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Applying the Efficiency Gap to Wisconsin Politics

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ABSTRACT

APPLYING THE EFFICIENCY GAP TO WISCONSIN POLITICS

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Gerrymandering is a plague on modern democracy, blatantly violating the democratic principle of "one person, one vote." Here we will methodically examine the 2018 Wisconsin state assembly election, and using a metric known as the efficiency gap demonstrate the extent to which gerrymandering played a role. Through this metric, and a probabilistic simulation of our own, we will show that in this election the Republican party benefited from systematic partisan gerrymandering. Additionally, we will use these findings to suggest methods for correcting this undemocratic practice that both parties utilize in order to disenfranchise opposition voters.

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1 Introduction

The drawing of congressional district lines has long been a topic of heated debate at both the state and national level. At face value how district lines are drawn may seem unimportant as it would appear that a higher proportion of votes would always correspond to a higher seat share. However, this is not always the case due to a political tactic known as gerrymandering. Gerrymandering is defined as the manipulating of district boundaries in such a way to maximally benefit one's political party, even if this comes at the cost of extreme voter disenfranchisement.

1.1 Background on Gerrymandering

There are several ways to effectively gerrymander to a partisan advantage, but the two most popular methods are known as dispersal and concentration gerrymanders, sometimes called "cracking" and "packing." The idea of a dispersal gerrymander is to draw district lines in such a way to spread the opposing party's voters across districts so that they get close, but fail to get 50% of the vote in as many districts as possible. Dispersal gerrymandering can come with substantial risk for the party doing the gerrymandering as a single bad election cycle could lead to a landslide victory for the opposition. The other primary, and perhaps most popular and well known form of gerrymandering is the concentration gerrymander. A good concentration gerrymander concentrates as many opposition votes as possible into as few districts as possible. The opposition party will easily win these districts by huge margins. However, this gives the party responsible for the gerrymander the opportunity to sweep up the leftover districts, by solid, albeit not as enormous margins. Between these methods of gerrymandering the party responsible for drawing district boundaries can often gain a disproportionate number of seats in an election.

1.2 A Brief History of Gerrymandering and Related Court Cases

Gerrymandering is not a modern concept by any means. The term was originally coined in 1812 when the *Boston Gazette* ran a political cartoon titled “The Gerrymander” which portrayed a misshapen Massachusetts voting district as a lizard-like creature. The political cartoon criticized a recent redistricting that had been approved by Massachusetts Governor Elbridge Gerry that seemingly strongly benefitted the Jeffersonian Republican party, of which Gerry was a member. As the 20th century rolled around, redistricting continued to be a topic of not only political debate, but of legal debate as well. In 1962, the landmark Supreme Court case of *Baker v. Carr* ruled that Federal courts had the jurisdiction to rule on the constitutionality of state legislative redistricting plans, which opened the door to numerous Supreme Court cases. In 1964 another landmark Supreme Court case, *Reynolds v. Sims*, determined that both houses in a bicameral state legislature must be apportioned by population, that is, it became federally required for state legislative districts to have approximately equal populations. Further cases would more concretely establish how much state legislative districts could vary in population. Although federal courts still have the ability to rule on the constitutionality of a state’s redistricting plan they cannot do so on the basis of partisanship, as the 2004 Supreme Court case of *Vieth v. Jubelirer* ruled that partisan gerrymandering cases should be kept out of the federal courts. This ruling was confirmed in the 2019 case of *Rucho v. Common Cause*. This means that currently federal courts have no jurisdiction over cases of partisan gerrymandering, forcing any legal challenges to redistrictings to be settled at the state level.

1.3 Purpose of Research

Although the case that will be studied in the coming sections is an example of Republicans gerrymandering against Democrats, it should be noted that this research is not an attempt to attack the Republican party. Rather, our criticisms will be aimed at the practice of gerrymandering itself, a practice that both major parties utilize. Additionally, we will keep our analysis and criticisms as objective as possible. We will examine possible methods for quantifying partisan gerrymandering, before selecting the efficiency gap as a method to apply in our case study of the 2018 Wisconsin State Assembly election. Using this method and a simulation of our own we will attempt to examine the extent to which gerrymandering impacted this election, and allowed certain votes to be worth more than others.

2 Analyzing Gerrymandering

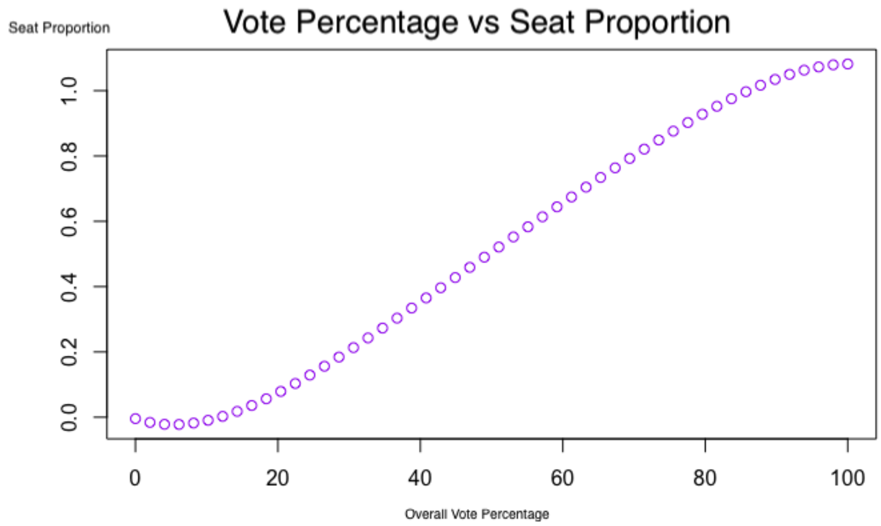
Two big questions that are often asked with regard to gerrymandering are “What makes a redistricting fair with regard to partisanship?” and “How do we quantify the extent to which gerrymandering impacted an election?” Certain organizations have already done extensive research on the question of fairness. One example is Princeton’s redistricting report card, which grades every map on an A-F scale in terms of partisan fairness before it is even used. The report card does this by taking into account the shape of districts, and the projected party vote share in certain regions of a given state. Our research will be primarily focused on answering the second question, retroactively focusing on how partisan gerrymandering impacts elections through a case study of Wisconsin politics. However, we will first provide some of our own insights into what makes a redistricting fair.

