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HUME'S CONCEPTION OF GEOMETRY AND THE ROLE OF CONTRADICTION

by

Sofia Paz

A Thesis Submitted in

Partial Fulfillment of the

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ABSTRACT

HUME'S CONCEPTION OF GEOMETRY AND THE ROLE OF CONTRADICTION

by

Sofia Paz

The University of Wisconsin-Milwaukee, 2019
Under the Supervision of Professor Miren Boehm

David Hume's account of geometry can seem puzzling as he claims that geometry is inexact and demonstrable. Graciela de Pierris argues for an interpretation that explains why Hume sees geometry as inexact and, yet, demonstrable. However, she doesn't consider Hume's description of relations of ideas found in the *Enquiry*. Hume distinguishes between matters of fact and relations of idea by checking to see if there is a contradiction with the denial of a proposition. Geometry is categorized as relations of idea, so the denials of geometric propositions cannot be conceivable and must imply a contradiction. I will argue that De Pierris' account depicts definitions of geometric objects in such a way as to leave open the possibility for some relations of ideas where the denial of their proposition does not imply a contradiction, something Hume clearly did not intend.

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To

my fiancé

and

my family

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1. Introduction

When we think of mathematics as being demonstrable, we think of its being "clearly apparent or capable of being logically proven"[1]. This seems to require mathematics to be clearly understood and reliable. If mathematics were vague or "1+1" wasn't reliably always "2", then it would seems strange to suggest its theorems were demonstrable. However, David Hume's account of geometry claims that it is both inexact and demonstrable. The bulk of Hume's account of geometry is found in *A Treatise of Human Nature* and *An Enquiry Concerning Human Understanding*. Between the two works we get a detailed discussion of the inexact and imprecise nature of geometry, but also its demonstrability. Graciela de Pierris argues for an interpretation that explains why Hume sees geometry as inexact and, yet, demonstrable. In her interpretation of the tension between inexactness and demonstrability we see her focus on the *Treatise.* However, her interpretation calls on many fundamental principles of Hume's philosophy which are present in the *Enquiry.* So, her interpretation should, at the very least, be compatible with the *Enquiry* as well as the *Treatise.* Not bringing in the *Enquiry* explicitly into her interpretation is a mistake as she does not consider Hume's fork—a distinction made in the *Enquiry* between relations of ideas and matters of fact.

One way Hume distinguishes between matters of fact and relations of idea is by checking to see if there is a contradiction with the denial of a proposition. If a denial implies a contradiction and the denial is inconceivable by the mind, then the proposition is an expression of a relation of idea. If there is no contradiction and the denial of the proposition is conceivable, then the proposition is a matter of fact. Geometry is categorized as relations of idea, so the

[1] Oxford English Dictionary

denials of geometric propositions cannot be conceivable and must imply a contradiction. I will argue that De Pierris' account depicts definitions of simple geometric objects in such a way as to leave open the possibility for the denial of a proposition expressing a relation of idea that does not imply a contradiction—something Hume clearly would not have intended.

In this paper, I will begin with Hume's account of geometry as inexact and demonstrable as outlined in the *Treatise* and the *Enquiry.* In the *Treatise,* Hume characterizes geometry as inexact and imprecise mainly because of its reliance on finite indivisible points, but also goes on to describe why geometry is nonetheless reliable and demonstrable. In the *Enquiry,* Hume classifies geometry along side algebra and arithmetic as relations of ideas and thus reliable while nonetheless maintaining a kind of skepticism with regards to geometrical proofs. I will describe Hume's criteria for being classified as a relation of idea and I will introduce a new criterion supported by Hume's Fork. Afterwards, I argue that de Pierris' interpretation suffers from a serious problem in that it is unable to meet this new criterion and consequently cannot classify a geometric proposition as expressing a relation of idea.

