May 2013

Essays on Asset Return and Housing Market

Swati Kumari

University of Wisconsin-Milwaukee

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ESSAYS ON ASSET RETURN AND HOUSING MARKET

by

Swati Kumari

A Dissertation Submitted in
Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY
in
ECONOMICS

at

The University of Wisconsin–Milwaukee
May 2013
The last two decades have witnessed substantial amount of research on time variation in asset returns. It has been found that macroeconomic variables contain useful information about asset returns. This dissertation consists of three essays that study the link between the macroeconomy and financial markets. A central idea behind the link is that households adjust their consumption spending in anticipation of variations in the return on household assets.

The first essay proposes a latent-variables approach to estimate expected returns on total household assets and expected growth rate of excess consumption (consumption in excess of labor income) within a present-value model of consumption. The present-value model of consumption implies that the ratio of consumption- aggregate wealth reflects information about future asset returns and consumption growth. Since expected returns and expected excess consumption growth are unobserved variables, the current literature uses lagged excess consumption-assets ratio or other proxies for estimation. This essay goes beyond the existing literature by using an unobserved component approach to filter these unobserved variables from the observed history of realized returns and realized excess consumption growth. Results suggest that both filtered returns and filtered excess consumption growth rate are significant and better predictors of realized returns and realized excess consumption growth rate than the one obtained by lagged excess consumption-assets ratio.
The second essay focuses on estimating expected return on housing by exploiting the information from the movements in consumption, income, and observable assets. To do so, a present-value model of consumption is combined with an unobserved component model. Kalman filter is then applied to extract expected housing returns from the observed history of realized returns and realized excess consumption growth. Results suggest that the filtered housing returns does a significantly better job in predicting realized housing returns than other popular predictors like mortgage rate and price-rent ratio.

The third essay uses an unobserved components model with heteroskedastic disturbances to measure the time-varying importance of permanent and transitory components in the U.S. and U.K. house prices. Estimation results suggest that the movement in house prices in the two economies is mainly transitory in nature from its trend path.
This dissertation is dedicated to my husband Ashutosh Sharma, my son Akshat Sharma, and to my parents Veena and Sudhir Chauhan for their unconditional love, guidance and support.
# Table of Contents

1 Introduction 1
   1.1 Significance of the dissertation ............................................. 1
   1.2 Structure of the dissertation .................................................. 1
   1.3 Contribution to the literature ............................................... 3

2 First Essay: Consumption and Expected Asset Returns: An Unobserved Component Approach 5
   2.1 Introduction .............................................................................. 5
   2.2 Brief Literature Review ............................................................ 9
   2.3 Model Specification .................................................................. 10
      2.3.1 Present-Value Model ......................................................... 10
      2.3.2 Variance Decomposition .................................................... 14
   2.4 Data Description ..................................................................... 15
   2.5 Model Estimation .................................................................... 17
   2.6 Estimation Results .................................................................. 20
      2.6.1 Comparison with the Benchmark Model ............................... 22
      2.6.2 Stock Return Predictability ............................................... 24
   2.7 Conclusion ............................................................................. 25

3 Second Essay: Consumption-Wealth Ratio and Expected Housing Return 27
   3.1 Introduction .............................................................................. 27
   3.2 Brief Literature Review ............................................................ 29
   3.3 The Present-Value model and Expected Return on Housing ........... 31
      3.3.1 Data Description ................................................................. 36
   3.4 Model Estimation .................................................................... 38
      3.4.1 State Space Representation .................................................. 38
   3.5 Estimation Results .................................................................. 40
   3.6 Quarterly Forecasting Regressions ............................................. 44
      3.6.1 Forecasting Housing Asset Return ....................................... 44
      3.6.2 Forecasting Real House Price Growth ................................. 48
      3.6.3 Out-of-Sample Evidence ..................................................... 51
   3.7 Conclusions ........................................................................... 56

4 Third Essay: The Relative Importance of Permanent and Transitory Components in the US and UK House Prices 58
   4.1 Introduction .............................................................................. 58
   4.2 The Unobserved Components Model ........................................... 61
      4.2.1 Model with Heteroskedastic Disturbances ............................ 62
      4.2.2 Model with Non Heteroskedastic Disturbances .................... 63
   4.3 State-Space Model ................................................................... 64
LIST OF FIGURES

2.1 Excess consumption-assets ratio .................................................. 17

3.1 Comparison of realized housing returns with forecasts from the unobserved component approach and the OLS approach ............... 43

3.2 Comparison of change in Real HPI with forecasts from the unobserved component approach and the OLS approach ..................... 44

4.1 Real HPI and Trend Component from the Unobserved Components Model with Heteroskedastic Disturbances-USA .......................... 68

4.2 Stationary Cyclical Component from the Unobserved Components Model with Heteroskedastic Disturbances-USA .......................... 69

4.3 Standard Errors of Transitory Shocks from the Unobserved Components Model with Heteroskedastic Disturbances-USA .................... 70

4.4 Standard Errors of Permanent Shocks from the Unobserved Components Model with Heteroskedastic Disturbances-USA .................... 70

4.5 Ratios of Standard Errors of Transitory Shocks to those of Permanent Shocks from the Unobserved Components Model with Heteroskedastic Disturbances-USA ........................................... 71

4.6 Stationary Cyclical Component from the Unobserved Components Model with Non Heteroskedastic Disturbances-USA ..................... 72

4.7 Real HPI and Trend Component from the Unobserved Components Model with Heteroskedastic Disturbances-UK .......................... 74

4.8 Stationary Cyclical Component from the Unobserved Components Model with Heteroskedastic Disturbances-UK .......................... 74

4.9 Standard Errors of Permanent Shocks from the Unobserved Components Model with Heteroskedastic Disturbances-UK .................... 75

4.10 Ratios of Standard Errors of Transitory Shocks to those of Permanent Shocks from the Unobserved Components Model with Heteroskedastic Disturbances-UK ........................................... 76

4.11 Stationary Cyclical Component from the Unobserved Components Model with Non Heteroskedastic Disturbances-UK ..................... 77
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Variance Decomposition of Excess Consumption-Assets Ratio</td>
<td>15</td>
</tr>
<tr>
<td>2.2</td>
<td>Maximum Likelihood Estimates of Hyperparameters</td>
<td>21</td>
</tr>
<tr>
<td>2.3</td>
<td>Implied Present-Value Model Parameters</td>
<td>21</td>
</tr>
<tr>
<td>2.4</td>
<td>R-squared values</td>
<td>22</td>
</tr>
<tr>
<td>2.5</td>
<td>OLS predictive regressions</td>
<td>24</td>
</tr>
<tr>
<td>2.6</td>
<td>Stock return predictability regressions</td>
<td>24</td>
</tr>
<tr>
<td>3.1</td>
<td>Maximum Likelihood Estimates of Hyperparameters</td>
<td>40</td>
</tr>
<tr>
<td>3.2</td>
<td>Implied Present-Value Model Parameters</td>
<td>42</td>
</tr>
<tr>
<td>3.3</td>
<td>Forecasting Quarterly Housing Asset Return</td>
<td>46</td>
</tr>
<tr>
<td>3.4</td>
<td>Forecasting Quarterly Growth in Housing Prices</td>
<td>49</td>
</tr>
<tr>
<td>3.5</td>
<td>Forecasting Quarterly Growth in Housing Prices: 1971Q2-1996Q4</td>
<td>50</td>
</tr>
<tr>
<td>3.6</td>
<td>One-Quarter-Ahead Forecasts of Housing Assets Returns: Nonnested</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>Comparisons</td>
<td></td>
</tr>
<tr>
<td>3.7</td>
<td>One-Quarter-Ahead Forecasts of House Price Growth: Nonnested</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>Comparisons</td>
<td></td>
</tr>
<tr>
<td>3.8</td>
<td>One-Quarter-Ahead Forecasts of House Price Growth: Nonnested</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Comparisons(1971Q2-1996Q4)</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Maximum Likelihood Estimates of the Unobserved Components</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>Model with Heteroskedastic Disturbances-USA</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>Maximum Likelihood Estimates of the Unobserved Components</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>Model with Non-Heteroskedastic Disturbances-USA</td>
<td></td>
</tr>
<tr>
<td>4.3</td>
<td>Maximum Likelihood Estimates of the Unobserved Components</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>Model with Heteroskedastic Disturbances-UK</td>
<td></td>
</tr>
<tr>
<td>4.4</td>
<td>Maximum Likelihood Estimates of the Unobserved Components</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>Model with Non-Heteroskedastic Disturbances-UK</td>
<td></td>
</tr>
</tbody>
</table>
### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>Akaike Information Criterion</td>
</tr>
<tr>
<td>AR</td>
<td>Autoregressive</td>
</tr>
<tr>
<td>ARCH</td>
<td>Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>ARIMA</td>
<td>Autoregressive Integrated Moving Average</td>
</tr>
<tr>
<td>BIC</td>
<td>Bayesian Information Criterion</td>
</tr>
<tr>
<td>CAY</td>
<td>Consumption, Asset Wealth and Labor Income</td>
</tr>
<tr>
<td>CPI</td>
<td>Consumer Price Index</td>
</tr>
<tr>
<td>GARCH</td>
<td>Generalized Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>GDP</td>
<td>Gross Domestic Product</td>
</tr>
<tr>
<td>HPI</td>
<td>House Price Index</td>
</tr>
<tr>
<td>IGARCH</td>
<td>Integrated Generalized Autoregressive Conditional Heteroskedasticity</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Squared Error</td>
</tr>
<tr>
<td>NIPA</td>
<td>National Income and Product Account</td>
</tr>
<tr>
<td>OFHEO</td>
<td>Office of Federal Housing Enterprise Oversight</td>
</tr>
<tr>
<td>OLS</td>
<td>Ordinary Least Squares</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>Standard&amp;Poor's</td>
</tr>
<tr>
<td>VAR</td>
<td>Vector Autoregression</td>
</tr>
</tbody>
</table>
This dissertation would not have been possible without the constant support, guidance and help of various individuals who extended their valuable assistance in the preparation and completion of this study and made my time working for my PhD an unforgettable experience.

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Chapter 1

Introduction

1.1 Significance of the dissertation

The last decade has brought forth an active research in predictability of time variation in asset returns. The predictability of asset returns is of considerable importance to the investor community that is interested in improving the predictability of asset returns over the short run. This improved predictability can allow investors to make better investment decisions without taking additional risks. However, in order to overcome the unobservability of expected asset returns, the current literature uses proxies for future asset returns. Two chapters of my dissertation show that this approach doesn’t efficiently use all available information and an unobserved components methodology can be used to significantly improve predictability of both total and housing asset returns. Moreover, the dissertation sheds potentially valuable insights into the permanent vs. transitory impact of movement in house prices in the U.S. and the U.K. housing markets. This has significance for the government and investors. It would help government make better policies and international property investors create more effective property management strategy.

1.2 Structure of the dissertation

This dissertation consists of three essays. The first essay proposes an unobserved component approach to estimate expected asset returns and expected excess consumption (consumption in excess of labor income) growth rate in a present-value
model of consumption. The expected returns based on the unobserved component approach uses larger information set which is based on the information from the whole history of observed returns and observed excess consumption. The findings suggest that expected returns and expected excess consumption growth rate estimated from the unobserved component approach do a better job in explaining realized returns and realized excess consumption growth rate than the lagged proxies used in the existing approach for future asset returns and future excess consumption growth rate.

The second essay estimates housing asset returns by utilizing the information in consumption, income, and observable assets. To do so, a modified present value model of consumption is combined with an unobserved component model. Acknowledging that expected housing asset returns are unobservable, the Kalman filter technique is used to extract them from the observed history of realized returns and realized excess consumption growth. The constructed predictor outperforms the other popular predictors like mortgage rate, price-rent ratio and GDP growth rate both in and out-of-sample, providing statistically and economically significant forecasts.

The third essay examines the behavior of the US and the UK housing market by allowing the permanent and transitory component of house prices to vary with time. It also focuses on the impact of the recent housing market collapse on the US and the UK markets. An unobserved component model with heteroskedastic disturbances is used to measure the permanent and transitory components of house prices. More specifically, it measures the ratio of standard errors of transitory shocks to those of permanent shocks over the entire sample period and examine whether the movements in house prices will have a long term or a short term impact on the US and the UK markets. The essay also investigates whether the impact of the decline in the housing market in 2007 in the UK and the US markets is a transient
deviation from the trend path.

1.3 Contribution to the literature

In the first essay, the existing approach in the literature includes consumption-wealth ratio or excess consumption-assets ratio as proxies for future asset returns. These proxies have provided some evidence that households adjust their consumption in anticipation of the variations in the return on household assets. However, the usual approach is not optimal as it does not use all the information efficiently. The main contribution of the first essay is to use an unobserved component approach to estimate the expected asset returns and expected excess consumption growth directly from a version of present value model of consumption. The unobserved component approach allows us to expand the information set by using the information from the whole history of observed returns and observed excess consumption growth. The use of more information provides us estimates of expected return and expected excess consumption that in theory should yield better forecasts of realized returns and realized excess consumption growth rate. The approach also enables decomposition of the movements in excess consumption-assets ratio into portion arising due to movements in excess consumption growth and movements due to expected returns.

The second essay goes beyond the traditional literature that has mainly focused on estimating the consumption response to changes in the housing market wealth to estimate housing asset returns. Rather than estimating the consumption response to changes in housing wealth, this essay utilizes the information in consumption, income, and observable assets to estimate expected housing asset returns. This essay contributes to the literature by estimating future housing market returns from a present value model of consumption using an unobserved component approach. This approach provides an estimate of expected housing return that uses more information than the conventional finite lag approach and therefore should yield a
better forecast of realized housing returns.

The third essay decomposes the movement in house prices into the permanent and transitory components by allowing the shocks to house prices to have a time-varying permanent and transitory effects. This is different from the traditional literature that decomposes house prices into trend and cycle by assuming the shocks to the permanent and transitory components to have same distribution. The data suggests time variation in house prices. The real HPI has been much more volatile at some times than at others. Therefore, I apply GARCH or IGARCH effect to conditional variance of the innovation in trend and cycle component. I then observe the ratio of standard errors of transitory shocks to those of permanent shocks over the sample period, and examine whether the movements in house prices will have a long term or short term impact on the US and the UK housing markets. More importantly, the essay also analyzes the impact of the recent housing bubble crisis on the housing market.
Chapter 2

First Essay: Consumption and Expected Asset Returns: An Unobserved Component Approach

2.1 Introduction

The relationship between financial markets and the macroeconomy has received widespread attention over the last few years. One of the models that has been used widely as a link between the macroeconomy and the financial markets is the present-value model of consumption. The central idea behind the present-value model of consumption is that the predictable fluctuations in asset returns may be reflected in the current consumption decision of a household in a manner consistent with rational expectations. Recently, Whelan (2008) proposed a modified present-value model of consumption that takes the following form:

\[ x_t - a_t \approx E_t \sum_{j=1}^{\infty} \rho^j (r_{t+j}^a - \Delta x_{t+j}) \]

where \( x_t \) is the log of consumption minus labor income, \( a_t \) is the log of observable household assets, \( r_t^a \) is the return on these assets, and \( \rho \) is a known constant slightly less than one. The above model suggests that log ratio of excess consumption (defined as consumption in excess of labor income) to observable assets can be expressed as an expected discounted sum of future returns on household assets minus future growth rates of excess consumption. Therefore an upward surprise in excess consumption today must correspond to an unexpected return on assets today or to news that future returns will be higher or to a downward revision in expected
excess future consumption growth rate.

