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TECHNIQUES FOR OPTIMUM DESIGN OF ACTIVELY CONTROLLED STRUCTURES INCLUDING TOPOLOGICAL CONSIDERATIONS

by

Arjumand Ali

A Dissertation Submitted in

Partial Fulfillment of the

Requirements for the Degree of

Doctor of Philosophy

In

Engineering

at

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ABSTRACT

TECHNIQUES FOR OPTIMUM DESIGN OF ACTIVELY CONTROLLED STRUCTURES INCLUDING TOPOLOGICAL CONSIDERATIONS

by

Arjumand Ali

The University of Wisconsin-Milwaukee, 2013
Under the Supervision of Professor Anoop Dhingra

The design and performance of complex engineering systems often depends on several conflicting objectives which, in many cases, cannot be represented as a single measure of performance. This thesis presents a multi-objective formulation for a comprehensive treatment of the structural and topological considerations in the design of actively controlled structures.

The dissertation addresses three main problems. The first problem deals with optimum placement of actuators in actively controlled structures. The purpose of control is to reduce the vibrations when the structure is subjected to a disturbance. In order to mitigate the structural vibrations as quickly as possible, it is necessary to place the actuators at locations such that their ability to control the vibrations is maximized. Since the actuator locations are discrete (0-1) variables, a genetic algorithm based approach is used to solve the resulting optimization problem.

The second problem this dissertation addresses is the multi-objective design of actively controlled structures. Structural weight, controller performance index and energy dissipated by the actuators are considered as the objective functions. It is assumed that a hierarchical structure exist between the actuator placement and structural-control design objective functions with the actuator placement problem considered being more important. The resulting multi-objective optimization problem is solved using Stackelberg game and cooperative game theory approaches. The exchange of information between different levels of the multi-level problem is done by constructing the rational reaction set of follower solution using design of experiments and response surface methods.

The third problem addressed in this dissertation is the optimization of structural topology in the context of structural/control system design. Despite the recognition that an optimization of topology can significantly improve structural performance, most of the work in design of actively controlled structures has been done with structures of a known topology. The combined topology and sizing optimization of actively controlled structures is also considered in this thesis. The approach presented involves the determination of optimum topology followed by a sizing and control system optimization of the optimum topology. Using two numerical examples, it is shown that a simultaneous consideration of topological, control and structural aspects yields solutions that outperform designs when topological considerations are neglected.

To

my parents, my loving husband

and

my daughter

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Chapter 1

Optimum Design of Actively Controlled Structures — Problem Overview

This thesis deals with the design of actively controlled structures. The approach to structural design entails that the structural integrity is insured, i.e., the stresses due to imposed loads should remain below the specified limit. Further, when disturbance(s) occur, the controller should damp out the structural vibrations quickly to bring the structure back to its equilibrium position. The design of an active control system, the placement of actuators within the structure as well as a determination of optimum structural topology are major design challenges which are the subject of this thesis and are briefly described in this chapter.

1.1 Active Structure and Control Design

Conventional approaches to design of actively controlled structures treat the structural and control system design aspects of the problem separately. Each design is optimized based on its objective function but the overall design is not system optimal. It is therefore necessary to solve the problem in such a way that the final structural design meets the requirements of weight, control effort and performance. This can be done by simultaneous optimization of control system design and structural design. In this method, either the structure and control objective functions have been optimized by linking them through constraints related to control performance, structural performance, or sometimes by

combining the structure and control objective functions into a single objective function.

While many approaches have been proposed for integrated/simultaneous design of structure and control systems, most of them deal with single or sometimes with multi-objective optimization problems with continuous design variables. Also, in most cases, the controller is designed using Linear Quadratic Regulator (LQR) theory with fixed state and control weighting matrices. In this work it is proposed that the optimum values of the state and control weighting matrices be determined as part of overall solution process to improve control system performance.

1.2 Actuator Placement

An important aspect of the active control design is the optimum placement of actuators. The number and locations of the actuators directly affect the dynamic response of the system. Further, the amount of energy consumption depends on the number of actuators used and their placement on the structure. The actuator placement problem is a discrete variable problem. The presence and absence of actuators at a location or in a member can be represented as discrete 1 and 0 variables. The studies on optimum placement of actuators have primarily been done in the context of control optimization only. This thesis proposes the optimum placement of actuators in the context of both structural and control optimization where the structural objective is minimization of the weight with cross-sectional areas of the members of the structure as continuous design variables and the control objective as the maximization of energy

dissipated by actuators with actuators locations as discrete design variables. To date, not much literature is available on solving multi-objective problems with mixed discrete continuous design variables. A brief overview of the available literature on this problem is discussed in Chapter 2.

1.3 Multi-level and Multi-objective Optimization

Most engineering systems are complex and the system performance depends on multiple and sometimes conflicting objectives. Multi-objective optimization, therefore, has become an important and essential aspect of design optimization. The approaches proposed for simultaneous structural and control optimization essentially solve a single objective optimization problem, and work well for simple structures. These methods are also applicable to large complex structures, but require more computational time and effort and the problem size may not always be manageable. In order to simplify the problem and to make the problem size manageable, the problem could be divided into multiple sub levels. The relationships between the sub levels could be either hierarchical or decentralized. In the case of a hierarchical relationship, the sub levels are integrated and coordinated at a higher level and this is a multi-level problem (Fig. 1.1). In case of a decentralized relation, the problem is a multi-objective optimization (Fig. 1.2). Stackelberg game theory method is used for solving hierarchical whereas cooperative game theory is used for solving decentralized multi-objective optimization problems in this thesis.

1.4 Topology Optimization

Another important aspect of structural/control system design is optimization of topology, and once again, not much literature is available on the determination of optimum topology in the context of active control of structures. The limited available literature indicates that the performance of a controlled structure could be significantly improved by optimization of the topology.

The optimum topology depends on the criteria selected as the objective function. A minimization of structural compliance (or strain energy) is commonly used as an objective function. This criterion is also used in this thesis. Once the optimum topology is determined, then each optimum topology is further considered for sizing and shape optimization. Such a solution approach, however, may not always lead to a globally optimum solution. A better approach may involve sizing optimization of each candidate topology; however, since many candidate topologies are considered for a given problem domain, performing shape and sizing optimization for each candidate topology is computationally very expensive. Further, when solving the sizing and shape optimization problem, the design variables are continuous whereas in the case of topology optimization, the variables generally are discrete. Combining the variables from these two optimization problems results in a problem with mixed discrete continuous variables. Computationally efficient approaches for solving problems with mixed discrete-continuous variables do not exist. Therefore, the approach presented in this thesis involves determination of optimum topology which is followed by sizing and control system optimization of the optimum topology. For

simplicity, the topology optimization is performed using a single objective function but the sizing and control optimization problems are allowed to have multiple objective functions.

1.5 Thesis Organization

In this thesis, the techniques for optimum design of actively controlled structures are organized as follows: Chapter 2 presents an overview of the available literature in the context of the simultaneous structural and control optimization of actively controlled structures as well as topology optimization. Chapter 3 introduces the concept of simultaneous structure and control system design. The effect of changing the number and locations of the actuators on the performance of the control system is also discussed in this chapter. The problem of determination of optimum number as well as optimum locations of the actuators is formulated in chapter 4. This design problem has mixed discrete and continuous design variables. Since gradient based search procedure cannot solve problems with discrete variables, a genetic algorithm based approach is used in this thesis to solve this problem.

Chapter 5 presents a multi-objective formulation for design of actively controlled structures with mixed discrete-continuous design variables. Stackelberg game theory and cooperative game theory are used to deal with multiple objectives in the formulation. The topological aspects of the design in the context of active control are presented in Chapter 6. The solution approach presented first determines the optimum topology followed by a simultaneous structural and control optimization of the optimum topology. The main findings of

this research as well as potential topics for future research are discussed in Chapter 7.

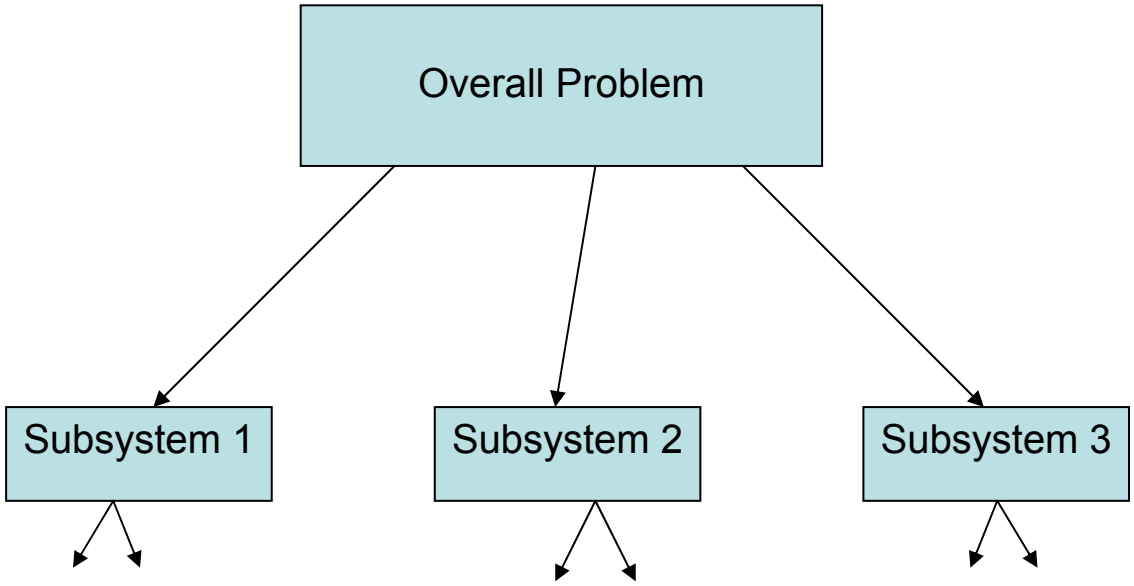

Figure 1.1 A Generic Hierarchic Multi-objective Problem

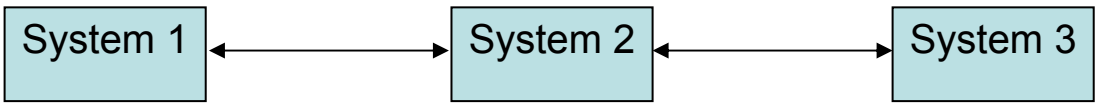

Figure 1.2 A Decentralized Multi-objective Problem

Chapter 2

Literature Summary

A simultaneous optimization of structure and control systems has attracted significant attention over the years. A number of approaches have been proposed for the simultaneous design of structure and control systems. A brief overview of the available literature on the design of actively controlled structures, multi-objective optimization, actuator placement problem as well as topology optimization is presented in this chapter.

2.1 Active Control and Structural Optimization

As mentioned in Sec. 1.1, traditionally, the structure and control systems have been designed separately with minimization of structural weight considered as an objective from a structural perspective and a minimization of the control energy as an objective function from a controls perspective. Both systems result in an optimum design, but the combined system might not be system-optimal. Therefore, in order to obtain the best overall performance with minimum cost, studies have been done on simultaneous optimum design of structure and control system.

Fonseca and Bainum (1995) proposed two approaches, combined and sequential integrated, to solve the simultaneous structural/control optimization problem. The combined approach uses a cost function that includes both control and structure design considerations whereas the sequential integrated approach uses two separate cost functions for control and structure, but they are matched

through constraints. Both these approaches yielded very similar transient performance in terms of response time and control efforts. Khot et al. (1986) use weight minimization of the structure as objective function with constraints on the distribution of the eigenvalues and/or damping ratio of the closed loop system. Onoda and Haftka (1987) formulated the combined structures/control optimization by minimizing the combined total cost of structure and control system with constraints on the magnitude of the response. The cost of the structure is taken to be proportional to its mass and the cost of control system is assumed to be a function of the magnitude of control force required for the actuators.

Instead of combining the structure and control objectives as one cost function or relegating one of them to a constraint, the simultaneous control/structural design problem has also been treated as a multi-objective problem. Lee (1993) proposed a multi-objective formulation to the integrated structure/control problem using structural weight, control energy, energy dissipated by active controller and stability robustness index as the objective functions. This multi-objective problem is solved using a cooperative game theoretic approach.

Usually the methods proposed for simultaneous structural and control optimization work well for simple structure with few design variables, which in most cases are continuous in nature. Further, in majority of these cases, the controller is designed using LQR theory with fixed Q and R matrices. For problems where the number of design variables and constraints is large, the

optimization process becomes costly. Therefore, this thesis proposes multi-level optimization techniques to solve the simultaneous structural and control design problem with discrete and continuous design variables. For simplicity, the LQR theory is used for control design, but the optimum values of Q and R matrices are determined as part of the solution process.

2.2 Determination of Weighting Matrices

Several methods have been developed for the simultaneous design of structure and control system as mentioned in Sec. 2.1. For control system design, the most commonly used method is the linear quadratic regulator (LQR) theory. Since the weighting matrices in LQR design directly affect the optimal control performance, some studies have been done for optimal selection of these matrices.

Sunar and Rao (1993) proposed a methodology for selecting the state and input weighting matrices, Q and R, when using linear quadratic regulator in the integrated structure and control system design. The optimum values of Q and R result in minimizing the performance index and reduced control effort. According to the proposed scheme, the performance index is significantly affected by the changes in the diagonal entries of Q and R matrices, therefore, the diagonal entries of Q and R are chosen as design variable to minimize the quadratic performance index. The design was done using a substructure decomposition scheme (for large structures) in order to save the computational cost with little loss in accuracy.

Ohta et al (1991) have presented a method for selecting weighting matrices in linear quadratic regulator with some diagonal weights that achieve a specified pole location. The proposed method uses a polynomial as a desirable pole specification and the weighting matrices are derived in an analytical form. Ochi and Kanai (1993) proposed a new way of pole placement by finding a weighting matrix which gives desired locations of the closed loop poles. These poles can then be placed arbitrarily and exactly at the desired positions but the method does not guarantee the positive definiteness of the weighting matrix. The problem of eigen vector assignment is not considered in the paper and the proposed method is computationally expensive.

Hiroe et al (1993) proposed a method called zero addition decoupling (ZED) for selecting weighting matrices of linear quadratic regulators which gives desired closed loop response. Choi and Seo (1999) presented an LQR design method which has the flexibility of exact eigen structure assignment with stability-robustness properties. The proposed method guarantees that the desired eigen values are assigned exactly and the desired eigen vectors are assigned in the least-square sense. Ang et al (2002) presented a weighted energy method for selecting the weighting matrices for vibration control of smart composite plates. The quadratic function is selected as a relative measure of strain, kinetic and input energy and their significance is represented through their relative weight factors. The effect of the weight factors on the active modal damping is predicted by modal control method.

Mansouri and Khaloozadeh (2002) proposed a genetic algorithm based approach for an optimal linear quadratic tracking problem. Proper choice of weighting matrices is necessary for satisfying the design specification and this difficulty is overcome by using genetic algorithm. Li et al (2008) presented a multi-objective evolution algorithm based approach for optimal design of weighting matrices in linear quadratic regulator. By establishing the multi-objective optimization model of LQR, the weighting matrices, Q and R, are designed which makes control system meet multiple performance indices simultaneously. Ghoreishi et al (2011) carried out a comparative study of different optimization methods for an optimal design of LQR weighting matrices. Closed-loop pole locations, speed of response and maximum level of control effort are combined into an objective function and this multi-objective problem is solved by a weighted sum method and the results for different optimization algorithms are then compared.

Almost all of the referred papers discussed above consider only the control optimization problem for the optimum selection of the weighting matrices. In this thesis, a combined approach to structural and control optimization is presented which not only considers structural design aspects, but also considers controller design, selection of suitable weighting matrices as well as proper actuator placement in an integrated manner. The proposed method results in an improved structural weight and control system performance of the overall structural-control system.

2.3 Actuator Placement

Different cost functions have been used by the researchers to find the optimum locations of actuators (sensors) to minimize the control energy required by maximizing a controllability criterion, maximizing the control forces transmitted by the actuators to the structure or optimizing a cost function based on linear quadratic regulator framework. Mirza and Van Niekerk (1999) proposed a method to determine the optimal location of actuators based on the disturbance sensitivity grammian matrix. Hakim and Fuchs (1996) compared the performance of different heuristic search techniques to determine their effectiveness in optimal actuator placement design for large truss structures. The techniques considered are simulated annealing, single-location iterative minimization and exhaustive single- point substitution.

Yan and Yam (2002) proposed a method for finding the optimal number and locations of actuators based on the eigenvalue distribution of energy correlative matrix of control input. Braunt and Proslier (2005) presented a modified approach for the usual approaches of minimizing control energy and maximizing control force to insure good controllability and observability of each mode of structure. The authors also considered the residual modes in the objective function to limit the spill over effects.

Gawronski (1997) dealt with non collocated actuators and disturbance inputs as well as non collocated performance and sensor outputs. Maghami and Joshi (1993) proposed a scheme that approximates the discrete nature of sensor and actuator placement problem by spatially continuous functions and reduces

the problem to a nonlinear programming optimization. Some literature uses linear quadratic regulator (LQR) framework to find the optimal locations of actuators/sensors. Demetriou (2000) considered minimizing the optimum value of a performance index to find the optimum locations of actuators and sensors. Different options for placing the sensors were presented. Pan (1989) proposed sequential-best-adding method, penalty function method and genetic algorithm, for solving the actuator/sensor location selection problem for maximizing the dissipation energy of the controller.

Liu et al. (2004) proposed a method for actuator placement on a reduced order model. The authors proposed a scheme based on H_2 norm of the transfer function from disturbance to controlled output in order to find the optimum locations of sensors and actuators for vibration control.

Khot et al. (1992) dealt with the effect of changing number and locations of actuators on optimum structure and control design. Lee (1993) proposed a similar approach but instead of using weight minimization as the objective function, the maximization of energy dissipated by the controller was used as the performance criteria. This mixed discrete continuous design variable problem was solved by using hybrid optimization method. Li et al. (2004) proposed a three level optimal design problem for finding the optimal number and locations of actuator in actively controlled structure using a two-level genetic algorithm.

These studies on actuator placement deal with finding the optimum locations of actuators; the number of actuators is assumed to be fixed. The design variables are discrete, i.e, the locations of actuators, and the problem is

treated as a control optimization problem with minimization of the performance index or controllability as objective functions. The approach presented in this thesis treats the actuator placement problem as a mixed discrete and continuous variables problem wherein both the structural and control optimization aspects are addressed simultaneously.

2.4 Stackelberg Game Theory

Stackelberg game theory is a technique for solving bi-level optimization problems and is used in this work. Several approaches such as the rational reaction set (Lewis and Mistree 1998), monotonicity analysis (Rao et al. 1997), sensitivity analysis (Ghotbi and Dhingra 2012) have been proposed for the computation of Stackelberg solutions. Simaan and Cruz (1973) introduced the concept of a rational reaction set in the context of Stackelberg games. For some simple problems arising in mechanical design such as the pressure vessel problem considered in Rao et al. (1997), design of a nonprismatic bar considered by Badhrinath and Rao (1996), closed form expressions for Stackelberg solutions can be obtained using the principles of montonicity analysis (Papalambros and Wilde, 2000). However, in general, numerical techniques are needed to approximate the rational reaction set (RRS). A design of experiments based approach (Montgomery 2005) coupled with response surface methodology (Myers and Montgomery 2002) has been proposed by Lewis and Mistree (1998), Marston (2000), and Hernandez and Mistree (2000) to approximate RRS for the players. Lewis and Mistree (1998) showed application of the Stackelberg game in the context of aircraft design, while Hernandez (2000) showed the application in

design of absorption chillers. Lewis and Mistree (1998) compared the solution of Stackelberg game with cooperative game and Nash solution (non-cooperative game) in design of a pressure vessel and a passenger aircraft. Sobieski (1982) presented the sensitivity of optimal design variables with respect to parameters existing in the problem. Ghotbi and Dhingra (2012) have developed a sensitivity based approach to approximate RRS in the design of flywheel problem. The method has been shown to be more general than DOE-RSM or monotonicity analysis based approaches.

2.5 Topology Optimization

An optimization of topology is usually considered in the context of structural design. Topology optimization problems are more challenging than sizing optimization problems because members can be added to or removed from the initial structure; therefore, the finite element model of the structure, number of design variables and constraints change from one iteration to the next. A number of approaches such as the ground-structure method (Xu et. al, 2003), integer programming using 0-1 variables (Ohsaki and Katoh, 2005), genetic algorithms (Liu et. al, 1998), and simulated annealing (Dhingra and Bennage, 1995) have been used for solving the topology optimization problem. All of these approaches are based on discretizing the problem domain at a finite number of nodal points; consequently, the resulting optimum topologies are dependent on the underlying distribution of nodes.

Xu et. al (2003) proposed a method for determination of optimum structural topology by choosing member cross-sectional areas and some

geometry parameters as topology design variables. The topology was changed by deleting elements with very small cross-sectional areas from the ground structure and combining overlapping elements into a single element. Ohsaki and Katoh (2005) formulated the topology optimization problem as a mixed integer programming problem (with 0-1 variables indicating the existence of nodes and members) with the local constraints on nodal instability and intersection of members. Liu et al. (1998) proposed a genetic algorithm based method for integrated structural topology/control optimization which includes robustness and controllability considerations. With a given structural weight, the proposed method yielded considerable improvements in performance in terms of vibration level, robustness and controllability.

Dhingra and Bennage (1995) proposed a method for topology optimization of trusses using simulated annealing in which the search for an optimum topology is simulated as a relaxation of stochastic structural system. The problem with this approach is that geometry of each candidate topology needs to be optimized, and thus the solution process involves significant computational effort. An integrated optimization of structural topology/actuator placement is carried out by Liu et al. (1997) using simulated annealing to deal with discrete design variables. The linear quadratic regulator cost index is considered as the objective function with constraints on weight and stability of the system. The method is computationally expensive and does not guarantee convergence to a global minima.