2.1 The Problem with Expecting Proportional Representation

One seemingly obvious answer to the question of fairness is to simply expect maps to produce proportional representation. If proportional representation does not occur then we can compare the proportion of seats won to the proportion of votes received and examine the extent to which these numbers differ in order to quantify a gerrymander. A higher difference would suggest substantial gerrymandering is present, and a lower difference would suggest little to no gerrymandering is present. While the general idea of this concept may be somewhat correct, proportional representation is an unrealistic expectation regardless of whether gerrymandering is present or not. Professor Jordan Ellenberg notes in a chapter of his book “Shape: The Hidden Geometry of Information, Biology, Strategy, Democracy, and Everything Else ” that when any given state’s population is two thirds or more in favor of one political party then it is likely that any reasonably sized geographic chunk of that state will favor that political party, and thus it becomes increasingly difficult for the minority party to gain any representation (Ellenberg 373). For example, West Virginia has two congressional districts and consistently votes for the Republican party at a popular vote percentage of around 65% to 70% across U.S. house elections. It would be almost impossible to draw a map in such a way to yield one Republican and one Democratic representative, even though at the state level this is closer to true proportional representation than both districts simply going to Republican candidates.

2.2 Simulating Expected Representation

We can further demonstrate that proportional representation does not occur naturally using a relatively straightforward simulation that requires no geographical coding. Assume an arbitrary state has two parties, A and B, and five congressional districts. Using a random number generator we assume that in any given district election all vote outcomes between 0% and 100% are equally likely, for example, Party A receiving 95% of the vote is just as likely as Party A receiving 10% of the vote in a given district, where Party B receives the entirety of the leftover vote share in any case. In any given district the party receiving more than 50% of the vote wins the district. We can repeat this for all 5 districts and compare the number of districts won by Party A to the total proportion of votes received by Party A across all five districts. We then run this simulation over and over again thousands of more times, each simulation giving us a single data point where x represents the percent of votes won by Party A, and y represents the proportion of districts won by Party A. Plotting all data points and then fitting a curve yields the following graph:



(F.1)

We see that when Party A receives 50% of the total popular vote, it expects to win approximately 2.5 out of 5 districts (50%). This is not particularly surprising. However as Party A's popular vote ticks up it tends to overperform in the number of districts it expects to win and conversely, as its popular vote goes down it expects to lose a disproportionate number of districts. For example, when Party A wins 70% of the popular vote in an election, on average it wins about 4 out of 5 districts (80%), alternatively when party A receives 30% of the popular vote, it wins an average of about only 1 out of 5 districts (20%). This phenomenon is fairly well known and is sometimes referred to as a "winners bonus," as the party receiving the higher proportion of the popular vote should naturally overperform in the number of districts it wins. Thus, under truly random conditions with no bias at all, proportional representation does not occur naturally and therefore it is a questionable metric when it comes to quantifying gerrymandering and an unreasonable standard to hold redistricting plans to. If simply examining the difference between proportional representation and actual representation isn't effective, then how can we more effectively quantify and examine cases of extreme gerrymandering?

3 The Efficiency Gap

Developed in 2014 by Professor Nicholas Stephanopoulos and fellow researcher Eric McGhee in their paper "Partisan Gerrymandering and the Efficiency Gap," the efficiency gap is a relatively new method of quantifying gerrymandering. The key idea behind the efficiency gap is to quantify how well a party converted its vote share into seats. The efficiency gap does this by examining wasted votes. Using the efficiency gap in a two-party system, wasted votes are defined as any votes cast for the losing candidate in a given district, as well as any votes in excess of 50% of the overall vote that the winning candidate receives (McGhee and Stephanopoulos 14-15). Thus in one fell swoop the efficiency gap is able to cover situations of both concentration and dispersal gerrymanders simultaneously, as in both of these types of gerrymandering the goal is to waste fewer votes than the opposing party. Furthermore, the efficiency gap can be calculated regardless of a region's partisan bias. The efficiency gap in a given election is calculated using the below formula (I.1), and represents an expected percentage of seats gained by the party with the advantage relative to if the efficiency gap had been 0.

$$\textit{Efficiency Gap} = 100(\textit{Wasted Votes Party1} - \textit{Wasted Votes Party2})/(\textit{Total votes}) \quad \text{(I.1)}$$

McGhee and Stephanopoulos note that, perhaps ironically, the above formula can be inefficient as it requires examining each individual district in a given election and summing wasted votes. However, through some algebraic manipulation (see appendix) and the assumption that all districts are equal in number of voters (legally, every district is required to have approximately the same population) the above equation can be converted to:

$$\textit{Efficiency Gap} = \textit{Seat Margin} - 2(\textit{Vote Margin}) \quad \text{(I.2)}$$