The existing approach in the literature that includes Whelan (2008) and Lettau and Ludvigson (2001) use excess consumption-assets ratio and consumption-wealth ratio as proxies for future asset returns and future excess consumption growth rate\(^1\). Their results have provided some evidence that households adjust their consumption spending in anticipation of variations in the return on household assets. In this essay, an alternative approach is proposed to estimate expected asset returns and expected excess consumption growth rate in a present-value model of consumption. Instead of using excess consumption-assets ratio as a proxy for expected asset returns and expected excess consumption growth rate, they are estimated directly using an unobserved component approach. This approach is based on recent research in stock returns literature, where Binsbergen and Koijen (2010) and Rytchkov (2008) have used an unobserved component approach to estimate expected returns in a present-value model of stock prices. To estimate the expected return on assets and expected consumption growth rate, I follow Campbell’s (1991) strategy and model both expected returns and expected excess consumption growth rate as a first order autoregressive process. It should be noted that this low order autoregressive process in a state-space setting admits an infinite order VAR representation in terms of excess consumption growth and excess consumption-assets ratio, as shown by Cochrane (2008). This implies that expected return based on the unobserved component approach uses a larger information set which is based on the information from the whole history of observed returns and observed excess consumption. The assumption of low order autoregressive specification for unobserved variables allows me to write log excess consumption-assets ratio as a linear function of expected return and expected excess consumption growth rate, which can be used as a measurement equation in a state-space system. The state-space system also includes

\(^1\)Lettau and Ludvigson (2001) use estimated residual from a cointegrating regression of consumption, labor income and wealth (cay) as a proxy for expected asset returns.
another measurement equation linking the observed $\Delta x_t$ with the unobserved expected excess consumption growth rate and one transition equation involving the AR process for expected excess consumption growth. This state-space model is estimated using maximum likelihood via the Kalman filter.

The results indicate that expected returns are more persistent than expected excess consumption growth rate, which is consistent with what other researchers have found in the finance literature for expected stock returns and expected dividend growth. This approach also allows decomposition of the movements in excess consumption-assets ratio into portion arising due to movements in excess consumption growth and movements due to expected returns\(^2\). The results suggest that expected return accounts for 92 percent of the variations in the excess consumption-assets ratio, whereas excess consumption growth rate accounts for only 8 percent of the variations. This is similar to the findings of Binsbergen and Koijen (2010), who find that expected return accounts for most of the variations in price-dividend ratio.

The findings suggest that expected returns and expected excess consumption growth rate estimated from the unobserved component approach do a better job in explaining realized returns and realized excess consumption growth rate than lagged excess consumption-assets ratio. Estimated expected returns explain 8.3 percent of the variations in realized asset returns, whereas estimated expected excess consumption growth rate explains 9 percent of the variations in realized excess consumption growth rate. The corresponding R-squared values are 3.9 percent and 1.9 percent when lagged excess consumption-wealth ratio is used as a predictor of realized asset returns and realized excess consumption growth rate. The approach improves upon the predictive power of lagged excess consumption-assets ratio in predicting asset returns because the unobserved component approach allows us to expand the information set by using the information from the whole history of

\(^2\)This is similar to the idea of Fama and French (1988), who point out that the price-dividend ratio is only a noisy proxy for expected returns when the price-dividend ratio also moves due to expected dividend growth rate variation.
observed returns and observed excess consumption. The use of more information provides us estimates of expected return and expected excess consumption growth rate that in theory should yield better forecasts of realized returns and realized excess consumption growth rate.

As pointed out by Lettau and Ludvigson (2001), return on total household assets is highly correlated with the stock returns. Therefore I also examine whether estimated expected return on assets from the proposed approach has significant explanatory power for future movements in real stock returns. The predictive power of the measure of expected returns is then compared with the predictive power of \( cay \) and the excess consumption-assets ratio. The results suggest that the measure improves upon the predictive power of the excess consumption-assets ratio. 5.8 percent of the variations in one quarter ahead real stock returns are explained by expected asset returns, whereas the excess consumption-assets ratio explains about 2.5 percent of the variation. This suggests that decomposing the movements in excess consumption-assets ratio into expected asset growth and expected excess consumption growth rate led to a reduction in the noise of the predictive ability of excess consumption-assets ratio, and helped improve the predictive power. In terms of comparison with \( cay \), \( cay \) explains 5.9 percent of the variation in one quarter ahead stock returns, which is marginally better than our measure of expected returns.

The plan of this essay is as follows: section 2.2 provides a brief literature review; section 2.3 proposes an unobserved component model to estimate the present-value model of consumption. Section 2.4 describes the data. Section 2.5 provides the empirical methodology; section 2.6 explains the empirical results; and section 2.7 concludes.
2.2 Brief Literature Review

The use of the present-value model of consumption to establish a link between the macroeconomy and the financial market goes back to Campbell and Mankiw (1989), who showed that log of consumption-aggregate wealth ratio reflects information about expected returns on wealth and expected consumption growth rate. The empirical implementation of this model, however, was limited by the unobservability of human capital component of aggregate wealth. To get around this problem of the unobservability of human capital, Lettau and Ludvigson (2001) used a set of approximating assumptions that link the unobservable total wealth series to observable series on assets and income. They found that stationary deviations from the shared trend of consumption, labor income and wealth, defined as \( cay \), is a good predictor of future asset returns. However, Whelan (2008) argued that the estimation of \( cay \) is based on the point estimate of cointegrating vector, which is subject to uncertainty.

Whelan (2008) introduced a new approach that eliminates the need to estimate any parameter and showed that the ratio of excess consumption (consumption in excess of labor income) to assets can be expressed as an expected discounted sum of future returns on household assets minus future growth rates of excess consumption. This model implies that an upward movement in excess consumption-assets ratio can occur either due to lower future expected excess consumption growth rate or/and due to higher-than-average expected asset returns. As a result, Whelan (2008) suggests to use lagged excess-consumption assets ratio instead of \( cay \) as a predictor of future asset returns.

The present-value approach used in this essay is closely related to literature on present-value models in finance. Rychkov (2008), Binsbergen and Koijen (2010), Pastor and Stambaugh (2009), and Brandt and Kang (2004) have also applied filter-

2.3 Model Specification

2.3.1 Present-Value Model

In this section I build on the present-value model of Whelan (2008) and combine it with the latent variable approach of Binsbergen and Koijen (2010). In doing so, I first briefly explain the present-value model used in this essay.

Whelan (2008) considers the following budget constraint that describes the evolution of total observable assets:

\[ A_{t+1} = R_{t+1}^a (A_t + Y_t - C_t) \] (2.1)

where \( A_t \) is total household assets, \( R_{t+1}^a \) is the gross return on assets, \( Y_t \) is labor income and \( C_t \) is consumption. Dividing across by \( A_t \) and taking logs we get:

\[ \Delta a_{t+1} = r_{t+1}^a + \log \left( 1 - \frac{C_t - Y_t}{A_t} \right) \] (2.2)

Define, excess consumption as \( X_t = C_t - Y_t \)^4

^4For the US data series used in this study, which rely on a standard definition of labor income, consumption always exceed labor income. Therefore, \( X_t \) is always positive. One of the interpre-
Equation (2.2) can be rewritten as:

\[ \Delta a_{t+1} = r_{t+1}^a + \log(1 - \exp(x_t - a_t)) \]  

where \( \log(1 - \exp(x_t - a_t)) \) is a non-linear function. Taking a first order Taylor expansion around the mean \((\bar{x} - \bar{a})\), the budget constraint in equation (2.3) can be approximated as:

\[ \log(1 - \exp(x_t - a_t)) \approx \log(1 - \exp(\bar{x} - \bar{a})) - \left( \frac{\bar{x}}{\bar{a} - \bar{x}} \right) (x_t - a_t - \bar{x} + \bar{a}) \]  

Equation (2.4) can be simplified to:

\[ \log(1 - \exp(x_t - a_t)) \approx \kappa + (1 - \rho^{-1})(x_t - a_t) \]  

where \( \kappa \) is a constant and equals \( \log(1 - \exp(\bar{x} - \bar{a})) - (1 - \rho^{-1})(\bar{x} - \bar{a}) \).\(^5\)

Substituting equation (2.5) into equation (2.3), we obtain:

\[ \Delta a_{t+1} = r_{t+1}^a + \kappa + (1 - \rho^{-1})(x_t - a_t) \]  

Rearranging and solving equation (2.6) forward via repeated substitution and imposing the condition that \( \lim_{j \to \infty} \rho^{-j}(x_{t+j} - a_{t+j}) = 0 \), yields the following expression:

\[ x_t - a_t \approx \frac{\rho \kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^j (r_{t+j}^a - \Delta x_{t+j}) \]

\(^5\)In Whelan’s (2008) model, \( \kappa \) is dropped from the budget constraint.
The algebraic steps applied to the above equation are similar to the methodology applied in Campbell and Mankiw (1989) model.

The above equation can also be considered as a log linear equivalent of nonlinear present-value budget constraint:

\[
\sum_{j=0}^{\infty} \frac{C_{t+j}}{\prod_{k=0}^{j} R_{t+k}^{a}} = A_t + \sum_{j=0}^{\infty} \frac{Y_{t+j}}{\prod_{k=0}^{j} R_{t+k}^{a}}
\]

Equation (2.7) holds \textit{ex post} and \textit{ex ante}. Taking conditional expectations of the equation yields the following expression for the excess consumption-assets ratio:

\[
x_t - a_t \approx \frac{\rho \kappa}{1 - \rho} + E_t \sum_{j=1}^{\infty} \rho^j (r_{t+j}^{a} - \Delta x_{t+j})
\]

An upward surprise in excess consumption today must correspond to an unexpected return on assets today or to news that future returns will be higher or to a downward revision in expected excess future consumption growth rate.

The model is in spirit of models by Campbell and Mankiw (1989) and Lettau and Ludvigson (2001). A key advantage of this approach is that it does not require any assumptions about unobservable variables such as human capital variable. Also, it does not require estimation of unknown parameters to arrive at a forecasting variable.

I build upon this model to filter expected asset returns from the observed history of realized returns and excess consumption growth rate. Expected returns and expected excess consumption growth rate are assumed as unobserved variables. This assumption is also justified by the finding that \(x_t - a_t\) is a noisy indicator for expected returns. We only observe realized returns and excess consumption growth rate. Since future returns and future excess consumption growth rate are unobserved, an unobserved component model is more suitable to model the present-value model.
We need to construct the most efficient estimates of unobservable expectations given available data that improves upon the predictive regressions.

Expected returns and expected excess consumption growth rate are assumed as latent variables and are modeled as AR(1) process. Similar approach has been applied in a series of papers such as Binsbergen and Koijen (2010) that estimate the expected stock returns and dividend growth by assuming an AR(1) process for the two variables. Fama and French (1988), Campbell (1991), Pastor and Stambaugh (2009) among others have argued that expected returns are likely to be persistent.

\[ r_{t+1}^e = \delta_0 + \delta_1 (r_t^e - \delta_0) + \epsilon_{t+1}^{re} \]  

\[ \Delta x_{t+1}^e = \gamma_0 + \gamma_1 (\Delta x_t^e - \gamma_0) + \epsilon_{t+1}^{xe} \]  

where \( r_t^e \equiv E_t(r_{t+1}^a) \) and \( \Delta x_t^e \equiv E_t(\Delta x_{t+1}) \). The shocks \( \epsilon_{t+1}^{re} \) and \( \epsilon_{t+1}^{xe} \) are independent and identically distributed over time. Realized asset return is equal to expected asset return plus an idiosyncratic shock\(^6\):

\[ r_{t+1}^a = r_t^e + \epsilon_{t+1}^r \]

The realized excess consumption growth rate is equal to expected excess consumption growth rate plus an idiosyncratic shock:

\[ \Delta x_{t+1} = \Delta x_t^e + \epsilon_{t+1}^x \]

Plugging equations (2.10) and (2.11) into equation (2.9) and solving, we get:

\[ x_t - a_t = A + B_1 (r_t^e - \delta_0) - B_2 (\Delta x_t^e - \gamma_0) \]

\(^6\)Return on total household assets is: \( r_{t+1}^a = \log\left( \frac{A_{t+1}}{A_t + Y_t - C_t} \right) \)
where $A = \rho \kappa + \frac{\rho(\delta_0 - \gamma_0)}{1 - \rho}$, $B_1 = \frac{\rho \delta_1}{1 - \rho \delta_1}$, and $B_2 = \frac{\rho \gamma_1}{1 - \rho \gamma_1}$. The above equation implies that the log of excess consumption-assets ratio is linear in the expected consumption growth rate and expected returns. There are three shocks in the above model: shock to expected excess consumption growth rate ($\epsilon^{re}_{t+1}$), shock to expected returns ($\epsilon^{re}_{t+1}$), and shock to realized excess consumption growth rate ($\epsilon^{xe}_{t+1}$). These shocks have a mean zero and have the following variance-covariance matrix:

$$\sum = \text{var} \begin{bmatrix} \epsilon^{re}_t \\ \epsilon^{xe}_t \\ \epsilon^{x}_t \end{bmatrix} = \begin{bmatrix} \sigma_{re}^2 & \sigma_{rexe} & \sigma_{rex} \\ \sigma_{rexe} & \sigma_{xe}^2 & \sigma_{xex} \\ \sigma_{rex} & \sigma_{xex} & \sigma_x^2 \end{bmatrix}$$

In this general correlation structure some of the parameters may be unidentified. Cochrane (2008) and Morley et al. (2003) suggest that we need to impose restrictions on covariance structure in the state space model to achieve identification. For our purpose, we assume that the covariance between shocks to expected returns and expected excess consumption is zero.

