleq g_2(I, d_2) \tag{3.3}$$

Proof. Since the cost of each doctor's visit is a deterministic constant, thus $L(d) = N(d)$, and $N(d) = Geo(\beta(d))$, then $E[M_{L(d_1)}(\gamma)^{-n}] = E[M_{N(d_1)}(\gamma)^{-n}]$, $N(d_1) = Geo(\beta(d_1))$, the pmf of $N(d_1)$ is $p(k) = (\frac{\beta(d_1)}{1+\beta(d_1)})^k(\frac{1}{\beta(d_1)})$, and the moment generating function of $N(d_1)$ is $M_{N(d_1)}(\gamma) = \frac{e^{\gamma}}{1+\beta(d_1)-\beta(d_1)e^{\gamma}}$.

$$\begin{aligned} M_{L(d_2)-(d_1-d_2-I)}(\gamma) &= E\left[e^{\gamma(L(d_2)-(d_1-d_2-I))}\right] \\ &= E\left[e^{\gamma L(d_2)}\right] e^{-(d_1-d_2-I)\gamma} \\ &= \frac{e^{\gamma}}{1+\beta(d_2)-\beta(d_2)e^{\gamma}} e^{-(d_1-d_2-I)\gamma} \\ &= \frac{e^{\gamma-(d_1-d_2-I)\gamma}}{1+\beta(d_2)-\beta(d_2)e^{\gamma}} \end{aligned}$$

Therefore,

$$z = \frac{1+\beta(d_2)-\beta(d_2)e^{\gamma}}{1+\beta(d_1)-\beta(d_1)e^{\gamma}} e^{(d_1-d_2-I)\gamma}$$

$$E[z^M] = E\left[\left(\frac{1+\beta(d_2)-\beta(d_2)e^{\gamma}}{1+\beta(d_1)-\beta(d_1)e^{\gamma}} e^{(d_1-d_2-I)\gamma}\right)^M\right]$$

By the property of the probability generating function of the binomial distribution,

$$E[z^M] = \left[\lambda(I)\left(\frac{1+\beta(d_2)-\beta(d_2)e^{\gamma}}{1+\beta(d_1)-\beta(d_1)e^{\gamma}} e^{(d_1-d_2-I)\gamma}\right) + 1 - \lambda(I)\right]^n$$

We set $\lambda(I) = 1 - (1 - \frac{I}{A})^{\alpha}$ for $\alpha > 1$ and such that $I < A$ for all I.

$$E[z^M] = \left[\left(1 - (1 - \frac{I}{A})^{\alpha}\right)\left(\frac{1+\beta(d_2)-\beta(d_2)e^{\gamma}}{1+\beta(d_1)-\beta(d_1)e^{\gamma}} e^{(d_1-d_2-I)\gamma}\right) + (1 - \frac{I}{A})^{\alpha}\right]^n$$

Thus, maximizing the expected utility is equivalent to

$$\max_{(I,d_2)\in D} \left(1-(1-\frac{I}{A})^{\alpha}\right)(1+\beta(d_2)-\beta(d_2)e^{\gamma})\,e^{(d_1-d_2-I)\gamma}+(1-\frac{I}{A})^{\alpha}$$

□

Proposition 3.2.3. *If $\beta(e^{\gamma}-1)<\frac{1}{p_\infty}$, $g_2(I,d_2)$ increases on $d_2\le d_g^*$, decreases on $d_2>d_g^*$, where $d_g^*=-\ln\frac{\gamma+\gamma\beta p_\infty(1-e^{\gamma})}{\beta(e^{\gamma}-1)(1+\gamma)(1-p_\infty)}$. The special case is if $\gamma(1+\beta p_\infty(1-e^{\gamma}))\geqslant\beta(e^{\gamma}-1)(1+\gamma)(1-p_\infty)$, then $g_2(I,d_2)$ decreases by d_2 for all $d_2\geqslant 0$.*

Proof. Taking the derivative of $g_2(I,d_2)$ with respect to d_2, we obtain

$$(1-(1-\frac{I}{A})^{\alpha})\left[\beta'(d_2)(1-e^{\gamma})e^{(d_1-d_2-I)\gamma}+(1+\beta(d_2)-\beta(d_2)e^{\gamma})e^{(d_1-d_2-I)\gamma}(-\gamma)\right]>0$$

$$-\beta(1-p_\infty)e^{-d_2}(1-e^{\gamma})>\gamma(1+\beta(d_2)(1-e^{\gamma}))$$

$$-\beta(1-p_\infty)e^{-d_2}(1-e^{\gamma})>\gamma(1+\beta(p_\infty+(1-p_\infty)e^{-d_2})(1-e^{\gamma}))$$

$$-\beta(1-p_\infty)e^{-d_2}(1-e^{\gamma})>\gamma+(\gamma\beta p_\infty+\gamma\beta(1-p_\infty)e^{-d_2})(1-e^{\gamma})$$

$$-\beta(1-p_\infty)e^{-d_2}(1-e^{\gamma})>\gamma+\gamma\beta p_\infty(1-e^{\gamma})+\gamma\beta(1-p_\infty)e^{-d_2}(1-e^{\gamma})$$

$$-\beta(1-p_\infty)e^{-d_2}(1-e^{\gamma})(1+\gamma)>\gamma+\gamma\beta p_\infty(1-e^{\gamma})$$