Some other recently developed alternatives (Huang and Xie 2007, Rong and Liang 2008, Bruggi and Verani 2011, Eom et al. 2011, Jia et al. 2011) which treat the problem domain as a continuum instead of a finite collection of nodal points include the homogenization method, SIMP, and evolutionary methods for topology optimization. The homogenization method (Bendsoe, 1989) is based on a discretization of the solution domain into micro structural centroids and redistributing the material using an optimality criteria approach. The SIMP method treats the density of each element as a variable and a heuristic relationship is defined between the Young's modulus and the density. The evolutionary approaches use the sensitivity of structural compliance to member addition and deletion to guide the search and arrive at the optimum topology.

Recently some works (Diaz and Mukherjee, 2006, Xu et al. 2007, Molter et al. 2010 and Silveira et al. 2010) have appeared which address topological and control considerations simultaneously. These include finding best locations of external forces to transfer energy from unmodeled modes to controlled modes and optimum actuator placement with constraints on controller performance. The solution approach involves first finding the optimum topology followed by optimum actuator placement according to optimum distribution of piezo electric material. It may be noted that while these works address control considerations, structural issues such as constraints on stresses, frequencies, etc. are not addressed.

A review of the available literature indicates that topology optimization has primarily been considered in the context of structural design. The problem of

topology optimization in the context of structural control has received limited attention. In case of simultaneous structural and control system design, generally structures with known topology are considered. This thesis presents a comprehensive approach to an integrated treatment of topology, structural and control optimization aspects for the design of actively controlled structures.

2.6 Summary

Though a lot of research has been done in developing methods dealing with active control of structures, there are some gaps that still need to be filled. As discussed in previous sections, most of the literature on simultaneous structural and control design deals with problems with continuous design variables, single objective function, and structures with known topologies. The actuator placement problem has been considered only in the context of control design. This thesis is an attempt to fill in these gaps by presenting a comprehensive treatment of structural, control and topological considerations in the context of actively controlled structures.

Chapter 3

Simultaneous Structure and Control Design of Actively Controlled Structures

This chapter presents basic concepts in simultaneous structure and control design of actively controlled structures. The approach to simultaneous structural and control design considered herein is that structure and control objective functions can be optimized by linking them through constraints related to structural and control performance. Linear quadratic control theory is used to design a controller for the structure under consideration. The effect of changing the state and control weighting matrices as well as the number and locations of the actuators on the performance of the control system is also discussed. An application of solution approaches presented in this chapter is illustrated through a 12 member 3-D space structure.

3.1 Introduction

Large size, light weight and ease of assembly are some of the desirable attributes in design of space structures. The compromise between a large size and low weight results in a structure that is very flexible, but it makes the control of the structure and its components very difficult. Because these structures are large and flexible, they are very sensitive to environmental effects. Further, these structures posses inherently low damping. Therefore, active control schemes are needed to quickly bring the structure back to its equilibrium position when it is subjected to a disturbance. The purpose of control is to damp out structural

vibrations to initial excitations. Linear quadratic regulator (LQR) control method is used for control system design in this thesis. Though a majority of the work on integrated structure and control design uses a linear quadratic regulator (LQR) for controller design, the influence of state and control weighting matrices on controller performance is ignored. It is proposed herein that the performance of the control system can be improved by selecting optimum values of the cross sectional areas of the members as well as the entries of the state and control weighting matrices used in the LQR design

Sensor and actuator placement is also an integral part of a control design. A number of studies have been done on vibration control of flexible structures. In these studies, the actuators are placed at some specific locations on the structure. Placing a sensor or an actuator at the correct location is important because it directly affects the observability and controllability of the structure. The location of actuators also influences the control of a vibration mode. For example, if an actuator is placed near a nodal point of a mode, then that mode cannot be controlled, or large forces are required to control that mode. The amount of energy consumption by the actuators also depends on the actuator placement and is a major concern in vibration control because actuator size depends on energy requirements. In order to improve the control system performance and minimize the energy consumption, the actuators should be placed at the optimum locations. The influence of actuator location on actuator efficacy is also studied in this chapter.

3.2 Dynamic Model

Control system design requires a mathematical model of the system being controlled. State-space models are commonly used for control system design and are used herein. The starting point for state-space models are the differential equations governing the structural dynamics. These equations are converted into state space form for control system design.

The finite element dynamical equations governing the motion of a controlled structural system are given as:

$$[M][\ddot{x}]+[C_D][\dot{x}]+[K][x]=[D][F_c] \tag{3.1}$$

where $[x]$ is a $n\times 1$ vector of physical coordinates, $[F_c]$ is $m\times 1$ control vector, $[M]$, $[C_D]$ and $[K]$ are $n\times n$ mass, damping and stiffness matrices respectively. The matrix $[D]$ is the $n\times m$ applied force distribution matrix which relates the input control force to the coordinate system. For actuator forces acting along the members of the structure, $[D]$ is defined using direction cosines of the constituent members.

Using the coordinate transformation $[x]=[\phi][y]$, Eq. (3.1) can be represented in state space form as:

$$[\dot{u}]=[A][u]+[B][F_c] \tag{3.2}$$

where $[y]$ is the vector of modal coordinates, $u=[[y],[\dot{y}]]^T$, is $2n\times 1$ state variable vector, $[\phi]$ is $n\times n$ modal matrix, $[A]$ is $2n\times 2n$ plant matrix, and $[B]$ is $2n\times m$ input matrix. The plant matrix $[A]$ and input matrix $[B]$ in Eq. (3.2) are given as:

$$[A]=\begin{bmatrix} 0 & I \\ -\omega_i^{\,2} & -2\xi_i\omega_i \end{bmatrix} \tag{3.3}$$

$$[B]=\begin{bmatrix} 0 \\ \phi^T D \end{bmatrix} \tag{3.4}$$

where ξ_i and ω_i denote the damping factor and natural frequency of the i^{th} mode respectively.

A controller for the system governed by Eq. (3.2) is designed using linear quadratic regulator (LQR) theory. The optimum control force $[F_c]$ is selected to minimize the quadratic performance index, PI, which is a compromise between minimum control energy and minimum error requirements, and is defined as:

$$PI=\int_0^\infty ([u]^T[Q][u]+[F_c]^T[R][F_c])dt \tag{3.5}$$

where $[Q]$ is a positive semi definite state weighting matrix and $[R]$ is a positive definite control weighting matrix. The optimum feedback control law is given as $[F_c]=-[\kappa][u]$ where $[\kappa]$ is the feedback gain matrix defined as $[\kappa]=[R]^{-1}[B]^T[P]$ and $[P]$ is the solution to matrix Riccati equation

$$[A]^T[P]+[P][A]+[Q]-[P][B][R]^{-1}[B]^T[P]=[0] \tag{3.6}$$

$[P]$ is a $2n\times 2n$ positive definite matrix called the Riccati matrix .The minimum value of the quadratic performance index (Eq. 3.5) is given as:

$$PI^*=u^T(0)[P]u(0) \tag{3.7}$$

where $u(0)$ is the initial state vector. The result in Eq. (3.7) depends on the initial state $u(0)$ which can vary or may not always be known. It has been found that

the expected value of PI^* over a set of possible initial states $u(0)$ is equivalent to trace of P. Therefore, it can be shown that the minimization of the quadratic control effort PI^* is proportional to trace$[P]$.

$$ev(J^*) = trace[P] \tag{3.8}$$

A minimization of trace$[P]$ will be considered as one of the objective functions in this thesis. Substituting the value of $[F_c]$ in Eq. (3.2) yields:

$$\dot{u} = ([A] - [B][\kappa])u = [A_{cl}]u \tag{3.9}$$

The eigenvalues of the closed-loop matrix $[A_{cl}]$ are a set of complex conjugate pairs given as:

$$\lambda_i = \alpha_i \pm j\beta_i \qquad i = 1.....n \tag{3.10}$$

where $j = \sqrt{-1}$ and $|\lambda_i| = \sqrt{\alpha_i^2 + \beta_i^2}$. The closed-loop damping ratios ξ_i associated with λ_i is given as:

$$\xi_i = -\frac{\alpha_i}{\sqrt{\alpha_i^2 + \beta_i^2}} \qquad i = 1.....n \tag{3.11}$$

The solution to Eq. (3.9) for a given initial condition $u(0)$, is given as:

$$u(t) = e^{[A_{cl}]t}u(0) \tag{3.12}$$

This equation can be used to find the dynamic response of the structure when it is subjected to some initial disturbance $u(0)$. The MATLAB function ode45 can be used to solve the first order differential equation given in Eq. (3.9).

3.3 Solution Procedure

The first problem considered involves solving a simultaneous structural and control design problem for minimizing the weight of the structure by fixing the actuators at some specific locations and fixing the $[Q]$ and $[R]$ matrices (see Eq. 3.5) as identity matrices. Next the effect of changing the $[Q]$ and $[R]$ matrices is studied by using a minimization of trace$[P]$ as the objective function. Two cases are considered: (i) the cross-sectional areas of members are fixed and $[Q]$ and $[R]$ matrices are varied; (ii) the member cross-sectional areas as well as entries of $[Q]$ and $[R]$ matrices are varied. Lastly the influence of the number and locations of the actuators on overall structural-control design is studied by performing a parametric study in which the actuators are placed at all possible locations and the effect of removing one (least effective) actuator at a time is studied. A solution methodology to find the optimum number and locations of actuators is presented in Chapter 4.

3.4 Influence of Weighting Matrices on Optimum Design

The effect of changing the weighting matrices is presented in this section with actuators fixed at some specific locations. Two cases are considered. In the first case, the weighting matrices are assumed to be fixed and cross-sectional areas are varied to optimize the controller performance index. The second case involves varying both the cross-sectional areas and weighting matrices to optimize the controller performance index.

3.4.1 Baseline Design — Weighting Matrices fixed

A design example is presented next for studying the effect of using optimum values for weighting matrices on the optimum design of structure. Towards this end, a baseline design is established first. In this design, only member cross-sectional areas are varied to optimize the controller performance index, the weighting matrices are assumed to be fixed.

3.4.1.1 Design Example

The 12-member ACOSS four structure is shown in Fig. 3.1 (Jin and Schmit 1993). This structure, designed by Draper Labs, is the simplest non-planar geometry representing a large space structure. All physical and geometric properties of the structure are nondimensionalized. The edges of the truss consist of six elements (1 through 6) of length 10 units each and six bipod legs (7 through 12) of $2\sqrt{2}$ units each. The nodal coordinates of the system are given in Table 3.1. The structure has twelve degrees of freedom, three at each of the four free nodes. The Young's modulus of the members is taken as 1.0 and the weight density of the material is assumed to be 0.001. The size of $[Q]$ matrix is $2n \times 2n$ and $[R]$ matrix is $m \times m$ and they are assumed to be identity matrices. The values of n and m here are 12 and 6 respectively. The cross-sectional areas of the members are treated as design variables. A total of six actuators are present in elements 7 through 12.

The dynamic response of the structure to an initial disturbance is also studied by measuring the displacement associated with the line of sight (LOS).

Node 1 represents the antenna feed, and its motion measures the deviation from the LOS. The square root of the sum of the squares of displacement at node 1 in x and y direction is defined as LOS error and it should be damped out in order to fall within a certain range in a specified time interval. The dynamic response of the optimum structure is initiated by a unit displacement at node 2 in the x-direction at t=0.

3.4.1.2 Optimization Problem Formulation

A minimization of the controller performance index (trace$[P]$) is considered as the objective function with the cross-sectional areas of the elements of the structure as design variables. Mathematically, the optimization formulation is stated as:

Minimize trace$[P]$

by varying A_i

subject to

$$0.16434 - \xi_1 \le 0$$

$$1.3374 - \beta_1 \le 0 \qquad (3.13)$$

$$1.5 - \beta_2 \le 0$$

$$10 \le A_i \le 2000$$

The optimization problem is solved using the Method of Feasible Directions and the solution steps are outlined in Fig. 3.2.

3.4.1.3 Results

The starting values of the cross-sectional areas, closed-loop damping ratios, closed-loop eigenvalues and square of the natural frequencies are given in Table 3.2. The value of the weight at this starting design is 43.69 and trace$[P]$ is 1763.2. The LOS error for the transient response is given in Fig. 3.3. The transient response is simulated by finding the solution to Eq. (3.12) for 60 seconds at 0.05 seconds time intervals. The magnitude of LOS error is calculated at each interval.

Using the nominal values of the areas as starting design for the optimization problem, the optimum values of the areas, closed-loop damping ratios and closed-loop eigenvalues are given in Table 3.3. The optimum trace $[P]$ is 715 and the weight of the structure at this design is 22.9. A 60% reduction in trace $[P]$ and 48% reduction in weight is obtained at the optimum design. The LOS error at the optimum solution is 1.52 and is shown in Fig. 3.4.

When comparing the nominal and optimum designs, it is seen that in the case of nominal design, the frequencies associated with modes 3 and 4 and modes 7 and 8 are close to each other. However, in the case of optimum design, the frequencies are spread out and no two frequency values are as close as in the nominal design case.

3.4.2 Effect of Changing the Weighting Matrices

In order to see the effect of changing the weighting matrices on the controller performance, the same ACOSS four structure (Fig. 3.1) is considered

for the optimization problem. A minimization of trace$[P]$ is considered as the objective function with diagonal entries of the state and control weighting matrices, $[Q]$ and$[R]$, treated as design variables. The design constraints imposed on the problem are given by Eq. (3.13) with one additional constraint that all the diagonal terms of $[Q]$ and $[R]$ matrices should be greater than or equal to 1. The controls toolbox in Matlab is used for solving Riccati equation and finding the control gains used in the LQR control method.

3.4.2.1 Results

Two scenarios are considered next for studying the effect of varying weighting matrices on the optimum controller performance with (i) member cross-sectional areas at fixed values and (ii) optimum values determined for member cross-sectional areas.

3.4.2.1.1 Areas fixed at nominal values

The only problem variables are entries of $[Q]$and $[R]$ matrices. Two different starting designs are considered. When starting value of $[Q]$ and $[R]$are taken as$[I]$, where $[I]$ is an identity matrix, the minimum trace $[P]$ is found to be 1843.06. The optimum values of entries of $[Q]$ matrix are: Q_1 =13.5 and Q_{13} =7.05 All others $[Q]$ values are at the lower bound which is 1.0. All optimum $[R]$ values converge to the lower bound of 1.0. The second starting design used the value $[Q]$ =10$[I]$ and $[R]$=$[I]$. In this case, the minimum value of trace$[P]$ is 1844.86 and only Q_1=25.06 and rest of $[Q]$ values are all at 1.0. Also all entries of $[R]$

matrix are 1.0 at the optimum solution. Some other starting points are also considered and they are shown in Table 3.4 with the corresponding weights, trace $[P]$ and LOS error values. It can be seen from Table 3.4, different values for the starting design results in different values for the optimum design variables. This indicates there are several local optima and the results are not globally optimum.

3.4.2.1.2 Areas and Q and R Matrices as design variables

In order to improve upon the results reported in the previous section, the optimization problem is solved by considering the member cross-sectional areas and the diagonal entries of $[Q]$ and $[R]$ as design variables. The design variables in this case are 42 (12 cross sectional areas, 24 diagonal entries of $[Q]$ and 6 diagonal entries of $[R]$). At the starting design of $[Q]=[R]=[I]$, the optimum value of trace $[P]$=553.46 with optimum Q_1=3.21 and Q_{13} =3.54, all other Q's and R's converge to lower bound of 1.0. The optimum weight of the structure is 15.2 and the LOS error for the optimum design is 1.88.

By changing the starting design as $[Q]$=10$[I]$ and $[R]$=$[I]$, optimum value of trace $[P]$=550.06 with optimum Q_1=7.47 all other Q's and R's at 1. The optimum weight of the structure is 15.14 and the LOS error is 1.52 and is shown in Fig. 3.5. Some other starting points are also considered and they are shown in Table 3.5 with the corresponding weights, trace $[P]$ and LOS error values. The optimum values of the cross- sectional areas are given in Table 3.6. It can be seen from the results presented herein that a 34% reduction in weight and 23%

reduction in trace $[P]$ can be achieved by considering the member cross-sectional areas and diagonal entries of $[Q]$ and $[R]$ matrices as design variables. Therefore in order to improve the overall performance of structure, member cross-sectional areas along with entries of $[Q]$ and $[R]$ matrices should be considered as design variables.

3.5 Effect of Changing the Number and Locations of Actuators

In the previous section, the effect of state and control weighting matrices on the optimum design of the structure is presented. However, it should be noted that in this study, the number and locations of the actuators are assumed to be fixed. Since the placement of actuators is a very important design aspect in the context of actively controlled structures and it directly affects the control performance, it is therefore necessary to examine the effect of changing the number and locations of the actuators on the optimum design, which is considered in this section. A parametric study is first performed to see the effect of the number and locations of the actuators on the optimum design of the structure.

3.5.1 Parametric Study

A parametric study dealing with the effect of number and locations of the actuators on the minimum weight structural design is performed. The structure chosen for the parametric study is again the ACOSS four structure shown in Fig. 3.1. The problem is solved by initially placing the actuators in all twelve members, i.e., at all available locations and solving the optimization problem for

weight minimization by varying the cross-sectional areas of the members. The constraints imposed on the problem are the same as given in Eq. (3.13). Next, the least effective actuator is removed from consideration and the problem is solved again. The least effective actuator is defined as the one doing least amount of work. The work done by an actuator is calculated as $\int_0^t F_i \dot{x}_i dt$ where F_i is the force exerted by actuator i over time interval t and $\dot{x}$ denotes the nodal velocities.

The cross-sectional areas of the members for designs with varying number of actuators are given in Table 3.7. The first row in Table 3.7 indicates the number of actuators present in the structure. The performance index, actuator work and structural weight values for these cases are given in Table 3.8. The first row corresponds to the non-optimum nominal design with twelve actuators. For the 12 actuator design, actuator seven does the maximum work and actuator six does the least work as shown in Table 3.9. Therefore, actuator six is removed and the structure is re-optimized with eleven actuators. The process is continued as long as a feasible design satisfying all the constraints is obtained. From Table 3.8, it can be seen that the structural weight is minimum for the six actuator design. It can also be noted that as number of actuators decreases, the total work done by all actuators also decreases until the number of actuators fall below 7, then the actuator work starts to increase. The LOS error for the cases with 12, 10, 8 and 6 actuators are shown in Figs. 3.6-3.9 respectively.

The results in Table 3.8 indicate that placing the actuators in all 12 elements results in a decrease in the weight of the overall structure from 20.5 (when actuators were present in elements 7-12) to 13.4. Also, by looking at the weight values in Table 3.8, it can be seen that 6-actuator case gives the least weight but the control energy increases in this case. Comparing the dynamic response, the 12-actuator case (Fig.3.6) damped out the induced disturbance faster than the other designs (see Fig. 3.7-3.9). Therefore depending on the objective function chosen, the optimum designs could be different. If weight minimization is considered more important than control energy minimization then 6-actuator design is better. On the other hand if minimization of control energy is important, then the 12-actuator design is a better design.

3.6 Conclusions

The approach to simultaneous structural and control design is presented in this chapter. From the design example presented, it is shown that great savings in the control energy as well as structural weight is possible by using both the cross-sectional areas and entries of weighting matrices as design variables. It has also been shown that changing the number and locations of actuators has a significant effect on the design of an actively controlled structure. The results in Table 3.8 do not show any fixed pattern in control energy and weight values as the number of actuators goes down. The best design for a structural engineer is a minimum weight design that is with 6 actuators. On the other hand, a control engineer prefers a 12 actuator design for minimum control energy. Since both the performance measures (weight and control energy)

constitute important aspects of design of actively controlled structures, there is a need to formulate the problem as a multi-objective function problem to simultaneously incorporate different objective functions in the optimization procedure. This multi-objective formulation is presented in Chapters 4 and 5.