### 2.3.2 Variance Decomposition

Since both $r^e_t$ and $\Delta x^e_t$ follow an AR(1) process, the variance of excess consumption-assets ratio can be decomposed using equation (2.12) as:

$$\text{var}(x_t - a_t) = B_1^2 \text{var}(r^e_t) + B_2^2 \text{var}(\Delta x^e_t) - 2B_1B_2 \text{cov}(\Delta x^e_t, r^e_t)$$

$$\text{var}(x_t - a_t) = \frac{(B_1\sigma_{re})^2}{1 - \delta_1^2} + \frac{(B_2\sigma_{xe})^2}{1 - \gamma_1^2} - \frac{2B_1B_2\sigma_{xere}}{1 - \gamma_1\delta_1}$$ (2.13)

The above formula implies that proportion of variation in excess consumption-assets ratio explained by expected returns equals $B_1^2 \text{var}(r^e_t)$, and percentage of variation explained by excess consumption equals $B_2^2 \text{var}(\Delta x^e_t)$. 
Table 2.1: Variance Decomposition of Excess Consumption-Assets Ratio

<table>
<thead>
<tr>
<th>Term</th>
<th>Estimate</th>
<th>Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{var}(x_t-a_t) )</td>
<td>0.0242</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{(B_1\sigma_x)^2}{1-\delta_1^2} )</td>
<td>0.0237</td>
<td>0.918</td>
</tr>
<tr>
<td>( \frac{(B_2\sigma_x)^2}{1-\gamma_1^2} )</td>
<td>0.0021</td>
<td>0.081</td>
</tr>
</tbody>
</table>

Table 2.1 presents the variance decomposition results for excess consumption-assets ratio from the estimated state space model. As reported in Table 2.1, most of the variation in \( x_t - a_t \) is explained by asset returns. The result show that expected returns explain 92% of the variation in the excess consumption-assets ratio. However, variations in expected excess consumption explains about 8% of the overall variation in the ratio. For identification of the state-space system, we assume that the correlation between the expected return and expected excess consumption equals zero.

2.4 Data Description

The data in this essay includes excess consumption-assets ratio, return on assets, and excess consumption\(^7\). Quarterly data has been used starting in the first quarter of 1952. The sample period runs through the last quarter of 2006. The data on total household assets is based on the Federal Reserve Board’s Flow of Funds net worth series. We subtract the value of consumer durables from the net worth series because our measure of consumption includes outlays on durable goods\(^8\). Consumption data has been obtained from National Income and Product Account (NIPA) Tables. Labor income has been constructed using the data from NIPA, and according to the procedure defined in Lettau and Ludvigson (2001). Labor income is wages and wages.

\(^7\)In NIPA, consumption and labor income series are reported on an annualized basis. Therefore, the excess consumption series constructed using data from NIPA is divided by four. This adjusts the excess consumption series to arrive at the correct figure for the average reduction in assets per quarter due to consumption in excess of labor income. However, the chart in Fig.1 reports the series for excess consumption on an annualized basis.

\(^8\)Our measure of consumption is based on Whelan’s (2008) approach.
salaries plus transfer payments plus other labor income minus personal contributions for social insurance minus labor taxes. Labor taxes are defined by imputing a share of personal tax and non-tax payments to labor income with the share calculated as the ratio of wages and salaries to the sum of wages and salaries, proprietors’ income, and rental, dividend, and interest income. All data on asset valuation, consumption, and labor income is in nominal terms. Consumption, assets, and income series is deflated by the price index of the total personal consumption expenditure to obtain real consumption, asset returns, and income\(^9\). I also examine the predictive ability of expected asset returns in explaining variations in real stock returns and compare it with the predictive ability of the consumption based predictors such as \(cay\)\(^10\) and \(x_t - a_t\). The data on real stock returns has been obtained from the Standard & Poor’s (S&P) composite Index for which quarterly earnings data are available\(^11\).

---

\(^9\)Whelan (2008) also deflates asset returns, consumption and labor income series by the price index of personal consumption expenditure. Also, see Palumbo, Rudd, and Whelan (2006).

\(^10\)The data on \(cay\) has been obtained from Martin Lettau’s website http://faculty.haas.berkeley.edu/lettau/data.

\(^11\)The S&P index data has been obtained from S&P’s website: http://www.standardandpoors.com/indices.
Figure 2.1: Excess consumption-assets ratio

The above figure describes the ratio of excess consumption to total household assets where excess consumption is expressed at an annual rate.

2.5 Model Estimation

State Space Representation

The model has two latent variables: expected returns $r_t^e$ and expected excess consumption growth rate $\Delta x_t^e$. Demeaned state variables are defined as:

$$\Delta x_t^e = \gamma_0 + \Delta \hat{x}_t^e$$

$$r_t^e = \delta_0 + \hat{r}_t^e$$

There are two transition equations associated with the demeaned latent variables:

$$\hat{r}_{t+1}^e = \delta_1 \hat{r}_t^e + \epsilon_t^{re}$$

$$\Delta \hat{x}_{t+1}^e = \gamma_1 \Delta \hat{x}_t^e + \epsilon_t^{xe}$$
and two measurement equations are:

\[
\Delta x_{t+1} = \gamma_0 + \Delta \hat{x}_t^e + \epsilon_{t+1}^x
\]

\[
x_t - a_t = A + B_1 \hat{r}_t^e - B_2 \Delta \hat{x}_t^e
\]

In the above measurement equations system, second equation does not contain any error term. Therefore, we can use the method employed by Binsbergen and Koijen (2010) and substitute out the latent variable \( r_t^e \). This makes the state space system smaller by reducing the number of transition equations. The final state space system has one transition equation and two measurement equations:

\[
\Delta \hat{x}_t^e = \gamma_1 \Delta \hat{x}_t^e + \epsilon_{t+1}^{x_e} \tag{2.14}
\]

Two measurement equations are:

\[
\Delta x_{t+1} = \gamma_0 + \Delta \hat{x}_t^e + \epsilon_{t+1}^x \tag{2.15}
\]

\[
x_{t+1} - a_{t+1} = (1 - \delta_1)A - B_2(\gamma_1 - \delta_1)\Delta \hat{x}_t^e + \delta_1(x_t - a_t) + B_1\epsilon_{t+1}^{r_e} - B_2\epsilon_{t+1}^{x_e} \tag{2.16}
\]

The measurement equation for excess consumption growth rate and the excess consumption-assets ratio implies the measurement equation for returns.

Transition equation 2.14 is represented as:

\[
\begin{pmatrix}
\Delta \hat{x}_t^e \\
\epsilon_{t+1}^x \\
\epsilon_{t+1}^{r_e} \\
\epsilon_{t+1}^{x_e}
\end{pmatrix} =
\begin{pmatrix}
\gamma_1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\Delta \hat{x}_{t-1}^e \\
\epsilon_t^x \\
\epsilon_t^{r_e} \\
\epsilon_t^{x_e}
\end{pmatrix} +
\begin{pmatrix}
0 \\
\epsilon_{t+1}^x \\
\epsilon_{t+1}^{r_e} \\
\epsilon_{t+1}^{x_e}
\end{pmatrix}
\]

Measurement equations (2.15-2.16) are represented as:
\[
\begin{pmatrix}
\Delta x_{t+1} \\
x_{t+1} - a_{t+1}
\end{pmatrix}
= \begin{pmatrix}
\gamma_0 \\
A(1 - \delta_1)
\end{pmatrix}
+ \begin{pmatrix}
0 & 0 \\
0 & \delta_1
\end{pmatrix}
\begin{pmatrix}
\Delta x_t \\
x_t - a_t
\end{pmatrix}
\]

\[
+ \begin{pmatrix}
1 & 1 & 0 & 0 \\
-B_2(\gamma_1 - \delta_1) & B_1 & -B_2
\end{pmatrix}
\begin{pmatrix}
\Delta x_e^e \\
\epsilon_{t+1}^x \\
\epsilon_{t+1}^{re} \\
\epsilon_{t+1}^{xe}
\end{pmatrix}
\]

We can estimate the above state space system using the maximum likelihood estimation via the Kalman filter. A detailed description of the Kalman filter can be found in Clark (1989), Harvey (1989), and Stock and Watson (1991).

The Kalman filter is a recursive procedure for computing the optimal estimate of the unobserved-state vector \([\Delta \hat{x}_t]'\) given the hyperparameters of the model. It consists of two steps: prediction and updating. Let \(\beta_t = [\Delta \hat{x}_t]', \gamma = [\epsilon_{t+1}^x, \epsilon_{t+1}^{re}, \epsilon_{t+1}^{xe}]'\), and the covariance of \(\beta\) be \(P_{t|T} = \text{E}((\beta_{t|T} - \beta_t)(\beta_{t|T} - \beta_t)')\). The prediction equations are:

\[
\beta_{t|t-1} = F\beta_{t-1|t-1}
\]

\[
P_{t|t-1} = FP_{t-1|t-1}F' + Q
\]

\[
f_{t|t-1} = \text{E}[v_{t|t-1}^2]
\]

The updating equations are:

\[
\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1}Z'G_t^{-1}v_t
\]

\[
P_{t|t} = P_{t|t-1} - P_{t|t-1}V'G_t^{-1}v_t
\]

where the prediction error is \(v_{t|t-1} = Y_t - Y_{t-1}\), the covariance of the prediction error
is \( f_{t|t-1}, G_t = E(v_t, v'_{t}) = VP_{t|t-1}V' \), the initial estimates of F, V, and Q are given. The estimation problem is then to maximize the Gaussian log likelihood function:

\[
\log L(\theta) = -T \log(2\pi) - \frac{T}{2} \log(\det f_t) - \frac{1}{2} \sum_{t=1}^{T} v'_{t} f_{t}^{-1} v_t
\]

### 2.6 Estimation Results

Table 2.2 reports the maximum likelihood estimates of the parameters of the present-value model described in equations (2.13-2.15). The estimated AR parameters for expected returns is persistent with a coefficient of 0.964. AR parameter for excess consumption growth rate is 0.681. The high persistence of expected returns is consistent with a variety of economic models in which the expected return varies over time. Binsbergen and Koijen (2010) found that expected return on stock is persistent with a coefficient of 0.932. Our finding is also consistent with Fama and French (1988), Campbell and Cochrane (1999), Pastor and Stambaugh (2009), and Rytchkov (2008). The unconditional mean for expected returns and expected excess consumption growth rate is 1.7% and 0.95% respectively. The estimated correlation structure between expected returns and expected excess consumption growth rate provides us some interesting results.\(^{12}\) The correlation between excess consumption and expected return is positive and equals 0.622. One interpretation of this positive and significant correlation is that an upward rise in excess consumption today must correspond to news that future returns will be higher. This result conforms with the theoretical budget constraint derived by Campbell and Mankiw (1989), Lettau and Ludvigson (2001) and Whelan (2008). I also find that expected excess consumption growth rate and realized excess consumption growth are negatively correlated with

\(^{12}\)It should be noted that different combinations of correlation structure between shocks can be used to identify the state space model. For example, we can allow non-zero correlation between shock to expected return and shock to expected excess consumption growth. We choose the correlation structure based on likelihood value.
a correlation coefficient of -0.685. This result suggests that an upward revision in excess consumption today corresponds to a downward revision in expected future excess consumption growth.

Table 2.2: **Maximum Likelihood Estimates of Hyperparameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x$</td>
<td>0.0443</td>
<td>0.0031</td>
</tr>
<tr>
<td>$\sigma_{\tau e}$</td>
<td>0.0019</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\sigma_{x e}$</td>
<td>0.0162</td>
<td>0.0044</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.0177</td>
<td>0.0032</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.9641</td>
<td>0.0153</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>0.0095</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.6812</td>
<td>0.0821</td>
</tr>
<tr>
<td>$\rho_{x e x}$</td>
<td>-0.6855</td>
<td>0.1537</td>
</tr>
<tr>
<td>$\rho_{e x x}$</td>
<td>0.6219</td>
<td>0.1493</td>
</tr>
</tbody>
</table>

Table 2.3 reports the estimated value of implied present value parameters. $B_1$ and $B_2$ are the loadings on expected returns and expected excess consumption growth, which depend on constant $\rho$ and respective persistence parameters $\delta_1$ and $\gamma_1$. High persistence of expected return leads to a much higher loading on expected return in the present-value relationship.

Table 2.3: **Implied Present-Value Model Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>-4.8091</td>
<td>0.472</td>
</tr>
<tr>
<td>$B_1$</td>
<td>21.5368</td>
<td>7.698</td>
</tr>
<tr>
<td>$B_2$</td>
<td>2.0792</td>
<td>0.771</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.991</td>
<td></td>
</tr>
</tbody>
</table>
2.6.1 Comparison with the Benchmark Model

Using the expected asset returns and expected excess consumption growth rate from our approach, the R-squared values for asset returns and excess consumption growth rate is computed\(^\text{13}\) and compare this with the benchmark model which uses the predictive OLS regressions\(^\text{14}\).

\[
R^2_{\text{returns}} = 1 - \frac{\text{var}(r_{t+1} - r_t^a)}{\text{var}(r_{t+1})}
\]

\[
R^2_{\Delta x} = 1 - \frac{\text{var}(\Delta x_{t+1} - x_t^e)}{\text{var}(\Delta x_{t+1})}
\]

The R-squared value for expected returns is 8.3\% and for excess consumption growth rate it equals 9.17 \% \(^\text{15}\). The results are reported in Table 2.4.

<table>
<thead>
<tr>
<th>(R^2_{\text{returns}})</th>
<th>0.083</th>
<th>(R^2_{\Delta x})</th>
<th>0.091</th>
</tr>
</thead>
</table>

Next, we compare the result of our model to a benchmark model where only lagged excess consumption-assets ratio is used as an explanatory variable\(^\text{16}\).

\[r_{t+1} = \alpha_1 + \beta_1(x_t - a_t) + \epsilon_{t+1}\]

\(^\text{13}\)Return on total household assets is: \(r_t^a = \log\left(\frac{A_{t+1}}{A_t + Y_t - C_t}\right)\)

\(^\text{14}\)This is in spirit of Binsbergen and Koijen (2010).

\(^\text{15}\)See also Harvey (1989).