2. Hume's Account of Geometry

2.1 Geometry as Inexact

The first substantial account of geometry is found in Book one of the *Treatise*. This discussion comes on the heels of his discussion of finite indivisible points which is situated in a larger discussion against the doctrine of infinite divisibility. Since complex ideas are made up of simple ideas and impressions (with those simple ideas originating from simple impressions themselves)[2], there is a limitation to the kinds of complex ideas our minds can have[3] (we can't have complex ideas that are infinite). So, Hume argues that extension must consist of "a finite number [of points], and these simple and indivisible" (T 1.2.4.1)[4]. Furthermore, these points "being nothing in themselves, are inconceivable when not fill'd with something real and existent" (T 1.2.4.2). Finite indivisible points, endowed with color or solidity, should not be confused with mathematical points which are "not fill'd with something real and existent" (T 1.2.1.5).

To help explain the sense in which these points are indivisible, Hume introduces a thought experiment. He says, "Put a spot of ink upon paper, fix your eye upon that spot, and retire to such a distance, that you lose sight of it; 'tis plain, that the moment before it vanish'd the image or impression was perfectly indivisible" (T 1.2.1.5). The ink spot is an example of an indivisible point, however, alone, the thought experiment does not explain why the existence of finite indivisible points is more veridical than the existence infinitely divisible points. To buttress his support for the existence of finite indivisible points, Hume turns to geometry to show that the system of indivisible points better conforms to geometrical axioms and definitions than that of infinite divisibility.

In geometry, the definition of a point—which "has neither length, breadth nor depth" (T 1.2.4.9)— is unintelligible without its being visible or tangible (i.e. a finite indivisible point), otherwise it's not clear that we are describing something existent. However, objectors might

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[2] T 1.1.1.6

[3] T 1.2.1.5

[4] Hume, David. A Treatise of Human Nature: Being an Attempt to Introduce the Experimental Method of Reasoning into Moral Subjects. Edited by Mary J. Norton and David Fate. Norton, Oxford University Press, 2000. Subsequent citations will contain a T for *Treatise* followed by the book, part, section, and paragraph.

claim that these geometrical objects (like points) “never did exist; for no one will pretend to draw a line or make a surface entirely conformable to the definition: They never can exist; for we may produce demonstrations from these very ideas to prove, that they are impossible” (T 1.4.2.10). For Hume, this response is already misguided because anything conceived “by a clear and distinct idea necessarily implies the possibility of existence” (T 1.2.4.11). Since we can clearly conceive of indivisible points, they must exist.

Another geometrical axiom which supports indivisible points is the termination of one geometrical object from another, for instance, when “a surface terminates a solid; a line terminates a surface; a point terminates a line” (T 1.2.4.14). In order to make sense of a point terminating a line, our imagination needs “ a concluding idea” (T 1.2.4.14). The system of indivisible points it able to supply a simple impression from which we get this concluding idea while infinite divisibility cannot and “'tis impossible we can ever conceive the termination of any figure; without which conception there can be no geometrical demonstration” (T 1.2.4.16).

However, there is nonetheless a “natural infirmity and unsteadiness both of our imagination and senses, when employ'd on such minute objects” (T 1.2.4.7) from which the system of indivisible points is not immune. Hume considers another geometrical example containing two lines that appear to be of equal length. Are they actually of equal length? The existence of indivisible points suggests a straightforward answer: “the lines or surfaces are equal, when the numbers of points in each are equal” (T 1.2.4.19). Unfortunately, this answer is “useless” because the indivisible points “are so minute and so confounded with each other, that ’tis utterly impossible for the mind to compute their number” (T 1.2.4.19). The unsteadiness and limitations of our imagination and senses make it impossible to know the exact number of indivisible points so “we seldom or never consider this the standard of equality or inequality” (T T 1.2.4.19). In other words, when looking at two lines, we don’t determine if they are equal by counting the indivisible points—we are unable to determine the exact number of points. Hume has shown the support the system of indivisible points lends geometry, however given our limitations regarding minute objects, when it comes to geometry, “we ought not to look for the utmost *precision* and exactness. None of its proofs extend so far” (T 1.2.4.19).