Table 3.1 Nodal Coordinates of ACOSS Four

Node	X	Y	Z
1	0	0	10.165
2	-5	-2.887	2
3	5	-2.887	2
4	0	5.7735	2
5	-6	-1.1547	0
6	-4	-4.6188	0
7	4	-4.6188	0
8	6	-1.1547	0
9	-2	5.7735	0
10	2	5.7735	0

Table 3.2 Nominal Areas, closed loop damping ratio, closed loop eigenvalues and squares of natural frequencies

Areas	Damping Ratio	Real Part	Imag. Part	Sq. of natural Frequencies
1000	0.0548	-0.0734	1.3375	1.79
1000	0.0655	-0.1088	1.6573	2.75
100	0.0738	-0.2121	2.8674	8.26
100	0.0802	-0.2357	2.9302	8.63
1000	0.084	-0.2837	3.3664	11.4
1000	0.0864	-0.362	4.1732	17.53
100	0.0761	-0.3536	4.6332	21.58
100	0.0723	-0.3421	4.72	22.39
100	0.0341	-0.2901	8.4986	72.31
100	0.0298	-0.2742	9.2062	84.83
100	0.0207	-0.2126	10.2456	105.02
100	0.0064	-0.0823	12.8504	165.14

Table 3.3 Optimum Areas, closed loop damping ratio, closed loop eigenvalues and squares of natural frequencies

Areas	Damping Ratio	Real Part	Imag. Part	Sq. of natural Frequencies
430.09	0.1635	-0.2218	1.336	1.79
424.82	0.0921	-0.0769	1.4926	2.25
306.03	0.0963	-0.2073	2.5533	6.57
397.06	0.0878	-0.197	2.8917	8.41
293.22	0.0655	-0.207	3.7632	14.21
222.21	0.0662	-0.2852	4.3519	19.02
122.85	0.0519	-0.2472	5.2807	27.94
304.48	0.0514	-0.3113	5.6312	31.79
27.89	0.0451	-0.3465	6.1208	37.56
50.53	0.0398	-0.2702	7.0125	49.25
142.49	0.0347	-0.315	8.008	64.17
120.54	0.0272	-0.2583	8.8766	78.81

Table 3.4 Areas fixed at nominal values

	Starting point				
	Q=R=I	Q=R=10I	Q=10I, R=I	Q^* $R_{1\text{-}3}$=10,$R_{4\text{-}6}$=1	Q^* $R_{1\text{-}3}$=1,$R_{4\text{-}6}$=10
Weight	43.70	43.70	43.70	43.70	43.70
trace $[P]$	1843.06	1843.60	1844.86	1853.82	1843.97
LOS	1.055	1.055	1.055	1.052	1.055
Optimum Q	Q_1=13.5 Q_{13}=7.05	Q_1=24.39	Q_1=25.06	Q_1=30.05	Q_1=21.98 Q_{13}=2.47

Q^* = $Q_{1\text{-}8}$=1, $Q_{9\text{-}16}$ =10 and $Q_{17\text{-}24}$ =5

Table 3.5 Areas and diagonal Q and R as design variables

Starting point	Q=R=I	Q=R=10I	Q=10I, R=I	Q^* R_{1-3}=1,R_{4-6}=10
Weight	15.23	15.13	15.14	15.29
trace $[P]$	553.46	550.24	550.06	554.92
LOS	1.88	1.89	1.52	1.87
Optimum Q	Q_1=3.21 Q_{13}=3.54	Q_1=7.83	Q_1=7.47	Q_1=1.47 Q_{13}=4.46

Q^* = Q_{1-8}=1, Q_{9-16} =10 and Q_{17-24} =5

Table 3.6 Optimum cross-sectional areas

Element	Q=R=I	Q=R=10I	Q=10I, R=I	Q^* R_{1-3}=1,R_{4-6}=10
1	271.58	310.42	235.8	277.45
2	209.78	192.14	247.74	211.33
3	205	208.39	240.68	205.6
4	217.4	201.48	216.41	216.07
5	220.88	202.78	217.91	219.72
6	228.03	232.24	182.6	228.89
7	66.16	160.95	195.33	67.28
8	187.68	76.75	84.15	185.8
9	107.37	50.51	91.89	105.79
10	96.99	99.4	70.11	96.22
11	53.09	107.61	122.31	55.04
12	93.64	90.56	50.17	93.09

Q^* = Q_{1-8}=1, Q_{9-16} =10 and Q_{17-24} =5

Table 3.7 Cross-sectional Areas of Members with Varying Actuators

Element	12 Nominal	12	11	10	9	8	7	6	5	4	3	2
1	1000	195.9	204.1	189.2	193.7	167.7	166.6	135.7	151.3	265	262.71	261.48
2	1000	144.5	146.6	144.4	137.2	135.4	141.4	199.9	241.1	238.8	182.34	443.37
3	100	183.5	178.4	186.8	176.9	213.9	223.5	183	147.7	300.6	247.32	164.5
4	100	134.8	132.6	133.2	148.6	142.4	134.5	185.1	218.9	160.6	189.21	350.34
5	1000	196.1	198	202	196.3	236.8	250.7	198.1	155.9	302.4	183.85	185.48
6	1000	205.3	199.5	207.1	209.9	181.9	177.6	144.9	158.3	197.9	240.61	246.67
7	100	179.4	189.5	184.9	176.5	203.2	195.4	162.8	92.97	31.73	170.33	95.14
8	100	135.1	128.5	131	132.4	99.01	83.14	169.3	207.8	120.4	199.3	28.37
9	100	127.4	133.3	131.3	110.7	133.3	138.9	185.1	209	194.1	151.02	313.52
10	100	193.5	188.9	188.6	202.4	179.7	173.2	129.6	123.3	148.6	34.78	199.56
11	100	158.8	171.1	166.8	158.8	198.2	212.2	180.3	152.9	205.9	162.12	197.1
12	100	180.4	171	174.6	189	141.2	132.5	131.1	171.5	121.1	203.9	231.07

Table 3.8 Performance index, total work and weight

Number of Actuators	$u^T Qu$	$F_c^T R F_c$	Actuator Work	Weight
12 nom	115.64	113.23	79.08	43.7
12	26.7	27.51	19.04	13.36
11	27.52	27.93	19.35	13.37
10	27.98	28	19.06	13.39
9	27.87	27.81	18.53	13.37
8	28.85	30	18.47	13.48
7	30.01	30.8	18.41	13.59
6	37.51	37.04	21.45	13.18
5	48.66	46.69	22.47	13.44
4	58.6	61.07	22.29	16.98
3	65.3	59.25	22.76	15.67

Table 3.9 Work done by each actuator

Act #	1	2	3	4	5	6	7	8	9	10	11	12
12 nom	4.39	5.27	2.8	2.98	1.13	1.02	22.83	10.08	19.33	5.63	1.83	1.72
12 opt	3.76	2.93	1.26	0.72	0.58	0.08	5.88	1.01	1.16	0.72	0.5	0.39
11 opt	4.11	3.07	1.15	0.67	0.43		5.84	1.09	1.14	0.82	0.55	0.434
10 opt	3.81	3.12	1.25	0.67			6.05	1.2	1.19	0.77	0.53	0.43
9 opt	3.61	3.064	1.472	0.643			5.985	1.04	1.29	0.781	0.633	
8 opt	4.07	2.88	1.32	1.14			5.46	0.7	1.69	1.17		
7 opt	3.938	2.629	1.362	1.557			5.542		1.868	1.509		
6 opt	3.928	1.994		4.022			8.552		1.417	1.53		
5 opt	2.741	3.293		4.154			10.71			1.565		
4 opt	2.045	4.656		3.729			11.86					
3 opt		5.552		1.229			15.97					
2 opt		4.296					25.74					

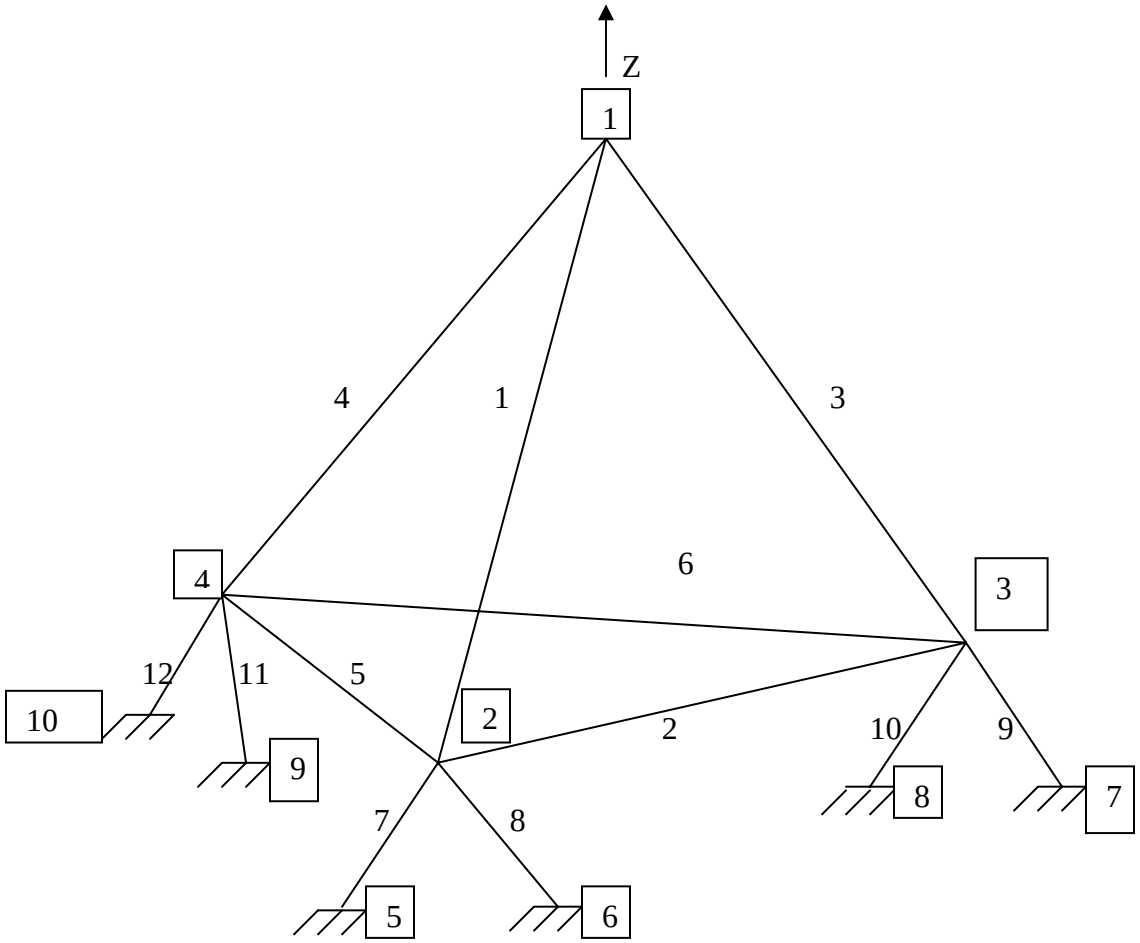

Figure 3.1 ACOSS-FOUR Structure

The numbers in boxes represent nodes while the others represent elements/members. Nodes 5 through 10 are fixed.

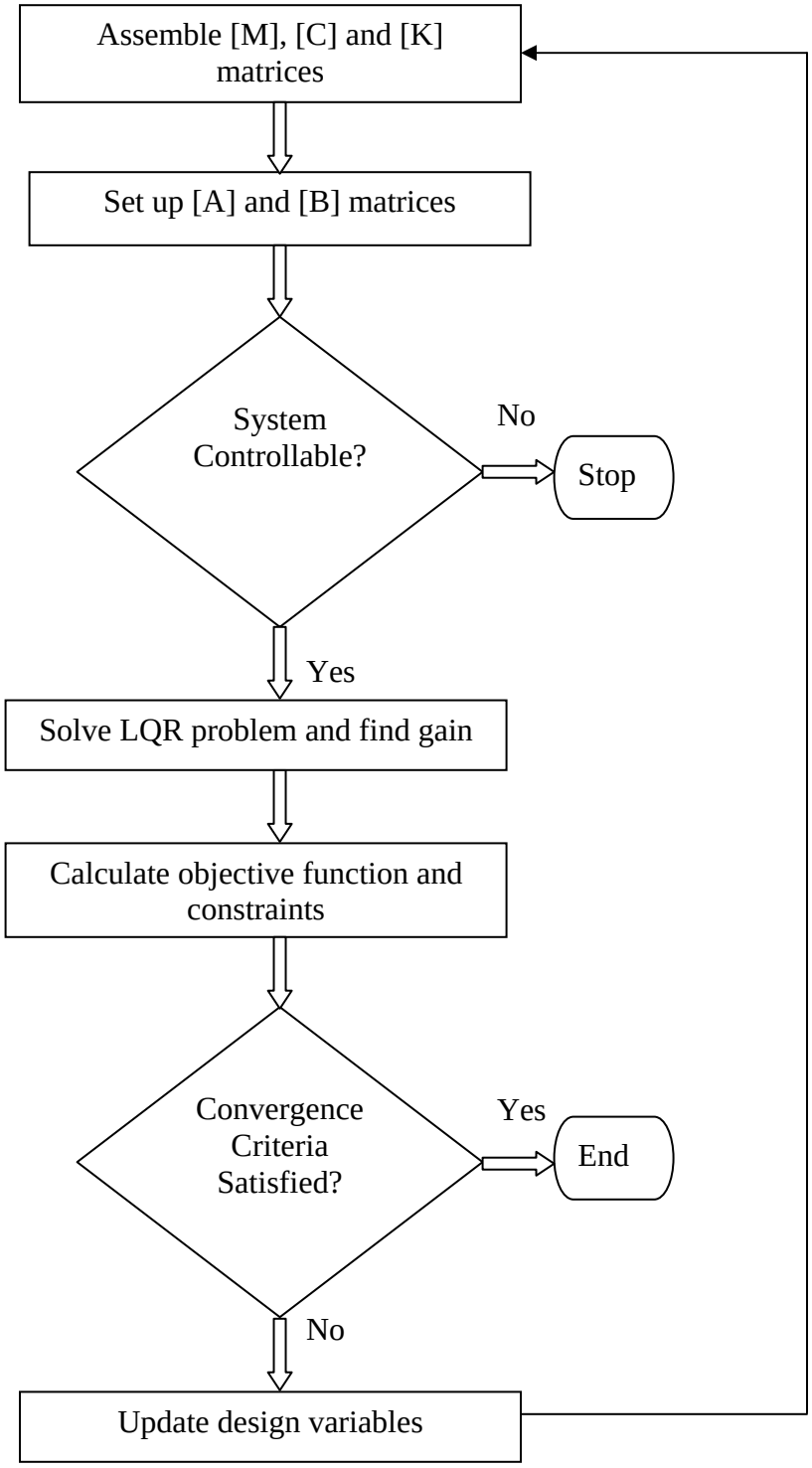

Figure 3.2 Steps in the optimization process

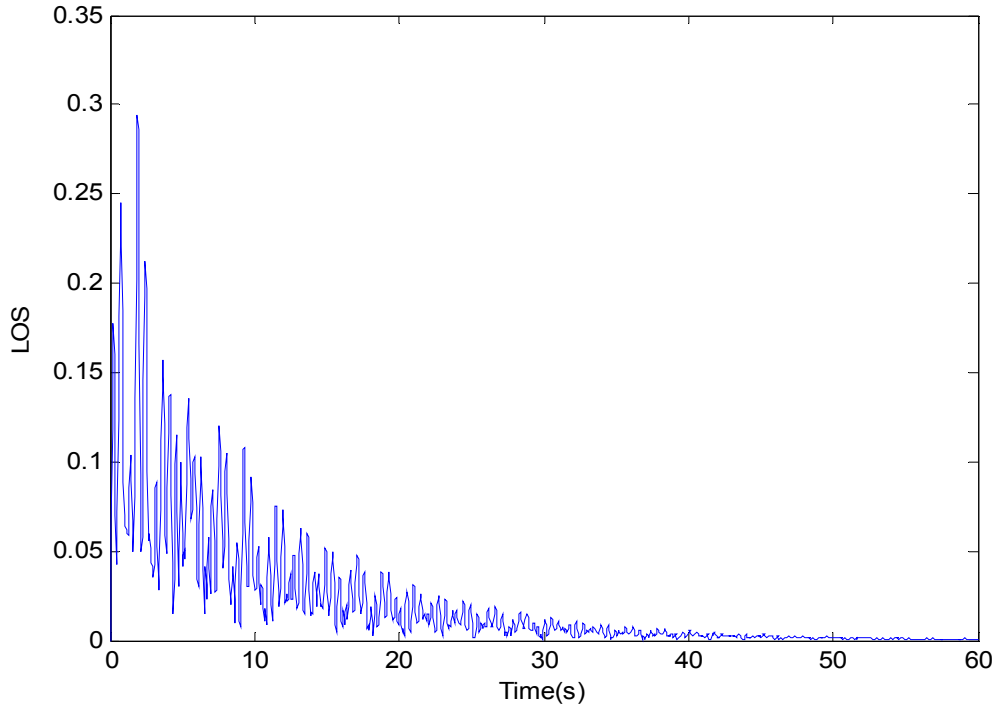

Figure 3.3 Transient response of structure at nominal design (LOS 1.3)

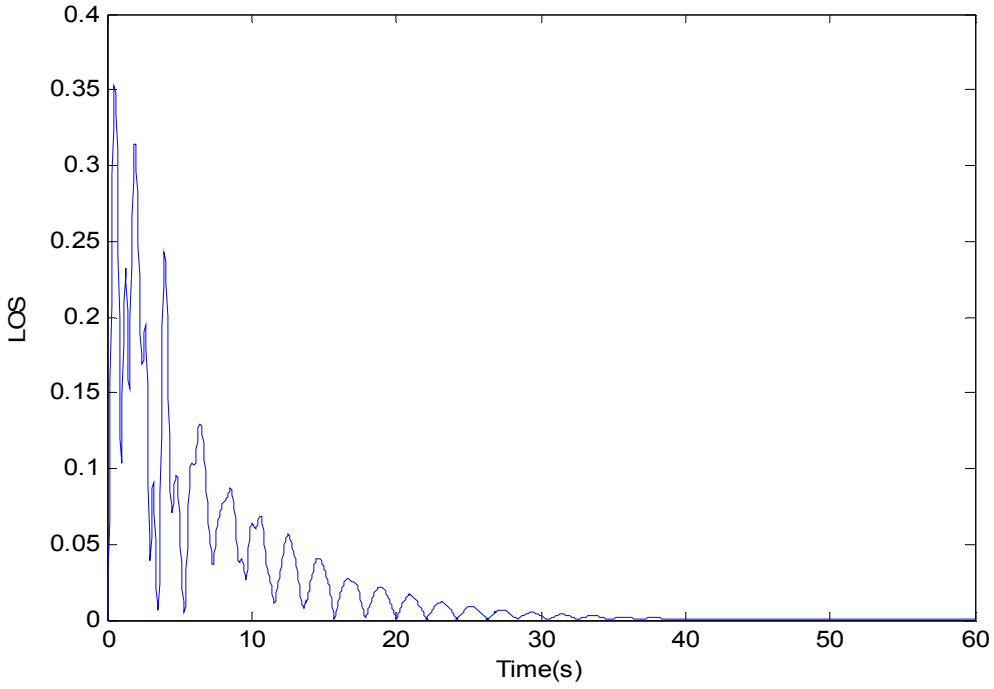

Figure 3.4 Transient response of structure at optimum design (LOS 1.52)

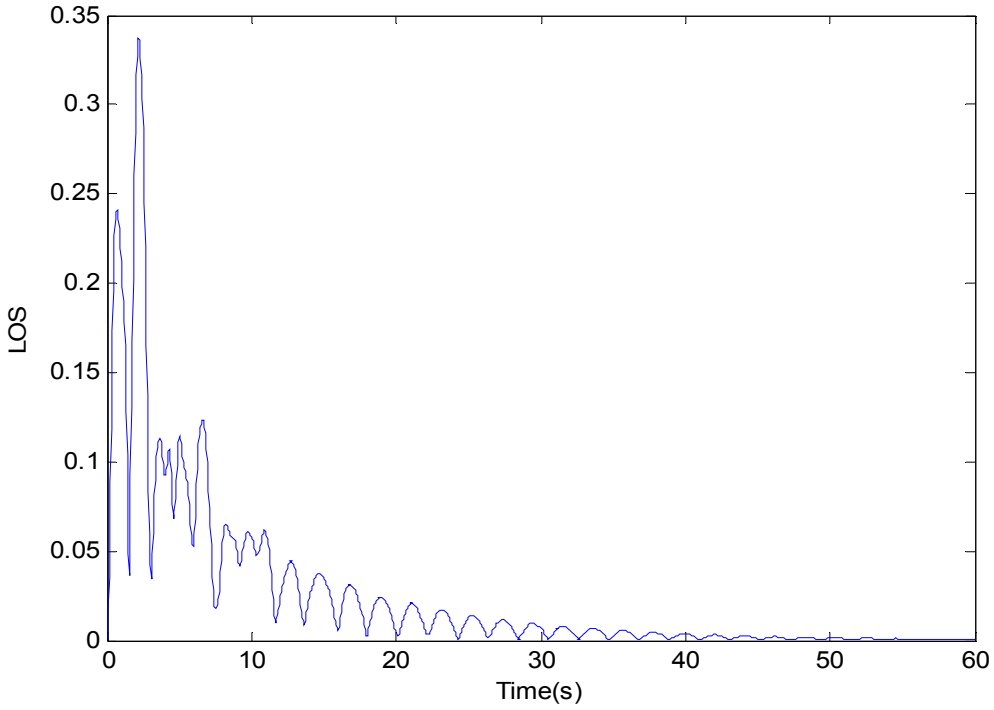

Figure 3.5 LOS error when areas and Q and R are varied (1.52)

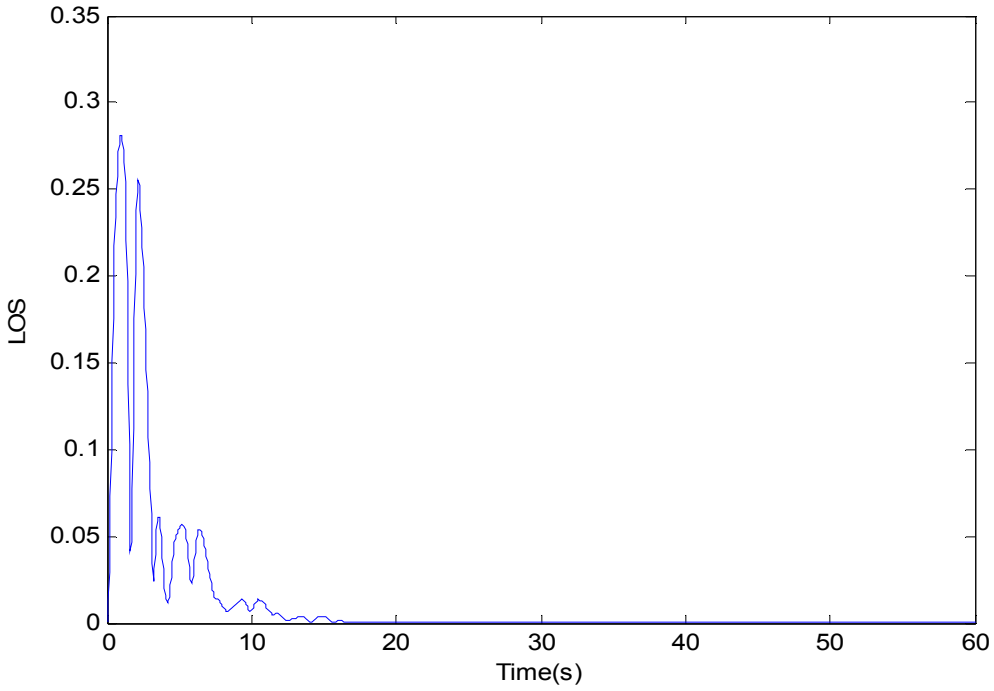

Figure 3.6 Transient response of structure with 12 actuators (LOS 0.77)