as $\gamma>0$, then $e^{\gamma}-1>0$,

$$e^{-d_2}>\frac{\gamma+\gamma\beta p_\infty(1-e^{\gamma})}{\beta(e^{\gamma}-1)(1+\gamma)(1-p_\infty)}$$

$$-d_2>ln(\frac{\gamma(1+\beta p_\infty(1-e^{\gamma}))}{\beta(e^{\gamma}-1)(1+\gamma)(1-p_\infty)})$$

$$d_2<-ln(\frac{\gamma(1+\beta p_\infty(1-e^{\gamma}))}{\beta(e^{\gamma}-1)(1+\gamma)(1-p_\infty)})$$

where,

$$1+\beta p_\infty(1-e^{\gamma})>0$$

$$\beta(e^{\gamma}-1)<\frac{1}{p_\infty}$$

Thus, when $0<d_2<-\ln(\frac{\gamma(1+\beta p_\infty(1-e^{\gamma}))}{\beta(e^{\gamma}-1)(1+\gamma)(1-p_\infty)})$ and $\beta(e^{\gamma}-1)<\frac{1}{p_\infty}$, $g_2(I,d_2)$ increases by

d_2. Similarly, when $d_2 > -\ln(\frac{\gamma(1+\beta p_\infty(1-e^\gamma))}{\beta(e^\gamma-1)(1+\gamma)(1-p_\infty)})$and $\beta(e^\gamma-1) < \frac{1}{p_\infty}$, $g_2(I, d_2)$ decreases by d_2. Thus, we can conclude that $d_g^* = -\ln(\frac{\gamma(1+\beta p_\infty(1-e^\gamma))}{\beta(e^\gamma-1)(1+\gamma)(1-p_\infty)})$ is where $g_2(I, d_2)$ achieves the maximum expected utility, under the condition $\beta(e^\gamma-1) < \frac{1}{p_\infty}$.

A special case occurs when $d_g^* \leqslant 0$. Then, $g_2(I, d_2)$ decreases by d_2 for all $d_2 \geqslant 0$.

$$
\begin{aligned}
d_g^* \leqslant 0 &\Leftrightarrow \ln(\frac{\gamma(1+\beta p_\infty(1-e^\gamma))}{\beta(e^\gamma-1)(1+\gamma)(1-p_\infty)}) \geqslant 0 \\
&\Leftrightarrow \frac{\gamma(1+\beta p_\infty(1-e^\gamma))}{\beta(e^\gamma-1)(1+\gamma)(1-p_\infty)} \geqslant 1 \\
&\Leftrightarrow \gamma(1+\beta p_\infty(1-e^\gamma)) \geqslant \beta(e^\gamma-1)(1+\gamma)(1-p_\infty)
\end{aligned}
$$

□

Proposition 3.2.4. *Assume the natural illness count N follows a geometric distribution with mean β. The expense for each visit is a deterministic constant. Assume the insured is transferred from full insurance to a high deductible insurance with deductible d_2. The probability that the insured visits the doctor during each illness is $p(d) = p_\infty + (1-p_\infty)e^{-\gamma_p d}$. Assuming $\beta(1-p_\infty) > 1$, there are 2 roots of of $I_m(d_2)$.*

Proof.

$$I_m(d_2) = E[L(d_1)] + d_1 - E[L(d_2)] - d_2$$

where since

$$E[L(d)] = \beta p(d) = \beta \left(p_\infty + (1-p_\infty)e^{-\gamma_p d}\right)$$

Since $d_1 = 0$,

$$
\begin{aligned}
I_m(d_2) &= \beta - \beta \left(p_\infty + (1-p_\infty)e^{-\gamma_p d_2}\right) - d_2 \\
&= \beta(1-p_\infty)(1-e^{-\gamma_p d_2}) - d_2
\end{aligned}
$$

$$I'_m(d_2) = \beta(1-p_\infty)\gamma_p e^{-\gamma_p d_2} - 1$$

Under the assumption of $\beta(1-p_\infty) > 1$, I_m first increases and then decreases on $[0, \infty)$ and eventually decreases below zero. To find the roots of I_m, set,

$$\beta \left(p_\infty + (1-p_\infty)e^{-\gamma_p d_2}\right) - d_2 = 0$$

$$d_2 = 0, d_2 = \beta(1-p_\infty) + \frac{W[-\gamma_p\beta(1-p_\infty)e^{-\gamma_p\beta(1-p_\infty)}]}{\gamma_p} \triangleq d_I^0$$

where, $W[z]$ gives the principal solution for w in $z = we^w$. □

Lemma 3.2.5. *$I_m(d_2) = \pi(d_1) + d_1 - \pi(d_2) - d_2$ has the following properties:*

(i) $I_m(d_2)$ is concave and obtains its maximum at d_I^*.

(ii) For any $I \in [0, I_m(d_I^*))$, there are two and only two solutions to $I_m(d_2) = I$, denoted as d_I^L and d_I^R. In particular, $I_m(d_2) = 0$ has two solutions: 0 and $d_I^0 > 0$. Apparently, $0 \leqslant d_I^L \leqslant d_I^R \leqslant d_I^0$ for any $I \in [0, I_m(d_I^*))$.