We must judge equality or inequality from general appearances. When we are presented with two lines, we can look and determine whether or not they are equal. Hume asserts, “the only useful notion of equality, or inequality, is deriv’d from the whole united appearance and the comparison of particular objects” (T 1.2.4.22). If we were to look at a yard stick and hold it up next to a foot long ruler, we don’t need to know the number of indivisible points in order to be able to see they are unequal lengths. The only reliable means to determine equality is by using our senses and relying on our senses must be used for all concepts, terms, objects in geometry. This reliance on the senses is what ultimately leads to the inexactness of geometry.

Hume says, “The reason why I impute any defect to geometry, is, because its original and fundamental principles are deriv'd merely from appearances. . .” (T 1.3.1.6). That said, Hume qualifies this conclusion saying “But since these fundamental principles depend on the easiest and least deceitful appearances, they bestow on their consequences a degree of exactness, of which these consequences are singly incapable. . .And this is the nature and use of geometry, to run us up to such appearances, as, by reason of their simplicity, cannot lead us into any considerable error” (T 1.3.16).

We see further support for this “degree of exactness” in the *Enquiry* where we see Hume’s discussion of the contrast between propositions in mathematics and the moral. He says, “An oval is never mistaken for a circle. . . distinguished by boundaries more exact that vice and virtue, right and wrong. . .” (E 7.1)[5]. Comparatively, since the difference between an oval and a circle can be seen in an instant, geometrical objects are awarded a higher degree of exactness which is denied from moral terms which do not depend on phenomenological appearances.

However, despite this reliability, it is important to note that at the end of the *Enquiry,* we see Hume claim that “nothing can be more convincing and satisfactory than all the conclusions concerning the properties of circles and triangles; and yet, when these are once received, how can we deny, that the angle of contact between a circle and its tangent is infinitely less than any rectilineal angle. . .Reason here seems to be thrown into a kind of amazement and suspense. . .” (E 12.18). Hume seems to argue that despite their certainty, the conclusions concerning the properties of circles and triangles can lead to “contradiction and absurdity” (E 12.8) which reaffirms Hume’s claim that geometry in inexact and imprecise.

2.2 Geometry as demonstrable

That said, we can see that while geometry is inexact and imprecise it is nonetheless reliable. However, Hume does not just want to say that geometry is reliable, but also claims that geometry is demonstrable. Because geometry relies on “the easiest and least deceitful appearances” (E 12.8) the appearances can be used to demonstrate other geometrical propositions.

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[5] An Enquiry Concerning Human Understanding.: a Letter from a Gentleman to His Friend in Edinburgh U.a. Hackett, 1993. Subsequent citation will include an “E” for *Enquiry* followed by the section and paragraph.

Geometry is clearly described as demonstrative in the *Enquiry.* The first mention of geometry in the *Enquiry* comes with the introduction of Hume's fork. Hume's fork refers to a division of "all the objects of human reason or enquiry" (E 4.1) into relations of ideas and matters of fact. Matters of fact are obtained through experience and cannot be disproven by appealing to logic. Relations of ideas, on the other hand, are intuitively or demonstratively certain. As a result, the negation of a relation of idea implies a contradiction and cannot be distinctly conceived. No such contradiction is implied with the negation of a matter of fact. In the *Enquiry,* we see Hume explicitly classify geometry as a relation of idea[6] and praise it for "all that accuracy of reasoning which it is so justly celebrated" (E 4.13).

We can also see a defense of geometrical propositions over the moral which further allows us to see geometry as demonstrable and reliable. Hume argues that geometrical terms are "always clear and determinate" (E 4.13) and "the same terms are still impressive of the same ideas, without ambiguity or variation" (E 4.13) which is different than vice and virtue. One reason is because "even when no definition is employed, the object itself may be presented to the senses, and by that means steadily and clearly apprehended" (E 4.13). In other words, we can understand the concept of a square by looking at a picture of a square even without knowing the formal definition which is something we cannot do with terms like "virtue" and "vice". This description of apprehending an object can be better understood when we consider Hume's account of obtaining general ideas.