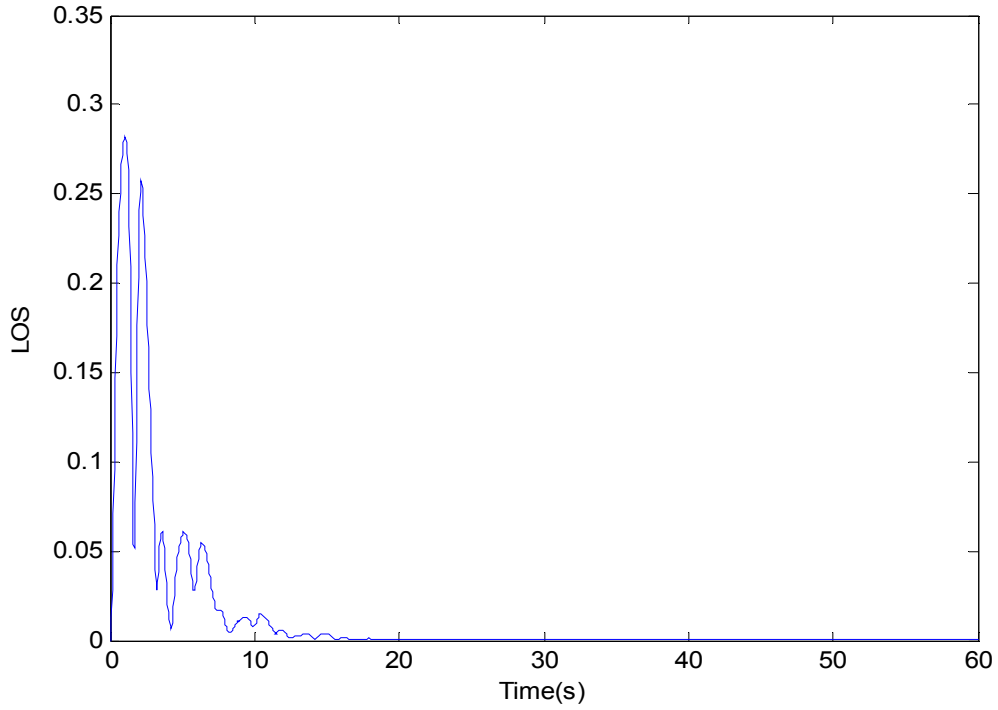

Figure 3.7Transient response of structure with 10 actuators (LOS 0.77)

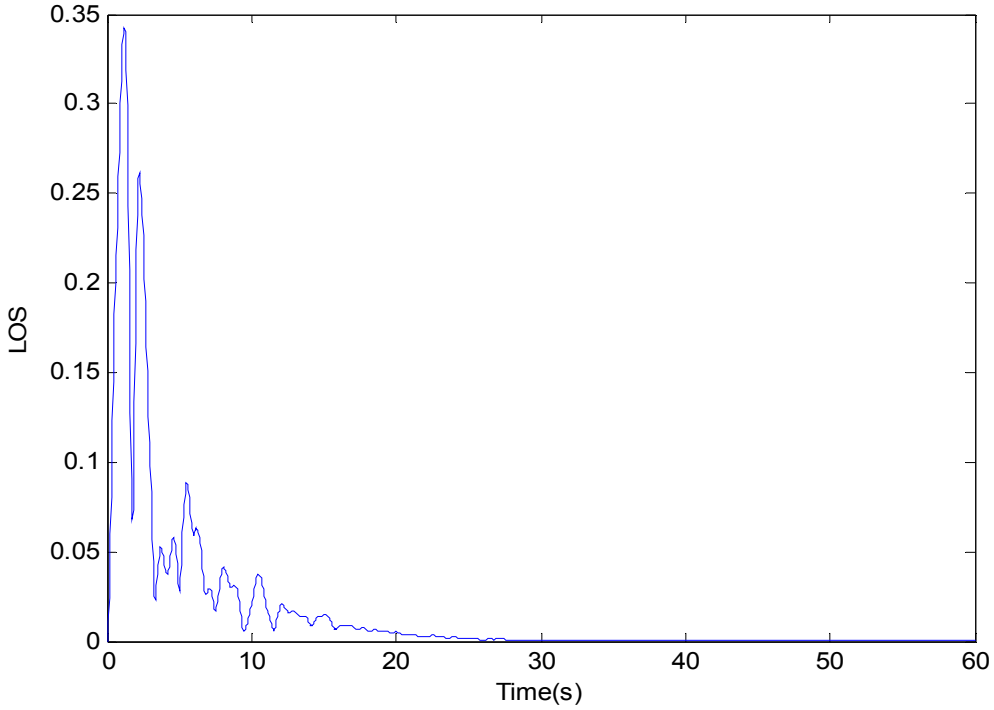

Figure 3.8 Transient response of structure with 8 actuators (LOS 1.01)

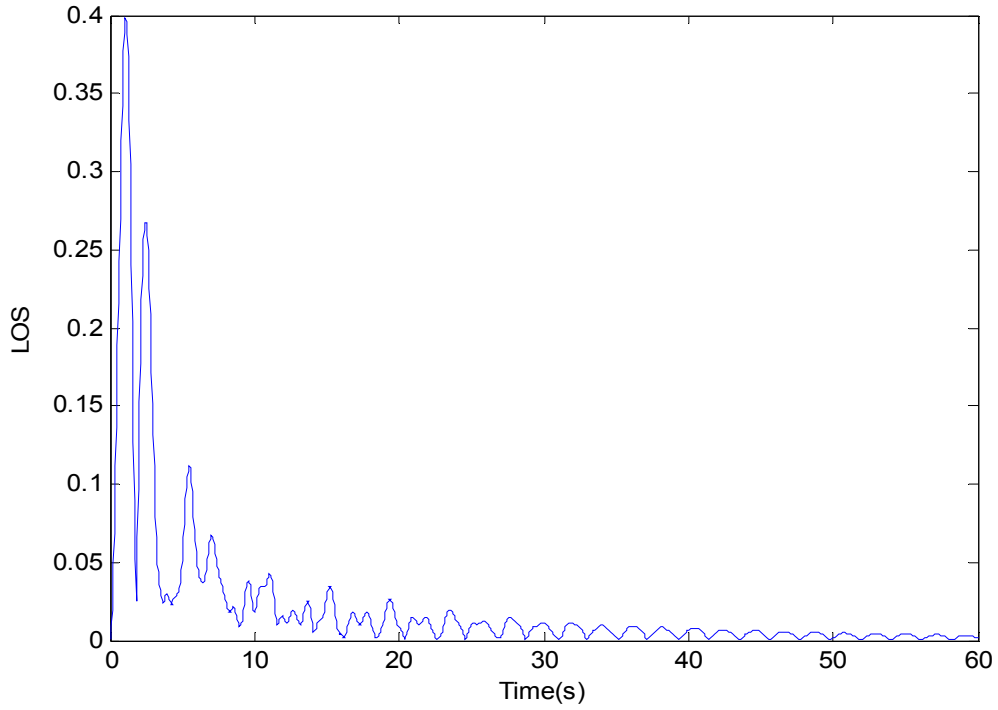

Figure 3.9 Transient response of structure with 6 actuators (LOS 1.31)

Chapter 4

Optimum Placement of Actuators in Actively Controlled Structures

The parametric study presented in Chapter 3 was performed to see the effect of changing the number and locations of actuators. This involved placing the actuators at certain adhoc locations and optimizing the controller and the structure. In order to efficiently reduce the vibrations of a structure, it is necessary to place the actuators at positions such that their ability to control the vibrations is maximized. Therefore, to get the optimum control performance with minimum control cost, the actuator locations should be optimized. An approach for determining the optimum placement of actuators is presented in this chapter. A genetic algorithm based approach is used to solve the optimization problem since the actuator locations are discrete (0-1) in nature.

4.1 Introduction

The placement of actuators is one of the important aspects of structural control design. The determination of the number and location of actuators and sensors in active vibration control of flexible structures is an important issue. Actuator placement has a significant effect on the dynamic response of the structure. Misplaced actuators and sensors lead to the problem of controllability and observability, and the desired system performance may not be achieved with any choice of control law.

Many of the studies on actuator placement deal only with the determination of optimum locations of actuators, and the number of actuators is

assumed to be fixed. Also in these studies, the design variables which are the positions of actuators are discrete (0-1), and the problem is treated only as a control optimization problem with minimization of performance index or controllability as objectives. In other words, structural design considerations are largely ignored. The approach presented in this thesis treats the actuator placement problem as a mixed discrete and continuous variable problem wherein both structural and control optimization aspects are addressed simultaneously. A determination of the optimum number as well as optimum positions of actuators along with optimum member cross-sectional areas helps to simultaneously optimize both structure and control aspects of structure design. The absence or presence of an actuator is defined using 0 and 1 discrete variables. The gradient-based optimizer which was used to solve the structural-control problem in chapter 3 can handle only continuous variables. For the mixed discrete and continuous variable problem presented in this chapter, genetic algorithm is used as an optimizer. The design variables are the cross-sectional areas of the elements as well as the number and locations of the actuators. The actuator placement problem is considered in the context of both single objective and multi-objective optimization. For the single objective optimization formulation, the objective function considered is the maximization of the vibrational energy dissipated by the actuators (trace$[H]$). In case of multi-objective formulation, cooperative game theory method is used to maximize the bargaining function between maximizing trace$[H]$ and minimizing weight of the structure.

4.2 Actuator Placement

The total energy stored in the system defined by Eq. (3.1) is the sum of kinetic and potential energies and can be written as:

$$E = K.E + P.E$$

$$= \frac{1}{2}\dot{x}^T M\dot{x} + \frac{1}{2}x^T Kx \tag{4.1}$$

$$= \frac{1}{2}\dot{y}^T \phi^T M\phi\dot{y} + \frac{1}{2}y^T \phi^T K\phi y$$

$$= \frac{1}{2}\dot{y}^T [I]\dot{y} + \frac{1}{2}y^T [\Delta]y \tag{4.2}$$

where $[x] = [\phi][y]$, $\phi^T M\phi = I$, $\phi^T K\phi = [\Delta]$ and $[\Delta] = diag(\omega_i^2)$

Differentiating Eq. (4.2) with respect to time gives the energy dissipation rate as:

$$\frac{dE}{dt} = \dot{y}^T [I]\ddot{y} + \dot{y}^T [\Delta]y \tag{4.3}$$

Integrating Eq. (4.3) from $t = 0$ to $t = \infty$ gives the total energy dissipated in the system due to internal damping as well as the damping induced by the control system.

$$\int_0^\infty \frac{dE}{dt}dt = \int_0^\infty (\dot{y}^T [I]\ddot{y} + \dot{y}^T [\Delta]y)dt \tag{4.4}$$

$$E = \int_0^\infty (\dot{y}^T [\phi^T M\phi]\ddot{y} + \dot{y}^T [\phi^T K\phi]y)dt$$

$$= \int_0^\infty (\dot{y}^T \phi^T M\phi\ddot{y} + \dot{y}^T \phi^T K\phi y)dt$$

$$= \int_0^\infty (\dot{x}^T M\ddot{x} + \dot{x}^T Kx)dt \tag{4.5}$$

$$= \int_0^\infty \dot{x}^T (M\ddot{x} + Kx)dt$$

$$= \int_0^\infty \dot{x}^T (DF_c - C_D\dot{x})dt$$

$$= \int_0^\infty \dot{x}^T DF_c dt - \int_0^\infty \dot{x}^T C_D \dot{x} dt$$

Considering the total energy dissipated in the system due to the damping induced by the control system which is:

$$E_c = \int_0^\infty \dot{x}^T DF_c dt \tag{4.6}$$

Substituting the optimal feedback control law $F_c = -\kappa u$ and the feedback gain $\kappa = [R^{-1}][B^T][P]$ in above equation yields:

$$E_c = \int_0^\infty \dot{x}^T D(-\kappa u)dt$$

$$= \int_0^\infty \dot{x}^T D(-R^{-1}B^T P)udt \tag{4.7}$$

From Eq. (3.4), $[B] = \begin{bmatrix} 0 \\ \phi^T D \end{bmatrix}$ and also $u = [[y],[\dot{y}]]^T$, therefore:

$$E_c = \int_0^\infty \dot{x}^T D(-R^{-1}B^T P)\begin{bmatrix} \phi^{-1}x \\ \phi^{-1}\dot{x} \end{bmatrix} dt \tag{4.8}$$

$$E_c = \int_0^\infty (-\dot{x}^T DR^{-1}\begin{bmatrix} 0 \\ \phi^T D \end{bmatrix}^T \begin{bmatrix} P_I & P_{II} \\ P_{III} & P_{IV} \end{bmatrix}\begin{bmatrix} \phi^{-1}x \\ \phi^{-1}\dot{x} \end{bmatrix})dt \tag{4.9}$$

In Eq. (4.9), the $2n \times 2n$ Riccati matrix $[P]$ is written in terms of four $n \times n$ block partitioned matrices $P_I - P_{IV}$

$$E_c = \int_0^\infty (-\dot{x}^T D R^{-1} \begin{bmatrix} 0 & D^T \phi \end{bmatrix} \begin{bmatrix} P_I & P_{II} \\ P_{III} & P_{IV} \end{bmatrix} \begin{bmatrix} \phi^{-1} x \\ \phi^{-1} \dot{x} \end{bmatrix}) dt \tag{4.10}$$

$$E_c = \int_0^\infty (-\dot{x}^T D R^{-1} \begin{bmatrix} D^T \phi P_{III} & D^T \phi P_{IV} \end{bmatrix} \begin{bmatrix} \phi^{-1} x \\ \phi^{-1} \dot{x} \end{bmatrix}) dt \tag{4.11}$$

Since $[x] = [\phi][y]$, Eq. (4.11) can be rewritten as:

$$E_c = \int_0^\infty (-\dot{y}^T \phi^T D R^{-1} \begin{bmatrix} D^T \phi P_{III} & D^T \phi P_{IV} \end{bmatrix} \begin{bmatrix} \phi^{-1} \phi y \\ \phi^{-1} \phi \dot{y} \end{bmatrix}) dt \tag{4.12}$$

Ignoring the minus sign since we are interested only in magnitude of energy dissipation, Eq. (4.12) can be written as:

$$E_c = \int_0^\infty \dot{y}^T \begin{bmatrix} \phi^T D R^{-1} D^T \phi P_{III} & \phi^T D R^{-1} D^T \phi P_{IV} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix}) dt \tag{4.13}$$

$$E_c = \int_0^\infty \begin{bmatrix} y & \dot{y} \end{bmatrix}^T \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} \phi^T D R^{-1} D^T \phi P_{III} & \phi^T D R^{-1} D^T \phi P_{IV} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} dt \tag{4.14}$$

$$E_c = \int_0^\infty \begin{bmatrix} y & \dot{y} \end{bmatrix}^T \begin{bmatrix} 0 & 0 \\ \phi^T D R^{-1} D^T \phi P_{III} & \phi^T D R^{-1} D^T \phi P_{IV} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} dt \tag{4.15}$$

$$E_c = \int_0^\infty \begin{bmatrix} y & \dot{y} \end{bmatrix}^T D_c \begin{bmatrix} y \\ \dot{y} \end{bmatrix} dt \tag{4.16}$$

where E_c is the energy dissipated by controller, $[y, \dot{y}]$ is the state vector, D_c is the damping matrix induced by the active controller and is defined as:

$$D_c = \begin{bmatrix} 0 & 0 \\ \phi^T D R^{-1} D^T \phi P_{III} & \phi^T D R^{-1} D^T \phi P_{IV} \end{bmatrix} \tag{4.17}$$

Using Eq. (3.12), Eq. (4.16) can be written as:

$$E_c = [y(0) \quad \dot{y}(0)]^T \int_0^\infty e^{[A_{cl}]^T t} D_c e^{[A_{cl}]t} dt \begin{bmatrix} y(0) \\ \dot{y}(0) \end{bmatrix} \tag{4.18}$$

where $[A_{cl}]$ is the stability matrix (Eq. 3.9). Using $[A_{cl}]$, a unique solution $[H]$ to the Lyapunov equation is given as:

$$\int_0^\infty e^{[A_{cl}]^T \tau} D_c e^{[A_{cl}]\tau} d\tau = H \tag{4.19}$$

$$[A_{cl}]^T H + H[A_{cl}] = -D_c \tag{4.20}$$

Now Eq. (4.18) becomes:

$$E_c = [y(0) \quad \dot{y}(0)]^T H \begin{bmatrix} y(0) \\ \dot{y}(0) \end{bmatrix} \tag{4.21}$$

Since the energy dissipated by the controller depends on the initial state which is not always known, Eq. (4.21) is not very useful. However, if the initial state is assumed to be a random variable distributed uniformly over the surface of a $2n$ dimensional unit sphere, maximization of expected value of E_c over the set of possible initial states is the same as maximizing trace$[H]$. Therefore,

$$ev[E_c] = trace[H] \tag{4.22}$$

For an efficient controller, trace $[H]$ can be maximized by treating the actuator locations as design variables. In addition, constraints can also be placed on closed-loop eigenvalues and damping ratios to specify natural frequencies of the controlled system as well as time required to damp out the vibrations. From a structural viewpoint, the designer may also want to minimize the weight of the structure by treating cross-sectional areas of the members as design variables.

Two variations of the structural-control optimization problem are considered next. The first approach involves maximization of trace$[H]$ with actuator locations and member cross-sectional areas as design variables. The second approach

considers a multi-objective problem where both control and structural objectives are considered simultaneously using a game theoretic approach and is presented in Sec. 4.4.

4.3 Optimization Using Genetic Algorithms.

The actuator placement optimization problem has mixed discrete (actuator locations) and continuous (members cross-sectional areas) design variables; therefore, it cannot be solved using conventional gradient based optimization methods. A genetic algorithm based approach is used in this work to solve this problem with mixed discrete-continuous variables.

4.3.1. Genetic Algorithms

Genetic Algorithms are a guided random search technique derived from the natural genetics of populations. The design variables are coded as a string of binary bits which correspond to the chromosome in natural genetics. A simple genetic algorithm involves copying strings and swapping partial strings between two mating strings. The three basic operators used in genetic algorithms are: reproduction, crossover and mutation. They are used to produce new generations as the search progresses and are briefly described below.

4.3.1.1 Reproduction

Reproduction is a randomized selection process in which individual strings are copied according to their objective function (fitness) value. Strings (population members) with a higher fitness value have a higher chance at

reproduction. The probability of reproduction is calculated by dividing the individual fitness by the sum of fitness values of the entire population.

4.3.1.2 Crossover

Crossover is the primary operator in the mating process which generates new individuals in the population. It consists of two steps. First, a crossover point is randomly selected between the mating couple. The second is swapping of genetic information between these two mating couples past the crossover point. Therefore, the mechanics of reproduction and crossover involves making copies of strings in proportion to their fitness values and exchange of genetic information between members in the mating pool.

4.3.1.3 Mutation

Mutation is the occasional random alteration (with small probability) of the gene value in a chromosome (string), that is, it involves changing a particular bit of a coded string from 0 to 1 and vice versa. Mutation is a random walk through the string space. Mutation rates are usually quite small, and it is considered as a secondary mechanism/operator of genetic algorithm.

Since genetic algorithms are primarily suited for solving unconstrained optimization problems, some simple modifications are needed to adapt the techniques for solving constrained optimization problems. In this work, the constraints in the problem are handled using a penalty function method and the objective function is defined as:

$$\phi(X, r_k) = f(X) + r_k \sum_{j=1}^{m} \left\langle g_j(X) \right\rangle^2 \tag{4.23}$$

where r_k is a positive penalty parameter, $j = 1,.....m$, is the total number of inequality constraints, and the bracket function $\left\langle g_j(X) \right\rangle$ is defined as:

$$\left\langle g_j(X) \right\rangle = \max \left(g_j(X), 0 \right)$$

$$= \begin{cases} g_j(X) & g_j(X) > 0 \\ 0 & g_j(X) \le 0 \end{cases} \tag{4.24}$$

4.4 Multi-objective Optimization Using Game Theory

Multi-objective optimization (MOO) problems requiring a simultaneous consideration of two or more conflicting objective functions frequently arise in design. A general MOO problem has the following form:

$$Min \ f(x) = [f_1(x), f_2(x), ... f_k(x)]$$

subject to

$$\begin{aligned} g_i(x) &\le 0 && i = 1.....m \\ h_j(x) &= 0 && j = 1.....p \\ x_i^{\min} &\le x_i \le x_i^{\max} && i = 1.....n \end{aligned} \tag{4.25}$$

where $f_1(x), f_2(x), ... f_n(x)$ are k different objective functions, $g_i(x)$ and $h_j(x)$ are inequality and equality constraints and x_i denotes the set of design variables. In a MOO problem, it is not possible to find an optimum point where all objective functions are simultaneously minimized. Therefore, the concept of a Pareto-optimal (PO) is frequently used in solving a MOO problem. Frequently, the set of

PO solutions contains more than one solution. Different methods have been used to determine an optimal compromise solution from the set of PO solutions. Game theory is one such approach which helps determine a compromise solution acceptable to all objective functions (players).

4.4.1 Game Theory Method

In the game theory method, the MOO problem is viewed as a game where each player corresponds to an objective function being optimized. These players are competing with each other to improve their overall position subject to some constraints.

There are three types of games in the context of engineering design: cooperative game, non-cooperative (Nash) game, and an extensive game. In a cooperative game, the players have knowledge of the strategies chosen by other players and collaborate with each other to find a Pareto-optimal solution. In a non-cooperative game, each player has a set of variables under his control and optimizes his objective function individually. The player does not care how his selection affects the payoff functions of other players. The players bargain with each other to obtain an equilibrium solution, called the Nash solution. Extensive games refer to situations in which the players make their decisions sequentially. Extensive games with two players have been used in engineering design and are called Stackelberg games. There are two groups of players in this game; one called the leader which dominates the other group called the follower. The leader makes its decision first and according to its decision, the follower optimizes its objective function.