Proposition 3.2.6. *The maximum expected utility of the profit, $g_1(I, d_2)$, over the interval $I \leqslant I_m(d_2)$ can be summarized by the following:*
if $d_g^* > d_B^*$, then

$$g_1(I, d_2) = \begin{cases} g(I, d_g^*), & I \leqslant I(d_g^*) \\ g(I, d_I^R), & I > I(d_g^*) \end{cases} \tag{3.4}$$

if $d_g^* < d_B^*$, then

$$g_1(I, d_2) = \begin{cases} g(I, d_g^*), & I \leqslant I(d_g^*) \\ g(I, d_I^L), & I > I(d_g^*) \end{cases} \tag{3.5}$$

Proof. From Proposition 3.2.3, we know that for any fixed I, there exists d_g^*, such that $g_2(I, d_2)$ increases on $d_2 \leq d_g^*$, decreases on $d_2 > d_g^*$. From the Lemma 3.2.5. $I_m(d_2)$ is concave in d_2, thus $I \leqslant I_m(d_2)$ is equivalent to say $d_2 \in [d_I^L, d_I^R]$.
Let's check whether $d_g \in [d_I^L, d_I^R]$. This is equivalently to check whether $I_m(d_g^*) \geq I$.
If $d_g^* \in [d_I^L, d_I^R]$, then the $g_1(I, d_2)$ is maximized when $d_2 = d_g^*$, thus, $\underset{0 \leq I \leq B(d_2)}{max} g_1(I, d_2) = g_1(I, d_g^*)$
If $d_g^* \notin [d_I^L, d_I^R]$, we need to compare d_g^* and d_B^*.

If $d_g^* > d_B^*$, then $[d_I^L, d_I^R]$ lies to the left of d_g^*, which is the increasing part of $g_1(I, d_2)$. In the other words, $\underset{d_2 \in [d_I^L, d_I^R]}{max} g_1(I, d_2) = g_1(I, d_I^R)$.

Otherwise, if $d_g^* < d_B^*$, then $[d_I^L, d_I^R]$ lies to the right of d_g^*, which is the decreasing part of $g_1(I, d_2)$; thus, $\underset{d_2 \in [d_I^L, d_I^R]}{max} g_1(I, d_2) = g_1(I, d_I^L)$.

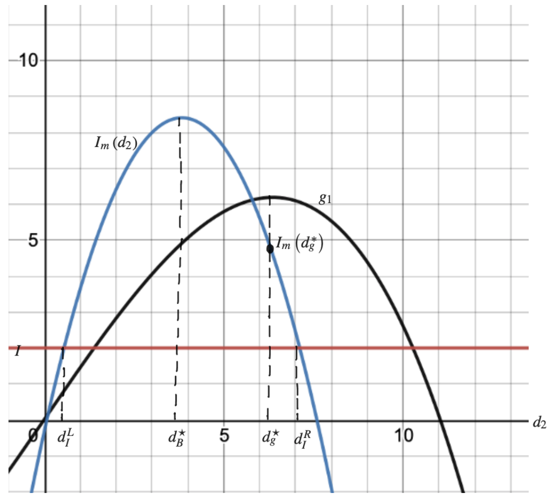

Figure 3.2: The case of $max\ g_1(I, d_2) = g_1(I, d_g^*)$

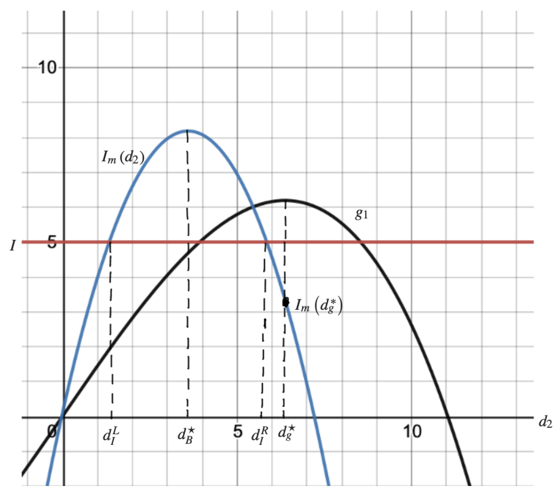

Figure 3.3: The case of $max\ g_1(I, d_2) = g_1(I, d_I^R)$

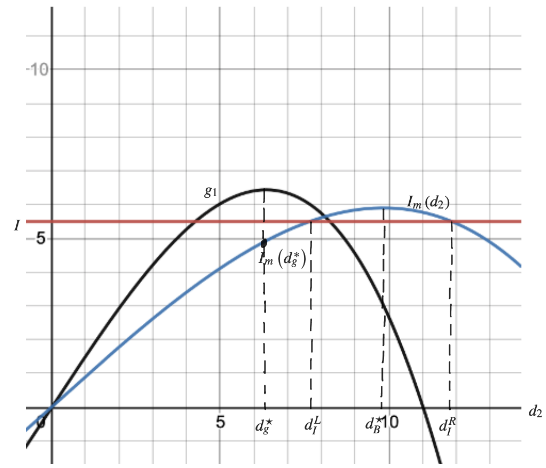

Figure 3.4: The case of $max\ g_1(I, d_2) = g_1(I, d_I^L)$

$\square$

In Proposition 3.2.6, we first fix incentive I to find the maximum profit based on the random variable d_2; then, according to the properties of I_m, we optimize I to achieve the maximum expected utility profit $g_1(I, d_2)$.