Hume endorses Berkeley's view "that all general ideas are nothing but particular ones, annexed to a certain term, which gives them a more extensive signification, and makes them

[6] Ibid.

recall upon occasion other individuals, which are similar to them" (T 1.1.7.1). So, the idea of an object—like a triangle—is particular and "the image in the mind is only of that particular object" (T 1.1.7.6) (i.e. a specific triangle in our mind). However, when we see a triangle that is an isosceles, a scalene, small, large, blue, etc. we are nonetheless able to arrive at a general representation of the particular idea of a triangle. Hume describes the beginning of this process by explaining that "when we have found a resemblance among several objects, that often occur to us, we apply the same name to all of them, whatever differences we may observe. . ." (T 1.1.7.7). Then, "after we have acquired a custom of this kind, the hearing of that name revives the idea of one of these objects, and makes the imagination conceive it with all its particular circumstances and proportions" (T 1.1.7.7). Obtaining the general idea of geometrical objects like a triangle, begins with a sensory impression of a particular triangle. Graciela de Pierris argues that this way of acquiring geometrical concepts/objects helps us to see why geometry can be inexact and demonstrable.

3. Graciela de Pierris' Interpretation

3.1 Diagrammic Reasoning

De Pierris' interpretation, of the aforementioned passages in the *Treatise*[7], relies on what she calls diagrammatic reasoning. By "diagrammatic", De Pierris is referring to "actually drawn spatial figures occupying small spatial regions whose properties can be perceived in a single act

[7] De Pierris only focuses on the tension between inexact and demonstrable in the *Treatise*. However, in footnote 17 she notes similarities in T 1.3.1.6 and E 7.2 which suggests that she sees the two accounts as compatible.

of apprehension"[8]. To appreciate why diagrammic reasoning is important for Hume's conception of geometry, de Pierris argues it is important to recall what Hume says about indivisible points.

She argues that Hume's conception of these points "is itself purely phenomenological"[9]. The ink spot thought experiment indicates that "Hume is not saying that there is a fixed finite number of minima already waiting to be discovered independently of our phenomenological apprehension"[10]. In other words, Hume is arguing that it is impossible to ever know the exact number of minima. There isn't a limitation that if we could overcome it, we could discover the exact number. These minima—or points—are not present when the ink spot is very close to our face—there the points "constitute the appearance of extension"[11]. So, when we are looking at the ink spot from afar or when we are looking at a line, we are looking at the confounding of all the minima—which make up these images. It is impossible to see the individual minima while looking at the line because if we are seeing the line, the minima must be confounded. For this reason, "the exact (finite) number of minima in a given whole of extension is completely indeterminate, not simply unknown"[12].

Hume's ink spot experiment illustrates what it would take to "see" a minimum. This occurs when the ink spot "appears as the last member of a temporal sequence of closely resembling visual appearances of ever-smaller parts of the original spot, ending at a threshold

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[8] De Pierris 170

[9] Ibid. 173

[10] Ibid.

[11] Ibid. 174

[12] Ibid.

immediately before the appearance vanishes"[13]. The ink spot, in the instant before it vanishes, is a minimum because at this point it cannot be broken up anymore. However, we cannot see a line, for instance, *and* a minimum simultaneously. If we are looking at a line, then the minima must be confounded, but in order to see a minimum, it cannot be confounded with other minima. This is why it is impossible to determine the exact number of minima present in a line.

This impossibility, de Pierris argues, explains why geometry is more inexact than algebra and arithmetic. Arithmetic, for instance, deals with discrete units which are then added, subtracted, etc. Dividing these units "is entirely irrelevant to enumerating the collection"[14]. Geometry, as we have been discussing, depends on diagrams which do not contain a determinable number of indivisible points, but rather a confounding of these points. De Pierris notes that "precisely because the minima appear confounded in any phenomenologically given extension, and geometry is based on nothing but such phenomenological appearances, then geometry can never demonstrate exact equality. . ."[15]. To demonstrate an exact equality like that of arithmetic and algebra requires determining the number of indivisible points—which is impossible.