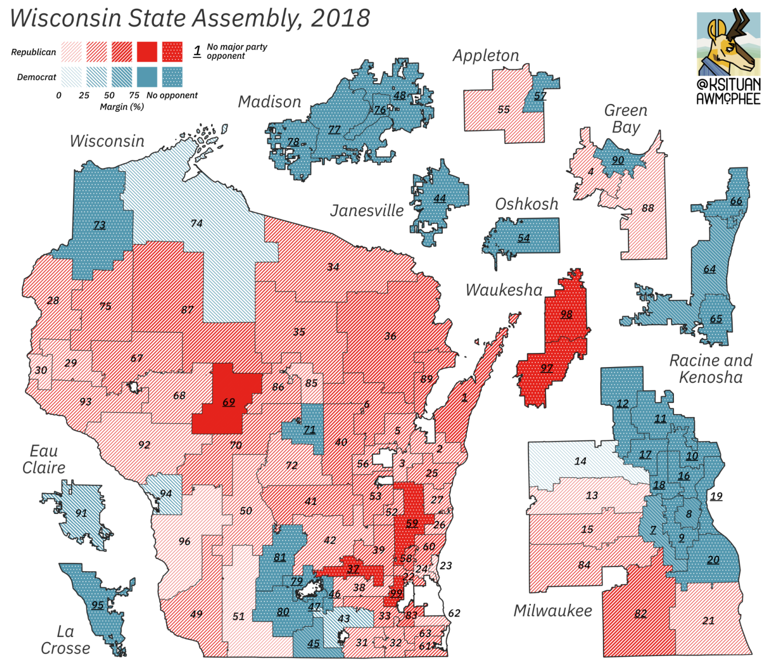
One limitation that equation (I.2) does not account for is the fact that despite all districts having approximately equal population, not all districts will necessarily have the same number of voters in any given election, and therefore we must be careful when generalizing wasted votes using this formula. Just one example of this phenomenon would be Wisconsin's 2022 U.S. House elections. In these elections, Wisconsin's 2nd congressional district had over 125,000 more voters than Wisconsin's 4th congressional district. We will address why districts might vary so much in voting populations in the next section, and discuss one method of accounting for this as we do our calculations. While the focus of this paper is not on the legal validity of the efficiency gap as a metric, it should be noted that McGhee and Stephanopoulos argue that at the state level, an efficiency gap in excess of 8% in favor of either party should be considered the legal threshold when determining the constitutionality of a legislative plan. The McGhee and Stephanopoulos paper examines efficiency gaps for congressional and state house plans between 1987 and 2012. Here, we will now examine a more recent election from 2018, calculate the efficiency gap, and then attempt to replicate the results using a simulation of our own.

4 Calculating the Efficiency Gap: A Case Study

Wisconsin is widely considered to be one of the most centrist states, voting for the Democratic presidential candidate in 2012 and 2020, and swinging for the Republican presidential candidate in 2016. Furthermore, since the 2010 election, Wisconsin's U.S. Senate delegation has consisted of one Republican and one Democrat. Despite Wisconsin being a seemingly middle-of-the-road state politically, the Wisconsin State Assembly has been consistently and dominantly controlled by the Republican party over the last decade, and this is no coincidence. Redistricting occurs every 10 years in the year following a census. Maps are drawn by the state legislature and then subject to Governor approval. In 2011, redistricting occurred in Wisconsin, the state assembly map that was drawn by the Republican-controlled legislature and ultimately approved by Republican Governor Scott Walker would become the modern gold standard for partisan gerrymandering, the effects of which can most clearly be seen in the 2018 Wisconsin State Assembly election.

4.1 The 2018 Elections in Wisconsin

On election night 2018, the Democrats flipped the Wisconsin governor's office defeating two-term Republican incumbent Scott Walker. Additionally the Democrats swept the other statewide offices earning victories in the Secretary of State, Attorney General, and Treasurer's elections. Furthermore, they won the overall popular vote for state assembly by approximately eight percentage points or about 200,000 total votes. Despite the seemingly incredible night for the Democratic party, there was one huge silver lining for the Republicans. Despite massive popular vote losses, they retained a 63-36 majority in the state assembly, losing only one seat from the previous cycle. Although Republicans had won less than 45% of the total state assembly popular vote, they had won almost 64% of the seats, thus maintaining a firm grasp on the lower house of the state legislature. Below a map of the 2018 Wisconsin State Assembly election can be seen along with the party vote margins.



(F.2)

There are a couple of notable things about the above map. First, although the focus of this paper is not on the geometry of the districts, it should be noted that many districts in this map appear to be misshapen, with unnatural or illogical boundaries. Second, nearly every district won by the Democratic party in this election appears in dark blue on the above map. Meaning that when the Democrats won a district, it would seem that they generally did so by large margins, which does not appear to be the case for the Republican party in this election. In order to more objectively demonstrate the extent to which gerrymandering impacted the 2018 Wisconsin State Assembly election we will calculate the efficiency gap for this election in three different ways. First we will use the theoretical equation (I.2), then we will use the standard equation (I.1) by running code that sums wasted votes in every district. Finally we will take recommendations from the McGhee and Stephanopoulos paper and manipulate the data to attempt to produce a best estimate for what the efficiency gap would have been had uncontested races been accounted for.

4.2 The Theoretical Calculation

We will start with a brief theoretical calculation of the efficiency gap for the 2018 Wisconsin State Assembly election (which used the map drawn in 2010) using the simplified efficiency gap equation (I.2). Although hypothetically the theoretical equation (I.2) should produce the same result as equation (I.1), there are some real-world constraints that prevent this from being true in actuality. As previously stated, although all districts are legally required to have approximately the same population, the districts still vary greatly in voting population. The varying voting populations from district to district could be due to a number of factors such as the perceived competitiveness of the election, poverty rate in the district, education level of the district, etc. Thus this calculation can only be used as an initial estimation for the efficiency gap in the 2018 Wisconsin State Assembly election. In order to keep this calculation simple, we will only look at the proportion of votes each party received out of votes cast for one of the two major parties. In other words, we will ignore third-party voters. In the 2018 Wisconsin State Assembly election the Republican party won 63.6% of the seats (63/99) but only 45.8% of the two party popular vote. Using the second equation for efficiency gap we have:

$$\textit{Efficiency Gap WI Assembly 2018} = \textit{Seat Margin} - 2(\textit{Vote Margin})$$

$$= (63.6\% - 50\%) - 2(45.8\%-50\%)$$

$$= 13.6\% - 2(-4.2\%)$$

$$= 13.6\% + 8.4\%$$

$$= 22\% \text{ (In favor of the Republican party)}$$

Thus the theoretical equation estimates an efficiency gap of 22% in favor of the Republican party, much larger than the 8% threshold for legal significance stated by McGhee and Stephanopoulos.