Consider two players, 1 and 2, who select strategies x_1 and x_2 where $x_1 \in X_1 \subset R^{n_1}$ and $x_2 \in X_2 \subset R^{n_2}$. Here X_1 and X_2 are the set of all possible strategies each player can select. The objective functions $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ represent the cost function for players 1 and 2, respectively.

In a Nash (non-cooperative) game, each player determines its optimum solution based on the choices made by other player(s). The set of solutions for each player is called the rational reaction set (RRS). The RRS for players 1 and 2 are defined as follows:

$$f_1\left(x_1^N, x_2\right) = \min_{x_1 \in X_1} f_1(x_1, x_2) \rightarrow \quad x_1^N(x_2) \tag{4.26}$$

$$f_2\left(x_1, x_2^N\right) = \min_{x_2 \in X_2} f_2(x_1, x_2) \rightarrow \quad x_2^N(x_1) \tag{4.27}$$

where x_1^N is the optimum solution of player 1 which varies depending on the strategy x_2 chosen by player 2. The functions $x_1^N(x_2)$ and $x_2^N(x_1)$ denote the RRS for players 1 and 2 respectively. The intersection of these two sets, if it exists, is the Nash solution for the non-cooperative game.

In a cooperative game, the players have knowledge of the strategies chosen by other players and collaborate with each other to find a Pareto-optimal solution. Unlike Nash and Stackelberg games, where players do not cooperate, it is not uncommon for players to improve their non-cooperative solution by cooperating. The cooperative game captures the effect of competition between the players in a bargaining situation.

4.4.2 Cooperative Game Theory Method

Consider a cooperative game-theory problem with two players. Let $U_i(X)$ is a utility (payoff) function associated with each player $i=1,2$ such that if strategy X is selected from a set of alternative strategies $S(X \in S)$, player i will have payoff $U_i(X)$. The two players compromise to select a mutually beneficial strategy such that their payoffs are as high as possible. It is assumed that if the players decide not to cooperate, their payoffs will be u^* and v^* where $u^* = U_1(X_w)$ and $v^* = U_2(X_w)$ and X_w is a status-quo point $X_w \in S$. The players want to maximize their distance from X_w.

The bargaining model that determines a compromise solution using the bargaining function $B(X)$ defined as:

$$B(X) = (u-u^*)(v-v^*) = \prod_{i=1}^{2}[U_i(X) - U_i(X_w)] \tag{4.28}$$

for all $X \in S^* \subset S$, where $S^* = [X \mid X \in S, U_i(X) - U_i(X_w) \geq 0]$

An optimum compromise solution is now defined as:

$$B(X^{opt}) = \max B(X), X \in S^* \tag{4.29}$$

This bargaining function yields a pareto-optimal solution X^{opt} which maximizes the payoff for each player.

Next consider a multi-objective function problem with k objectives which need to be minimized (Eq. 4.25). A game theory formulation for this problem consists of k players where each player corresponds to an objective function to be minimized. The bargaining function $B(X)$ in this case is defined as:

$$B(X) = \prod_{i=1}^{k} [\, f_{iw} - f_i(X)] \tag{4.30}$$

where f_{iw} is the worst value of the objective function f_i that player i is willing to accept. The assumption in the above bargaining function is that all objective functions f_i's are equally important. Therefore the game theory formulation for a multi-objective problem becomes:

$$\max B(X) = \prod_{i=1}^{k} [\, f_{iw} - f_i(X)] \tag{4.31}$$

such that $X \in S^*$.

Presented next are two formulations of the structure-control optimization problem. The first formulation treats the problem as a single objective problem whereas the second formulation casts the problem as a multi-objective optimization problem.

4.5 Design Example

The ACOSS-four flexible space structure shown in Fig. 3.1 is considered again in this chapter. The nodal coordinates of the system are given in Table 3.1. Four lumped masses of 2 units each are attached at nodes 1 through 4. The actuators can be located in any one of the twelve members. Both the state weighting matrix $[Q]$ and the control weighting matrix $[R]$ are assumed to be identity matrices.

4.5.1 Single Objective Optimization Formulation

The actuator placement problem has mixed discrete-continuous design variables with member cross-sectional areas as continuous and actuator locations as discrete design variables. The presence or absence of actuators is denoted by discrete values 1 or 0. Since the structure under consideration has 12 members, therefore the problem has a total of 24 design variables (12 member cross-sectional areas and 12 potential actuator locations). A maximization of trace$[H]$ is considered as the objective function and the constraints imposed on the problem are: (i) The closed loop damping ratio $\xi_1 > 0.16434$; (ii) The imaginary part of the first closed loop eigenvalue $\beta_1 > 1.3374$; (iii) The imaginary part of the second closed loop eigenvalue $\beta_2 > 1.5$; (iv) The cross-sectional areas of the members are bounded between 10 and 2000. The optimization problem is given as:

Minimize trace$[H]$

by varying cross-sectional areas and actuator locations

subject to

$$\begin{aligned} & 0.16434 - \xi_1 \le 0 \\ & 1.3374 - \beta_1 \le 0 \\ & 1.5 - \beta_2 \le 0 \\ & 10 \le A_i \le 2000 \end{aligned} \tag{4.32}$$

The weight of the structures at the nominal areas (given in Table 3.2) is 43.69. By randomly placing the actuators in elements 6, 7, 9 and 11, the trace$[H]$ value is 265.36 and the LOS error is shown in Fig. 4.1. The weight of the structure at the optimum design is found to be 55.11, trace$[H]$ value is 11751 and the optimum number of actuators is four placed in element 2, 5, 7 and 8. The optimum areas, closed-loop damping ratios, closed-loop eigenvalues and squares of the natural frequencies are given in Table 4.1. The LOS error for this design is shown in Fig. 4.2. The response was simulated by subjecting the optimized structure to a disturbance at node 2 in the x-direction at t=0. Although the weight of the structure at the optimum design is higher than the weight at nominal design but it should be noted that weight is not the objective function in this case. The objective function is to maximize trace $[H]$ and therefore the optimum design has a very higher value of trace $[H]$ than the nominal design. By comparing Fig. 4.1 and Fig. 4.2, it is clear that by placing the actuators at the optimum locations, the response dies out faster than the case when the actuators are placed randomly at certain locations.

4.5.2 Multi-objective Formulation

The two objective functions considered in this work are minimizing the weight of the structure (f_1) and maximizing the energy (trace$[H]$) dissipated by the controllers (f_2). A bargaining function is constructed in between the two objectives as follows:

$$B(x) = \frac{f_{1worst} - f_1}{f_{1worst} - f_{1best}} * \frac{f_2 - f_{2worst}}{f_{2best} - f_{2worst}} \tag{4.33}$$

where f_{1best} and f_{2best} are the single objective function optimum values and f_{1worst} and f_{2worst} are their corresponding worst values. The bargaining function between the weight and trace$[H]$ is maximized. Again, the design variables are the member cross-sectional areas and actuator locations (12+12=24 design variables). The constraints imposed on the problem are the same as given in Eq. (4.32) except for $\beta_1 \geq 1.2$. The optimum value of the bargaining function is 0.51. The optimum weight of the structure is 40.7, the optimum value of trace$[H]$ is 9654.2 and the optimum locations of the actuators are in element 2, 5, 7 and 8. This result shows about 18% lower trace$[H]$ value than the single objective case because in this case trace$[H]$ has to cooperate with the other objective (weight). The optimum values of member cross-sectional areas, closed-loop damping ratios, closed-loop eigenvalues, and square of the natural frequencies are given in Table 4.2. The LOS error for multi-objective design is shown in Fig. 4.3.

4.6 Conclusions

The method presented in this chapter permits a simultaneous determination of optimum cross-sectional areas, optimum number and optimum locations of the actuators in actively controlled structures. The energy dissipated by the actuators is used as the performance criterion for the single objective problem. The problem variables include mixed discrete-continuous design variables. The solution approach involves solving the problem using genetic

algorithms. The optimum number of actuators, for both single objective and multi-objective problems, is four with actuators present in elements 2, 5, 7 and 8 (Table 4.1 and Table 4.2). In the case of multi-objective problem, the bargaining function between structural weight and trace$[H]$ is maximized. The optimum value of weight is 40.7 which is lower compared to the single objective value of 55.1. This result makes sense as weight was not the objective for the single optimization problem. Since trace $[H]$ was the only objective considered for the single objective case, the resulting design has a better value for trace$[H]$ when compared to the multi-objective case. The multi-objective optimization problem results in a better value for weight, but this improvement is at the expense of a lower trace$[H]$ value.

Table 4.1 Cross-sectional areas, closed-loop damping ratios, closed-loop eigenvalues and natural frequencies with optimum actuator placement (Single Objective)

Element	Actuator	Areas	Damping Ratio	Real Part	Imag. Part	Sq. of natural freq.
1		749.60	0.1711	-0.2325	1.3387	1.7512
2	X	80.40	0.0569	-0.0858	1.5045	2.3332
3		96.00	0.0086	-0.0256	2.9664	8.7997
4		250.10	0.0552	-0.2085	3.768	14.2378
5	X	1535.90	0.0084	-0.0525	6.2334	38.8575
6		110.90	0.03	-0.3082	10.2812	105.7966
7	X	1681.60	0.0002	-0.0038	15.7195	247.0667
8	X	1399.40	0.0166	-0.2605	15.727	247.4106
9		1837.80	0.0181	-0.3114	17.1611	294.6292
10		1859.10	0.0005	-0.0087	18.052	325.8032
11		1363.70	0.0207	-0.3746	18.079	327.0376
12		1362.20	0.0005	-0.0096	18.1576	329.7372

trace$[H]$ = 11751 Weight= 55.11

Table 4.2 Cross-sectional areas, closed-loop damping ratios, closed-loop eigenvalues and natural frequencies with optimum actuator placement (Multi-objective)

Element	Actuator	Areas	Damping Ratio	Real Part	Imag. Part	Sq. of natural freq.
1		250.00	0.1736	-0.2112	1.1987	1.4283
2	X	248.20	0.0939	-0.1575	1.6707	2.8292
3		825.30	0.0327	-0.1128	3.445	11.796
4		168.70	0.068	-0.2572	3.7725	14.3362
5	X	54.60	0.0292	-0.1235	4.2327	18.0031
6		198.20	0.0281	-0.1715	6.1114	37.3813
7	X	1897.50	0.0016	-0.0127	8.0068	64.1063
8	X	1885.60	0.0024	-0.0261	10.8812	118.3959
9		240.40	0.001	-0.0183	18.3085	334.78
10		422.00	0.0233	-0.4279	18.3216	336.2055
11		1886.60	0.0003	-0.0051	18.3851	338.0179
12		1894.00	0.0193	-0.3557	18.4638	341.0945

trace$[H]$ = 9654.2 Weight= 40.7

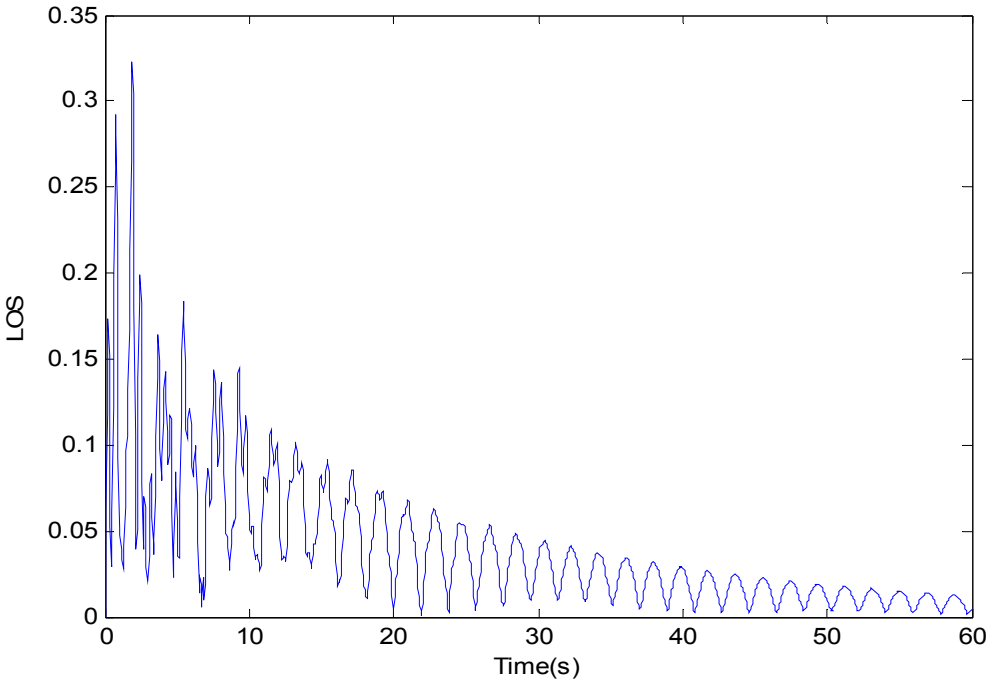

Figure 4.1 Structure response at nominal areas with 4 actuators randomly placed in elements 6, 7, 9 and 11 (LOS 2.4)

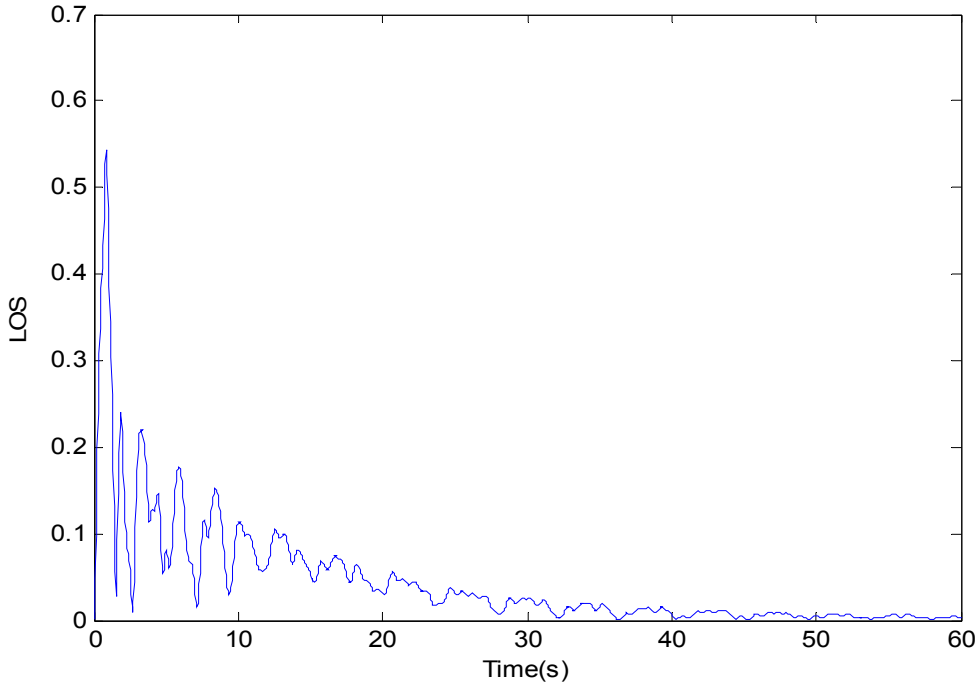

Figure 4.2 Structure response with 4 actuators present at optimum locations-single objective formulation (LOS 2.6)

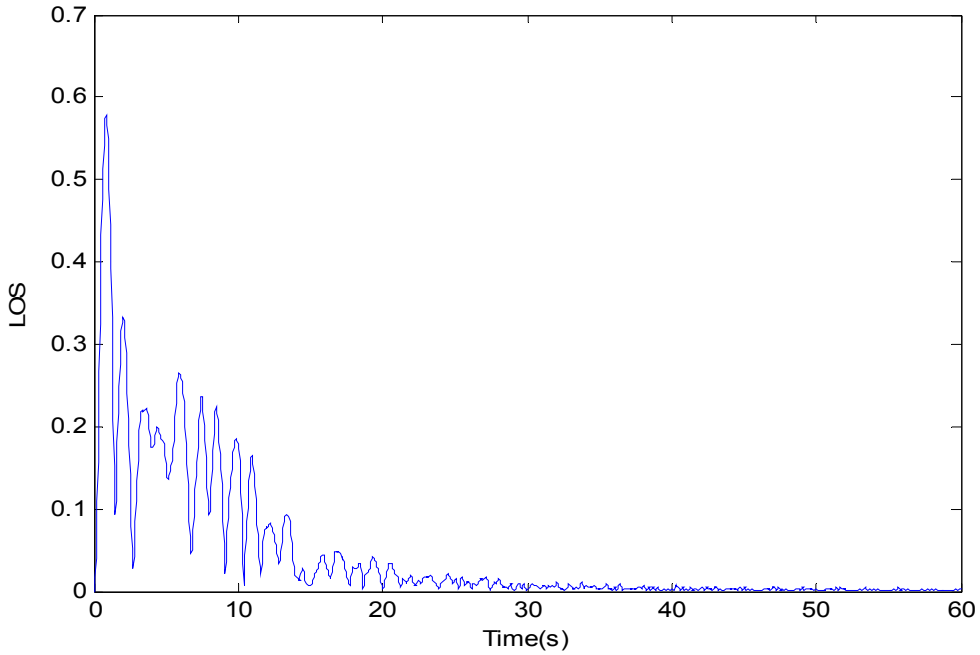

Figure 4.3 Structure response with 4 actuators present at optimum locations-multi-objective formulation (LOS 2.6)

Chapter 5

Multi-objective Optimization of Actively Controlled Structures

This chapter addresses the design of actively controlled structures wherein both the actuator placement and controller design aspects are addressed simultaneously. It is assumed that a hierarchical structure exists between the actuator placement and controller design objective functions with the actuator placement problem considered as being more important. The resulting multi-objective design problem is solved as a bi-level Stackelberg game. A computational procedure based on variable updating using response surface methods is developed for exchanging information between the two levels (leader and follower). The optimization problem has mixed discrete-continuous variables with discrete variables corresponding to actuator placement and continuous variables associated with the structural and controller design problems. The solution approach includes a blend of genetic algorithms and sequential quadratic programming techniques.

5.1 Introduction

Some of the important aspects of structural-control optimization include minimum weight design, minimum control energy design, maximum energy dissipated by the actuators, and fast damping of vibrations. The weight of the structure is controlled by the cross-sectional areas of the elements. A minimization of control energy required is dependent on the proper choice of state and control weighting matrices (Q and R). Varying the number of actuators

as well as their locations has significant effect on the dynamic response of the structure. Therefore, the optimum values of Q and R should be selected with actuators placed at optimum locations in order to achieve optimum control performance with minimum control cost. The approach proposed in this chapter presents a solution to the multi-objective, integrated structural and control optimization problem using the Stackelberg game theory approach.

The weight of the structure, the energy dissipated by the controller (trace$[H]$) and the quadratic performance index of LQR controller (trace $[P]$) are all considered as the objective functions. The cross-sectional areas of the structural members, diagonal entries of the state weighting matrix, and actuator locations are treated as the design variables. The problem has mixed discrete-continuous design variables. To meet the stability requirements for the active controller, constraints are placed on the closed-loop damping ratios and closed-loop eigenvalues. To date, not much literature is available on solving multi-objective problems with mixed discrete-continuous design variables. Because of the mixed discrete-continuous nature of problem variables, the structural and control optimization problem cannot be solved using conventional gradient based optimization methods. The proposed solution approach partitions the discrete and continuous design variables into different levels each with their own objective function. A computational procedure based on variable updating using DOE-RSM approach is developed for exchanging information between the two levels (leader and follower).

5.2. Multi-level Design Optimization

A number of methods have been proposed over the years to solve the multi-objective optimization problem (Marler and Arora, 2004). These include the utility function method, bounded objective function method, lexicographic method, goal programming, and game theory based approaches. Many of the proposed approaches for multi-objective optimization essentially convert a multi-objective optimization into a single objective problem through a weighted combination of objective functions. If the objective functions have varying degree of importance such that a hierarchical structure exists, then a scalarization of objectives is not possible and multi-level optimization techniques are needed. In this chapter, the Stackelberg method is used to solve the multi-level optimization problem. If more than one objective function is present at the leader or the follower level, then either cooperative or non-cooperative game techniques are used to combine these objective functions. Both cooperative and non-cooperative game theoretic approaches were discussed in Sec. 4.4.

Presented next is the solution procedure when a hierarchical structure exists in the multi-objective optimization problem. This problem is modeled as a Stackelberg game and solved using the solution approach outlined in the next section.

5.2.1 Stackelberg Game Theory Method

Consider two players, A and B, who can select strategies x_1 and x_2 where $x_1 \in X_1 \subset R^{n_1}$ and $x_2 \in X_2 \subset R^{n_2}$. Here X_1 and X_2 are the set of all possible

strategies each player can select. The objective functions $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ account for the cost (or loss) functions of players 1 and 2, respectively. The game theory models deal with finding the optimum strategy (x_1, x_2) which corresponds to the decision protocol of the specific game model. The goal of each model is to minimize the loss function for each player.

If there exists a mapping (function) $R_1 : x_2 \rightarrow x_1$ such that for any fixed x_2, $f_1(R_1(x_2), x_2) \leq f_1(x_1, x_2)$ for all x_1, then R_1 is the Rational Reaction Set (RRS) for player 1. Similarly, the RRS for player 2, R_2, can be defined. The Nash solution (x_1^N, x_2^N) for players 1 and 2 is the intersection of R_1 and R_2 and indicates that (x_1^N, x_2^N) satisfies R_1 and R_2 simultaneously.