Proposition 3.2.7. *If $d_g^* > d_B^*$, then,*

$$g_1(I, d_2) = \begin{cases} g_1(I, d_g^*), & I \leqslant I_m(d_g^*) \\ g_1(I_m(d_2), d_2), & d_B^* < d_2 < max\,\{d_g^*, d_B\} \end{cases} \tag{3.6}$$

If $d_g^* < d_B^*$, then,

$$g_1(I, d_2) = \begin{cases} g_1(I, d_g^*), & I \leqslant I_m(d_g^*) \\ g_1(I_m(d_2), d_2), & d_g^* < d_2 < d_B^* \end{cases} \tag{3.7}$$

Proof. By Proposition 3.2.6. when $I_m(d_g^*) \geq I$, $\underset{0 \leq I \leq I_m(d_g^*)}{max} g_1(I, d_2) = g_1(I, d_g^*)$, and we know $d_g^* = -\ln \frac{\gamma + \gamma\beta p_\infty (1 - e^\gamma)}{\beta(e^\gamma - 1)(1+\gamma)(1-p_\infty)}$, from Proposition 3.2.3. In the case of $d_g^* > d_B^*$, when $I_m(d_g^*) < I$, $\underset{I_m(d_g^*) < I < I_m(d_B^*)}{max} g_1(I, d_I^R) = \underset{d_B^* < d_2 < max\,\{d_g^*, d_B^0\}}{max} g_1(I_m(d_2), d_2)$, where $d_B^* = \frac{1}{\gamma_p} ln(\beta(1 - p_\infty)\gamma_p)$. In the case of $d_g^* < d_B^*$, $\underset{I_m(d_g^*) < I < I_m(d_B^*)}{max} g_1(I, d_I^L) = \underset{d_g^* < d_2 < d_B^*}{max} g_1(I_m(d_2), d_2)$ □

3.3 Maximize expected utility under the aggregate loss model

Assume the natural illness count N has a geometric distribution as in the previous section. However, the medical expenses X_i are different for each doctor's visit, which is much more realistic. Assume the medical expenses X_i follow an exponential distribution with mean θ, $\{X_i, i = 1, 2, ...\} \overset{i.i.d.}{\sim} \text{Exp}(\theta)$. The total profit B is:

$$B = \sum_{i=1}^{n-M} (\pi(d_1) - L_i(d_1)) + \sum_{j=1}^{M} (\pi(d_1) + d_1 - d_2 - I - L_j(d_2))$$

where $L_i(d_1) = \sum_{k=1}^{N(d_1)} X_k$, $L_j(d_2) = \sum_{k=1}^{N(d_2)} X_k$, $X_i \sim Exp(\theta)$, and $N(d)$ is the geometric over the exponential model. Therefore, we have $L(d) \sim Exp(\theta(1 + \beta p(d)))$. Assuming that the insurer has utility function u, the optimization problem can be formulated as maximizing the expected utility of the profit B under the aggregate loss model as follows:

$$\max_{(I,d_2)\in D} E\left[u\left(\sum_{i=1}^{n-M}(\pi(d_1)-L_i(d_1))+\sum_{j=1}^{M}(\pi(d_1)+d_1-d_2-I-L_j(d_2))\right]\right) \tag{3.8}$$

Proposition 3.3.1. *Assume the natural illness count N follows a geometric distribution with mean β. The medical expense X_i has an exponential distribution with mean θ. Set u to be an exponential utility function, $u(x)=1-e^{-\gamma_u x}$, and $\lambda(I)$ is the probability policyholders switch from the LDHP to the HDHP, $\lambda(I)=1-(1-\frac{I}{A})^{\alpha}$, where A is a constant number. Maximizing the expected utility is equivalent to maximizing the following:*

$$(1-(1-\frac{I}{A})^{\alpha})(1-\theta(1+\beta p(d_2))\gamma_u)e^{(d_1-d_2-I)\gamma_u}+(1-\frac{I}{A})^{\alpha}\triangleq g_4(I,d_2) \tag{3.9}$$

Proof. By Proposition 3.2.1, we know that maximizing the expected utility is equivalent to,

$$\max E[M_{L(d_1)}(\gamma_u)^{-n}]\cdot E[z^M], z=\frac{M_{L(d_1)}(\gamma_u)}{M_{L(d_2)-(d_1-d_2-I)}(\gamma_u)}$$

Here,

$$M_{L(d_1)}(\gamma_u)=\frac{1}{1-\theta(1+\beta p(d_1))\gamma_u}$$

Thus,

$$\begin{aligned}M_{L(d_2)-(d_1-d_2-I)}(\gamma_u)&=E[e^{\gamma_u L(d_2)}]e^{-(d_1-d_2-I)\gamma_u}\\&=\frac{e^{-(d_1-d_2-I)\gamma_u}}{1-\theta(1+\beta p(d_2))\gamma_u}\end{aligned}$$

Thus,

$$z=\frac{1-\theta(1+\beta p(d_2))\gamma_u}{1-\theta(1+\beta p(d_1))\gamma_u}e^{(d_1-d_2-I)\gamma_u}$$