4.3 Summing Wasted Votes

Now that we have a baseline estimation of a gap of 22% in favor of the Republican party, we can attempt to acquire a more exact value for the efficiency gap by using equation (I.1). This can be done by summing the total number of wasted votes in every district for each respective party and dividing by the total number of votes for the two parties in the election. Remember, wasted votes are considered to be any votes for the losing candidate in a district, and any votes for the winning candidate in excess of 50% of the overall vote in the district. It should be noted that the number of wasted votes in a district is always equal to half of the total turnout in that district. In order to calculate the efficiency gap in the 2018 Wisconsin State Assembly election we imported a table (see appendix) containing the total votes for the Democratic and Republican parties in each of the 99 districts. Using the Rstudio software we can quickly calculate the total number of wasted votes for each party across all districts (see code section). The totals come out to approximately 923,000 wasted Democrat votes and 292,000 wasted Republican votes. Summing total two party votes (About 2,430,000 votes between the two parties) and using equation (I.1) this corresponds to an efficiency gap of approximately 25.9% in favor of the Republican party, even more extreme than the efficiency gap found in the theoretical calculation.

4.4 Adjusting for Uncontested Races

One factor that greatly impacts the number of wasted votes in a given election and thus impacts the efficiency gap as well, is uncontested races. In the 2018 Wisconsin State Assembly election thirty Democrats and six Republicans ran uncontested. Uncontested races are not so simple to account for when it comes to calculating the efficiency gap as we must take into account voters who would've voted had there been a candidate of their preferred party to vote for. For example, if the Republican party had run a candidate in each of the thirty uncontested districts carried by Democrats, those candidates would have received some minimum number of votes, all of which would be wasted. This would increase the Republican party's wasted votes and simultaneously decrease the Democratic party's wasted votes as more votes would suddenly become necessary to win the district. McGhee and Stephanopoulos address this exact issue, and their simplest solution is to assume each party would have received some minimum percentage of votes had the election been contested, giving an example value of 25%.

Let's reapply equation (I.1) to the 2018 Wisconsin State Assembly data, but this time we'll assume the losing party received 25% of the vote in all uncontested races. Overall we expect this will substantially decrease the number of wasted Democratic votes, and substantially increase the wasted Republican votes, as well as increasing the overall voters in the election. Running our code in Rstudio using the adjusted table, we find the totals come out to approximately 863,000 wasted Democratic votes, and 477,000 wasted Republican votes, with a total voting population of approximately 2,680,000. This corresponds to an efficiency gap of approximately 14.4% in favor of the Republican party. This is obviously still substantial, but not nearly as large as the 22% found in the theoretical calculation or the 25.9% found in the pure data calculation. What does this 14.4% figure actually represent? In simple terms it means that the Republican party won an expected 14.4% more seats (about 14) than they would have if the election had an efficiency gap of 0. That is, this metric would predict that the Republicans won 14 "underserved" seats. We will consider the 14.4% value to be our official efficiency gap value for the 2018 Wisconsin State Assembly election and use it in the next section as we attempt to simulate elections.

5 Simulating The Efficiency Gap

In the previous section we estimated the efficiency gap for the 2018 Wisconsin State Assembly election to be 14.4% in favor of the Republican party. Logically, the next question that must be asked is “How significant is this?” and “What are the chances of this occurring randomly?” In order to answer these questions we will run a couple of our own simulations, keeping in mind that McGhee and Stephanopoulos argue in their paper that for state house plans, an efficiency gap of 8% in favor of either party should be the threshold for determining the constitutionality of partisan gerrymandering. In order to further examine the significance of a 14.4% efficiency gap, we will attempt to determine the probability of an efficiency gap in excess of 14.4% in favor of either party occurring by random chance. We will do this by running a simulation that samples district vote percentages from a uniform distribution and then calculates efficiency gap values from those percentages.