The Stackelberg game is a bi-level game in which each level has its own player, with one player dominating other. The two players are referred to as the leader and the follower. The follower's solution depends on the choices made by the leader. The leader first chooses a value of its design variables and then the follower selects best possible value for its variables (Rational Reaction Set, RRS) based on the values of leader's design variables. The leader then optimizes its problem, over its variables, based on the rational reaction set provided by follower. In other words, the leader always optimizes its model over the optimum design model of the follower.

The non linear programming (NLP) formulation for a bi-level game is defined as:

Minimize $f_1(l_1, l_2, x)$

by varying l_1

subject to

$$(l_2, x) = X_2^R(l_1) \tag{5.1}$$

where f_1 is the leader's objective function and $X_2^R(l_1)$ is the rational reaction set (RRS) of the follower which is defined as solution of following problem:

$$\begin{aligned} &\text{Minimize} \quad f_2(l_1, l_2, x) \\ &\text{by varying} \quad (l_2, x) \end{aligned} \tag{5.2}$$

where f_2 is the follower's objective function.

For simple optimization problems, it may be possible to obtain the RRS analytically. Otherwise, approximation techniques such as the response surface method (RSM) or a sensitivity based approach (Ghotbi and Dhingra, 2012) can be used to construct a RRS. The RSM utilizes design of experiments (DOE) techniques to construct various experiments and a response surface is then fitted to the experiment outcomes. In this work, since the leader's design variables are discontinuous (0-1 variables), the sensitivity based approach cannot be applied. Therefore, the RSM method is used to construct the rational reaction set of the follower problem.

5.2.2 Design of Experiments and Response Surface Method

Design of experiments plays an important role in engineering. In an experiment, some input x transform into an output that has one or more observable response variables Y. Therefore useful results are drawn by

conducting experiments. In this thesis, the relationship between leader and follower design variables is approximated using the Response Surface Method. The Response Surface Method is a collection of statistical and mathematical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables, and the objective is to optimize this response (Montgomery, 2005).

In case of two independent variables x_1 and x_2, the mathematical relationship between the response Y and variables x_1 and x_2 is given as:

$$Y = f(x_1, x_2) + e \tag{5.3}$$

The response Y is a function of the variables x_1, x_2, and the experimental error is denoted as e. The error term represents any measurement error or other variations not accounted in *f*. It is a statistical error that is assumed to be normally distributed with a zero mean and a finite variance.

If the response is defined by a linear function of the independent variables, then the approximating function is a first-order model and is defined as:

$$Y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + ...\alpha_n x_n + e \tag{5.4}$$

where n is the number of independent variables. If there is a curvature in the response surface, then a higher degree polynomial should be used; then the approximating function is a second-order model. In case of two variables, the approximating function is:

$$Y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_{11} x_{11}^2 + \alpha_{22} x_{22}^2 + \alpha_{12} x_1 x_2 + e \tag{5.5}$$

A model with several independent variables is a multiple-regression model and the $\alpha_i's$ are the regression coefficients. Since the independent variables for the actuator placement problem considered in this work are zeros and ones, the second-order model converges to the first-order model because the higher order terms simply reduce to zeros and ones. Therefore, the first order model is used in this work for finding the RRS.

5.2.3 Multiple Regression Model

Regression Model is a mathematical model which determines the relationship between a set of independent variables, $x's$, and the response y. When there are more than two independent variables, the model is referred to as a multiple-regression model. The mathematical formulation of a multiple-regression model with n experimental runs and q independent variables is defined as:

$$y_i = \alpha_0 + \alpha_1 x_{i1} + \alpha_2 x_{i2} + ... + \alpha_q x_{iq} + e_i \qquad \text{where } i = 1, 2.....n \qquad (5.6)$$

The data structure for multiple-regression-model is shown below:

$$\begin{array}{cccc} y & x_1 & x_2.... & x_q \\ y_1 & x_{11} & x_{12}.... & x_{1q} \\ y_2 & x_{21} & x_{22}.... & x_{2q} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ y_n & x_{n1} & x_{n2}.... & x_{nq} \end{array}$$

The multiple-regression model can be expressed in a matrix from:

$$Y = X\,\mathrm{A} + \varepsilon \qquad (5.7)$$

where $Y = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{bmatrix}_{n\times 1}$ $\mathrm{A} = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ . \\ . \\ \alpha_q \end{bmatrix}_{q\times 1}$ $\varepsilon = \begin{bmatrix} e_1 \\ e_2 \\ . \\ . \\ e_n \end{bmatrix}_{n\times 1}$ $X = \begin{bmatrix} 1 & x_{11} & x_{12}\ldots & x_{1q} \\ 1 & x_{21} & x_{22}\ldots & x_{2q} \\ . & . & . & . \\ . & . & . & . \\ 1 & x_{n1} & x_{n2}\ldots & x_{nq} \end{bmatrix}_{n\times q}$

Y is an $n\times 1$ vector of observations, X is an $n\times q$ matrix of levels of independent variables, A is a $q\times 1$ vector of regression coefficients and ε is an $n\times 1$ vector of random errors. (Montgomery 2005). The multiple-regression model given by Eq. (5.7) is used to construct the RRS for the follower problem. The MATLAB function “regress” is used to solve for the regression coefficients.

5.3 Solution Procedure

The actuator placement and control system design problem presented here is solved as a bi-level Stackelberg game. The two levels correspond to the leader’s and follower’s objective functions. In case of two objective functions, the objective function of the leader is a maximization of the energy dissipated by the actuators with actuator locations as the design variables. The objective function of the follower is the minimization of the weight of the structure with cross-sectional areas of the members as design variables.

In case of three objective functions, the objective function of the leader is the maximization of energy dissipated by the actuators with actuator locations as design variables. The objective function of the follower is the maximization of the bargaining function (Eq. 4.31) between minimization of the weight of the structure (with cross-sectional areas of the members as design variables) and

minimization of trace$[P]$ (with diagonal entries of state weighting matrix as design variables).

One critical point in solving a bi-level problem as a Stackelberg game is obtaining the RRS of the follower. The rational reaction set of the follower gives the change of optimum solution of the follower problem while the leader's variables are varying. Finding the RRS of the follower involves solving the follower problem using various combinations of leader's design variables, which are discrete 0 or 1, actuator locations. If the number of possible actuator locations is x, then each potential actuator location variable has two possibilities 0 or 1 (presence or absence). Therefore, a total of 2^x combinations of design variables are possible. These combinations are used to construct the RRS for the follower. Since the follower problem has continuous variables, that is, the cross-sectional area of structural elements, a sequential quadratic programming method is used in this problem.

Once the RRS of the follower is found, it is inserted into the leader problem to find the optimum solution to the leader problem. The leader's problem has discrete variables and will be solved using a genetic algorithm. The flowchart of the complete solution process is shown in Fig. 5.1.

5.4 Design Example

Once again, the ACOSS-four flexible space truss structure shown in Fig. 3.1 is considered for the multi-level optimization problem considered in this chapter. The multi-level, multi-objective problem is solved with two objectives as well as with three objective functions. For the case with two objective functions,

the objective functions considered are: i) maximize trace$[H]$, and (ii) minimize structural weight, whereas in the case of three objective functions, the objective functions considered are: i) maximize trace$[H]$, (ii) minimize structural weight, and (iii) minimize trace$[P]$.

5.4.1 Case 1 - Two Objective Functions

The two objective functions considered include maximizing trace$[H]$ and minimizing the weight of the structure. Player 1 (trace$[H]$) wishes to maximize the energy dissipated by the controller by controlling variables $x_1 - x_6$ which are actuator locations in elements 2, 5, 6, 7, 9 and 11. Player 2 (weight) wants to minimize the weight of the structure with control over variables $y_1 - y_6$ which are the cross-sectional areas of elements 1-6. The cross-sectional areas of the other six bipod legs (elements 7-12) are all fixed at 100 units. Since there are 6 possible actuator locations with two possibilities for each member, 0 or 1 (presence or absence), a total of $2^6 = 64$ combinations of design variables are possible. These combinations are used to construct the RRS for the follower. The problem constraints include:

1. The closed-loop damping ratio corresponding to the 1st mode must be greater than 0.16434.
2. The imaginary part of the first closed-loop eigenvalue should be greater than 1.2.
3. The imaginary part of the second closed-loop eigenvalue should be greater than 1.5.

4. The cross-sectional area of the members must lie between 10 and 2000.

The optimization problem is given as:

Leader:

Maximize trace $[H]$

by varying $(x_1 - x_6)$

subject to

$$\begin{aligned} &\beta_1 \geq 1.2 \\ &\beta_2 \geq 1.5 \\ &\xi_1 \geq 0.16434 \\ &10 \leq A_i \leq 2000 \end{aligned} \tag{5.8}$$

Follower:

Minimize Weight

by varying $(y_1 - y_6)$

subject to the same constraints in Eq. (5.8).

5.4.1.1 Results

For each actuator location $(x_1, x_2, x_3, x_4, x_5, x_6)$ combination, an optimum solution for cross-sectional areas, $y_1 - y_6$, is obtained. From the 64 solutions, a response surface regression yields the following approximation function for the RRS for the follower objective function.

$$y_1 = 627.3 + 0.84x_1 - 9.83x_2 + 9.92x_3 - 203.56x_4 - 187.39x_5 - 181.95x_6$$

$$y_2 = 212.65 + 17.32x_1 - 3.01x_2 - 0.74x_3 + 132.57x_4 - 29.62x_5 - 35.08x_6$$

$$y_3 = 595.87 - 37.21x_1 - 5.95x_2 - 25.29x_3 - 137.77x_4 - 158.27x_5 - 147.34x_6 \tag{5.9}$$

$$y_4 = 562.13 - 11.83x_1 - 11.75x_2 - 3.35x_3 - 125.06x_4 - 129.70x_5 - 119.04x_6$$

$$y_5 = 222.53 - 39.91x_1 - 5.13x_2 - 25.88x_3 - 16.92x_4 - 84.75x_5 + 93.901x_6$$

$$y_6 = 184.27 + 18.61x_1 + 10.38x_2 + 25.90x_3 - 110.07x_4 + 85.97x_5 - 62.12x_6$$

where $y(x)$ approximates the optimum vector of the weight minimization problem for varying values of $x_1 - x_6$. Next, this RRS is used to obtain the Stackelberg solution.

5.4.1.2 Stackelberg Solution

With players trace$[H]$ as the leader and weight as the follower, the Stackelberg game problem is solved by substituting Eq. (5.9), which is RRS of the follower problem, into the leader's problem. The optimum solution to the leader problem results in an optimum value of trace$[H]$=151.55, the optimum weight of the structure is 15.90, and the LOS error for this design is 2.11. The optimum cross-sectional areas are listed in Table 5.1. The optimum number of actuators is three corresponding to placement of actuators in elements 2, 6 and 11. The system response to an external disturbance is shown in Fig. 5.2.

The two next best solutions with three actuators include actuators placed in elements 5, 7 and 11, and elements 6, 9 and 11. These two solutions are compared with the optimum solution as shown in Table 5.1 and Fig. 5.3. It is seen that if the actuators are placed in elements 5, 7 and 11, the minimum weight of the structure is 20.5, trace$[H]$ is 167.3 and the LOS error is 3.62. Likewise, actuator placement in elements 6, 9 and 11 yields a weight of 20.25,

trace$[H]$ equaling 166.89 and the LOS error is 4.96. It may be noted that while moving from an optimum to a sub-optimal design results in a better value for trace$[H]$, but this improvement is at the expense of a higher weight and a higher LOS error. Therefore, an integrated determination of the optimum number and locations of the actuators as well as optimum structural weight is critical in determining the overall optimum solution.

5.4.2 Case 2 - Three Objective Functions

The bi-level structural-control optimization problem is modeled using Stackelberg game and cooperative game theory. The three objective functions considered are (i) maximize trace$[H]$, (ii) minimize structural weight, and (iii) minimize trace$[P]$. Player 1 (leader) wishes to maximize trace$[H]$ by varying $x_1 - x_6$, which are the actuator locations in elements 2, 5, 6, 7, 9 and 11. The follower level contains two objective functions, minimize weight and minimize trace$[P]$. It is assumed that a cooperative game scenario exist between these two functions. These two objective functions are combined using a bargaining function. Therefore, player 2 (follower) maximizes the bargaining function $(F_{b\arg})$ between weight and trace$[P]$ by varying member cross-sectional areas, $area_1 - area_{12}$, and diagonal entries of state weighting matrix, $[Q]$, namely Q_1, Q_2, Q_3 and Q_{13}. The other entries of diagonal entries of $[Q]$ are fixed at 1.0. The control weighting matrix $[R]$ is assumed to be identity matrix.

Finding the RRS of the follower involves solving the follower's problem for various combinations of leader's design variables, which are discrete 0 or 1

actuator locations. Once again, there are six leader design variables with two possibilities either 0 or 1, the follower problem is solved $2^6 = 64$ times to construct the RRS. Once the RRS for the follower is found, it is inserted into the leader problem to find the optimum solution to the leader problem. The problem constraints are same as given by Eq. (5.8) with one additional constraint on the diagonal entries of the state weighting matrix which should all be greater than 1.0.

The optimization problem is stated as:

Leader:

Maximize trace $[H]$

by varying $(x_1 - x_6)$

subject to

$$\begin{aligned} &\beta_1 \geq 1.2 \\ &\beta_2 \geq 1.5 \\ &\xi_1 \geq 0.16434 \\ &Q_{jj} \geq 1.0 \\ &10 \leq A_i \leq 2000 \end{aligned} \tag{5.10}$$

Follower:

Maximize $F_{b\arg} = \dfrac{(f_{1w} - f_1)(f_{2w} - f_2)}{(f_{1w} - f_{1b})(f_{2w} - f_{2b})}$

by varying $(y_1 - y_{16})$

subject to the constraints in Eq. (5.10). Here f_{1w}, f_{2w}, f_{1b} and f_{2b} denote the worst and best values of weight and trace$[P]$. For the problem under consideration the best and worst values of weight are found to be 2.7 and 33.1

respectively. Similarly, the best and worst values of trace $[P]$ are found to be 421.4 and 1.11x10^4 respectively.

5.4.2.1 Stackelberg Solution

For each $(x_1, x_2, x_3, x_4, x_5, x_6)$ combination, an optimum solution for $y_1 - y_{16}$ is obtained. In this case, the response surface regression yields the following RRS for the follower.

$$
\begin{aligned}
y_1 &= 173.26 - 14.17x_1 - 22.30x_2 - 21.93x_3 - 28.68x_4 - 30.78x_5 - 40.77x_6 \\
y_2 &= 129.81 + 17.89x_1 - 14.41x_2 - 4.57x_3 - 8.85x_4 - 42.61x_5 - 18.64x_6 \\
y_3 &= 136.17 - 12.81x_1 - 0.15x_2 - 11.15x_3 - 19.05x_4 - 23.44x_5 - 8.88x_6 \\
y_4 &= 132.35 + 6.88x_1 - 12.49x_2 - 3.33x_3 - 6.73x_4 - 20.80x_5 - 11.96x_6 \\
y_5 &= 118.55 + 0.07x_1 - 6.47x_2 - 5.77x_3 - 21.64x_4 - 27.80x_5 - 1.35x_6 \\
y_6 &= 154.07 + 1.79x_1 - 20.94x_2 - 14.54x_3 - 20.70x_4 - 18.87x_5 - 30.68x_6 \\
y_7 &= 98.94 - 44.75x_1 - 3.34x_2 - 12.10x_3 - 24.97x_4 - 11.89x_5 - 8.70x_6 \\
y_8 &= 86.26 - 5.44x_1 - 4.92x_2 - 6.26x_3 - 10.17x_4 - 19.51x_5 - 41.71x_6 \\
y_9 &= 68.00 + 2.07x_1 - 29.91x_2 - 3.41x_3 + 7.41x_4 - 18.94x_5 - 4.87x_6 \\
y_{10} &= 121.66 - 6.07x_1 - 19.89x_2 - 25.67x_3 - 18.61x_4 - 25.11x_5 - 32.36x_6 \\
y_{11} &= 70.51 - 0.61x_1 + 2.00x_2 - 306.61x_3 - 18.06x_4 - 6.04x_5 - 0.60x_6 \\
y_{12} &= 89.33 - 3.51x_1 - 15.27x_2 - 10.40x_3 - 10.80x_4 - 5.83x_5 - 15.55x_6 \\
y_{13} &= 215.66 + 0.54x_1 - 5.98x_2 - 4.91x_3 + 1.18x_4 - 4.43x_5 + 3.36x_6 \\
y_{14} &= 163.58 + 4.32x_1 + 2.20x_2 + 7.74x_3 - 0.84x_4 - 3.51x_5 - 3.58x_6 \\
y_{15} &= 16.45 + 8.54x_1 + 0.43x_2 + 2.11x_3 - 6.29x_4 + 5.12x_5 + 3.98x_6 \\
y_{16} &= 5.69 + 0.77x_1 + 0.41x_2 + 1.83x_3 - 0.03x_4 - 0.38x_5 - 0.61x_6
\end{aligned}
\tag{5.11}
$$

where $y(x)$ approximates the optimum vector which maximizes the bargaining function between weight and trace$[P]$ for varying values of $x_1 - x_6$. Note that $y_1 - y_{12}$ are the member cross-sectional areas and $y_{13} - y_{16}$ corresponds to $Q_1 - Q_3$ and Q_{13}. Next, this RRS is used to obtain the Stackelberg solution. Substituting

the RRS of the follower problem into the leader's problem, the leader's problem is solved. Since the leader problem variables are discrete, a genetic algorithm based approach is used to solve the leader's problem. The optimum solution of the leader's problem results in an optimum value of trace $[H]$=98.68 with actuators located in elements 2 and 6. The weight of the structure is 6.98 and trace $[P]$ is 1452.4 and the LOS error is 1.48. The optimum cross-sectional areas are listed in Table 5.2 and the LOS error shown in Fig. 5.4. It should be noted that the three objective function problem results in about 57% improvement in the weight and 30% improvement in the LOS but at the same time about 35% reduction in trace $[H]$ value.

5.5 Conclusions

A multi-objective problem for design of actively controlled structures is solved using a bi-level game theoretic formulation. The optimization problem is modeled as a Stackelberg game. The leader corresponds to maximization of energy dissipated by the controller. At the follower level either the structural weight is minimized or both the structural weight and controller performance index are minimized. A RSM based computational procedure is developed for generating the RRS of follower's variables as a function of leader's variables. The RRS facilitates information exchange between the two levels. The proposed method can be applied to problems with conflicting objectives and with discrete and continuous design variables. From the example problem considered in this work with two objective functions, the proposed approach results in a 30%

reduction in weight and about 40% improvement in LOS error when compared with designs where the actuator locations are not optimum. It is shown that the proposed approach yields an optimum controller which minimizes the weight of the structure while simultaneously maximizing the energy dissipated by the controllers needed to bring the structure to its equilibrium position.

Table 5.1 Cross-sectional Areas of Members and Actuator Locations at Optimum Design-two objectives

Optimum Result			Two next best Actuator Locations			
Element	Actuator	Areas	Actuator	Areas	Actuator	Areas
1		246.27		456.1		440
2	X	314.86		194.15		179.28
3		274.54		386.03		406.36
4		304.02		427.91		417.33
5		94.98	X	250.64		106.77
6	X	186.07		166.67	X	306.52
7		100	X	100		100
8		100		100		100
9		100		100	X	100
10		100		100		100
11	X	100	X	100	X	100
12		100		100		100
Trace $[H]$		151.55		167.31		166.89
Weight		15.9		20.51		20.25
LOS		2.11		3.62		4.96

Table 5.2 Cross-sectional Areas of Members, Diagonal entries of Q and Actuator Locations at Optimum Design-three objectives

Element	Actuator	Areas	Q*
1		113.79	212.41
2	X	78.34	159.22
3		93.67	15.28
4		104.82	5.28
5		69.11	
6	X	114.49	
7		62.07	
8		56.57	
9		56.47	
10		77.93	
11		46.4	
12		72.69	
Trace $[H]$	98.68		
Weight	6.79		
Trace $[P]$	1452.4		

Q* =First, second, third and thirteenth diagonal entry of Q matrix

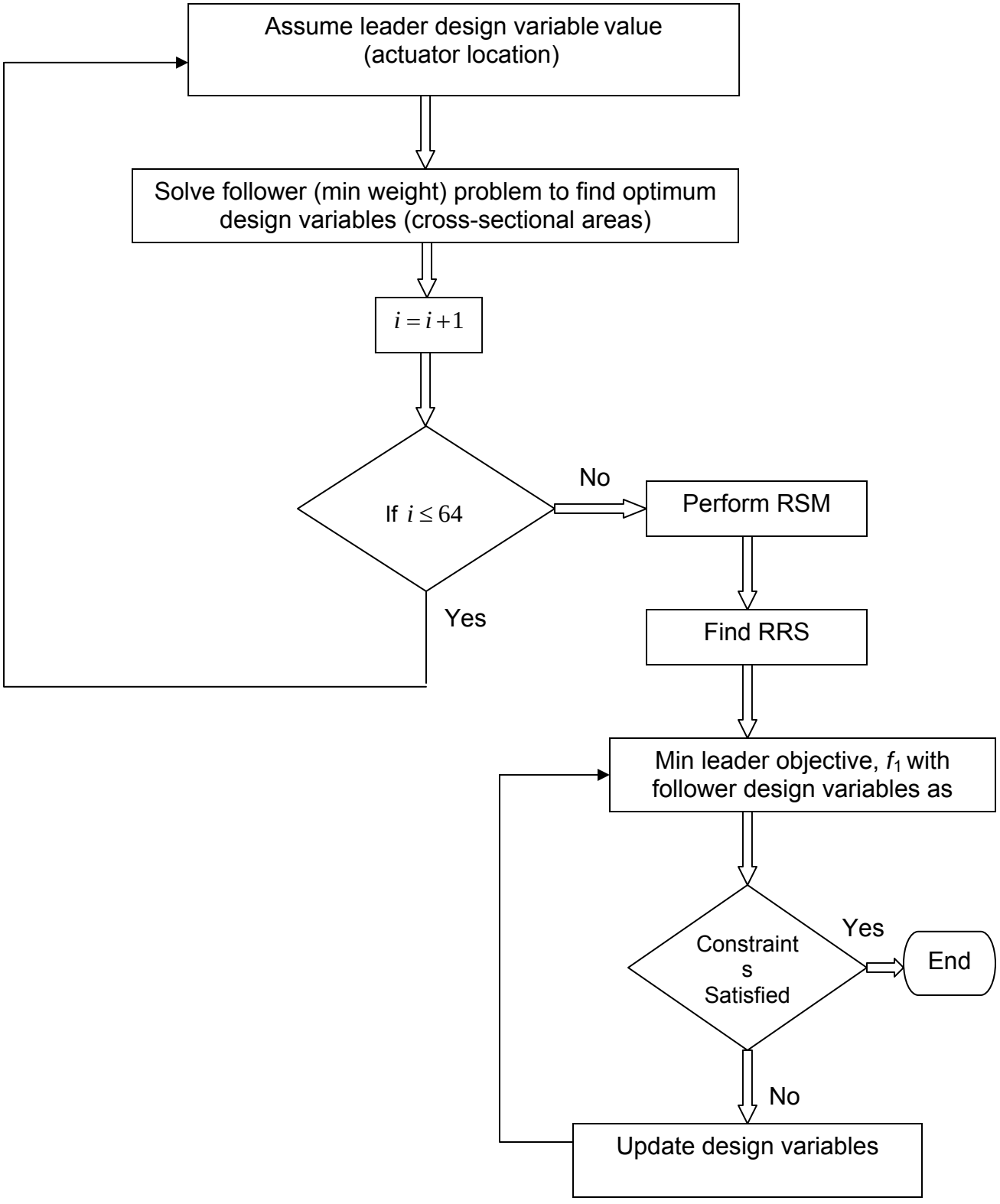

Figure 5.1 Flow Chart for determining Stackelberg Solutions.