\(^\text{16}\)Whelan (2008) studies regressions of the form \(\sum_{k=1}^{N} \rho_k^a(r_{t+1+k}^a - \Delta x_{t+k}) = \gamma(x_t - a_t) + \epsilon_{t+N}\). In addition, he also examines separate forecasting regressions for \(r_t\) and \(\Delta x_t\).
\[
\Delta x_{t+1} = \alpha_2 + \beta_2 (x_t - a_t) + \epsilon_1'_{t+1}
\]

The results are reported in Table 2.5. For the first regression, returns on excess consumption-assets ratio has a predictive coefficient of \( \beta_1 = 0.03 \) with a R-squared value of 3.9%. The slope coefficient is positive and significant with a t-statistic of 2.33. The second regression of excess consumption growth rate has a predictive coefficient of \( \beta_2 = -0.045 \) with a R-squared of 1.9%. The slope coefficient is negative and weakly significant with a t-statistic of -1.77. The result suggests that the estimated expected return on household assets and expected excess consumption growth rate obtained from state-space approach perform significantly better than when only lags of excess consumption-assets ratio are used as predictors. The predictive superiority arises from the fact that the unobserved component approach uses more information and aggregates the past information in a parsimonious way. In addition to the lagged excess consumption-assets ratio, the state space approach uses the entire history of excess consumption growth rates and excess consumption-assets ratio to predict future returns and excess consumption growth rates.

\[
\Delta x_t = \alpha^x + \sum_{j=0}^{\infty} \beta^x_{1j} (x_{t-j-1} - a_{t-j-1}) + \sum_{j=0}^{\infty} \beta^x_{2j} \Delta x_{t-j-1} + \epsilon_t^x
\]

Therefore, latent variable approach is more flexible than a simple OLS regression. The Kalman filter allows the data to form the best linear predictor, making the forecast more precise.
Table 2.5: OLS predictive regressions

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>$r_t$</th>
<th>$\Delta x_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0.160</td>
<td>-0.208</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.088)</td>
</tr>
<tr>
<td>$x_t - a_t$</td>
<td>0.030</td>
<td>-0.045</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.039</td>
<td>0.019</td>
</tr>
</tbody>
</table>

2.6.2 Stock Return Predictability

As suggested by Lettau and Ludvigson (2001), most of the variation in total asset returns can be explained by real stock market returns. Therefore, I try to analyze the ability of expected returns to forecast stock returns and compare it with the benchmark model of predictive regressions of stock returns on $x_t - a_t$ ratio, and $cay$ respectively. The results are summarized in Table 2.6 below. 5.83 percent of the variation in the one quarter ahead movement in real stock returns is explained by expected asset returns, whereas $x_t - a_t$ ratio explains about 2.5 percent of the variation. $cay$ explains 5.97 percent of the variation in one quarter ahead real stock returns. The results suggest that the filtered returns significantly improves upon the predictive power of $x_t - a_t$ ratio. This is not surprising since one of the objectives of this approach is to reduce the noise associated with $x_t - a_t$ as a proxy for expected returns.

Table 2.6: Stock return predictability regressions

<table>
<thead>
<tr>
<th>Returns on stocks</th>
<th>$r_t^C$</th>
<th>$x_t - a_t$</th>
<th>$cay$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>0.024(0.25)</td>
<td>10.45(0.00)</td>
<td></td>
<td>0.0583</td>
</tr>
<tr>
<td>Model 2</td>
<td>1.467(0.045)</td>
<td>0.30(0.05)</td>
<td></td>
<td>0.0251</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.024(0.24)</td>
<td>4.77(0.00)</td>
<td></td>
<td>0.0597</td>
</tr>
</tbody>
</table>
2.7 Conclusion

In this essay, a latent-variables approach has been proposed to estimate expected asset returns and excess consumption growth rate within a present-value model of consumption. Acknowledging that expected returns and excess consumption growth rate are unobservable, the Kalman filter technique is used to extract them from the observed history of realized return on assets and realized excess consumption growth rate. To apply the Kalman filter to the unobserved component model, expected returns and expected excess consumption growth rate are assumed to follow a parsimonious autoregressive progress. Since returns is the residual in accounting identity, we can trace the time variation in returns once we estimate the expected excess consumption growth rate. This approach is combined with Whelan’s (2008) version of the present-value model of consumption that implies that the excess consumption-assets ratio can be expressed as a function of present discounted value of expected excess consumption growth rate and expected asset returns. This model also has some practical advantages over the earlier models by Campbell and Mankiw (1989) and Lettau and Ludvigson (2001). Notably, it does not rely on untestable assumptions about unobserved variables or require estimation of unknown parameters to operationalize the forecasting equation.

Filtered series for returns and excess consumption from this approach is a good predictor for future returns and future excess consumption growth rate. Almost 9 percent of the variation in excess consumption growth rate is explained by filtered series on excess consumption. The constructed predictor for asset returns can explain about 8.3 percent of the variation in asset returns. In contrast, the predictive regression of excess consumption on excess consumption-assets ratio lacks the power to predict future consumption, with an R-squared of 1.9 percent. The predictive regression of returns on excess consumption-assets ratio explains about 3.9 percent of the variation in asset returns. The variance decomposition of the
excess consumption-assets ratio suggest that expected returns account for about 92 percent variation in excess consumption-assets ratio and 8 percent of the variation of the excess consumption-assets ratio is related to expected excess consumption growth rate variation.

The estimated expected returns also has significant explanatory power for future movements in real stock returns. The results suggest that filtered series for returns improve upon the predictive power of the excess consumption-assets ratio.
Chapter 3

Second Essay: Consumption-Wealth Ratio and Expected Housing Return

3.1 Introduction

One of the channels through which the housing market affects the overall macroeconomic activity is through its impact on the household balance sheet. There is a consensus in the economic literature and policymaking about housing wealth being one of the determinants of consumption expenditure\(^1\). The linkage between housing wealth and consumption suggests that changes in household consumption should contain information about expected changes in housing market wealth.

The traditional literature on the relationship between housing market wealth and consumption has mainly focused on estimating the consumption response to changes in the housing market wealth. We take a different approach in this essay. Rather than estimating consumption response to changes in housing wealth, we utilize the information in consumption, income and observable assets to estimate expected housing returns. To do so, we combine a modified version of a present-value model proposed by Whelan (2008) with an unobserved component model. Whelan’s modified present-value model suggests that an upward surprise in excess consumption-assets ratio—a modified measure of consumption-wealth ratio—today must correspond to lower than average excess consumption growth or higher than average asset returns in future\(^2\). In simple words, if representative household’s


\(^2\)Excess consumption is defined as consumption in excess of labor income.
consumption increases relative to its housing wealth, it may be either due to higher than expected housing wealth growth or lower than average consumption growth. Therefore changes in expected housing return should be reflected in the current consumption decision of the households. The challenge, therefore, is to estimate the expected return on housing assets using the present information set that contains information about current and past excess consumption, return on financial assets, and return on housing assets. One simple method that can be applied to estimate expected housing returns is the standard VAR approach, where lags of consumption-wealth ratio and past asset returns can be used as predictors of housing return. However, the application of traditional VAR or univariate autoregression approach in the present context is fraught with limitations, as has been suggested by Binsbergen and Koijen (2010) and Rytchkov (2008) in the context of stock returns literature. In particular, the VAR approach only uses finite lags to predict the variable of interest, and may miss individually small but possibly important moving average terms in the long run as pointed out by Cochrane (2008).

In this essay, we use Kalman filter to extract expected housing asset returns from the present-value model. This approach allows us to expand the information set by using the information from the whole history of observed housing asset returns, financial asset returns, and excess consumption growth rather than using just finite lags of excess consumption-assets ratio. The state space approach provides us an estimate of expected housing return that uses information from an expanded information set. Since the estimate of expected housing returns in our model uses more information than the conventional finite lag approach, it should yield a better forecast of realized housing returns and house price growth. The results obtained in this essay broadly support this hypothesis. In particular, we find that the filtered returns explains 18% of the variation in one-period ahead housing asset returns and

---

3Note that housing asset or housing wealth has been used interchangeably in this essay. Section 4 explains how housing asset or housing wealth is calculated.

4The correlation coefficient between housing assets return and house price growth is 0.51.
22% of the variation in one-period ahead house price growth rate. Our results show that filtered series of expected housing return obtained from the present-value model is a superior in-sample and out-of-sample predictor of realized housing asset returns compared to other predictors like mortgage rate, price-rent ratio, yield spread, $cay^5$, and real GDP growth. The superiority of the predictive power of the filtered return is also statistically significant.

The rest of the essay is organized as follows. The next section provides a brief literature review; section 3.3 proposes an unobserved component model to estimate the present value model of consumption. Section 3.4 describes the data. Section 3.5 provides the empirical methodology. Section 3.6 documents the main findings on the predictability of housing asset returns. Section 3.7 concludes.

### 3.2 Brief Literature Review

This essay combines the literature on the present-value models of consumption with the literature on the housing market predictability. Present-value models have been applied extensively in macroeconomics and finance literature. For example, Campbell and Mankiw (1989) show that consumption-wealth ratio reflects information about expected returns on wealth and expected consumption growth rate. Lettau and Ludvigson (2001) also use consumption-wealth ratio as a proxy for future asset returns and show that whenever consumption-wealth ratio moves above/below its long-run value, wealth adjusts to correct for the disequilibrium. The literature on the estimation of marginal propensity to consume out of wealth is also based on the present-value model of consumption that states that current consumption depends on the present discounted value of life-time income and current wealth. A number of empirical studies have used this model and analyzed the impact of changes in $cay$

---

5Lettau-Ludvigson (2001) use estimated residual from a cointegrating regression of consumption, labor income and wealth ($cay$) as a proxy for expected asset returns.
housing wealth on consumption. Poterba (2000) finds that the traditional wealth effect estimate implies that for every dollar increase in wealth, consumption should increase by 2-10 cents. Case, Quigley and Shiller (2005, 2011) find strong evidence that variations in housing market wealth have important effects upon consumption. According to them, housing wealth effect on consumption is especially important in recent decades as institutional innovations have made it simple to extract cash from housing equity. Benjamin, Chinloy and Jud (2004) have shown that an additional dollar of real estate wealth increases consumption by 8 cents. They point out that with the availability of home equity loans and low-cost tax deductible refinancing, homeowners can access their housing to finance consumption.

The literature on the forecastability of housing market is substantial, starting with Case and Shiller (1989), who show that unlike stock returns, there is a significant predictive component in house prices. Other papers like Crawford and Fratantoni (2003) utilize ARIMA, GARCH, and regime switching univariate time series model to estimate the behavior of home price growth rates in California, Florida, Massachusetts, Ohio, and Texas. They find that regime-switching models perform better in-sample, while ARIMA and GARCH perform better in out-of-sample forecasting. Rapach and Strauss (2007) analyze the forecasting ability of a large number of potential predictors of state real housing price growth using an autoregressive distributed lag model framework. Guirguis, Giannikos, and Anderson (2005) use estimation methodologies where the estimated parameters are allowed to vary overtime. They find that the forecasts generated by the Kalman Filter and rolling GARCH techniques outperform the forecasts of all the other specifications considered. Our approach contributes to the literature by deriving an estimate of future housing market returns from a version of widely used present-value model of consumption.

In this essay, we estimate the expected return on housing assets from a present-
value model using an unobserved component approach. Using this filtered return, we examine the predictive power of our measure in forecasting realized housing asset returns and compare the forecasting performance with other popular predictors like mortgage rate, price-rent ratio, yield spread, and GDP growth, among others.

3.3 The Present-Value model and Expected Return on Housing

This section presents a modified present value model of consumption that is based on Whelan (2008) and Binsbergen and Koijen (2010). The present value of consumption links excess consumption-asset ratio to expected housing asset returns, expected financial asset return and expected excess consumption growth. We modify this present value model and apply Kalman filter to extract expected housing return. The household budget constraint can be described as follows:

\[ A_{t+1} = R^a_{t+1}(A_t + Y_t - C_t) \]  

(3.1)

where \( A_t \) is total household assets and equals sum of household assets and financial assets, \( R^a_{t+1} \) is the gross return on assets, \( Y_t \) is labor income, and \( C_t \) is consumption. Dividing across by \( A_t \) and taking logs we get:

\[ \Delta a_{t+1} = r^a_{t+1} + \log \left( 1 - \frac{C_t - Y_t}{A_t} \right) \]  

(3.2)

Define, excess consumption as \( X_t = C_t - Y_t \).

\footnote{For the US data series used in this study, which rely on a standard definition of labor income, consumption always exceed labor income. Therefore, \( X_t \) is always positive. One of the interpretations of this positive sign may arise from the fact that in addition to after tax labor income \( Y_t \), consumption is financed out of total wealth.}
Equation (3.2) can be rewritten as:

$$\Delta a_{t+1} = r_{t+1}^a + \log(1 - \exp(x_t - a_t))$$  \hspace{1cm} (3.3)

Total wealth is sum of housing wealth $H_t$ and stock market wealth $S_t$ i.e. $A_t = H_t + S_t$. The logarithm of total assets may be approximated as:

$$a_t = \omega h_t + (1 - \omega)s_t$$  \hspace{1cm} (3.4)

where $a_t$ is log of total asset, $h_t$ is the log of housing wealth, and $s_t$ is the log of stock of net financial assets, $\omega$ is the steady state share of housing assets in total assets and $(1 - \omega)$ is the steady state share of financial assets in total assets\footnote{This logarithmic approximation is widely used in economics and finance. For example, see page 820 Lettau and Ludvigson (2001). As long as the share of a particular component doesn’t explode over time, $\omega$ and $(1 - \omega)$ refer to the long-run averages of the share of different types of wealth.} \footnote{The sample average is used as the steady state share. In our case, the data suggests that $\omega = 0.25$ and $(1 - \omega) = 0.75.$}

The return on total assets can be decomposed into the return of its two components:

$$r_t^a \approx \omega r_t^h + (1 - \omega)r_t^s$$  \hspace{1cm} (3.5)

Substituting (3.4) and (3.5) into (3.3) gives

$$\Delta(\omega h_{t+1} + (1 - \omega)s_{t+1}) = (\omega r_t^h + (1 - \omega)r_t^s) + \log(1 - \exp(x_t - (\omega h_t + (1 - \omega)s_t)))$$  \hspace{1cm} (3.6)

In the above equation, $\log(1 - \exp(x_t - (\omega h_t + (1 - \omega)s_t)))$ is a non-linear function. Taking a first order Taylor expansion around the mean $(x - (\bar{\omega h} + (1 - \bar{\omega})\bar{s}))$ results in the following approximation:

$$\log(1 - \exp(x_t - (\omega h_t + (1 - \omega)s_t)) \approx \kappa + (1 - \rho^{-1})(x_t - (\omega h_t + (1 - \omega)s_t))$$  \hspace{1cm} (3.7)
where \( \rho \equiv 1 - \exp(\bar{\pi} - (\omega \bar{h} + (1 - \omega) \bar{s})) \) and \( \kappa \) is a constant and equals \( \log(1 - \exp(\bar{\pi} - (\omega \bar{h} + (1 - \omega) \bar{s}))) - (1 - {\rho}^{-1})(\pi - (\omega \bar{h} + (1 - \omega) \bar{s})) \). \(^9\)

Substituting equation (3.7) into equation (3.6) and rearranging we obtain:

\[
x_t - a_t \approx \rho((\omega r_{t+1}^h + (1 - \omega)r_{t+1}^s) + \kappa - \Delta x_{t+1}) + \rho(x_{t+1} - (\omega h_{t+1} + (1 - \omega)s_{t+1}))
\] (3.8)

Solving forward via repeated substitution and imposing the transversality condition \( \lim_{j \to \infty} \rho^{-j}(x_{t+j} - (\omega h_{t+j} + (1 - \omega)s_{t+j})) = 0 \) we obtain:

\[
x_t - (\omega h_t + (1 - \omega)s_t) \approx \frac{\rho\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^j(\omega r_{t+j}^h + (1 - \omega)r_{t+j}^s - \Delta x_{t+j})
\] (3.9)