5.1 Methodology

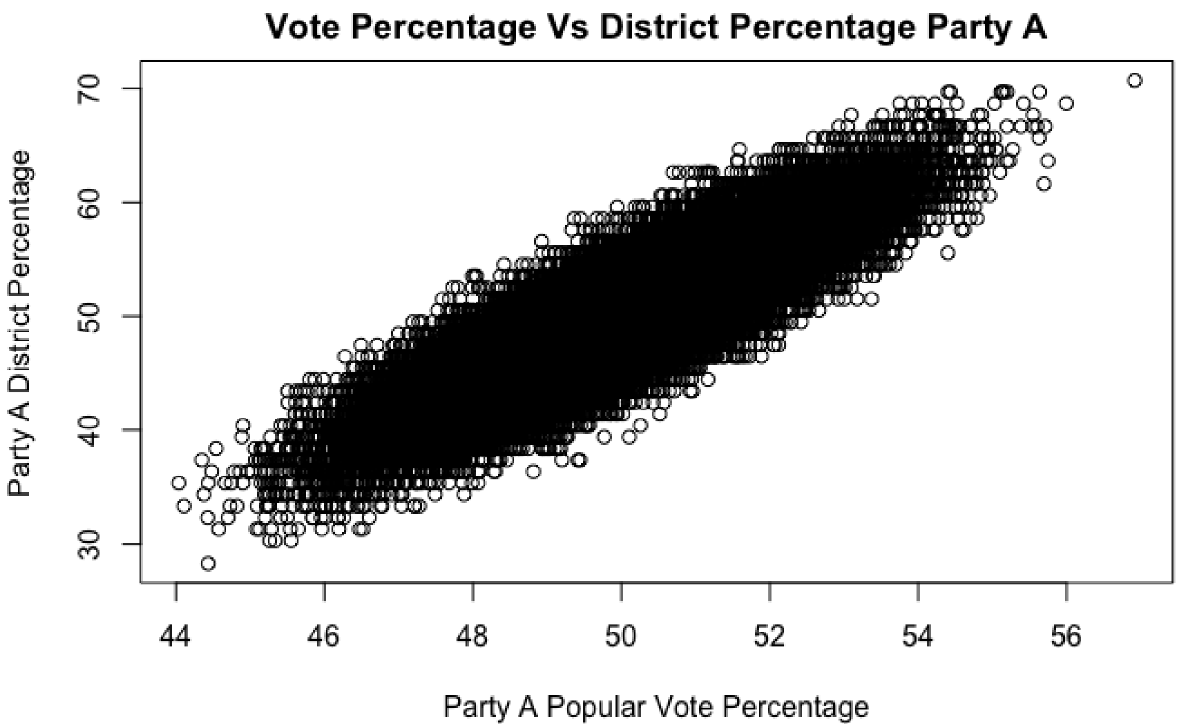
Let’s go through the methods that will be used to construct this simulation. We will only be looking at a two-party race, calling the parties A and B. In each simulation we will randomly sample 99 numbers from a uniform(25,75) distribution, this will be done using the random vector function in Rstudio. Each of these random numbers will represent Party A’s vote percentage in a single district in a 99-district election, with Party B receiving all of the leftover vote. If party A receives greater than 50% of the vote in a district, it will be counted as having won that district, if it fails to get 50%, that district would go to Party B. For each time the simulation is run, we will sum up the total percentage of the overall vote Party A received and the number of districts Party A won, then using equation (I.2) we can attribute an efficiency gap score to that single simulation. Using a loop, we can repeat this simulation thousands of times and examine the distribution of efficiency gap scores in order to determine the probability of an efficiency gap in excess of 14.4% occurring randomly under these conditions.

5.2 Assumptions

The above methodology makes a number of assumptions that should be addressed. First and foremost, there is a major assumption being made that all possible vote outcomes are equally likely as they are randomly selected from a uniform distribution. That is, a 70%-30% election in favor of Party A in a district is just as likely as a 52%-48% victory for Party B in that same district in any given running of the simulation. Second, we of course are assuming that we only have two parties in the race, A and B, i.e. no third party candidates. Thus any vote proportion that Party A doesn't receive goes directly to Party B. In the 2018 Wisconsin State Assembly election, third-party candidates altogether received approximately 2% of the overall vote, not substantial enough to have much effect. Third, we assume that support for each party is approximately equal across the state, as the uniform distribution is centered at 50. Thus while individual district elections could lean one way or another, across thousands of elections we would expect that support for parties would be approximately equal, this is not an unreasonable assumption given the general centrism of Wisconsin politics. Fourth, we are assuming that neither party can receive less than 25% nor greater than 75% of the vote in a single district. This assumption relates back to the idea of uncontested seats, as we are assuming some minimum vote share for each party in any district election, even if the election isn't competitive. Finally, since we are using the second equation for efficiency gap, the key assumption being made is that all districts are weighted equally in terms of the number of voters, i.e. every district has the same number of voters. As previously stated, in theory we would expect all districts to have the same number of voters as they all have the same population, but this doesn't end up being true in reality.

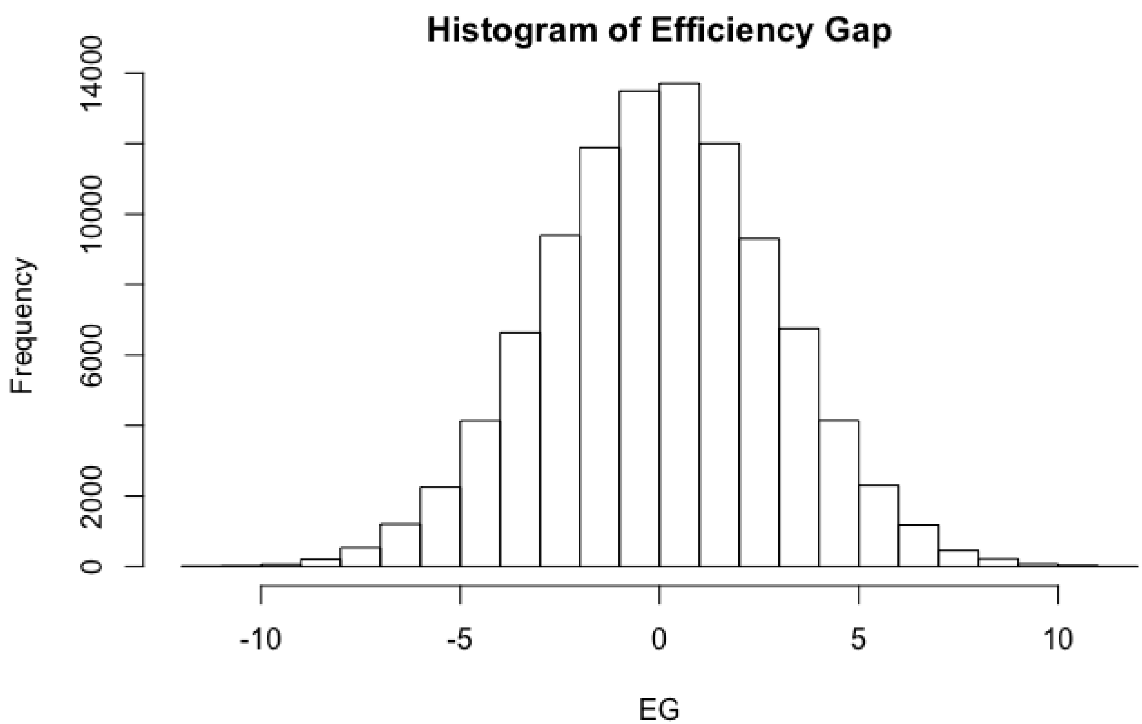
5.3 Results

Below is a graph showing 100,000 election simulations run under the conditions above, with each point representing a single election. The x coordinate shows the overall popular vote percentage Party A received in a given election, and the y coordinate shows the percentage of the 99 districts that Party A won in that election.