Therefore,

$$\begin{aligned}E[z^M]&=E\left[\left(\frac{1-\theta(1+\beta p(d_2))\gamma_u}{1-\theta(1+\beta p(d_1))\gamma_u}e^{(d_1-d_2-I)\gamma_u}\right)^M\right]\\&=\left(\lambda(I)\frac{1-\theta(1+\beta p(d_2))\gamma_u}{1-\theta(1+\beta p(d_1))\gamma_u}e^{(d_1-d_2-I)\gamma_u}+1-\lambda(I)\right)^n\end{aligned}$$

Maximizing the expected utility is equivalent to

$$\underset{(I,d_2)\in D}{max}(1-(1-\frac{I}{A}))^{\alpha}\left[1-\theta(1+\beta p(d_2))\gamma_u\right]e^{(d_1-d_2-I)\gamma_u}+(1-\frac{I}{A})^{\alpha}$$

$\square$

Lemma 3.3.2. *If* $d_2 \leq -\frac{\ln\left[\frac{1-\theta\gamma_u(1+\beta p_\infty)}{\theta\beta(1-p_\infty)(\gamma_p+\gamma_u)}\right]}{\gamma_p}$, *$g_4(I,d_2)$ increases by d_2. Otherwise, $g_4(I,d_2)$ decreases by d_2, where $p(d_2)=p_\infty+(1-p_\infty)e^{-\gamma_p d_2}$.*

Proof. For $g_4(I,d_2)$ to be increasing in d_2, we need,

$$-\theta\beta p'(d_2)\gamma_u e^{(d_1-d_2-I)\gamma_u}+(1-\theta(1+\beta p(d_2))\gamma_u)e^{(d_1-d_2-I)\gamma_u}\cdot(-\gamma_u)>0$$

$$-\theta\beta p'(d_2)\gamma_u e^{(d_1-d_2-I)\gamma_u}>\gamma_u\left(1-\theta(1+\beta p(d_2))\gamma_u\right)e^{(d_1-d_2-I)\gamma_u}$$

$$-\theta\beta p'(d_2)>1-\theta(1+\beta p(d_2))\gamma_u$$

Since $p(d_2)=p_\infty+(1-p_\infty)e^{-\gamma_p d_2}$, thus $p'(d_2)=-\gamma_p(1-p_\infty)e^{-\gamma_p d_2}$. Therefore,

$$\theta\beta\gamma_p(1-p_\infty)e^{-\gamma_p d_2}>1-\theta\gamma_u\left[1+\beta(p_\infty+(1-p_\infty)e^{-\gamma_p d_2})\right]$$

$$\theta\beta\gamma_p(1-p_\infty)e^{-\gamma_p d_2}>1-\theta\gamma_u-\theta\beta\gamma_u\left(p_\infty+(1-p_\infty)e^{-\gamma_p d_2}\right)$$

$$\theta\beta(1-p_\infty)e^{-\gamma_p d_2}(\gamma_p+\gamma_u)>1-\theta\gamma_u(1+\beta p_\infty)$$

$$-\gamma_p d_2>\ln\left[\frac{1-\theta\gamma_u(1+\beta p_\infty)}{\theta\beta(1-p_\infty)(\gamma_p+\gamma_u)}\right]$$

$$d_2<-\frac{\ln\left[\frac{1-\theta\gamma_u(1+\beta p_\infty)}{\theta\beta(1-p_\infty)(\gamma_p+\gamma_u)}\right]}{\gamma_p}$$

We can conclude that If $d_2 \leq -\frac{\ln\left[\frac{1-\theta\gamma_u(1+\beta p_\infty)}{\theta\beta(1-p_\infty)(\gamma_p+\gamma_u)}\right]}{\gamma_p}$, $g_4(I,d_2)$ increases by d_2. Otherwise, $g_4(I,d_2)$ decreases by d_2 $\square$

Proposition 3.3.3. *Assume the natural illness count N has a geometric distribution with mean β. The medical express X_i follow an exponential distribution with mean θ. The proba-*

bility that the insured visits the doctor for each illness is $p(d) = p_\infty + (1-p_\infty)e^{-\gamma_p d}$. *Assume the insured is transferred from full insurance to high deductible insurance with deductible* d_2. *Under the assumption of* $\theta\beta(1-p_\infty) > 1$, *there are 2 roots of* $I_m(d_2)$.

Proof.

$$I_m(d_2) = E[L(d_1)] + d_1 - E[L(d_2)] - d_2$$

Since $N(d) \sim Geo(\beta p(d))$, $X_i \sim Exp(\theta)$, then $L(d) \sim Exp(\theta(1+\beta p(d)))$, therefore, $E[L(d)] = \theta(1+\beta p(d))$. $E[L(0)] = \theta(1+\beta p(0)) = \theta(1+\beta)$.

$$\begin{aligned} I_m(d_2) &= \theta(1+\beta) - \theta(1+\beta p(d_2)) - d_2 \\ &= \theta(1+\beta) - \theta\left(1+\beta(p_\infty + (1-p_\infty e^{-\gamma_p d_2}))\right) - d_2 \end{aligned}$$

$$\begin{aligned} I'_m(d_2) &= -\theta\beta(1-p_\infty)e^{-\gamma_p d_2}(-\gamma_p) - 1 \\ &= \theta\beta(1-p_\infty)\gamma_p e^{-\gamma_p d_2} - 1 \end{aligned}$$