The above equation implies that log ratio of excess consumption-asset ratio is stationary since the right hand side is stationary. This suggests that if the excess consumption-asset ratio goes above its long-run value, either expected excess consumption growth will decline in future or expected housing return or expected stock return will go up in future. If one takes expectations at time \( t \), it yields the following expression:

\[
x_t - (\omega h_t + (1 - \omega)s_t) \approx \frac{\rho\kappa}{1 - \rho} + E_t \sum_{j=1}^{\infty} \rho^j(\omega r_{t+j}^h + (1 - \omega)r_{t+j}^s - \Delta x_{t+j})
\] (3.10)

The above model is central to the estimation of expected housing return. There are two ways to estimate the unobserved expected return: the conventional approach and the unobserved component approach. The conventional approach uses finite lags of excess consumption growth, realized housing return and realized stock return or estimate cointegrating relationship between consumption, labor income, housing

\(^9\)For detailed derivation see Kishor and Kumari (2011).
wealth and stock market wealth and use the cointegrating residual. The second approach is to use the unobserved component model to estimate the unobserved returns on assets and expected excess consumption growth. Since expected returns and future expected excess consumption growth rate are unobserved, an unobserved component model is more suitable to model the present value model of consumption. Since the estimate of expected housing returns in the unobserved component model uses more information than the conventional finite lag approach, it should yield a better forecast of realized housing returns. Following Campbell (1991), we model expected housing asset returns, expected financial asset returns, and expected excess consumption growth rate as AR(1) process.

\[
\omega_{r_{t+1}} = \delta_0 + \delta_1 (\omega_{r_t} - \delta_0) + \epsilon_{r_{he}^t+1} \tag{3.11}
\]

\[
(1 - \omega) r_{t+1}^{se} = \psi_0 + \psi_1 ((1 - \omega) r_t^{se} - \psi_0) + \epsilon_{r_{se}^t+1} \tag{3.12}
\]

\[
\Delta x_t^{e} = \gamma_0 + \gamma_1 (\Delta x_t^{e} - \gamma_0) + \epsilon_{x_t}^{e} \tag{3.13}
\]

where \( r_{t}^{he} \equiv E_t(r_{t+1}^{h}) \), \( r_{t}^{se} \equiv E_t(r_{t+1}^{s}) \), and \( \Delta x_t^{e} \equiv E_t(\Delta x_{t+1}) \). The shocks \( \epsilon_{r_{he}^t+1} \), \( \epsilon_{r_{se}^t+1} \), and \( \epsilon_{x_t}^{e} \) are independent and identically distributed. Realized housing asset return and financial asset return is equal to expected housing asset return and expected financial asset return plus an idiosyncratic shock.

\[
\omega_{r_{t+1}} = \omega_{r_{t}^{he}} + \epsilon_{r_{t+1}}^{rh} \]

\[\text{Similar approach has been applied in a series of papers such as Binsbergen and Koijen (2010), Fama and French (1988), and Pastor and Stambaugh (2009) among others.}\]

\[\text{The one question that immediately comes to mind is whether AR(1) assumption is sufficient to capture the dynamics of these variables. To that end, we also perform ARIMA modeling of the realized housing asset return, realized financial asset return and realized excess consumption growth and the results suggest that AR(1) model is sufficient to capture the dynamics.}\]

\[\text{In order to convert the present value model in equation (3.10) into a linear measurement equation of a state space system, it is important that the unobserved variables follow a stationary process. Therefore, we model unobserved variables as an autoregressive process instead of a random walk process.}\]
\[(1 - \omega)r_{t+1}^s = (1 - \omega)r_t^{se} + \epsilon_{t+1}^{rs}\]

The realized excess consumption growth rate is equal to expected excess consumption growth rate plus an idiosyncratic shock.

\[\Delta x_{t+1} = \Delta x_t^e + \epsilon_{t+1}^x\]

Substituting equations (3.11-3.13) into equation (3.10) and solving we get:

\[
x_t - a_t = \frac{\rho}{1 - \rho}\kappa + \frac{\rho(\delta_0 + \psi_0 - \gamma_0)}{1 - \rho} + \frac{\rho \delta_1}{1 - \rho \delta_1}(\omega r_t^{he} - \delta_0) + \frac{\rho \psi_1}{1 - \rho \psi_1}((1 - \omega)r_t^{se} - \psi_0) - \frac{\rho \gamma_1}{1 - \rho \gamma_1}(\Delta x_t^e - \gamma_0) \tag{3.14}\]

Let \(A = \frac{\rho}{1 - \rho}\kappa + \frac{\rho(\delta_0 + \psi_0 - \gamma_0)}{1 - \rho}\), \(B_1 = \frac{\rho \delta_1}{1 - \rho \delta_1}\), \(B_2 = \frac{\rho \psi_1}{1 - \rho \psi_1}\), and \(B_3 = \frac{\rho \gamma_1}{1 - \rho \gamma_1}\).

Equation (3.14) can be rewritten as:

\[
x_t - a_t = A + B_1(\omega r_t^{he} - \delta_0) + B_2((1 - \omega)r_t^{se} - \psi_0) - B_3(\Delta x_t^e - \gamma_0) \tag{3.15}\]

The above equation is a linear relationship between log of excess consumption-assets ratio, expected housing asset returns, expected financial asset returns, and expected excess consumption growth rate. There are five shocks in the above model: shock to expected excess consumption growth rate \(\epsilon_{t+1}^{xe}\), shock to expected housing asset returns \(\epsilon_{t+1}^{rhe}\), shock to expected financial asset returns \(\epsilon_{t+1}^{rse}\), shock to realized excess consumption growth rate \(\epsilon_{t+1}^x\) and shock to realized return on housing assets \(\epsilon_{t+1}^h\). These shocks have a mean zero and have the following variance-covariance

\[\text{\textsuperscript{13}}\text{Even though }\omega\text{ enters the measurement equation, it doesn’t affect the estimated dynamics of expected return on housing and stock.}\]
matrix:

\[
\begin{bmatrix}
\epsilon_{xt}^e \\
\epsilon_{t}^{rhe} \\
\epsilon_{t}^{rse} \\
\epsilon_{t}^x \\
\epsilon_{t}^{rh}
\end{bmatrix}
\begin{bmatrix}
\sigma_{xe}^2 & \sigma_{xeh} & \sigma_{xse} & \sigma_{xhx} & \sigma_{xeh} \\
\sigma_{xeh} & \sigma_{hhe}^2 & \sigma_{hse} & \sigma_{hxh} & \sigma_{hhe} \\
\sigma_{xse} & \sigma_{hse} & \sigma_{sse}^2 & \sigma_{shx} & \sigma_{sesh} \\
\sigma_{xhx} & \sigma_{hxh} & \sigma_{sesh} & \sigma_{xhx}^2 & \sigma_{xhx} \\
\sigma_{xeh} & \sigma_{hhe} & \sigma_{sesh} & \sigma_{xhx} & \sigma_{hhe}^2
\end{bmatrix}
\]

In the general correlation structure, some of the parameters may be unidentified. Following Cochrane (2008) and Morley et al. (2003) we impose restrictions on the covariance structure to achieve identification. We follow Binsbergen and Koijen (2010) identification strategy and assume that covariance between realized return on housing assets and realized excess consumption growth is uncorrelated with shocks to the unobserved state variables. This implies that \( \sigma_{xeh} = \sigma_{hhe} = \sigma_{sesh} = \sigma_{xhx} = \sigma_{sesh} = \sigma_{xhx} = 0 \). In addition, we assume that shocks to realized excess consumption growth and realized return on housing assets are uncorrelated, that is \( \sigma_{xrh} = 0 \).

To summarize, our approach converts the present-value model as represented by equation (3.10) into a linear measurement equation of state space system represented by equation (3.15). To do so, we assume a simple autoregressive structure for unobserved variables: expected return on housing \( r_{t}^{he} \), expected return on financial asset \( r_{t}^{se} \) and expected excess consumption growth \( \Delta x_{t}^{e} \).

### 3.3.1 Data Description

We use quarterly data starting in the first quarter of 1952. The sample period runs through the last quarter of 2006. The data in this essay includes excess consumption-assets ratio, return on housing assets, return on financial assets, and excess consump-

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[14] Alternatively, we can impose zero covariance restriction on one of the covariances between shocks to expected excess consumption growth, expected housing return and expected stock returns. In the empirical section, we also explore this approach.
Our measure of consumption includes outlays on durable goods\textsuperscript{16}. The data on consumption has been obtained from the National Income and Product Account (NIPA) Tables. Labor income has been constructed using data from the NIPA, and according to the procedure defined in Lettau and Ludvigson (2001)\textsuperscript{17}. Labor income is wages and salaries plus transfer payments plus other labor income minus personal contributions for social insurance minus labor taxes. Labor taxes are defined by imputing a share of personal tax and non-tax payments to labor income with the share calculated as the ratio of wages and salaries to the sum of wages and salaries, proprietors’ income, and rental, dividend, and interest income.

The data on housing asset returns and financial asset returns is based on the Federal Reserve Board’s flow of funds net worth series. Net housing assets equals real estate minus home and commercial mortgages and outlays on durable goods\textsuperscript{18}. Net financial assets equals financial assets minus financial liabilities. Asset returns are calculated as quarterly changes in log of asset values.

All data on asset valuation, consumption and labor income is in nominal terms. We deflate consumption, housing assets, financial assets, and labor income series by the price index of total personal consumption expenditure to obtain real consumption, asset returns, and income\textsuperscript{19}. We also examine the predictive ability of popular forecasting variables in explaining the variation in housing returns as compared to our filtered measure of expected housing asset returns. We include data on mortgage rates, price-rent ratio, real GDP growth rate, excess consumption-assets ratio, yield spread, and \textit{cay}\textsuperscript{20}. Data on mortgage rates and yield spread has been obtained

\textsuperscript{15}In NIPA, consumption and labor income series are reported on an annualized basis. Therefore, the excess consumption series constructed using data from NIPA is divided by four. This adjusts the excess consumption series to arrive at the correct figure for the average reduction in assets per quarter due to consumption in excess of labor income.

\textsuperscript{16}Our measure of consumption is based on Whelan’s (2008) approach.

\textsuperscript{17}Whelan (2008) also follows the same approach.

\textsuperscript{18}We also include data on equipment and software owned by nonprofit organizations.

\textsuperscript{19}Whelan (2008) also deflates asset returns, consumption and labor income series by the price index of personal consumption expenditure. Also, see Palumbo, Rudd, and Whelan (2006).

\textsuperscript{20}Lettau-Ludvigson (2001) use estimated residual from a cointegrating regression of consumption, labor income and wealth (\textit{cay}) as a proxy for expected asset returns.
from the Federal Reserve Bank of St. Louis’s Fred data set\textsuperscript{21}. The data on the price-rent ratio has been obtained from Davis et. al (2008), who combine different data sources and provide a measure of the price-rent ratio for the US economy that goes back to 1961. We obtain real GDP data from the National Income and Product accounts data (NIPA). The data on cay has been obtained from Martin Lettau’s website\textsuperscript{22}. Data on nominal house price index has been obtained from the Case-Shiller house price index. We deflate nominal house price index by the price index of total personal consumption expenditure to obtain the real house price index.

3.4 Model Estimation

3.4.1 State Space Representation

The model has three latent variables: expected return on housing assets, expected return on financial assets, and expected excess consumption growth rate. The demeaned state variables are defined as:

\[ \Delta x_e^t = \gamma_0 + \Delta \hat{x}_e^t \]

\[ \omega r_{he}^t = \delta_0 + \omega r_{he}^t \]

\[ (1 - \omega)r_{se}^t = \psi_0 + (1 - \omega)r_{se}^t \]

There are three transition equations associated with the demeaned latent variables

\[ \Delta \hat{x}_e^{t+1} = \gamma_1 \Delta \hat{x}_e^t + \epsilon_{t+1}^{x_e} \] (3.16)

\[ \omega r_{he}^{t+1} = \delta_1 \omega r_{he}^t + \epsilon_{t+1}^{r_{he}} \] (3.17)

\textsuperscript{21}The data on mortgage rate is available from 1971 Q2
\textsuperscript{22}http://faculty.haas.berkeley.edu/lettau/data.
\[(1 - \omega)\hat{r}_{t+1}^{se} = \psi_1 (1 - \omega)\hat{r}_t^{se} + \epsilon_{t+1}^{se} \]  
(3.18)

Three measurement equations are:

\[\Delta x_{t+1} = \gamma_0 + \Delta \hat{x}_t^e + \epsilon_{t+1}^x\]  
(3.19)

\[\omega r_{t+1}^h = \delta_0 + \omega \hat{r}_t^{he} + \epsilon_{t+1}^h\]  
(3.20)

\[x_t - a_t = A + B_1 \omega \hat{r}_t^{he} + B_2 (1 - \omega) \hat{r}_t^{se} - B_3 \Delta \hat{x}_t^e\]  
(3.21)

The measurement equation for excess consumption growth rate, return on housing assets, and excess consumption-assets ratio implies the measurement equation for the return on financial assets. The above state space system can be estimated using the maximum likelihood estimation via the Kalman filter.

The transition equations in (3.16-3.18) is represented as:

\[
\begin{bmatrix}
\Delta \hat{x}_t^e \\
\hat{r}_t^{he} \\
\hat{r}_t^{se}
\end{bmatrix}
= \begin{bmatrix}
\gamma_1 & 0 & 0 \\
0 & \delta_1 & 0 \\
0 & 0 & \psi_1
\end{bmatrix}
\begin{bmatrix}
\Delta \hat{x}_{t-1}^e \\
\hat{r}_{t-1}^{he} \\
\hat{r}_{t-1}^{se}
\end{bmatrix}
+ \begin{bmatrix}
\epsilon_{t}^x \\
\epsilon_{t}^{rhe} \\
\epsilon_{t}^{rse}
\end{bmatrix}
\]

Measurement equation in equations (3.19-3.21) can be written as:

\[
\begin{bmatrix}
\Delta x_{t+1} \\
r_{t+1}^h \\
x_t - a_t
\end{bmatrix}
= \begin{bmatrix}
\gamma_0 \\
\delta_0 \\
A
\end{bmatrix}
+ \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-B_3 & B_1 & B_2
\end{bmatrix}
\begin{bmatrix}
\Delta \hat{x}_t^e \\
\hat{r}_t^{he} \\
\hat{r}_t^{se}
\end{bmatrix}
+ \begin{bmatrix}
\epsilon_{t+1}^x \\
\epsilon_{t+1}^{rhe} \\
0
\end{bmatrix}
\]

This model provides us estimates of expected return on housing assets ($r_{t}^{he}$), expected return on financial assets ($r_{t}^{se}$), and expected excess consumption growth ($\Delta x_{t}^e$).
3.5 Estimation Results

The estimated hyperparameters from the state space system described by equations (3.16-3.21) are shown in Table 3.1. The estimated AR parameter for return on financial assets is highly persistent with a coefficient of 0.97. The high persistence of expected financial asset returns is consistent with a variety of economic models in which the expected returns varies overtime. Binsbergen and Koijen (2010), Fama and French (1988), Campbell (1991), Campbell and Cochrane (1989), Pastor and Stambaugh (2009), and Rytchkov (2008) have also found expected return on stocks to be highly persistent. The corresponding estimate of the AR parameter for expected return on housing assets and excess consumption growth is 0.78 and -0.34 respectively. Our results suggest that expected return on financial assets and excess consumption growth rate are highly positively correlated at 0.93. This implies that households raise their excess consumption in anticipation of higher than expected

<table>
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<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
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<tr>
<td>$\sigma_x$</td>
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<td>$\sigma_{rh}$</td>
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<td>$\delta_1$</td>
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<td>0.0838</td>
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<tr>
<td>$\rho_{rhe,pe}$</td>
<td>-0.4295</td>
<td>0.0855</td>
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</table>
return on financial assets. The expected return on housing assets and financial assets are negatively correlated. The direction of such linkage could be due to the fact that the substitution effect and wealth effect point in opposite directions and substitution effect dominates the wealth effect during the sample period\textsuperscript{23}. There is a negative but insignificant correlation between expected excess consumption growth and expected return on housing assets. We also estimate correlation between shock to realized excess consumption growth and shock to realized housing return by restricting the correlation between expected excess consumption growth and expected return on housing assets to be zero. The estimated correlation between realized excess consumption growth and realized housing return is positive and insignificant and the likelihood value is lower than the original model. The other estimated parameters remain qualitatively similar. The insignificant correlation may arise due to the fact that the realized excess consumption growth is very close to a white noise, whereas realized housing return is highly persistent. Therefore a shock to realized excess consumption growth disappears immediately, whereas a shock to realized housing return may take some time to dissipate. Table 3.2 shows the estimated value of the present-value parameters, where $B_i$'s represent the loadings on expected returns on housing assets, financial assets, and excess consumption growth. These loadings depend positively on the persistence parameter of the unobserved state variables. As reported in table 3.2, these loadings are statistically significant.