(F.3)

Unsurprisingly there is a clear positive correlation between the percentage of votes party A received in each election and the percentage of the 99 districts it won. However, there is also clearly a substantial amount of variability. There are plenty of cases in this simulation where party A received more than 50% of the overall vote but failed to win a majority of districts, and conversely there are plenty of cases where party A failed to win 50% of the overall vote but still won a majority of the seats. Could this mean that the Democrats were simply unlucky in the 2018 Wisconsin State Assembly election? In order to further understand the variability of this simulation, we can create a histogram of the efficiency gaps produced by each election and examine the summary statistics to get a sense of the distribution. Then we can attempt to determine what a "reasonable" deviation in efficiency gap would be given the conditions of the simulation.



(F.4)

Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum	Standard Deviation
-11.73	-1.95	0.01	0	1.95	11.48	2.91

(F.5)

This histogram and its summary statistics provides a number of interesting insights. First, approximately half of the simulations produced an efficiency gap in the interval +/- 1.95%. Second, across the 100,000 simulations the very largest efficiency gap produced was 11.73% in favor of Party B. None of the 100,000 random simulations managed to produce an efficiency gap as high as the 14.4% we calculated for the 2018 Wisconsin State Assembly election. In order to determine the odds of an efficiency gap of 14.4% in favor of either party occurring randomly under the conditions above, we will perform a two tailed hypothesis test. We have a mean (0) and a standard deviation (2.91), running the skewness and kurtosis functions for the simulated efficiency gap data in Rstudio we confirm that the data is approximately normally distributed (as we would expect it to be under the Central Limit Theorem). Our efficiency gap value for the 2018 Wisconsin State Assembly election is 14.4%. This is 4.95 standard deviations away from the mean. For normally distributed data, the probability of a data point falling more than 4.95 standard deviations from the mean is approximately 0.0000007, or 7 in 10 million, which is statistically significant at all reasonable confidence levels. Additionally, we can construct an interval for which we would expect 99% of simulated data points to fall. The two tailed 99% confidence interval for normally distributed data is +/- 2.58 standard deviations from the mean, which in the case of our simulation corresponds to an efficiency gap range of +/- 7.51%, notably similar to the 8% value which McGhee and Stephanopoulos regard as the legal threshold in determining the constitutional validity of a given state legislative map.

6 Limitations

While our simulation suggests that the Republican party almost certainly benefited from gerrymandering in the 2018 Wisconsin State Assembly election, we must be careful when making broad assumptions due to a few potential limitations with regard to our simulation, and the efficiency gap as a calculation in and of itself.

First, because our simulation is based simply in random numbers it fails to take into account certain geographical and demographic factors surrounding gerrymandering, primarily the fact that Democrats tend to be concentrated in cities. Studies and simulations have shown that the urban concentration of Democrats tends to inherently put them at a disadvantage when it comes to redistricting. In the research paper "Unintentional Gerrymandering: Political Geography and Electoral Bias in Legislatures" authors Jowei Chen and Jonathon Rodden state that because of inefficient geographical concentration "[Democrats] can expect to win fewer than 50% of the seats when they win 50% of the votes" (Chen and Rodden 239). Thus it would seem that generally, before any gerrymandering even occurs, it is difficult to draw a "fair" plan that doesn't immediately tilt the efficiency gap against the Democratic party, through no fault or intent of the Republicans.

Additionally, the efficiency gap as a metric has been criticized extensively. In their paper "A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap" Mira Bernstein and Moon Duchin provide a number of examples demonstrating flaws in the efficiency gap as a metric. Among their most compelling points, Duchin and Bernstein note that the efficiency gap can be extremely volatile in competitive elections, stating "If, for instance, all districts are competitive but a last-minute trend pushes voters to one side systematically, then the plan itself will be rated as a gerrymander" (Bernstein and Duchin 1022). Thus it is entirely possible for even a well-drawn plan to be penalized under the efficiency gap simply due to external factors. In the 2018 Wisconsin State Assembly election, although the overall popular vote was close between the parties, very few individual races were actually competitive. In fact, only 5 of the 99 races in the 2018 Wisconsin State Assembly Election were decided by less than 4 percentage points, making this point about the volatility not particularly applicable in this specific election. In another criticism, Duchin and Bernstein note that the efficiency gap doesn't work particularly well in areas with few legislative districts, noting "for a particular voting split, a small state may have no outcome at all with a permissibly small efficiency gap" (Duchin and Bernstein 1022). Duchin and Bernstein attribute this to the non-granularity of the efficiency gap as a metric. This also isn't a particularly large problem in our example due to the fact that Wisconsin has 99 state assembly districts, which should leave room for plenty of acceptable outcomes.