Under the assumption $\theta\beta(1-p_\infty) > 1$, $I_m(d_2)$ first increases and then decreases on $[0,\infty)$ and eventually goes below zero. To find the roots of $I_m(d_2)$, set,

$$I_m(d_2) = 0$$

$$\theta\beta\left[1 - p_\infty - (1-p_\infty)e^{-\gamma_p d_2}\right] - d_2 = 0$$

$$d_2 = 0, d_2 = \theta\beta(1-p_\infty) + \frac{W[-\gamma_p\theta\beta(1-p_\infty)e^{-\gamma_p\theta\beta(1-p_\infty)}]}{\gamma_p} \triangleq d^0_{B_u}$$

where $W[z]$ gives the principal solution for w in $z = we^w$. □

Proposition 3.3.4. *The maximum expected utility of the profit,* $g_3(I, d_2)$, *over the interval* $I \leqslant I_m(d_2)$ *can be summarized by the following:*
if $d^*_g > d^*_B$, *then*

$$g_3(I, d_2) = \begin{cases} g(I, d^*_g), & I \leqslant I(d^*_g) \\ g(I, d^R_I), & I > I(d^*_g) \end{cases} \tag{3.10}$$

if $d_g^* < d_B^*$, *then*

$$g_3(I, d_2) = \begin{cases} g(I, d_g^*), & I \leqslant I(d_g^*) \\ g(I, d_I^L), & I > I(d_g^*) \end{cases} \tag{3.11}$$

3.4 Further investigation of $\lambda(I)$

At the beginning of this chapter, for convenience, the special form of the probability of policyholders switching from the LDHP to the HDHP is assumed to be $\lambda(I) = 1 - (1 - \frac{I}{A})^{\alpha}$. In this section, we introduce prospect theory to show that the function of this probability is increasing and concave by the incentive I.

From the insured's perspective, who is on a low-deductible health plan how does one plan? How can one decide whether to switch to the HDHP? Individuals benefit by switching to the HDHP; in addition to the lower insurance premium, they obtain incentives from employers. For a policyholder who purchases the LDHP with premium P_1 and deductible d_1, if she or he decides to switch to the HDHP, the premium is P_2, and the deductible is d_2, here $P_1 > P_2$, and $d_1 < d_2$. The policyholder also obtains incentive I by switching the HDHP, where $P_2 - P_1 = I + (d_2 - d_1)$.

In addition to the benefit, the policyholder needs to pay for the cost, for example, the cost of selection S_0. With the low deductible health plan, the policyholder is unlikely to consider the out of pocket cost, especially if covered under full insurance. With the high deductible health plan, the policyholder will consider the cost to see the doctor and the deductible before making the choice. At the beginning of the policy year, with the high deductible, the policyholder is not likely would like to see the doctor if they don't have to. For simplicity, we do not consider the cost of selection in this chapter. The other cost we consider is the cost of seeing the doctor. From Chapter 2, we assumed that the cost during each doctor's visit is a deterministic constant; moreover, we assume that the cost is the base unit 1. N is the random counts of natural illness of an insured person during the one-year policy year. The policyholder will not choose to visit the doctor for every illness; thus, assume N_1 is the annual number of doctor visits. Based on the benefit and the cost, the question is, how to make the decision, i.e., switch to the HDHP or stay with the LDHP.

The prospect theory (PT) is currently one of the most prominent descriptive theories of decision-making under risk. PT was developed by Kahneman and Tversky (2013) and Tversky and Kahneman (1992) to accommodate empirically observed violations of expected utility (EU). Prospect theory is an economic theory that tries to describe the way people will behave when given choices that involve probability. An essential feature of the prospect theory is that the carriers of value are changes in wealth or welfare, rather than final states. The value function v with $v(0) = 0$ is defined by gains and losses measured as deviations from the reference point and not by final wealth positions as the utility function in EU.

Empirical applications of PT often employ the following functional form of the value function proposed by Tversky and Kahneman (1992):

$$v(x) = \begin{cases} x^{\alpha}, & x \geq 0 \\ -(1+\gamma) \mid x \mid^{\alpha}, & x < 0 \end{cases} \tag{3.12}$$

where $\alpha < 1$ and $\gamma > 0$.

The policyholder prefers to switch from the LDHP to the HDHP when the value of the benefit and the value of the total cost together is at least larger than zero; otherwise, the policyholder prefer to stay with the LDHP. In other words, if at least $v(I + d_2 - d_1) + N_1 v(-1) > 0$, the policyholder is willing to switch. Some people decide to switch even though the value of the benefit approximately covers the value of the cost, for example, a young single healthy adult who has a low probability of seeing a doctor during the policy year. The policyholder believes during the one-year policy year that the total cost he or she spends is less than the deductible amount under the LDHP. Some people decide to stay with the LDHP even though the benefit is larger than the total cost; for example, an adult with a few children, which increases the number of doctor visits. Thus, we can classify the population with parameter Θ, and policyholders have a similar situation in the same group.

In Chapter 3, we assume the probability of switching from the LDHP to the HDHP is $\lambda(I)$, as we know one of the property of $\lambda(I)$ that it is increasing concave in I, so we want to demonstrate this situation with 2 different decision criteria.