\textsuperscript{23}A substitution effect predicts a negative relationship between the prices of the two assets, as the high return in one market tends to cause investors to leave the other market. A wealth effect, by contrast, predicts a positive relationship because the high return in one market will increase the total wealth of investors and their capability of investing in other assets.
Table 3.2: **Implied Present-Value Model Parameters**

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Figure 3.1 plots the realized housing asset return along with the filtered return from the latent-variables approach, as well as the fitted return from the excess consumption-asset ratio. It is evident from the graph that the filtered return from the latent variable approach tracks the realized returns much more closely than the predicted returns from excess consumption-asset ratio\(^{24}\). The simple contemporaneous correlation between expected housing return and realized return is 0.84, whereas the corresponding correlation between mortgage rate and the realized housing return is -0.16. In fact, the filtered return tracks realized return better than other predictors. Our analysis in the next section shows this result in a greater detail.

\(^{24}\)Rychkov (2008) and Binsbergen and Koijen (2010) also compare the filtered series of state variables with the forecasts based on conventional predictive regression.
Figure 3.1: Comparison of realized housing returns with forecasts from the unobserved component approach and the OLS approach

Figure 3.2 plots the change in Real HPI along with the filtered return from the latent-variables approach, as well as the fitted return from the excess consumption-asset ratio. Again, the graph suggests that the filtered return from the latent variable approach tracks the change in house price growth much more closely than the predicted returns from excess consumption-asset ratio.
3.6 Quarterly Forecasting Regressions

3.6.1 Forecasting Housing Asset Return

Our approach yields us an estimate of expected housing return that in theory should have better predictive power in forecasting realized housing return than finite lags of excess consumption-assets ratio. We test this hypothesis by comparing the forecasting performance of expected return on housing obtained from the present-value model with excess consumption-assets ratio. In addition, we also compare the forecasting performance of our measure with other popular predictors of housing market, for example, among others, mortgage rate, the price-rent ratio, GDP growth, and yield spread. Table 3.3 reports the results for this exercise.

The first row of the table is the regression of realized housing asset returns on its own lag. This model predicts about 13% of the next quarter’s variation in realized housing asset returns. If expected housing return is used as a predictor of realized housing asset returns, then we find that it explains 18% of the variation. This implies that our preferred measure contains an extra 5 percent explanatory power.
as compared to the lagged housing returns. It should be noted that our measure of expected housing return co-varies positively with future housing asset returns, and is procyclical. It is also positively correlated with GDP, with a correlation coefficient of 0.23. This measure tends to increase during expansions and decline in recessions. The statistical properties of expected housing return from our approach matches well with the overall developments in the housing market as well as the macroeconomic environment in the US.

Our results show lagged real GDP growth rate explains around 3% of the variation in realized return on housing wealth. The estimation results show that there is a positive relationship between GDP growth and realized return on housing.
Table 3.3: Forecasting Quarterly Housing Asset Return

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</table>

The table reports regression results of realized housing asset returns on lagged variables. $\hat{r}h$ is the expected return on housing assets computed via the filtering approach, mg is the mortgage rate, rgdp is the real GDP growth rate, xa is the excess consumption-assets ratio, yield is the spread between 10 year and 1 year Treasury bill rate, cay is the cointegrating residual between consumption, assets, and labor income, and prent is the price-rent ratio. The sample covers the period 1953 Q2-2006 Q4. The numbers in the parentheses are P-values.
Whelan (2008) and Kishor and Kumari (2011) find that excess consumption-assets ratio is a significant predictor of total asset returns, however, our findings suggest that it is an insignificant predictor for returns on housing wealth. We use the cointegrating residual between consumption, labor income and assets, $cay$, as one of the predictors. Our results suggest that the cointegrating residual, $cay$, is a significant predictor of one period ahead return on housing assets and explains around 5% of the variation. The mortgage rate explains only 2.47% of the variation in housing asset returns and is insignificant. Business cycle literature suggests that the yield spread, which is the difference between 10-year and 1-year treasury bond is a significant predictor of business cycle, though its predictive power has declined recently. We also use yield spread as one of the predictors of housing markets, but we don’t find any significant relationship between yield spread and one-period-ahead housing asset returns. Similarly, we also find that the price-rent ratio is not a significant predictor of one-period-ahead housing asset returns and explains only 1.5% of variation.

In addition to examining the in-sample predictive power of different predictors separately, we also examine whether the inclusion of lagged expected housing return in a prediction equation with other predictors improve the explanatory power of realized housing return. For example, as shown in row 9, if we augment the prediction model of realized housing return with its own lag by including lagged expected housing return, R-squared increases from 12.9% to 18.2% implying an increase of 5.3%. Similarly, if we augment the model of lagged mortgage rate with lagged expected housing return, R-squared increases by almost 24%. The results also suggest that in the presence of lagged expected return on housing, own lag of realized housing return and lag of mortgage rate becomes insignificant. Rows 11-15 of Table 3.3 represent results when lagged expected return on housing is included in a regression with lagged real GDP growth rate, excess consumption assets ra-
tion, yield, \( cay \), and price-rent ratio. Adding \( \hat{r}h \) significantly improves the degree of variation in 1-quarter ahead realized housing return.

Therefore, the results presented in this section suggest that the filtered expected returns obtained from the present-value model contains additional information about the future movements in housing returns that is not already present in the alternative predictors discussed here.

### 3.6.2 Forecasting Real House Price Growth

In addition to analyzing the role of expected housing return obtained from the present-value model in forecasting realized housing asset returns, we also explore its performance in forecasting house price growth and compare it with other competing predictors. Table 3.4 shows the estimation results for the predictive power of expected housing return and other predictors for house price growth.
Table 3.4: Forecasting Quarterly Growth in Housing Prices

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</table>

The table reports regression results of realized housing price growth on lagged variables. \( \hat{r}h \) is the expected return on housing assets computed via the filtering approach, mg is the mortgage rate, rgdp is the real GDP growth rate, xa is the excess consumption-assets ratio, yield is the spread between 10 year and 1 year Treasury bill rate, cay is the cointegrating residual between consumption, assets, and labor income, and prent is the price-rent ratio. The sample covers the period 1953 Q2-2006 Q4. The numbers in the parentheses are P-values based on the Newey-West standard errors.

We find that filtered expected housing return is a significant predictor of 1-period ahead house price growth, and explains 22% of the variation in house price growth. The mortgage rate and yield spread are also significant predictors of house price growth. Mortgage rate explains around 15% of the variation in house price growth. The estimated coefficient is negative, which is intuitive as it suggests that a higher mortgage rate entails higher amortization which in turn impinges on the cash flow of households. This reduces the affordability of new housing demand and pushes down house prices. Our results also suggest that yield spread has a positive and significant effect on house price growth. Increase in yield spread shows market expectation of higher interest rate in future, as it expects economic activity to pick up. This may also lead to an increase in house prices.

Real GDP growth rate and cay are also significant predictors of house price
growth. They explain 5.75% and 5% of the variation in dependent variable respectively. The increase in the growth rate of real GDP would be expected to lead overtime to higher house prices. The excess consumption-assets ratio is an insignificant predictor of house price growth. The price-rent ratio explains 4.8% of the variation in the dependent variable.

It is well known that the housing market witnessed an unprecedented rise in the price level between 1997-2006. To examine how much our results are affected by this period, we also estimate our forecasting regressions for the pre-1997 time period. Table 3.5 reports the results for the sub-sample 1970 Q2-1996 Q4. The filtered series of expected housing asset returns explains around 43% of the variation, showing that the predictive power was much higher in the pre-1997 time period. In contrast, mortgage rate is weakly significant and explains only 5% of the variation.

Table 3.5: Forecasting Quarterly Growth in Housing Prices: 1971Q2-1996Q4

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<td></td>
</tr>
<tr>
<td>6</td>
<td>0.007</td>
<td>-0.151</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.103</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.03)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.013</td>
<td>-0.044</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.146</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.008)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table reports regression results of realized housing price growth on lagged variables for the subsample 1971Q2-1996Q4. The numbers in the parentheses are P-values based on the Newey-West standard errors.
in change in house prices. The yield spread is an insignificant predictor for the pre-1997 period. Real GDP growth rate and $cay$ are significant predictors of house price growth explaining 13 percent and 10.3 percent of the variation in dependent variable respectively. The excess consumption-assets ratio remains an insignificant predictor of house price growth. The price-rent ratio is a negative and significant predictor for the subsample and explains around 15% of the variation. The results for the pre-1997 sample period suggests that the forecasting power of most of the predictors was higher in the pre-1997 sample period.

### 3.6.3 Out-of-Sample Evidence

The results presented in the previous section suggests that expected housing return obtained from the present-value model dominates other competing predictors in forecasting realized housing returns within the whole sample period. In this section, we examine one-period-ahead out-of-sample forecasting ability of estimated filtered returns and compare it with other alternative models. Each model is first estimated using data from the second quarter of 1953 through the third quarter of 1963. We use recursive regressions to re-estimate the forecasting model each period, adding one quarter at a time till the end of the sample and calculating a series of one-step-ahead forecasts. Our forecasting sample ends in 2006:04.

In our forecasting experiment, we compare the mean-squared error from a series of one-quarter-ahead out of sample forecasts obtained from a prediction equation that includes $\hat{rh}$ as the sole forecasting variable, to a variety of forecasting equations that use different predictors. Table 3.6 reports the ratio of MSE of the forecasts generated using filtered housing asset returns as a regressor in the forecasting equation to MSE of the forecasts generated using an alternative regressor. The ratio below unity represents superior forecasting performance of expected housing asset returns relative to the alternative predictors. The results in table 3.6 indicate that
MSE of forecasts generated with the expected housing asset returns does a superior job in predicting housing asset returns out-of-sample. For example, MSE-ratio of 0.828 implies that forecasts from the model with filtered housing asset returns has 17.2 percent lower MSE than the corresponding forecasts of housing asset returns obtained from the mortgage rate. The other alternative predictors are real GDP growth, excess consumption-assets ratio, yield spread, $cay$, and price-rent ratio. In all cases, the reduction in MSE is around 20%.

To test the significance of forecast accuracy, we perform a statistical forecast comparison test. Since the forecasts in question are non-nested, we use Diebold and Mariano (1995) and West (1996) type of forecast evaluation test. This test statistic is referred to as the modified Diebold-Mariano (MDM) test statistic and is estimated with the Newey-West corrected standard errors that allow for heteroskedastic/autocorrelated errors. The null hypothesis of this test implies that the forecast accuracy of our preferred model and the alternative forecasts are not significantly different from each other. The P-values for this test are reported in table 3.6.
Table 3.6: **One-Quarter-Ahead Forecasts of Housing Assets Returns: Nonnested Comparisons**

<table>
<thead>
<tr>
<th>Row</th>
<th>Model 1 vs Model 2</th>
<th>(\frac{MSE_1}{MSE_2})</th>
<th>MDM P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\hat{r}_h) vs. mg</td>
<td>0.828</td>
<td>0.045</td>
</tr>
<tr>
<td>2</td>
<td>(\hat{r}_h) vs. gdp</td>
<td>0.831</td>
<td>0.007</td>
</tr>
<tr>
<td>3</td>
<td>(\hat{r}_h) vs. xa</td>
<td>0.796</td>
<td>0.003</td>
</tr>
<tr>
<td>4</td>
<td>(\hat{r}_h) vs. yield</td>
<td>0.802</td>
<td>0.004</td>
</tr>
<tr>
<td>5</td>
<td>(\hat{r}_h) vs. cay</td>
<td>0.850</td>
<td>0.008</td>
</tr>
<tr>
<td>6</td>
<td>(\hat{r}_h) vs. prent</td>
<td>0.769</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The table reports the results of one-quarter-ahead nonnested forecast comparisons. The dependent variable is return on housing assets. In each case two models are compared. Model 1 always uses lagged \(\hat{r}_h\) as a predictive variable. Model 2 uses one of several alternate variables labeled in the second column. The column labeled \(MSE_1/MSE_2\) reports the ratio of the mean squared error of Model 1 to Model 2. The first row uses the lagged mortgage rate as a predictive variable; the model denoted gdp uses lagged real GDP growth rate as a predictive variable; the model denoted xa uses lagged excess consumption-assets ratio as a predictive variable; the fourth row uses lagged yield spread; the model denoted cay uses lagged value of cay and the last row uses change in price- rent ratio as a predictive variable. The fourth column reports the P-values of modified Diebold-Mariano test statistic. The null hypothesis is that Model 2 encompasses Model 1. Each model is first estimated using data from the second quarter of 1953 to the third quarter of 1963. It is recursively re-estimated each period until 2006 Q4, adding one quarter at a time and calculating a series of one-step-ahead forecasts.

The results suggest that the forecasts of housing asset returns generated using expected housing asset returns as a predictive variable significantly outperforms the forecasts from alternative predictors.