Geometers run into problems when their definitions require an exactness that is impossible to obtain. For instance, the definition of a straight line, as defined by Euclid, is "that which lies evenly between its extreme points"[16]. De Pierris points out "the order of the points is completely unknown because, as we have seen, we cannot reach all the minima

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[13] Ibid.

[14] de Pierris 178

[15] Ibid. 179

[16] Euclid Elements Book 1 Definition IV

simultaneously"[17]. Furthermore, since geometry relies on phenomenological appearances "there is no room for any intellectual process of *idealizing* these appearances so as to arrive at the supposed perfectly exact objects of geometry"[18].

However, geometry's reliance on diagrams is precisely what makes it reliable, especially when compared to the judgements of the vulgar. These diagrams—which are fundamental to geometry—"depend on the easiest and least deceitful appearances. . ."[19]. Hume says, "this is the nature and use of geometry, to run us up to such appearances, as, by reason of their simplicity, cannot lead us into any considerable error"[20]. De Pierris sees this as "the crucial advantage of Euclidean diagrammatic reasoning in the demonstrative science of geometry in comparison with our cruder estimations and reasonings in common life"[21]. We see two lines that look equal in length and, in general, this is a reliable judgement.

De Pierris describes a situation in which our judgements do not, at first glance, seem reliable. She considers the following proposition: "two right lines cannot have one common segment"[22]. Hume himself considers this example in Part II, Section 4 of the *Treatise* where he asks geometers "what infallible assurance he has, not only of the more intricate and obscure propositions of his science, but of the most vulgar and obvious principles?" (T 1.2.4.30). While the proposition might be true most of the time, when the angle between the two lines is

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17 de Pierris 180

18 de Pierris 180

19 Ibid.

20 Ibid.

21 De Pierris 184

22 Ibid.

imperceptible, de Pierris notes that "the appearance of the lines in a region very close to their intersection is phenomenologically indistinguishable from that of a single line. . ."[23].

Geometry, argues de Pierris, must be restricted to regions where appearances can be apprehended in an instant. It is in these regions we can be certain that we are not being led into "considerable error". However, once we are led into regions that are imperceptible, then there is no such assurance despite what geometers might want to claim. In fact, regarding our proposition in an imperceptible region, de Pierris says, "in this region the two lines have a common segment in the intuitive appearance, contrary to Postulate 1 of Euclid". In other words, the proposition appears to be false.

4. Problems for de Pierris' Interpretation

4.1 Definitions of Geometric Objects

De Pierris' response to the example of intersecting straight lines in an imperceptible region is problematic as it is not clear it expresses a relations of idea—though clearly it should. Recall that in the *Enquiry* Hume distinguishes between relations of ideas and matters of fact. He lays out clearly the criteria for relations of ideas[24] as follows:

- All mathematical truths
- Must be "intuitive or demonstrably certain" (E 4.1)
- Be discoverable "by the mere operation of thought, without dependence on what is any where existence in the universe" (E 4.1)

Matters of fact, on the other hand, must meet the following criteria:

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[23] de Pierris 184

[24] These lists in particular were used by Peter Millican, but all points can be explicitly found in Hume's works

- Are "not ascertained in the same manner" (E 4.2) as a relation of idea
- Both possibly true and possibly false
- Can be distinctly conceived to be true or to be false without contradiction
- Cannot be demonstrated to be true or be false.[25]