Proposition 3.4.1. *From the perspective of probability, $P(v(I+d_2-d_1)+N_1 v(-1) > 0|\Theta) \triangleq h_1(\Theta; I)$; then $\lambda(I) = P(h_1(\Theta; I))$. For simplicity, only assume the natural illness count N follows a geometric distribution with mean Θ, and $\Theta \overset{pdf}{\sim} \pi(\theta)$. Assume $d_1 = 0$, then $\lambda(I)$ is increasing in I. However, we need more evidence to show $\lambda(I)$ is concave.*

Proof.

$$\begin{aligned} h_1(\Theta; I) &= P\left(N_1 < -\frac{v(I+d_2)}{v(-1)} |\Theta\right) \\ &= P\left(N_1 < \frac{v(I+d_2)}{1+\delta} |\Theta\right) \end{aligned}$$

Thus,

$$
\begin{aligned}
\lambda(I) &= P\left(P\left(N_1 < \frac{v(I+d_2)}{1+\delta}\right)\middle|\Theta\right) \\
&= \int \pi(\theta)\cdot P\left(N_1 < \frac{v(I+d_2)}{1+\delta}\middle|\Theta=\theta\right) d\theta \\
&= \int \pi(\theta)\cdot P\left(N_1 < \frac{(I+d_2)^2}{1+\delta}\middle|\Theta=\theta\right) d\theta
\end{aligned}
$$

Since $N \sim Geo(\Theta)$, by the Example 2.1.2, $N_1 \sim Geo(\Theta \cdot p(d_2))$, then the pmf is $P(k) = \left(\frac{\Theta p(d_2)}{\Theta p(d_2)+1}\right)^k \left(\frac{1}{1+\beta}\right)$. Thus,

$$\lambda(I) = \frac{1}{1+\beta}\int \pi(\theta)\left(\frac{\theta p(d_2)}{\theta p(d_2)+1}\right)^{\frac{(I+d_2)^2}{1+\delta}} d\theta$$

It is clear that $\lambda(I)$ is increasing in I, but we need more evidence to show that $\lambda(I)$ is concave. □

Proposition 3.4.2. *If f and g are both increasing and concave, then its composite $f \circ g$ is increasing concave.*

Proof. Since f and g are both increasing and concave, then $f' > 0$, $g' > 0$, $f'' < 0$, and $g'' < 0$.

$$(f \circ g)' = f'(g)\cdot g' > 0$$

$$(f \circ g)'' = f''(g)\cdot (g')^2 + f'(g)\cdot g'' \leq 0$$

Thus, $f \circ g$ is increasing concave. □

Proposition 3.4.3. *If $E[N|\Theta]$ is an increasing function of Θ, then $\lambda(I)$ is increasing in I. Furthermore, if $E[N|\Theta]$ is a linear function of Θ and $F_\Theta(\theta) = P(\Theta \leqslant \theta)$ is concave, then $\lambda(I)$ is concave.*

Proof.

$$E[v(I+d_2) + N_1 \cdot v(-1)|\Theta] > 0$$

$$\Leftrightarrow v(I+d_2) + E[N_1 \cdot v(-1)|\Theta] > 0$$

$$\Leftrightarrow v(I+d_2) + v(-1) \cdot E[N_1|\Theta] > 0$$

$$\Leftrightarrow E[N_1|\Theta] < -\frac{v(I+d_2)}{v(-1)}$$

$$\Leftrightarrow E[N_1|\Theta] < \frac{v(I+d_2)}{1+\delta}$$

$$\Leftrightarrow \Theta p(d_2) < \frac{v(I+d_2)}{1+\delta}$$

$$\Leftrightarrow \Theta < \frac{v(I+d_2)}{(1+\delta)p(d_2)}$$

Thus,

$$\begin{aligned} \lambda(I) &= P\left(\Theta < \frac{v(I+d_2)}{(1+\delta)p(d_2)}\right) \\ &= F_\Theta\left(\frac{v(I+d_2)}{(1+\delta)p(d_2)}\right) \\ &= F_\Theta\left(\frac{(I+d_2)^2}{(1+\delta)p(d_2)}\right) \end{aligned}$$

Since $v(I+d_2)$ is increasing concave by the property of the value function, as we know $F_\Theta(\theta)$ is the increasing function, for $\lambda(I)$ to be increasing concave, we need $F_\Theta(\theta)$ to be concave by the Proposition 3.4.2.

□

Chapter 4

Conclusion and future work

4.1 Conclusion

In this thesis, we analyzed optimal health insurance designs from the perspectives of both the insured and insurer. In the context of health insurance, it is critical to properly treat moral hazard, which refers to the phenomenon that lowering the marginal cost of care to individuals may result in an increase in the use of health care. While the majority of the literature features only on qualitative or empirical studies of moral hazard, we conduct a quantitative analysis in this thesis, with the aim of not only establishing some theoretical models to quantify moral hazard, but also finding the optimal design of health insurance for both the insurer and the insured.