7 Conclusion

Using Wisconsin's 2018 State Assembly election as a case study, this research has shown the scope to which gerrymandering can influence elections and in turn undermine our democracy. Using McGhee and Stephanopoulos's efficiency gap metric, we would predict that had this election utilized a perfectly fair map, the Republicans likely would have won 14 fewer seats, and the Democrats 14 more. Additionally, the simulation we have constructed predicts that the probability of a map with an efficiency gap this lopsided occurring randomly is approximately 7 in 10 million. A couple of obvious questions stem from this research. First and foremost, what can be done to prevent the ongoing and undemocratic practice that is gerrymandering? And second, given that there has been another redistricting since 2018, what is the current state of gerrymandering in Wisconsin?

7.1 On the Prevention of Gerrymandering

Let's begin by addressing the first question. As previously mentioned, Wisconsin's state legislative maps are drawn and by the state legislature itself, then approved by the governor. This is not an uncommon practice as in 35 states the state legislature has some responsibility when it comes to redistricting. It doesn't take extensive research to argue that allowing legislators to draw maps that will then be used in their own elections is a blatant conflict of interest. Outlawing this practice would likely reduce partisan gerrymandering. In terms of alternatives, some states, such as California and New Jersey, have switched to having independent commissions draw district boundaries. This method isn't perfect either as the definition of "independent" is hazy, and members of these commissions often have party affiliations. Another idea that has been suggested is to do away with districts all together, and instead hold statewide elections for the legislature, and then apportion the seats using proportional representation. This method is probably the most equitable in terms of party representation, but it is not without its own flaws. Perhaps the greatest flaw in using this method is that it could put moderate voters in an awkward position, as voters who base their ballots off the candidates rather than partisan values would be forced to select a single party when voting in legislative elections. Regardless of its shortcomings, this method would solve the gerrymandering problem. Lastly, other metrics have been suggested to discourage gerrymandering at the redistricting stage, such as establishing concrete laws with regard to the concavity of districts, with exceptions only for state borders, and natural barriers.

Expanding on this point, could the efficiency gap itself be used as a legal standard to prevent partisan gerrymandering? One of our own criticisms of the efficiency gap as a metric is not explicitly in how it determines the bias of a given map, but rather that it tends to be a retroactive measure. That is, in this paper we have used it to demonstrate the extent to which gerrymandering impacted the 2018 Wisconsin state assembly election, and have shown that probability suggests the map drawn was undemocratic as it falls well outside of McGhee and Stephanopoulos's 8% threshold, but how does this help going forward as wasted votes are not officially known until after an election has occurred? One could respond to this criticism by saying that voting patterns of regions tend to be widely known when maps are being drawn as voting records are readily available, and thus an estimate of the efficiency gap could be constructed for a proposed map based off of past elections, still this response is based in hypothesis, as the efficiency gap will naturally vary from election to election, even if the same map is used.

7.2 The Current State of Gerrymandering in Wisconsin

Our focus thus far has been strictly on the 2018 Wisconsin State Assembly election because of how well it encapsulates a truly effective gerrymander through the eyes of the efficiency gap. However, this election is now five years past and utilized a map that has since been replaced. Now, we will briefly examine the current state of gerrymandering in Wisconsin, and attempt to examine how the issue has changed since the 2018 election.

Redistricting occurs once every 10 years in the year following a national census, and thus the most recent redistricting in Wisconsin came in 2021 following the 2020 census. The 2021 redistricting process proved to be a very different animal from the 2011 redistricting primarily due to the split government of Wisconsin. Unlike in 2011, the Republican party did not control the governorship, and thus the Democratic governor had the ability to veto any map. Knowing this, the state legislature still put forward a seemingly heavily gerrymandered map, which was indeed vetoed by the governor. The governor's office then proposed an alternative map, which the legislature subsequently rejected. This created a stalemate between the branches of government which was ultimately broken by the State Supreme Court. The court ruled in favor of the legislature in April of 2022, allowing their maps to proceed.

Two things should be noted about the new maps. First, the map put forward by the state legislature received an "F" on Princeton's partisan fairness scorecard, whereas the alternative map drawn by the governor's office received an "A." Second, although technically a non-partisan office, the State Supreme Court was considered to have a conservative majority at the time of the decision on the maps. The state legislature of course knew this when they drew the new maps, and felt safe in the knowledge that the court would side with them in the end. It also gave the Democratic party the ability to take the political "high road" and draw a fair map, knowing whatever map they put forth would just be rejected by the courts. The new state assembly map was used in the 2022 midterm elections where the Republican party very nearly captured a two-thirds supermajority, despite winning just over half of the popular vote. Thus, despite new maps, the issue of partisan gerrymandering is still very much alive in the state of Wisconsin.

7.3 Final Notes and Further Research

It should be reiterated that the purpose of this thesis was not to vilify the Republican party or give the appearance that gerrymandering is a problem specific to Wisconsin. In certain "blue" states such as Illinois, the Democratic party engages in the same practice. Just one example would be the most recent U.S. congressional map drawn by the Democratically controlled Illinois legislature which received an "F" on Princeton's partisan fairness report card.

The goal of this research has simply been to objectively and quantitatively analyze the presence of gerrymandering in one particular election. There is plenty more research that could be done on the subject, including deeper analysis of the 2022 Wisconsin State Assembly election or the legislative elections of other states where gerrymandering is also a problem. The practice of gerrymandering has become a national epidemic, and a mechanism both parties have weaponized in efforts to maintain and expand their power, regardless of the will of the citizens.