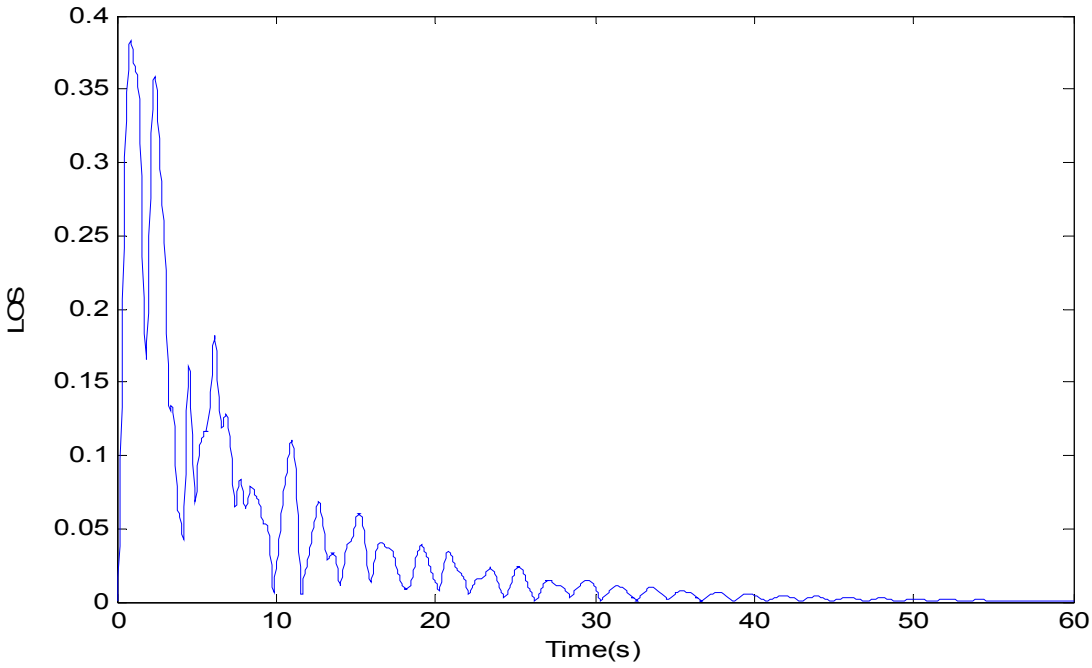

Figure 5.2 LOS error at the optimum design - two objectives

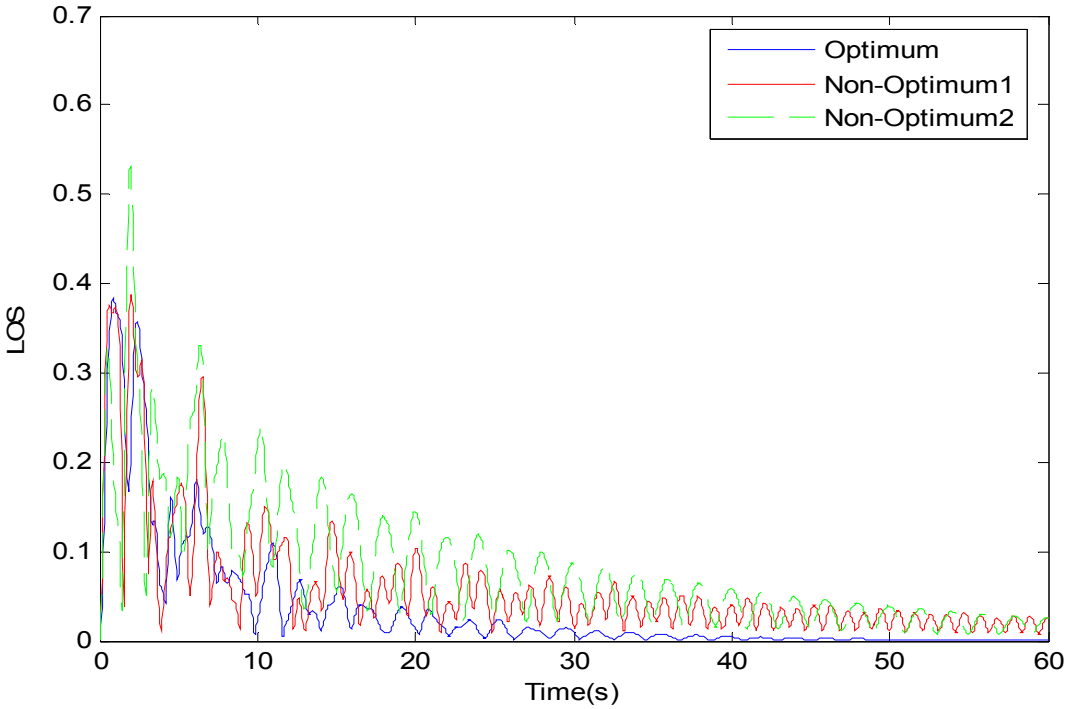

Figure 5.3 LOS error at the optimum and non-optimum design - two objectives

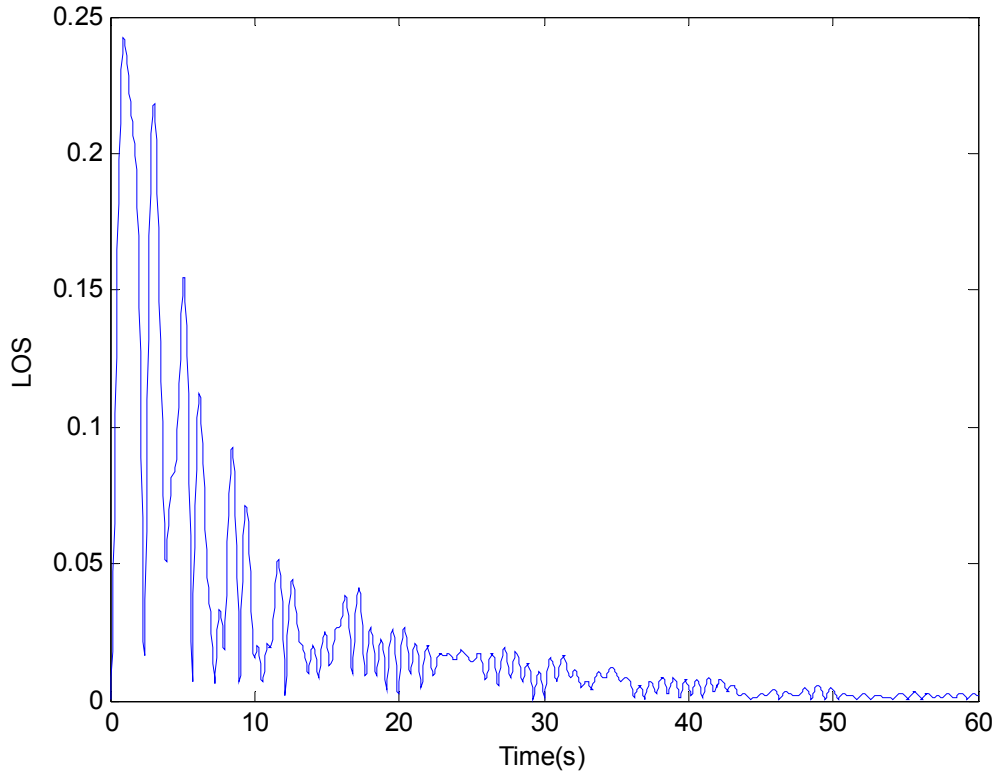

Figure 5.4 LOS error at the optimum design -three objectives

Chapter 6

Integrated Topology and Sizing Optimization of Actively Controlled Structures

A review of the available literature indicates that topology optimization has primarily been considered in the context of structural design. Further, most of the available literature for design of actively controlled structures deals with structures of a predetermined topology. It is recognized that the structural performance can be improved significantly by optimization of topology. This chapter presents a comprehensive treatment of structural and topological considerations in the context of actively controlled structures.

6.1 Introduction

The approach for solving the combined topology, structural and control optimization involves first determining the optimum topology followed by an iterated structural and control system optimization of the optimum topology. To reduce the computational burden involved with sizing and controller design of each candidate topology, the optimum topology is determined first. This is followed by a sizing and control system optimization of the predetermined optimum topology. The approach to finding an optimum structure topology involves defining a domain for the structure as well as the points of load application and supports. The optimum topology is created by minimizing the strain energy. Once the optimum topology is obtained, the next step involves a simultaneous sizing and control system optimization of the optimum topology.

Since the structural and control optimization is performed after topology optimization, the results may not be system-optimal. In spite of this simplifying assumption, it is shown through numerical examples that both structural and control system performance can be simultaneously improved if topological considerations are included in the problem formulation. In general, it is shown that a simultaneous reduction in structural weight and improvement in root mean square displacement (RMSD) error can be achieved when topological, control and structural aspects of design are considered simultaneously.

6.2 Topology Optimization

Topology optimization deals with finding the optimum layout of structure within a specified region when the only known quantities are applied loads, possible structural supports and the volume of the structure. The approach generally is to find optimum density distribution of material in a fixed domain modeled with a fixed finite element mesh, that is, finding the optimum placement of a given isotropic material in space by determining which points of space should be material points and which points should remain void. For a fixed domain, the topology design problem can be formulated as a sizing problem by modifying the stiffness matrix which can be expressed in terms of density of the material, which is the design variable. The optimization results in design consisting almost entirely of region of material or no material. This means that intermediate values of the density functions should be penalized in a manner analogous to other continuous optimization approximations to a 0-1 problem. The

popular and efficient SIMP (solid isotropic material with penalization) model (Bendsoe and Sigmund 2003) is used herein where:

$$E_{ijkl}(x) = \rho(x)^p E^o_{ijkl}$$
$$\int_\Omega \rho(x) d\Omega \le V; 0 \le \rho(x) \le 1, x \in \Omega \tag{6.1}$$

Here $\rho(x)$ is the relative density function and E^o_{ijkl} represents the stiffness tensor for the solid phase and $E_{ijkl}(x)$ denotes the tensor for the heterogeneous material. The density varies between the material properties 0 and E^o_{ijkl}. It has been shown that $\rho > 3$ helps minimize problem associated with intermediate values of density function. Reuss (iso-stress) and Voigt (iso-strain) mixing rules are commonly used to express E_{ijkl} as a function of the density.

The numerical approach to topological design adopted herein starts with a region of material meshed into small finite elements. External loads and boundary conditions are defined next. Every element is assumed to consist of a porous material of density ρ to which external loads and boundary conditions are applied. The purpose of optimization is to find optimum density distribution while maintaining a constant volume constraint. Topology optimization is done by creating design variables associated with the Young's Modulus and density of each element in the design space. The design variable value ranges between 0 and 1 where 0 indicates the element has no stiffness or mass and 1 indicated the element has its normal stiffness and mass. A power law interpolation penalizes intermediate densities to obtain nearly 0/1 material distribution. The solution process starts with a block of material formed by a large number of finite

elements and then the search procedure will take out from the block the unnecessary elements such that the volume constraint is met.

Mathematically, the topology optimization problem is formulated as follows: The design domain is divided into $N = N_x \times N_y$ elements where N_x denotes number of elements along x-axis and N_y denotes the number of elements along y-axis. The optimization problem to minimize the compliance is formulated as:

$$\min_{\rho} C(\rho) = \frac{1}{2}[F]^T[u] = [u]^T[K][u]$$

subject to

$$\frac{V(\rho)}{V_o} = f \tag{6.2}$$

$$[K][u] = [F]$$

$$0 < \rho_{\min} \le \rho \le 1$$

Here $[u]$ and $[F]$ denote the global nodal displacement and force vectors, respectively, $[K]$ is the global stiffness matrix, f is the prescribed volume fraction (VF), and the density $0 \le \rho_{xy} \le 1$ for each element. Depending on the finite element type selected to model the structural continuum, the entries in the stiffness matrix will change.

As members are added to and removed from a given topology, the strain energy of the structure changes. The changes to the strain energy of the structure can be computed as shown next.

In finite element analysis, the static equilibrium equations are given as:

$$[K][d]=[P] \tag{6.3}$$

where $[P]$ is the nodal load vector, $[d]$ is the nodal displacement vector and $[K]$ is the global stiffness matrix. Whenever an element is added to or removed from the structure, it will have an effect on the overall stiffness and the nodal displacements but the load vector remains unchanged. Let the resulting change in the stiffness matrix when i^{th} element is removed be given as:

$$[\Delta K]=[K^{-}]-[K]=-[K^{i}] \tag{6.4}$$

where $[K^{i}]$ denotes the stiffness matrix of the i^{th} element and $[K^{-}]$ is the stiffness matrix of the structure after the element is removed.

For a linear approximation, the resulting change in the displacement vector $[\Delta d]$ from Eq. (6.3) is given as:

$$[\Delta d]=-[K]^{-1}[\Delta K][d] \tag{6.5}$$

The strain energy of the structure can be expressed as:

$$C=\frac{1}{2}[P]^{T}[d] \tag{6.6}$$

From Eq. (6.5) and (6.6), the corresponding change in the strain energy is given as:

$$\Delta C^{-}=\frac{1}{2}[P]^{T}[\Delta d]=-\frac{1}{2}[P]^{T}[K]^{-1}[\Delta K][d] \tag{6.7}$$

$$=\frac{1}{2}[d^{i}]^{T}[K^{i}][d^{i}] \tag{6.8}$$

The equation above gives the change in strain energy due to the removal of i^{th} element. Here, $[d^{i}]$ is the element displacement vector containing the entries of

$[d]$ which are related to the i^{th} element. Similarly, the change in strain energy due to the addition of i^{th} element is given by:

$$\Delta C^{+} = -\frac{1}{2}[d^{i}]^{T}[K^{i}][d^{i}] \tag{6.9}$$

In topology optimization, the objective is to minimize the strain energy (which is equivalent to maximizing the stiffness) while keeping the volume constant. The strain energy of the structure is increased when the material is removed and decreased when material is added. The solution approach herein is to start with an initial structure with a fully connected grid meshed into a number of elements. In order to minimize the structural strain energy, it would be most effective to remove elements with minimum ΔC^{-} value and add elements with minimum ΔC^{+} value. To keep the structural volume constant, the material added should equal material removed.

Lastly, the sensitivity of response (displacements, strain energy etc.) with respect to the variables (ρ_i) is computed as follows:

$$[K][d] = [P] \tag{6.10}$$

$$\frac{\partial K}{\partial \rho_i}[d] + [K]\frac{\partial d}{\partial \rho_i} = 0 \tag{6.11}$$

$$C = \frac{1}{2}[d]^{T}[K][d] \tag{6.12}$$

$$\frac{\partial C}{\partial \rho_i} = \frac{1}{2}\left(2[d]^{T}[K]\frac{\partial d}{\partial \rho_i} + [d]^{T}\frac{\partial K}{\partial \rho_i}[d]\right) = \frac{1}{2}\left(-2[d]^{T}\frac{\partial K}{\partial \rho_i}[d] + [d]^{T}\frac{\partial K}{\partial \rho_i}[d]\right)$$

$$= -\frac{1}{2}[d]^{T}\frac{\partial K}{\partial \rho_i}[d] \tag{6.13}$$

Since the global stiffness matrix $[K]$ is assembled from element stiffness matrix, k_e^i, and each element stiffness matrix is a function of the density ρ_i, $w(\rho_i)$, the derivative in Eq. (6.13) at the element level is calculated as follows. The stiffness matrix for i^{th} element is defined as:

$$k_e^i = w(\rho_i)k_e^0 \tag{6.14}$$

where k_e^0 is the stiffness matrix with full material. Differentiating Eq. (6.14) yields

$$\frac{\partial k_e^i}{\partial \rho_i} = \frac{w'(\rho_i)}{w(\rho_i)} k_e^i \tag{6.15}$$

These derivatives can now be used to update the design variables (material density) when the topology optimization problem is solved to minimize the compliance.

6.3 Optimization Problem Formulation

Once the optimum topology is known by solving the optimization problem given in Eq. (6.2), a detailed sizing and control optimization is performed on the given topology. The control design theory has been explained in detail in Chapter 3 (Sec. 3.2). A minimization of the structural weight is considered as the objective function and is defined as:

$$\mathbb{F} = \sum_{i=1}^{n} \rho A_i l_i \tag{6.16}$$

where ρ is the weight density of the members, A_i is the cross sectional area of i^{th} element, l_i is the length of the i^{th} element, and n denotes the total number of members.

Some of the constraints that can be imposed on the problem include:

1. Stresses induced in each member should be less than the allowable stress, $\sigma_i <$ |S|.

2. The closed-loop damping ratio corresponding to the i^{th} mode must be greater than a specified value, $\xi_i > \xi_0$

3. The controlled system must be asymptotically stable (constraint on real part of closed loop eigenvalue λ_i)

4. The eigenvalues of the closed-loop system matrix must meet performance related requirements such as peak overshoot, settling time, etc.

5. The fundamental natural frequency of the open loop system must be greater than a specified value.

6. The cross-sectional area of the members must lie within prescribed bounds,

$$A_i^l \le A_i \le A_i^u$$

The above enumeration of constraints is by no means the only set of constraints that can be imposed. The nature and number of constraints varies depending on the desired system performance characteristics for open and closed-loop system. Mathematically, the optimization problem is formulated as follows:

Minimize Weight

by varying $(A_1 - A_i)$

subject to

$$\begin{aligned} &\sigma_i - S \le 0, \quad i = 1,...n \\ &\xi_o - \xi_i \le 0 \\ &\alpha_o - \alpha_i \le 0 \\ &\omega_o - \omega_1 \le 0 \\ &A_i^l \le A_i \le A_i^u, \quad i = 1,...n \end{aligned} \tag{6.17}$$

where n is the total number of elements, S is the allowable stress limit, ξ_i is the closed-loop damping ratio corresponding to the i^{th} mode, α_i is the real part of closed-loop eigenvalue corresponding to the i^{th} mode, ω_1 is the fundamental natural frequency of the open loop system, and A_i^l and A_i^u are the lower and upper bound on the member cross-sectional areas.

6.4 Solution Procedure

The complete solution procedure involves determination of optimum topology followed by sizing and control optimization of the optimum topology. A determination of optimum topology begins with defining an initial domain of the structure, i.e., the region occupied by the structure. This region of material is meshed using finite elements. External loads and boundary conditions are next specified with respect to this domain. The purpose of topology optimization is to find optimum density distribution while maintaining a constant volume fraction. The objective is to minimize the strain energy such that the final volume (or weight) of the structure should not be more than, say 20% of the initial volume of the structure.

Once the optimum topology is found, the resulting configuration is approximated using truss elements. It may be noted that for the problems considered herein, since the structural members are expected to carry only axial loads, truss elements are sufficient to approximate the structure. If lateral loads and/or moments are expected to be present, beam elements can be used to approximate the topology.

Next, support conditions are defined and loads are applied as defined in the topology optimization and a sizing optimization is performed. In this case, the cross-sectional areas of the elements are treated as design variables with the objective of minimizing the weight of the structure such that the stresses induced in the members are below the specified limits. Next, the optimum control problem is solved by adding controlled system performance constraints. The member cross-sectional areas are varied, the controller problem is re-solved; these iterations continue until the weight cannot be reduced any further. A complete flowchart of the solution process is given in Fig. 6.1.

For the two of the three example problems considered next, a sensitivity study was also performed to assess the influence of VF ratio on the optimum topology. This was done by changing the VF constraint limit to 25%, 30%, 35% and 40% of the initial volume. As discussed in the next section, for both the examples considered herein, it is seen that the optimum topology does not change significantly as the VF constraint value is varied. It may be noted that this somewhat low sensitivity of optimum topology to volume fraction ratio may not hold in general. For such cases, the designer needs to carefully select the prescribed value of VF ratio used in Eq. (6.2).

6.5 Numerical Examples

Three examples are presented next for solving the topology and control optimization problem. For all these examples, it is shown that an integration of topological considerations leads to final solutions which outperform fixed topology optima on both structural and control performance measures.

6.5.1 Example 1

The first example deals with sizing and control design for a 10-bar truss (fixed topology) followed by topology, sizing and control design for the same problem.