In Chapter 2, we first formulate the quasi-arbitrage condition. When policyholders switch from the LDHP to the HDHP, the decrease in the premium is steeper than the decrease in the insurance coverage, seemingly creating an arbitrage opportunity. However, a closer investigation into this result revealed the assumption that the underlying risk does not change when the policyholder switches from the LDHP to the HDHP, which does not align with reality. In this regard, quasi - arbitrage is not real arbitrage. Instead, it is a reasonable design to mitigate moral hazard. We then investigate specific models, namely, aggregate and non-aggregate models with binomial, Poisson, geometric frequencies, and derive sufficient conditions for the quasi - arbitrage condition. These results are summarized as below.

Table 4.1: Summary of results with non-aggregate models

Frequency of illness	Condition
$N \sim Poisson(\lambda)$	$-\lambda(1-p_\infty) - 1 \leqslant -\frac{1}{P(N(\lambda p_\infty)>d)}$
$N \sim Bin(n,q)$	$(nqp'(d) - 1) \leqslant -\frac{1}{P(N'(q')>d)}$, where $N' \sim Bin(n-1,q)$
$N \sim Geo(\beta)$	special case: (i) if $\beta \to \infty$, $\pi'(d) \leqslant -1$ (ii) if $d = 0$, $\beta > \beta_0$, $\pi'(d) \leqslant -1$

Table 4.2: Summary of results with aggregate models

Frequency of illness	Condition
$N \sim Poisson(\lambda)$	$E[X]\lambda p'(d) - 1 \leqslant -\frac{1}{P(\sum_{i=1}^{N(\lambda)} X_i>d)}$
$N \sim Bin(n,q)$	$E[X]nqp'(d) - 1 \leqslant -\frac{1}{P(\sum_{i=1}^{N'} X_i>d)}$, where $N' \sim Bin(n-1,q)$
$N \sim Geo(\beta)$	special case: (i) if $\beta \to \infty$, $\pi'(d) \leqslant -1$ (ii) if $d = 0$, for all β, $\pi'(d) \leqslant -1$

The results indicate that for the policyholder who gets sick more frequently, the policyholder has a larger chance to satisfy the quasi-arbitrage condition. In reality, the insured with an LDHP (set to a low deductible such as zero in this thesis) does not hesitate to visit the doctor during each illness because of no cost. However, the insured with the HDHP needs to consider the doctor's visiting cost and the deductible before making a choice. The number of increases in illness would decrease the remaining deductible in the policy, which affects the policyholder's decision. The probability that the insured decides to visit the doctor increases as the remaining deductible decreases. Therefore, we can conclude that the insured with an HDHP who get sick saves more money in premiums than the maximum possible loss in coverage.

In Chapter 3, optimization analysis is conducted from the insurer's perspective. We assumed that the probability that the insured decides to visit the doctor during each illness is a special function of the deductible. We also assumed that the probability that the insured decides to switch from the LDHP to the HDHP is also a special function of the incentive. We derived the level to set the incentive to maximize the expected profit and the expected utility. We found that the outcomes are similar whether the policyholder's medical cost is constant or random. The follow-up question concerns the insured's behavior. For example, at the same incentive level, one may choose to transfer from the LDHP to the HDHP, and one may not. Thus, We introduced prospect theory aimed at explicitly combining irrational behaviors in a more realistic manner. Even though, policyholders make decisions based on different incentives, we can classify the policyholders have the similar situation into the same group. Then, we could let policyholders in the same group make the same decision of whether to switch from the LDHP to the HDHP or not. With this idea, it is important to look back and analyze this probability, which was presented in Section 3.4. Overall, prospect theory incorporates policyholders' behaviors that are commonly used in decision-making. However, it is far more complicated to implement in practice.

4.2 Future work

In this thesis, we explained that the quasi-arbitrage condition is a reasonable design to mitigate moral hazard. What happens if quasi-arbitrage is not satisfied? It turns out to be another issue known as self-selection. For auto insurance, the insurer would achieve homogeneity by evaluating the risk status of individuals. However, for health insurance, insured individuals self-select and identify their own optimal plans; decisions are made considering individual risk attitudes toward a wide range of behavioral patterns and economic outcomes. It is also difficult to measure and verify individuals' health status; thus, homogeneity is unlikely to be achieved. Therefore, further research is needed to determine the effects of self-selection. In Chapter 2, we found that the expectation of the policyholder's number of illnesses per year is the most important factor in achieving the conditions of quasi-arbitrage. Can we obtain the same result if a more general frequency is considered? An example is the copay rate. These questions that arise from Chapter 2 can motivate future work.

In Chapter 3, we first fixed the amount of incentive to find the maximum expected profit and expected utility for the insured and then optimized the incentive based on the properties of the maximum incentive. The deductible is not the only factor that impacts the insurer's profit; the copay and coinsurance are also worth including in the sensitivity analysis. We would expect to find more general cases in different health insurance models. Prospect theory has been found to explain risk taking in different scenarios, which means that different policyholders could make different decisions depending on how situations are framed and how risks are perceived. Therefore, grouping policyholders who have the same risk attitudes then comparing risk sensitivity with respect to age, health, wealth, family or other factors will be the subject of our subsequent research. For example, healthy and unhealthy behaviors could be influenced. Furthermore, the risk preferences are not consistent over time; they vary depending on the individual and the situation. After rebuilding risk preferences, the policyholder might make a different decision in the next policy year, and risk attitudes could also make risk a multidimensional concept, which might vary with many different factors. These considerations are among the topics to consider in future research.

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