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Appendix

A.1 Deriving The Simplified Efficiency Gap Equation (I.2 from I.1)

(From McGhee's "Measuring Partisan Bias in Single-Member District Electoral Systems")

O = Opposition Party
G = Governing Party
DO = Opposition Party Districts
DG = Governing Party Districts
DT = Total Districts
SG = Governing party seat proportion
SO = Opposition party seat proportion

$$Surplus(o) = \sum_{1}^{do}(0.5 - V_i^g) = 0.5do - \sum_{1}^{do}(V_i^g)$$

$$Surplus(g) = \sum_{1}^{dg}(V_i^g - 0.5) = -0.5dg + \sum_{1}^{dg}(V_i^g)$$

$$Lost(o) = \sum_{1}^{dg}(1 - V_i^g) = dg - \sum_{1}^{dg}(V_i^g)$$

$$Lost(g) = \sum_{1}^{do}(V_i^g)$$

$$Waste(g) = Lost(g) + Sur(g) = \sum_{1}^{do}(V_i^g) - 0.5dg + \sum_{1}^{dg}(V_i^g)$$

$$Waste(o) = Lost(o) + Sur(o) = dg - \sum_{1}^{dg}(V_i^g) + 0.5do - \sum_{1}^{do}(V_i^g)$$

$$Efficiency\ gap = (Waste(o) - Waste(g))/\sum_{1}^{dt}V_i^t$$

$$Efficiency\ gap = ((dg - \sum_{1}^{dg}(V_i^g) + 0.5do - \sum_{1}^{do}(V_i^g)) - (\sum_{1}^{do}(V_i^g) - 0.5dg + \sum_{1}^{dg}(V_i^g)))/\sum_{1}^{dt}(V_i^t)$$

$$Efficiency\ gap = (0.5(do + dg) - 2(\sum_{1}^{do}(V_i^g) + \sum_{1}^{dg}(V_i^g)) + dg)/dt$$

$$dt = do + dg$$

$$Sg = dg/dt$$

$$Vg = (\sum_{1}^{do}(V_i^g) + \sum_{1}^{dg}(V_i^g))/dt$$

$$Ef = 0.5 - 2Vg + Sg$$

Let $Vmargin = Vg - 0.5$, $Smargin = Sg - 0.5$

$$Efficiency\ gap = 0.5 - 2(Vmargin + 0.5) + Sg = Sg - 0.5 - 2Vmargin$$

$$Efficiency\ gap = Smargin - 2(Vmargin)$$

A.2 RStudio Code

Wasted Votes Code:

```
```{r}
library(tidyverse)
library(readr)
#Efficiency Gap Calculations
election2018 <- X2018_State_Assembly_Data_Sheet1
summary(election2018$Republican)
summary(election2018$Democrat)
WVRep <- 0
WVDem <- 0
TotRep<-0
TotDem<-0
sum1<-c()
Rep18 <-c()
Dem18 <-c()
threshold <-c()
i=1
while(i<100){
Rep18[i] <- election2018$Republican[i]
Dem18[i] <- election2018$Democrat[i]
TotDem <- TotDem + Dem18[i]
TotRep <- TotRep + Rep18[i]
threshold[i] <- (Dem18[i] + Rep18[i])/2
i<-i+1
}
t = 1
while(t<100){

if(Rep18[t]>Dem18[t]){WVRep <- WVRep + (Rep18[t]-threshold[t])} else{WVDem <- WVDem + (Dem18[t] - threshold[t])}

if(Rep18[t]>Dem18[t]){WVDem <- WVDem + Dem18[t] } else {WVRep <- WVRep + Rep18[t]}

t<-t+1
}
WVDem
WVRep
((WVDem - WVRep)/(TotDem+TotRep))*100
TotDem+TotRep
```
```

```
Rep18[10]
Dem18[10]
```

Expected Representation Simulation:

```{r}
library(tidyverse)

d1 = 50
d2 = 50
d3 = 50
d4 = 50
d5 = 50
p<- c()
A<- c()
B<- c()
MA = 0
i = 0

while(i < 10000){

A1 = 0
B1 = 0
A2 = 0
B2 = 0
A3 = 0
B3 = 0
A4 = 0
B4 = 0
A5 = 0
B5 = 0
VA1 <- runif(0:100,1)
if(VA1 >= d1 ){A1<-1} else{B1<-1}
VA2 <- sample(0:100,1)
if(VA2 >= d2 ) {A2<-1} else{B2<-1}
VA3 <- sample(0:100,1)
if(VA3 >= d3 ) {A3<-1} else{B3<-1}
VA4 <- sample(0:100,1)
if(VA4 >= d4 ) {A4<-1} else{B4<-1}
VA5 <- sample(0:100,1)
if(VA5 >= d5 ) {A5<-1} else{B5<-1}
p[i]<-VA1+VA2+VA3+VA4+VA5
A[i]<-A1+A2+A3+A4+A5
B[i]<-B1+B2+B3+B4+B5
if(A1+A2+A3+A4+A5 > 2){MA = MA +1}
i<-i+1
```

```
}
i=1
x<-p/5
y<-A/5
plot(p/5,A/5)
fit5 <- lm(y~poly(x,5))
xx <- seq(0,100, length=50)
lines(xx, predict(fit5, data.frame(x=xx)), col="purple")
plot(xx, predict(fit5, data.frame(x=xx)), col="purple")
mean(A)
hist(A)
hist(p)
mean(p)
MA/10000
```

Efficiency Gap Simulation:

```{r}
library(tidyverse)
t=0
VA<-c()
DA<-c()
EG<-c()
while(t<100000){
WA=0 #Districts won by A
WB=0 #Districts won by B
VT=0 #Total votes for A
i=1
A <-runif(n=99, min=25, max=75)

while(i<100){

if(A[i]>=50){WA<-WA+1} else {WB<-WB+1}

VT<-VT+A[i]
i<-i+1
}

VA[t]<-(VT/99)
DA[t]<-(WA/99)*100
EG[t]<- (DA[t]-50)-2*(VA[t]-50)
t<-t+1
}


hist(EG, main = "Histogram of Efficiency Gap")
```

```
plot(VA,DA, main = "Vote Percentage Vs District Percentage Party A", xlab = "Party A Popular Vote Percentage",
ylab = "Party A District Percentage")
summary(EG)
sd(EG)
```