In the *Enquiry,* Hume makes the further claim that, "whatever is intelligible, and can be distinctly conceived, implies no contradiction, and can never be proved false by any demonstrative argument or abstract reasoning *a priori*" (E 4.18). Earlier in the description of a matter of fact (where the denials of propositions like "the sun will rise tomorrow" does not imply a contradiction) we see Hume make a claim about what would be the case if matters of fact were demonstrably false saying ,"were it demonstratively false, it would imply a contradiction, and could never be distinctly conceived by the mind" (E 4.2). There are those[26] who argue Hume uses inconceivability and contradiction interchangeably and does not necessarily mean logical contradiction. In other words, there might be something that is inconceivable that isn't a logical contradiction, but would still be considered a contradiction *because of* its inconceivability. This issue will be re-examined later on in the paper, for now, it will suffice that logical contradictions cannot be conceived in the mind and the discussion will be limited to these instances.

As a relation of idea, the denial of a proposition which implies a contradiction cannot be "distinctly conceived by the mind" (E 4.2). For instance, a five-sided square. This example implies a contradiction because the term "square" already contains the idea of a four sided shape. So, to try and conceive a five-sided and four sided figure is impossible. Its inconceivability

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[25] Millican 29

[26] See Atkinson, R. F. "Hume on Mathematics." The Philosophical Quarterly, vol. 10, no. 39, Apr. 1960, pp. 127–137., doi:10.2307/2960061.

shows it's a contradiction, but we can also think of "five-sided" as "not four-sided" and then it is more clearly a logical contradiction[27].

The same line of thinking can be applied to arithmetical examples. Arithmetic *is* exact and precise because it relies on what Hume refers to as "unites" (T 1.2.2.3). A unite "must be perfectly indivisible, and incapable of being resolved into any lesser unity" (T 1.2.2.3). In geometry we are unable to know the exact quantity of indivisible points in a line, but when dealing with arithmetical problems, like 1+1=2, I know the exact number of unites this problem is dealing with. So, if I have one unite and I add another unite next to it, then I have two unites. However, I can only conceive of having two unites. I am unable to conceive of different outcomes to "1+1" as I could with matters of facts, because anything other than "2" automatically implies a contradiction. Hence, why arithmetical examples are relations of ideas.

Now consider the proposition "all bachelors are unmarried men". It is only after the designation of "bachelor" that we get a contradiction with the negation. It might be tempting to think that the designation of terms seems to require experience—which relations of ideas seemingly are not meant to depend on. However, the justification for knowing that the negation of "all bachelors are unmarried men" implies a contradiction does not require any experience. This knowledge of whether the proposition implies a contradiction is not dependent on experience. Experience is only required in understanding the term "bachelor". So, there is no inconsistency in describing this statement as demonstrable or a relation of idea.

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[27] It is not clear whether Hume means to use "inconceivability" and "contradiction" interchangeably. Some like Atkinson have argued that Hume is not referring to only logical contradiction, but that a proposition is a contradiction because it is inconceivable and in this sense the terms are interchangeable. Whether or not there are cases of inconceivability which do not imply a logical contradiction is not going to be examined in this paper. For the purposes of this paper, it is sufficient that a logical contradiction (regardless of whether the terms are used interchangeably) is inconceivable and so our focus will be on cases of logical contradiction.

I would like to propose a criterion for a relation of idea which explains the role of contradiction in a relation of idea. Recall that Hume divides "all the objects of human reason or enquiry" (E 4.1) into relations of ideas or matters of fact. So, if an object of human reason is not a matter of fact, it is a relation of idea and vice versa. Since we know that the negation of proposition—which is a matter of fact—does not imply a contradiction, then the opposite is true for a proposition which is a relation of idea—its negation does imply a contradiction. Stated explicitly, the criterion is that "the negation of a proposition which is a relation of ideas cannot be distinctly conceived by the mind and implies a contradiction". Let us now begin to look at geometrical objects starting with triangles. For our purposes a triangle is "a plane figure with three sides and three angles". This definition of a triangle meets all the requirements for being a relation of idea as laid out by Millican and my new proposed criterion, i.e. we cannot conceive of the denial of the proposition "a triangle is a plane figure with three sides and three angles".