We also compare out-of-sample forecasting power of different predictors in forecasting change in house prices. As above, each model is first estimated using data from the second quarter of 1953 to the third quarter of 1963. We use recursive regressions to re-estimate the model. The results from a set of non-nested forecast comparison tests are given in Table 3.7. The alternative models are the same as above. More specifically, we compare the model with lagged value of expected housing asset returns to alternative models in which either the lagged mortgage rate, lagged yield spread, lagged real growth of GDP, lagged excess consumption-assets
ratio, lagged \( cay \), and lagged price-rent ratio is the only predictor. As indicated in

Table 3.7: **One-Quarter-Ahead Forecasts of House Price Growth: Nonnested Comparisons**

<table>
<thead>
<tr>
<th>Row</th>
<th>Model 1 vs Model 2</th>
<th>( \frac{MSE_1}{MSE_2} )</th>
<th>MDM P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \hat{r}h ) vs. mg</td>
<td>0.830</td>
<td>0.041</td>
</tr>
<tr>
<td>2</td>
<td>( \hat{r}h ) vs. gdp</td>
<td>0.731</td>
<td>0.012</td>
</tr>
<tr>
<td>3</td>
<td>( \hat{r}h ) vs. xa</td>
<td>0.694</td>
<td>0.003</td>
</tr>
<tr>
<td>4</td>
<td>( \hat{r}h ) vs. yield</td>
<td>0.772</td>
<td>0.064</td>
</tr>
<tr>
<td>5</td>
<td>( \hat{r}h ) vs. cay</td>
<td>0.747</td>
<td>0.001</td>
</tr>
<tr>
<td>6</td>
<td>( \hat{r}h ) vs. prent</td>
<td>0.716</td>
<td>0.025</td>
</tr>
</tbody>
</table>

The table reports the results of one-quarter-ahead nonnested forecast comparisons. The dependent variable is growth in house prices. In each case two models are compared. Model 1 always uses lagged \( \hat{r}h \) as a predictive variable. Model 2 uses one of several alternate variables labeled in the second column. The column labeled \( MSE_1/MSE_2 \) reports the ratio of the mean squared error of Model 1 to Model 2. The first row uses the lagged mortgage rate as a predictive variable; the model denoted gdp uses lagged real GDP growth rate as a predictive variable; the model denoted xa uses lagged excess consumption-assets ratio as a predictive variable; the fourth row uses lagged yield spread; the model denoted cay uses lagged value of cay and the last row uses change in price-rent ratio as a predictive variable. The fourth column reports the P-values of modified Diebold-Mariano test statistic. The null hypothesis is that Model 2 encompasses Model 1. Each model is first estimated using data from the second quarter of 1953 to the third quarter of 1963. It is recursively re-estimated each period until 2006 Q4, adding one quarter at a time and calculating a series of one-step-ahead forecasts.

Table 3.7, the MSE of forecasts generated with the expected housing returns does a superior job in predicting house price growth out-of-sample. For example, MSE-ratio of 0.830 implies that forecasts from the model with housing asset returns has 17 percent lower MSE than the corresponding forecasts of house price growth obtained from mortgage rate. The difference in the forecast accuracy of our preferred model and the alternative forecasts are statistically significant according to the MDM test statistic. P-values for this test are reported in Table 3.7.

We also test the forecasting performance of the filtered series and other alterna-
The filtered series of $\hat{r}h$ again produces a superior forecasts to all other variables. Table 3.8 reports the results.

Table 3.8: One-Quarter-Ahead Forecasts of House Price Growth: Nonnested Comparisons(1971Q2-1996Q4)

<table>
<thead>
<tr>
<th>Row</th>
<th>Model 1 vs Model 2</th>
<th>$\frac{MSE_1}{MSE_2}$</th>
<th>MDM P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\hat{r}h$ vs. mg</td>
<td>0.373</td>
<td>0.010</td>
</tr>
<tr>
<td>2</td>
<td>$\hat{r}h$ vs. gdp</td>
<td>0.478</td>
<td>0.003</td>
</tr>
<tr>
<td>3</td>
<td>$\hat{r}h$ vs. xa</td>
<td>0.386</td>
<td>0.007</td>
</tr>
<tr>
<td>4</td>
<td>$\hat{r}h$ vs. yield</td>
<td>0.328</td>
<td>0.005</td>
</tr>
<tr>
<td>5</td>
<td>$\hat{r}h$ vs. cay</td>
<td>0.536</td>
<td>0.020</td>
</tr>
<tr>
<td>6</td>
<td>$\hat{r}h$ vs. prent</td>
<td>0.628</td>
<td>0.016</td>
</tr>
</tbody>
</table>

The table reports the results of one-quarter-ahead nonnested forecast comparisons for the subsample-1971 q2-1996 q4. The dependent variable is growth in house prices. In each case two models are compared. Model 1 always uses lagged $\hat{r}h$ as a predictive variable. Model 2 uses one of several alternate variables labeled in the second column. The column labeled $MSE_1/MSE_2$ reports the ratio of the mean squared error of Model 1 to Model 2. The last column reports the P-values of modified Diebold-Mariano test statistic. The null hypothesis is that Model 2 encompasses Model 1. Each model is first estimated using data from the second quarter of 1971 to the first quarter of 1981. It is recursively re-estimated each period until 1996 Q4, adding one quarter at a time and calculating a series of one-step-ahead forecasts.

All of these encompassing tests indicate that the forecasting model that includes expected return on housing as a predictive variable contains information that produces superior and significant forecasts to those produced by any of the competitor models.
3.7 Conclusions

The literature on the relationship between housing market wealth and consumption has mainly focused on estimating the consumption response to changes in housing market wealth. Rather than estimating the impact of housing wealth on consumption spending, the goal of this essay is to exploit the information in consumption, income and observable assets to estimate expected return on housing wealth, as the life cycle model suggests that household’s consumption responds to expected changes in housing market wealth and financial market wealth. To do so, we combine a present-value model with an unobserved component model to write down the observed excess-consumption asset ratio as a linear function of unobserved expected return on housing assets and financial assets and unobserved excess consumption growth. By assuming a simple autoregressive process for the unobserved variables, we apply Kalman filter to extract the unobserved expected return on housing from the present-value model.

Our results show that expected housing return from a present-value model is a superior predictor of realized housing asset returns and house price growth rate as compared to the other popular predictors both in-sample and out-of-sample. The estimated expected housing return explains 18% of the variation in one-period ahead housing assets returns and 22% of the variation in one-period ahead house price growth rate. The results suggest that the filtered returns dominates popular predictors like mortgage rate, price-rent ratio, yield spread, cay and real GDP growth. The superior predictive ability of the filtered returns is not surprising since the unobserved component approach uses information from the whole history of the variables in the information set, as compared to the traditional approach where only finite lags are used for forecasting. To examine whether the differences in the forecasting performance of forecast obtained from our approach is significantly superior to that of the traditional predictors, we also perform non-nested forecast comparison.
tests. The modified Diebold-Mariano (MDM) test suggests that the mean squared errors of the forecasts generated from the expected return on housing asset from the present-value model is significantly lower than the mean squared error of the forecasts generated from predictors like the mortgage rate, price-rent ratio, among others.
Chapter 4

Third Essay: The Relative Importance of Permanent and Transitory Components in the US and UK House Prices

4.1 Introduction

The housing sector is one of the largest sectors of the US and the UK economy. Therefore, movement in house prices is likely to have a significant impact on both the economies. The US housing market has experienced a higher degree of volatility over the last 30 years due to major structural changes and economic fluctuations. Real house prices rose by only 3.7 percent between 1985-1995, but increased by 46 percent between 1995 and 2005. The subsequent correction in house prices led to a decline of 34 percent from the year end 2006 through the first quarter of 2009. House prices increased exponentially in the UK during 1995-2007 Q3. Increase in house prices during this period has surpassed its historic precedents in the late 1970s and the late 1980s in both its duration and scale. The condition of housing market has deteriorated since the third quarter of 2007.

In this essay, we decompose the movement in house prices into trend and cycle and assess the relative importance of trend and cyclical component in the US and the UK house prices. Trend-Cycle decomposition is important both theoretically and statistically. In fact, there is a great deal of literature on the decomposition of data into trend and cycle. Prior to the 1980s, the general approach to time series data was that an economic time series could be decomposed into a secular or
growth component and a cyclical component, which were deterministic functions of time. In this method, the cyclical component emerges as a residual from the trend line. However, researchers such as Beveridge and Nelson (1981) and Nelson and Plosser (1982) have a different view. They find that an economic time series are non-stationary stochastic processes. The deviation of the series from any deterministic path will grow without bound. Therefore, growth in a time series should be removed by first-differencing. Harvey (1985), Watson (1986), and Clark (1987) are also in line with the latter view. Harvey et al. (1982) apply the autoregressive conditional heteroskedasticity (ARCH) model proposed by Engle (1982) and generalized ARCH (GARCH) model developed by Bollerslev (1986) to time series model formulated in terms of unobserved components. They examined how ARCH and GARCH disturbances might be incorporated in time series models with unobserved components. According to Engle (1982), if the variance varies through time, then modeling the residual variance as a constant leads to consistent but inefficient parameter estimates and suboptimal forecasts.

The historical evidence on house prices in the US and the UK suggests that the volatility in house prices has changed over time. Therefore, the trend-cycle decomposition of house prices without taking into account time-varying volatility may provide us with misleading results. Therefore, we decompose the movement in house prices into the permanent and transitory components by allowing the shocks to house prices to have a time-varying permanent and transitory effects. We apply GARCH or IGARCH effect to conditional variance of the innovation in trend and cycle component. More specifically, we incorporate GARCH (1,1) and IGARCH (1,1) processes into the unobserved component model. We then observe the ratio of standard errors of transitory shocks to those of permanent shocks over the sample period, and examine whether the movements in house prices will have a long term or short term impact on the US and the UK markets. We also investigate
whether the impact of the decline in housing market in 2007 in the US and the UK markets is a transient deviation from the trend path. Movement in house prices is also decomposed using a conventional unobserved components model that assumes constant variance of the white noise processes. The model is then compared with the model with heteroskedastic disturbances. We perform the likelihood ratio test to compare the fit of two models.

We find that for the US economy, the standard errors of the transitory component reached peaks on three periods: around 1980, 2004-2006, and around 2008-2010. The standard errors of the permanent component have gradually reduced since 1984Q2. We also find that standard errors of transitory component were larger than those of permanent component on an average in US real house price index. This is especially true for the housing crisis of 2007. Thus it is plausible to conclude that the level of real house prices in US are transitorily lower than its trend path. The likelihood ratio test suggests that the model without heteroskedastic disturbances is significantly worse than the model with heteroskedastic disturbances. In the UK housing market, the standard errors of the permanent components have gradually reduced over the entire sample period employed, even though they had increased in three periods : 1998, 2002 and between 2008Q3-2009Q4. We also find that the ratios of standard errors of transitory components to those of permanent components were larger over the entire sample period.

This essay is organized as follows. In section 4.2, the unobserved components model with heteroskedastic and non heteroskedastic disturbances is specified. Section 4.3 discusses the state-space representation and estimation. Then, in section 4.4, estimations of parameters are assessed. Section 4.5 concludes.
4.2 The Unobserved Components Model

The purpose of this essay is twofold. First, to decompose the movement in house prices into trend and cycle and second, to analyze whether the movement has been permanent or transitory. The movement in house prices can be decomposed using an unobserved component model. We decompose the prices via two types of unobserved component model. First, by using an unobserved component model with heteroskedastic disturbances. Next, using a conventional unobserved component model that assumes constant variance of the white noise process. The first model allows us to measure the relative size of each shock, permanent and transitory. Each country is modeled based on the process the permanent and transitory shock follows. The heteroskedastic disturbances are permitted to follow either the generalized autoregressive conditional heteroskedasticity (GARCH) specification or Integrated GARCH(1, 1)\(^1\) specification, based on the characteristics of the data for each country.

We apply the UC model proposed by Watson (1986) and Clark (1987, 1989) and incorporate GARCH (1, 1) or IGARCH(1, 1) disturbances into the model. This is different from the conventional unobserved component model that assumes constant variance of the white noise processes\(^2\).

Data on nominal house price index (HPI) for the US has been obtained from the OFHEO house price index. Quarterly data runs from 1975 Q1 through the fourth quarter of 2011. National Statistical Office is the data source for the Nominal house price index for the UK. Quarterly data has been used starting in the second quarter of 1968. The sample period runs through the third quarter of 2011. Nominal House

\(^1\)IGARCH (1,1) model has the sum of ARCH and GARCH parameters equal to one
\(^2\)There is a great deal of literature on the decomposition of output data into trend and cycle. Prior to the 1980s, the general approach to time series data was that an economic time series could be decomposed into a secular or growth component and a cyclical component, which were deterministic functions of time. However, researchers such as Beveridge and Nelson (1981) and Nelson and Plosser (1982) have a different view. They find that economic time series are non-stationary stochastic processes and growth in economic activity should be removed by first-differencing. Harvey (1985), Watson (1986), and Clark (1987) are also in line with the latter view.
Price index is converted to real house price index by deflating it by CPI.

### 4.2.1 Model with Heteroskedastic Disturbances

The unobserved components model with heteroskedastic disturbances has the following structure:

\[
Y_t = T_t + C_t
\]  \hspace{1cm} (4.1)

\[
T_t = g_t + T_{t-1} + u_t, \quad u_t | \psi_{t-1} \sim N(0, h_{u,t})
\]  \hspace{1cm} (4.2)

\[
C_t = \phi_1 C_{t-1} + \phi_2 C_{t-2} + e_t, \quad e_t | \psi_{t-1} \sim N(0, h_{e,t})
\]  \hspace{1cm} (4.3)

where \( \psi_{t-1} \) refers to information up to \( t-1 \). \( Y_t \) is the log of real HPI. Equation (4.1) shows that \( Y_t \) (real HPI) is decomposed into two unobserved components consisting of the stochastic trend component \( T_t \), and the stationary cyclical component, \( C_t \).

The stochastic trend component follows a random walk with a constant drift term. \( T_t \) is subject to innovation to the level \( u_t \). The cyclical component is allowed to follow a second order autoregressive process given by equation (4.3) which is the most parsimonious way to generate boom and burst in the housing market. \( e_t \) is an innovation to the cyclical component.

We first assume heteroskedastic disturbances. This is due to the fact that the real HPI has been more volatile in recent times than at others. Therefore, instead of assuming the shocks to the permanent and transitory components to have same distribution, we apply GARCH (1,1) or IGARCH (1,1) effect to conditional variance of the innovation in equation (4.4) and (4.5)\(^3\). More specifically, if we apply GARCH

---

\(^3\)Engle(1982,1985) also found evidence that the disturbance variances for some kinds of time-series data were less stable than usually assumed, and so specified the model for the inflation rate allowing the uncertainty of inflation to change over time.
(1,1) effect, the conditional variance of the error term of the trend component consists of the intercept term \( \alpha_0 \), the ARCH parameter \( \alpha_1 \), and the GARCH parameter \( \alpha_2 \). Similarly, the conditional variance of the error term of the cyclical component consists of the intercept term \( \beta_0 \), the ARCH parameter \( \beta_1 \), and the GARCH parameter \( \beta_2 \). However, it turns out that for the two countries, the conditional variance of the permanent shocks follows an IGARCH (1, 1) process. IGARCH in the conditional variance of the permanent shocks, the error term of the trend component consists of the following structure:

\[
h_{u,t} = \alpha_1 u_{t-1}^2 + (1 - \alpha_1)h_{u,t-1}
\]

where \( \alpha_1 \) is the ARCH parameter and \( 1 - \alpha_1 \) is the GARCH parameter and the sum of the ARCH and GARCH parameters sum to one.