Recall under de Pierris' interpretation, the reliability of geometry stems from phenomenological appearances which are used to demonstrate all other propositions—or theorems. However, "precisely because the fundamental principles of geometry, for Hume, are drawn from 'appearances'—that is sensory impressions or images—there is no room for perfectly exact idealizations"[28]. We can ask at this point, whether or not our proposed definition of a triangle is a perfectly exact idealization.

4.2 Problems with the Definitions?

A "perfectly exact ideal" triangle would be one "which has no precise proportion of sides and angles" (T 1.1.7.6). It would be a triangle that is neither an isosceles nor a scalene. We would

[28] De Pierris fn 16

be trying to abstract away certain properties through a “pure and intellectual view” (T 1.3.1.7). Recall, that Hume does not think this is possible and instead argues that we must always see a particular image whenever the name of a general idea is called to mind. So, whenever we think of a triangle we will always call to mind a particular one. Geometers might be tempted to think that we can conceive of a triangle that is not a particular, but then these objects essentially become “spiritual” (T 1.3.1.7). In order to “destroy this artifice” (T 1.3.1.7) we must remember that “all our ideas are copy’d from our impressions” (T 1.3.1.7). We obtain the general idea for a triangle by looking at various different triangles and noticing all their shared properties, which, in turn, becomes the definition for a triangle.

On de Pierris’ account even the definitions of geometrical objects are limited to phenomenological appearances and limited to perceptible regions. So, consider the definition of a circle as proposed by Euclid: “a circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure equal one another”[29]. De Pierris could insist that Euclid’s definition is too precise to be useful. Euclid’s definition requires that if we pick two lines coming out from the center, they must be the same length. However, under the system of indivisible points, the exact and precise measure of equal length is having the same number of indivisible points—which is impossible to know. So, if the equal lengths of the two lines are determined by a quick judgement, then Euclid’s definition does not appear too precise and idealized. The same can be said for the proposed definition of a triangle. The definition is one that can be seen in an instant and describes characteristics shared by all triangles. In this sense, the definition does not appear too idealized or precise.

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[29] Euclid Book 1 Definition 15

4.3 Thought Experiment

Suppose we have a shape that looks like a triangle, but actually contains four sides. Neither our eyes nor our best, current instruments (like a microscope) can tell that there are four sides. On de Pierris' interpretation, this shape is a triangle because the fourth side falls outside of our perceptible region. Now, let's say time passes and we build better instruments that reveal to us the fourth side. This fourth side is now in the perceptible region and it seems clear that we were mistaken to think that shape was a triangle. However, this seems a bit strange on de Pierris' interpretation. Obtaining the definition of a triangle relied on phenomenological appearances and determining whether or not a shape meets the definition of a triangle should also rely on phenomenological appearances. So, in the first instance we have a shape that meets the definition and is classified as a triangle. In the latter instance we can see that the shape does not meet the definition and is not classified as a triangle. However, there is no difference in the shape, the only difference was the accuracy of the viewing instrument. For de Pierris the shape in the earlier instance is "phenomenologically indistinguishable" from a triangle even though in reality it is not a triangle. However, if all it takes to be classified as a triangle is to be "phenomenologically indistinguishable" from a triangle, then it is not clear how the definition of a triangle meets my newly articulated criterion for a relation of idea.

In my thought experiment, there is not a focus on treating the definition of a triangle as idealized or abstract. It is still reliant on phenomenological appearances. However, the new and more accurate instrument is widening the perceptible region so the focus remains on what we can see in an instant. Nonetheless, we are saying that the triangle in the earlier instance meets the criteria for being a triangle even though it does not.

Why couldn't de Pierris just say that it was a mistake to think the first shape was a triangle and was corrected later on? She can say this, but to say that at the time of the older instrument what we had was not a triangle seems to suggest a definition—or conception—of a triangle that is not dependent on our appearances. This does not seem like something Hume would want to admit nor de Pierris.

A definition that is so inexact like the one de Pierris is arguing for—which relies only on our senses—seems to leave open the possibility for a contradiction wherein something that is classified as a triangle is not a triangle. Bachelors cannot sometimes be married, they must, by definition, always be unmarried. The only way something could be classified as a bachelor is by conforming to the definition. The same is true for the definition of a triangle. There seems to be a difference between the triangle of the definition and the triangle of appearances—which is a result Hume would not have wanted. De Pierris' interpretation, then should show no difference between the definition and appearance, however, defining geometric objects that are so inexact and imprecise as de Pierris is suggesting doesn't leave open the possibility for meeting the new criterion.

5 Conclusion

As I have shown, Hume is very clear that geometry is more imprecise than algebra and arithmetic, but nonetheless demonstrable. It is more imprecise because it relies on the senses—its fundamental principles are obtained from general appearances. So, we learn about what a line is by seeing lines. However, our senses and imagination is limited and cannot extend to minute objects like finite indivisible points. Hume made it clear that in order for many geometric objects, definitions, principles, etc. to make sense there must exist finite indivisible points. Their existence suggest a precise and exact standard for geometry, for instance, we could tell if lines are equal if they contain the same number of indivisible points. However, we will never be able to determine the exact number of indivisible points making up geometrical objects because of the limitation of our senses and imagination. Nonetheless, the general appearances of many geometrical objects, like two lines which appear to be of equal length, is reliable and principles and theorems built upon these appearances are also reliable and what makes geometry demonstrable.

De Pierris has a very compelling interpretation of this relationship between inexactness and demonstrability. She argues that we need to be careful about how we understand the reason it is impossible to determine the exact number or indivisible points. Our senses and imagination are limited, but to leave it at this leaves open the possibility that it might be possible to determine the exact number if we overcame our limitations. De Pierris argues that it is never possible because when we see a line we are seeing the confounding of these indivisible points. As soon as we attempt to try and see some sort of indivisible point the image of the line disappears, so it would be impossible to try and count the number of points in a line—we must either see the line or attempt to see a minima, not both. Much of her interpretation is compelling and contains much textual support. However, her focus lies in the *Treatise* and when bringing in the *Enquiry* there appear to be a problem.

The problem arises when we consider that Hume explicitly classifies geometry as a relation of ideas. This suggests that the denials of geometric propositions ought to be inconceivable and imply a contradiction. However, de Pierris' interpretation requires that definitions and determinations of whether something meets a definition be limited to what is phenomenal. I hope to have shown that this leaves open the possibility that an object is able to be meet the definition of a triangle while nonetheless not being a triangle because each is phenomenally indistinguishable from each other. De Pierris' account seems to be suggesting something much stronger than a simple mistake occurred. The observer committed no mistake in designating the shape as a triangle because it is phenomenologically indistinguishable from a triangle. However, it is not clear how this designation implies a contradiction as it ultimately ends up being the negation of the definition.

Not meeting my proposed criterion is very problematic for de Perris because that means the definitions of geometric objects—like triangles—do not meet all the criteria for being classified as a relation of ideas. However, these definitions are meant to be geometrical truths and are, by Hume, clearly meant to be classified as a relation of idea. This suggests an oversight on part of de Pierris, for although her interpretation takes great care to consider the *Treatise* there are still elements of the *Enquiry*, namely Hume's fork, which de Pierris did not accommodate into her account. She could amend her interpretation for being one solely based on the *Treatise* and claim that something in Hume's account of geometry changes from the *Treatise* to the *Enquiry*. However, given that her interpretation relies on principles which still appear in the *Enquiry* and, in fact, are fundamental to Hume's philosophy—like ideas must be obtained from a corresponding impression—it seems that she would be committed to the idea that her interpretation is meant be compatible with both works. As it currently stands, however, her interpretation is at odds with Hume's fork which is another clear and explicit aspect of Hume's philosophy. Until the negation of the definitions of geometrical objects always imply a contradiction, then de Pierris must amend her interpretation.

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