The model enables us to observe the ratio of standard errors of the transitory shocks to those of permanent shocks over the sample period.

### 4.2.2 Model with Non Heteroskedastic Disturbances

Next, we assume the shocks \( u_t \) and \( e_t \) to the trend and cyclical component to be normally distributed, i.i.d, with mean zero and are not correlated. The conventional unobserved component model proposed by Watson (1986) and extended by Clark (1987, 1989) has assumed constant variance of the white noise processes.

The unobserved components model with non-heteroskedastic disturbances has the following structure:

\[
Y_t = T_t + C_t
\]

\[
T_t = \mu + T_{t-1} + u_t, \quad u_t|\psi_{t-1} \sim N(0, h_{u,t}) \tag{4.7}
\]

\[
C_t = \phi_1 C_{t-1} + \phi_2 C_{t-2} + e_t, \quad e_t|\psi_{t-1} \sim N(0, h_{e,t}) \tag{4.8}
\]
The trend is modeled as a random walk with drift and the cyclical component is modeled as an AR(2) process. We perform the likelihood ratio test to compare the fit of two models, model with heteroskedastic disturbances and model with constant disturbances.

4.3 State-Space Model

In this section, we present the state-space representation and estimation of the model given by equations (4.1-4.5). Measurement equation is:

\[ Y_t = H\beta_t \]  

(4.9)

where

\[ H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix} \]

\[ \beta_t = \begin{bmatrix} T_t \\ C_t \\ C_{t-1} \\ g_t \\ u_t \\ e_t \end{bmatrix} \]

Transition equation is represented as:

\[ \beta_t = F\beta_{t-1} + Gv_t \]  

(4.10)
where
\[
F = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \phi_1 & \phi_2 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad G = \begin{bmatrix}
1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
1 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}, \quad v = \begin{bmatrix}
ut_t \\
e_t \\
\end{bmatrix}
\]

Covariance matrix of the disturbance vector in the transition equation is represented as:
\[
E(Gv_t v'_t | G') = E \left[ \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
1 & 0 \\
0 & 1 & 0 \\
\end{bmatrix} \right] * \begin{bmatrix}
ut_t \\
e_t \\
0 \\
\end{bmatrix} * \begin{bmatrix}
ut_t & e_t & 0 \end{bmatrix} * \begin{bmatrix}
1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
\[
= \begin{bmatrix}
h_{u,t} & 0 & 0 & h_{u,t} & 0 \\
0 & h_{e,t} & 0 & 0 & h_{e,t} \\
0 & 0 & 0 & 0 & 0 \\
h_{u,t} & 0 & 0 & h_{u,t} & 0 \\
0 & h_{e,t} & 0 & 0 & h_{e,t} \\
\end{bmatrix} = Q_t^*
\]

Kalman Filter is then applied to the model given by equations (4.9-4.10). The Kalman filter provides us with prediction error and its variance as by-products. Based on this prediction error decomposition the Gaussian log likelihood function is maximized with respect to the unknown parameters of the model. However, to operate the Kalman filter, we need to calculate \(h_{u,t} = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 h_{u,t-1}\) and \(h_{e,t} = \beta_0 + \beta_1 e_{t-1}^2 + \beta_2 h_{e,t-1}\) in the \(Q_t^*\) matrix of the above model. Both \(h_{u,t}\) and \(h_{e,t}\) are functions of the squared lagged disturbance and the variance of lagged disturbance, which are unobserved. Harvey et al. (1992) solve the problem of the
unobserved $u_{t-1}^2$ and $e_{t-1}^2$ by replacing the squared lagged disturbances by their conditional expectations. Thus,

$$h_{u,t} = \alpha_0 + \alpha_1 E[u_{t-1}^2 | \psi_{t-1}] + \alpha_2 h_{u,t-1} \tag{4.11}$$

$$h_{e,t} = \beta_0 + \beta_1 E[e_{t-1}^2 | \psi_{t-1}] + \beta_2 h_{e,t-1} \tag{4.12}$$

To calculate $E[u_{t-1}^2 | \psi_{t-1}]$ and $E[e_{t-1}^2 | \psi_{t-1}]^4$, we use

$$E[u_{t-1}^2 | \psi_{t-1}] = E[u_{t-1} | \psi_{t-1}]^2 + E[(u_{t-1} - E(u_{t-1} | \psi_{t-1}))^2] \tag{4.13}$$

$$E[e_{t-1}^2 | \psi_{t-1}] = E[e_{t-1} | \psi_{t-1}]^2 + E[(e_{t-1} - E(e_{t-1} | \psi_{t-1}))^2], \tag{4.14}$$

where $E[u_{t-1} | \psi_{t-1}]$ and $E[e_{t-1} | \psi_{t-1}]$ are obtained from the last two elements of $\beta_{t-1|t-1}$, and where $E[(u_{t-1} - E(u_{t-1} | \psi_{t-1}))^2]$ and $E[(e_{t-1} - E(e_{t-1} | \psi_{t-1}))^2]$ are obtained from the last two diagonal elements of the covariance matrix of $\beta$, $P_{t-1|t-1}$.

### 4.3.1 Normally Distributed Error Terms

Following is the state-space representation and estimation of the model given by equations (4.6-4.8).

$$Y_t = H \beta_t \tag{4.15}$$

where

$$H = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

$$\beta_t = \begin{bmatrix} T_t \\ C_t \\ C_{t-1} \end{bmatrix}$$

\[ ^4 \text{We follow the approach used in Song (2011)} \]
Transition equation is represented as:

\[
\beta_t = F\beta_{t-1} + Gv_t \quad (4.16)
\]

where

\[
F = \begin{bmatrix}
1 & 0 & 0 \\
0 & \phi_1 & \phi_2 \\
0 & 1 & 0
\end{bmatrix}, \quad G = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0
\end{bmatrix}, \quad v = \begin{bmatrix}
u_t \\
e_t
\end{bmatrix}
\]

Covariance matrix of the disturbance vector in the transition equation is represented as:

\[
Q_t = E(v_tv_t') = \begin{bmatrix}
\sigma_\mu^2 & 0 & 0 \\
0 & \sigma_\epsilon^2 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

4.4 Estimation Results

4.4.1 USA

Table 4.1 contains parameter estimates of the unobserved component model with heteroskedastic disturbances over the sample period. The results show that the cycle is highly persistent at 0.989 (\(\phi_1 + \phi_2 = 0.989\)). Both ARCH parameters \(\alpha_1\) and \(\beta_1\) are statistically significant, but the GARCH parameter \(\beta_2\) is statistically insignificant. This implies that the volatility of the error term of the stationary cyclical component follows ARCH(1) process. In the conditional variance of the error term of the stochastic trend component, we first considered GARCH (1,1), but found that the sum of the ARCH and GARCH parameters approximately equal to one. Thus IGARCH (1,1) process is more suitable to the volatility of the error term of the stochastic trend component, and GARCH parameter \(\alpha_1\) and the initial value of error term \(h_{u,0}\) are estimated.
Table 4.1: Maximum Likelihood Estimates of the Unobserved Components Model with Heteroskedastic Disturbances-USA

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Standard Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.162</td>
<td>0.063</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.796</td>
<td>0.041</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.807</td>
<td>0.037</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.523</td>
<td>0.261</td>
</tr>
<tr>
<td>$h_{l_0}$</td>
<td>1.453</td>
<td>1.815</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.128</td>
<td>0.072</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.617</td>
<td>0.231</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.088</td>
<td>0.227</td>
</tr>
<tr>
<td>Likelihood Value</td>
<td>-161.64</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.1 shows that there was a large increase in the level of real House Price Index (HPI) during the period 2005-2007. The house prices peaked in 2007. There was a large fall-off of the level of real house prices following the house price bust and financial crisis in 2007. Figure 4.2 indicates that the cyclical component of house prices had a sharp increase during the housing crisis.

Figure 4.1: Real HPI and Trend Component from the Unobserved Components Model with Heteroskedastic Disturbances-USA
Figure 4.2: Stationary Cyclical Component from the Unobserved Components Model with Heteroskedastic Disturbances-USA

Figure 4.3 and 4.4 represent standard errors of innovations of the trend and cyclical components, namely, standard errors of the permanent component and those of the transitory component, respectively. One interesting finding is that the standard errors of the permanent component have gradually reduced since 1984Q2. The standard errors of the transitory component reached peaks on three periods: around 1980, 2004-2006, and around 2008-2010.
Figure 4.3: Standard Errors of Transitory Shocks from the Unobserved Components Model with Heteroskedastic Disturbances- USA

Figure 4.4: Standard Errors of Permanent Shocks from the Unobserved Components Model with Heteroskedastic Disturbances- USA

Figure 4.5 shows that standard errors of transitory component were larger than those of permanent component on average in US real house price index. It follows that the standard errors of transitory component were 2-10 times larger than those of the permanent component during the housing crisis of 2007.
This fact indicates that there is a transitory decrease in the level of house prices in the US. Therefore, it is likely that the level of real house prices are transitorily lower than its initial trend path. We also analyzed the simple trend-cycle decom-

Figure 4.5: Ratios of Standard Errors of Transitory Shocks to those of Permanent Shocks from the Unobserved Components Model with Heteroskedastic Disturbances—USA

position for the US economy and compared the fit of the two models: model with heteroskedastic disturbances and model without heteroskedastic disturbances using a likelihood ratio test. Table 4.2 contains parameter estimates of the unobserved components model with constant disturbances over the same sample period. Again, the cycle is highly persistent at 0.996. The likelihood ratio test suggests that the fit is significantly worse under the alternate, where the alternate is the unobserved component model with non-heteroskedastic disturbances.
Table 4.2: Maximum Likelihood Estimates of the Unobserved Components Model with Non-Heteroskedastic Disturbances-USA

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Standard Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_u$</td>
<td>0.631</td>
<td>0.062</td>
</tr>
<tr>
<td>$h_v$</td>
<td>0.428</td>
<td>0.080</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.157</td>
<td>0.110</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.875</td>
<td>0.043</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.879</td>
<td>0.041</td>
</tr>
<tr>
<td>Likelihood Value</td>
<td>-179.227</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.6: Stationary Cyclical Component from the Unobserved Components Model with Non Heteroskedastic Disturbances-USA

4.4.2 UK

For the UK housing market, the results in Table 4.3 suggest that the cycle of an unobserved component model with heteroskedastic disturbances is highly persistent at 0.983 ($\phi_1 + \phi_2 = 0.983$). ARCH parameter $\alpha_1$ is statistically significant, but the ARCH parameter $\beta_1$ is statistically insignificant. GARCH parameter $\beta_2$ is statistically significant implying that the volatility of the error term of the stationary cyclical component follows GARCH(1,1) process. IGARCH (1,1) process is incorporated in the volatility of the error term of the stochastic trend component, and
GARCH parameter $\alpha_1$ and the initial value of error term $h_{u,0}$ are estimated.

Table 4.3: Maximum Likelihood Estimates of the Unobserved Components Model with Heteroskedastic Disturbances-UK

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Standard Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.822</td>
<td>0.116</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.744</td>
<td>0.043</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.761</td>
<td>0.038</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.721</td>
<td>0.318</td>
</tr>
<tr>
<td>$h_{10}$</td>
<td>141.00</td>
<td>641</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.245</td>
<td>1.143</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.917</td>
<td>0.382</td>
</tr>
<tr>
<td>Likelihood Value</td>
<td>-353.997</td>
<td></td>
</tr>
</tbody>
</table>

The increase in house prices in the United Kingdom exceeded the level of house prices in the United States. Figure 4.7 shows that there was a large increase in the level of real House Price Index (HPI) during the period 1997-2007. The condition of the house prices has deteriorated since the third quarter of 2007. Figure 4.8 reflects the fact that there was a sharp increase in the cyclical component during 1997-2003 and a sharp decrease thereafter.
As represented by figure 4.9, the standard errors of the permanent component have gradually reduced over the entire sample period, even though they had increased over the three periods: 1998, 2002 and between 2008Q3-2009Q4. There is not much variation in the standard errors of the transitory component.
Figure 4.9: **Standard Errors of Permanent Shocks from the Unobserved Components Model with Heteroskedastic Disturbances- UK**

Figure 4.10 reflects the fact that the level of real house prices are transitorily lower than its initial trend path. The standard errors of transitory component were larger than those of permanent component on average in UK real house price index. They were especially higher during the housing crisis of 2007.
Table 4.4 contains parameter estimates of the unobserved components model with constant disturbances over the same sample period. Again, the cycle is highly persistent at 0.988. The likelihood ratio test suggests that the difference between the log likelihoods of the two models is not statistically significant. Given the volatility in the house prices of UK, it seems that the model with heteroskedastic disturbances is better suited for analysis.
Table 4.4: Maximum Likelihood Estimates of the Unobserved Components Model with Non-Heteroskedastic Disturbances-UK

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Standard Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_u$</td>
<td>1.0652</td>
<td>0.165</td>
</tr>
<tr>
<td>$h_v$</td>
<td>1.510</td>
<td>0.166</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.783</td>
<td>0.129</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.782</td>
<td>0.046</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.794</td>
<td>0.041</td>
</tr>
<tr>
<td>Likelihood Value</td>
<td>-355.278</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.11: Stationary Cyclical Component from the Unobserved Components Model with Non Heteroskedastic Disturbances-UK
4.5 Conclusions

In this essay, we analyze the relative importance of the permanent and transitory components of the house prices of the US and the UK economies. Acknowledging that volatility of house prices has changed over time, the movement in house prices is decomposed into the permanent and transitory components by allowing the shocks to house prices to have a time-varying permanent and transitory effects. GARCH (1,1) and IGARCH (1,1) processes is incorporated into the unobserved component model and then the ratio of standard errors of transitory component to those of permanent component is observed. We then examine the relative importance of permanent and transitory components in the US and UK house prices. Our findings suggest that the standard errors of transitory component were larger than those of the permanent component on an average in the US and the UK real house price index. For both the economies, the standard errors of transitory component were much larger than those of permanent component during the housing crisis. This implies that there is a greater degree of similarity in the evolution of house price volatility in the US and the UK.


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- B.A. Economics, University of Delhi, Delhi, India, 2002

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- Principles of Macroeconomics (Fall 2009, Fall 2010, Spring 2011(online))
- Intermediate Macroeconomics (Spring 2010)
- Principles of Microeconomics (Spring 2011)
- Introduction to International Economic Relations (Fall 2011)
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- “The Relative Importance of Permanent and Transitory Components in the US and UK House Prices”

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