6.5.1.1 Sizing and Control Design for a fixed Topology

The 10 bar truss shown in Fig. 6.2 is first considered for structural design followed by simultaneous structure and control system design to establish a base line design to be used for comparison purposes later. The structure has eight degrees of freedom, two at each of the four free nodes. The total length of the truss is 720 inches, equally divided between two bays. The width of the truss is 360 inches. Two loads, 5000 lbs each, are acting at nodes 2 and 4 in the y direction whereas nodes 5 and 6 are fixed. The Young's modulus of the members is $10x10^6$ psi and the weight density of the material is 0.1 lb/in^3. A sizing optimization on this structure is performed first to minimize the weight of the structure subject to the constraint that member stresses should not exceed 25,000 psi. The cross-sectional areas of the members are taken as design variables and are constrained to lie between 0.1-20 in^2. The minimum weight of the structure is found to be 88.38 lbs and the corresponding optimum cross sectional areas are listed in Table 6.1.

Next, a controller is designed for the 10-bar truss. A non-structural mass of 1.29 $lb\text{-}s^2/in$ is attached at nodes 1 through 4. A total of four actuators are present at the four free nodes and they are assumed to be acting along y-direction only. The passive (material) damping is taken to be 1.0 E-5. The control

weighting matrix [R] is a 4x4 identity matrix and the state weighting matrix [Q] is taken as 1000*I.

The cross-sectional areas of the members are taken as design variables and are assumed to lie between 0.1 and 20 in^2. The objective is to minimize the weight of the structure. The design constraints imposed on the problem include: i) The stress in each member should not exceed 25,000 psi ($g_1 - g_{10}$); ii) The closed-loop damping ratio corresponding to the first mode should be greater than 0.6 (g_{11}); iii) a stability margin of 5 is required corresponding to the second eigenvalue of the closed-loop system matrix (g_{12}). Thus the problem formulation has a total of twelve inequality constraints. The complete problem is as follows:

Minimize Weight

by varying $(x_1 - x_{10})$

subject to

$$\begin{aligned} \sigma_{1-10} - 25000 &\le 0 \\ 0.6 - \xi_1 &\le 0 \\ 5 - \alpha_2 &\le 0 \qquad i = 1,...10 \\ 0.1 - x_i &\le 0 \\ x_i - 20 &\le 0 \end{aligned} \tag{6.18}$$

where $x_1 - x_{10}$ are the cross-sectional areas of the elements. This optimization problem is solved using sequential quadratic programming. The integrated structure and control optimization problem yields an optimum structural weight of 93.69 lb and the corresponding cross-sectional areas are given in Table 6.1.

The dynamic response of the optimum structure to an initial disturbance is studied by measuring the root mean square displacement (RMSD) associated

with all free nodes. The square root of the sum of the squares of displacements at all free nodes (nodes 1-4) in x and y direction is called the RMSD error and it should be damped out to fall within a certain range in a specified time interval. The dynamic response of the optimum structure is initiated by a unit displacement at node 3 in the y-direction at t=0. The RMSD error for this design is given in Fig. 6.3, and is about 0.102 in.

In addition to the stress constraints, Euler buckling constraints are also imposed on the problem. The members are assumed to be tubular with a nominal diameter to thickness ratio of 100 and the buckling stress in member i is given as:

$$P_i = \frac{-100.01\pi E_i A_i}{8l_i^2} \qquad i = 1,...n \tag{6.19}$$

where E_i, A_i and l_i denote the Young's modulus, cross-sectional area and length of member i respectively. The optimum weight of the structure is found to be 314.52 lb. The optimum cross-sectional areas listed in Table 6.2. When control constraints are added to the problem with both stress and buckling constraints, the optimum weight of the structure is found to be 326.1 lb. The optimum cross- sectional areas for this design are also listed in Table 6.2.

6.5.1.2 Topology Optimization

Next, a topology optimization of this structure is performed. For this problem, the initial problem domain is defined as a rectangular grid of nodal points as shown in Fig. 6.4 The Young's modulus and material density are $E=10x10^6$ psi and ρ = 0.1 lb/in^3 respectively. Top and bottom nodes on the

extreme left are fixed while the nodes at the center and the bottom right are subjected to two loads of 5000 lbs acting simultaneously in the y-direction. A topology optimization is performed using pshell elements with the objective of minimizing strain energy such that the mass of the final structure should not be more than 20% of the initial structure. The resulting optimum topology is shown in Fig. 6.5. The topologies for 25% and 30% volume fraction constraint are shown in Figs. 6.6 and 6.7. It can be seen from Figs. 6.6 and 6.7 that the optimum topology does not change significantly as the volume constraint is varied.

6.5.1.3 Sizing and Control Design for Optimum Topology

The resulting optimum topology can be approximated as an 8-bar or a 6-bar truss as shown in Fig. 6.8 and Fig. 6.9. A sizing optimization of these structures is performed next. Keeping every thing same as in case of initial 10 bar truss (Sec. 6.5.1.1), the minimum weight of the structures are found to be 79.3 lbs and 79.2 lbs. So, an optimization of topology leads to a 10% reduction in the optimum weight of the structure. The optimum cross-sectional areas for the 6-bar truss are listed in Table 6.3. Since both these structures results in the same minimum weight, the 6-bar truss is selected for controller design.

Next, a controller is designed for the optimum 6-bar truss shown in Fig. 6.9. The material properties and the applied loading is kept the same as in case of 10-bar truss (Sec 6.5.1.1). A non-structural mass of 1.29 lb-s^2 /in is attached at nodes 1 through 3. A total of three actuators are present at the three free nodes and they are assumed to be acting along y- direction only. The control weighting

matrix $[R]$ is a 3x3 identity matrix and the state weighting matrix $[Q]$ is taken as 1000*I. The design constraints are also kept the same as those in ten bar truss. The optimum weight of the structure is 79.2 lbs and the cross sectional areas are listed in Table 6.3. This design has a 15% lower weight than the corresponding design given in Table 6.1.The RMSD error for this design is given in Fig. 6.10. The overall RMSD error over a 2 sec interval in this case is 0.019 in, which is one order of magnitude smaller than the non optimum topology case. It is evident from Fig. 6.10 that the optimum topology case has a response that damps out much faster than the non-optimum topology case (Fig. 6.3). This example illustrates that by integrating topological considerations in the design process, designs with improved structural and control system performance are obtained.

6.5.2 Example 2

The next example considers a topology optimization problem considered by Ohsaki and Katoh (2005) to analyze the influence of grid size on overall topology.

6.5.2.1 Topology Optimization

Consider first the topology optimization for 3x2 grid considered by Ohsaki and Katoh (2005), and shown in Fig. 6.11. The length of each member is 200 in. The structure is subjected to two loads, each equaling 1000 lbs, acting in the negative y-direction at x=400 in and x=600 in as shown in Fig. 6.11. The top and bottom left nodes are fixed.

A topology optimization is performed by considering a 600x400 rectangular region on the problem domain and meshing it using 90x45 elements (see Fig. 6.4). The objective function is to minimize the strain energy of the structure with the constraint that the final mass of the structure should not be more than 25% of the initial mass. The optimum topology is shown in Fig. 6.12.

This topology is approximated in two ways as shown in figures 6.13 and 6.14. In Fig. 6.13, nodes 4 and 6 are stretched to the original fixed positions as in a 3×2 grid whereas in Fig. 6.14, these nodes are retained at the respective position as shown in Fig. 6.12. A sizing optimization of these structures is performed with the objective of minimizing the weight of the structure. The optimization problem formulation is as follows:

Minimize Weight

by varying $(A_1 - A_i)$

subject to

$$\sigma_i - 25000 \le 0 \qquad (6.20)$$

$$0.001 \le A_i \le 20 \qquad i = 1.....n$$

where n is the total number of elements. The minimum weight for the structure in Fig. 6.13 is 13.62 lbs whereas the structure shown in Fig. 6.14 yields a minimum weight 13.2 lb. These results show that a 3% reduction in weight is possible if the nodes in the optimum topology are not stretched to conform to the grid shown in Fig. 6.11. A sizing optimization of the optimum topology reported in Ohsaki and Katoh (2005) yielded an optimum weight of 15.22 lbs, 13% higher than the result reported herein.

Controls optimization of the optimum topology shown in Fig. 6.14 is performed next by adding three actuators at nodes 2, 3 and 4 and they are assumed to be acting along y-direction only. A load of 1000 lb is applied downwards at nodes 2 and 3 and nodes 1 and 7 are fixed. In addition to the structural constraints, constraints on the first and second closed-loop damping ratios are also imposed on the problem. The complete optimization problem is given as follows:

Minimize Weight

by varying $(A_1 - A_i)$

subject to

$$\sigma_i - 25000 \le 0$$

$$1 - \xi_1 \le 0 \qquad (6.21)$$

$$0.5 - \xi_2 \le 0$$

$$0.001 \le A_i \le 20 \qquad i = 1.....n$$

The optimum weight of the resulting structure is 13.5 lb and the corresponding cross-sectional areas are listed in Table 6.4. The RMSD error for the optimum design is 0.017 in. A controller design for the optimum topology reported in Ohsaki and Katoh (2005) is performed by adding actuators and applying disturbance at the corresponding nodes results in an optimum weight of 15.2 lb and an RMSD error of 0.024 in. This example highlights that topologies based on grids corresponding to a predetermined distribution of nodal points are less efficient than topologies where nodal points as well as their connectivity is determined by the optimization procedure.

6.5.3 Example 3

Examples1 and 2 demonstrated the benefits of integrating topological considerations in the context of structure and control design of actively controlled structures. Example 3 considers a multi-objective optimization problem where topological, structural and control considerations are combined using a game theory approach.

6.5.3.1 Topology Optimization

Consider first the problem domain shown in Fig. 6.15 where a structure is required to support two loads of 1000 lb each acting in the negative y-direction. The top and bottom left nodes are fixed. A candidate topology for this problem is based on a 3x2 grid, shown in Fig. 6.11. The sizing optimization of this topology results in an optimum weight of 13.81 lb. In this work, topology optimization is performed by considering a 600x400 rectangular region on the problem domain and meshing it using 180x90 elements (see Fig. 6.15). The objective function is to minimize the strain energy of the structure with the constraint that final volume of the structure should not be more than 25% of the initial volume. The resulting optimum topology is shown in Fig. 6.16. A sizing optimization of this structure results in an optimum weight of 12.7 lb, which corresponds to a 9% reduction in optimum weight.

6.5.3.2 Structural and Control Optimization

A multi-objective structural and control optimization of the optimum topology obtained in Sec 6.5.3.1 is presented next. Stackelberg and Cooperative game theory formulations are used to solve the problem with multiple objectives.

6.5.3.2.1 Single Objective Optimization

The optimum topology of Fig. 6.16 is approximated as an eight bar truss shown in Fig. 6.17. This structure has eight degrees of freedom (DOF), two DOF at each of the four free nodes. The Young's modulus of the members is 10×10^6 psi and the weight density of the material is 0.1 lb/in^3. A load of 1000 lb is applied downwards at nodes 3 and 4.The $[R]$ and $[Q]$ matrices are 8x8 and 16x16 diagonal matrices. The single objective optimization problems are solved first to determine the best and worst values of the follower objective functions which are weight (f_1) and trace $[P]$ (f_2) with cross-sectional areas of members and diagonal entries of $[Q]$ and $[R]$ as design variables. It is seen that the best and worst values of weight are 14.7 lb and 61.15 lb respectively. Similarly the best and worst values of trace $[P]$ are found to be 43814 and 4.26×10^6 respectively.

6.5.3.2.2 Multi-objective Optimization

The multi-objective optimization is performed using Stackelberg and cooperative game theory as shown in Fig. 6.18. The three objective functions considered are (i) the maximization of energy dissipated by the actuators (trace$[H]$), (ii) minimization of the weight of the structures, and (iii) minimization

of trace$[P]$. Player 1 (leader, trace$[H]$) wishes to maximize the energy dissipated by controller by controlling variables $x_1 - x_8$, which are the actuator locations in all elements whereas player 2 (follower, $F_{b\arg}$) maximizes the bargaining function between weight and trace $[P]$ with control over variables, $A_1 - A_8$, a diagonal entry of state weighting matrix,$[Q]$ and first, second, third and sixth diagonal entries of control weighting matrix, $[R]$. The other entries of $[Q]$ and $[R]$ matrices are fixed at 0.1.

The rational reaction set (RRS) of follower gives the change of optimum solution of follower problem while the leader's variables are varying. Since there are eight leader design variables with two possibilities, either zero or one, the follower problem is run 2^8 = 256 times to find the RRS. The sequential quadratic programming method is used to solve the follower problem with continuous design variables. Once the RRS for the bargaining function for the follower is found, it is inserted into the leader problem to find the optimum solution to the leader problem. The problem formulation is stated as:

Leader:

Maximize $trace[H]$

by varying ($x_1 - x_8$)

subject to

$$0.03 - \xi_1 \le 0$$

$$200 \le \beta_i \le 2500 \qquad i = 1,...8$$

$$\sigma_i - 25000 \le 0 \tag{6.22}$$

$$0.001 \le area_i \le 20$$

$$0.1 \le Q_j \le 1000 \qquad j = 9$$

$$0.1 \le R_p \le 1000 \qquad p = 1, 2, 3, 6$$

Follower:

$$\text{Maximize } F_{b\arg} = \frac{(f_{1w} - f_1)(f_{2w} - f_2)}{(f_{1w} - f_{1b})(f_{2w} - f_{2b})} \tag{6.23}$$

by varying $(area, Q, R)$

subject to the same constraint in Eq. (6.22). Here f_{1w} and f_{1b} are the worst and best values of first follower objective function (weight) and f_{2w} and f_{2b} are the worst and best values of second follower objective function (trace$[P]$) as specified in Sec. 6.5.3.2.1.

6.5.3.3 Results

For each $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ combination, an optimum solution for $A_1 - A_8$, Q_9, R_1, R_2, R_3, R_6 is obtained. From the 256 solutions, a response surface regression yields the following approximation function for the RRS for the follower objective function $F_{b\arg}$.

$$A_1 = 0.1206 - 8.4041\times10^{-4}(x_1 + x_2 + x_3) + 0.9979\times10^{-4}x_4 - 8.4041\times10^{-4}(x_5 + x_6 + x_7) - 1.003\times10^{-4}x_8$$

$$A_2 = 0.0495 - 5.8829\times10^{-4}x_1 - 7.5156\times10^{-4}x_2 - 6.2875\times10^{-4}x_3 - 2.6468\times10^{-4}x_4 - 7.3374\times10^{-4}x_5 - 5.9925\times10^{-4}x_6 - 7.3127\times10^{-4}x_7 - 4.2876\times10^{-4}x_8$$

$$A_3 = 0.0749 - 1.6555\times10^{-5}(x_1 + x_2 + x_3) + 1.7269\times10^{-5}x_4 - 1.6555\times10^{-5}(x_5 + x_6 + x_7 + x_8)$$

$$A_4 = 0.0355 - 1.0606\times10^{-4}x_1 + 1.9600\times10^{-4}x_2 + 2.7567\times10^{-4}x_3 - 4.4676\times10^{-4}x_4 + 1.6085\times10^{-4}x_5 + 3.2073\times10^{-4}x_6 + 0.7784\times10^{-4}x_7 - 0.6327\times10^{-4}x_8$$

$$A_5 = 0.0657 - 2.9621\times10^{-4}x_1 - 3.8406\times10^{-4}x_2 - 3.0136\times10^{-4}x_3 - 3.1952\times10^{-4}x_4 + 1.9552\times10^{-4}x_5 - 2.0022\times10^{-4}x_6 - 3.8406\times10^{-4}(x_7 + x_8)$$

$$A_6 = 0.0362 - 1.6823\times10^{-3}x_1 - 2.2994\times10^{-3}x_2 - 2.1958\times10^{-3}x_3 + 0.4443\times10^{-3}x_4 - 2.6243\times10^{-3}x_5 - 2.2788\times10^{-3}x_6 - 2.3711\times10^{-3}x_7 - 0.7722\times10^{-3}x_8$$

$$A_7 = 0.1003 + 7.7238\times10^{-5}x_1 - 7.8801\times10^{-5}(x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8)$$

$$A_8 = 0.0473 - 2.5557\times10^{-5}(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8)$$

$$Q_9 = 2.3815 + 2.7677\times10^{-1}x_1 - 7.5077\times10^{-1}x_2 - 4.3118\times10^{-1}x_3 - 2.1125\times10^{-1}x_4 - 8.9931\times10^{-1}x_5 - 9.4868\times10^{-1}x_6 + 5.1807\times10^{-1}x_7 - 9.8433\times10^{-1}x_8$$

$$R_1 = 0.1275 + 4.2284\times10^{-2}x_1 - 4.2284\times10^{-2}x_2 + 3.0104\times10^{-2}x_3 + 4.1747\times10^{-2}x_4 - 4.2284\times10^{-2}(x_5 + x_6) + 4.2284\times10^{-2}x_7 - 4.2284\times10^{-2}x_8$$

$$R_2 = 0.1092 - 3.6764\times10^{-3}x_1 + 3.6764\times10^{-3}x_2 - 3.6764\times10^{-3}(x_3 + x_4 + x_5 + x_6 + x_7) + 3.6764\times10^{-3}x_8$$

$$R_3 = 0.1077 - 5.1429\times10^{-3}(x_1 + x_2) + 5.1429\times10^{-3}(x_3 + x_4) - 5.1429\times10^{-3}(x_5 + x_6) + 5.1429\times10^{-3}x_7 - 5.1429\times10^{-3}x_8$$

$$R_6 = 0.5415 - 1.7660\times10^{-1}(x_1 + x_2 + x_3) + 1.7660\times10^{-1}x_4 - 1.7660\times10^{-1}x_5 + 1.7660\times10^{-1}x_6 - 1.7660\times10^{-1}(x_7 + x_8)$$

(6.24)

Stackelberg Solution: With player trace $[H]$ as the leader and the bargaining function $F_{b\arg}$ between weight and trace $[P]$ as the follower, the Stackelberg game problem is solved by substituting Eq. (6.24), which is RRS of the follower

problem, into the leader's problem. The optimum solution to the leader problem results in an optimum value of trace $[H]$=5.99x10^6 with optimum actuator locations of in element 4, 7 and 8. The weight of the structure is 14.92 lb and trace $[P]$ is 6.27x10^4. The dynamic response of the optimum structure is initiated by a unit displacement at node 2 in the y-direction at t=0. The dynamic response of the optimum structure to an initial disturbance is studied by measuring the root mean square displacement (RMSD) error associated with all free nodes. The RMSD error for this design is given in Fig 6.19 and is about 0.0895 in. The optimum cross-sectional areas are listed in Table 6.5.

6.6 Conclusions

This chapter presented an approach for simultaneous topological and sizing optimization of actively controlled structures. Based on the results of the numerical examples, it is seen that irrespective of the fact that only structural optimization is performed or an integrated structural and control optimization is solved, the optimum topology formulation always yields better structural and control designs compared with a fixed topology formulation. For one of the examples considered herein, a 10-15% reduction in weight and about 80% improvement in RMSD error is obtained by optimizing the topology of the structure. The solution approach for optimizing the topology, structure and control system is not intense because control system optimization is performed once optimum topology is determined. It is seen that the proposed approach yields designs with improved structural and controller performance and the controller is

quickly able to bring the structure to its equilibrium position when subjected to an external disturbance.

Table 6.1 Cross-sectional areas for 10 bar truss

Design Variables	Starting Values	Optimum Areas Stress Constraints only	Optimum Areas Stress & Control Constraints
x1	1.0	0.1000	0.2511
x2	1.0	0.1000	0.1000
x3	1.0	0.1379	0.1121
x4	1.0	0.3379	0.3281
x5	1.0	0.1000	0.1000
x6	1.0	0.4621	0.4842
x7	1.0	0.1949	0.1585
x8	1.0	0.1000	0.1434
x9	1.0	0.3707	0.4018
x10	1.0	0.1949	0.1638
Weight	419.64	88.38	93.69

Table 6.2 Cross-sectional areas with stress, buckling and control constraints for 10 bar truss

Design Variables	Optimum areas with stress & buckling constraints	Optimum areas with stress, buckling & control constraints
x1	0.1	0.1
x2	0.1	0.1
x3	1.2832	1.1561
x4	0.5736	0.4531
x5	0.1	0.1
x6	1.5744	1.6918
x7	0.6525	0.2785
x8	0.1	0.9419
x9	0.142	0.2077
x10	2.6442	2.4305
Weight	314.515	326.1122

Table 6.3 Cross-sectional areas for optimum topology formulation (6-bar)

Design Variables	Starting Values	Optimum Areas Stress Constraints only	Optimum Areas Stress & Control Constraints
x1	1.0	0.400	0.400
x2	1.0	0.283	0.283
x3	1.0	0.200	0.200
x4	1.0	0.400	0.400
x5	1.0	0.283	0.283
x6	1.0	0.283	0.283
Weight	260.73	79.20	79.20

Table 6.4 Cross-sectional areas for optimum topology –example 2.

Design Variables	Optimum Areas Stress Constraints only	Optimum Areas Stress & Control Constraints
x1	0.080	0.080
x2	0.020	0.021
x3	0.045	0.045
x4	0.015	0.029
x5	0.075	0.075
x6	0.028	0.028
x7	0.030	0.030
x8	0.085	0.085
x9	0.040	0.040
x10	0.038	0.038
Weight	13.20	13.53

Table 6.5 Optimum results for example 3.

Element	Actuator	Areas	R*	Q_9
1		0.1198	0.1692	1.7
2		0.0481	0.1055	
3		0.0749	0.1128	
4	X	0.0351	0.3649	
5		0.0646		
6		0.0334		
7	X	0.1		
8	X	0.0473		
Trace H	5.99×10^6			
Weight	14.92			
Trace P	6.27×10^4			
RMSD	0.0895			

R^*=First, second, third and sixth diagonal entries of R matrix

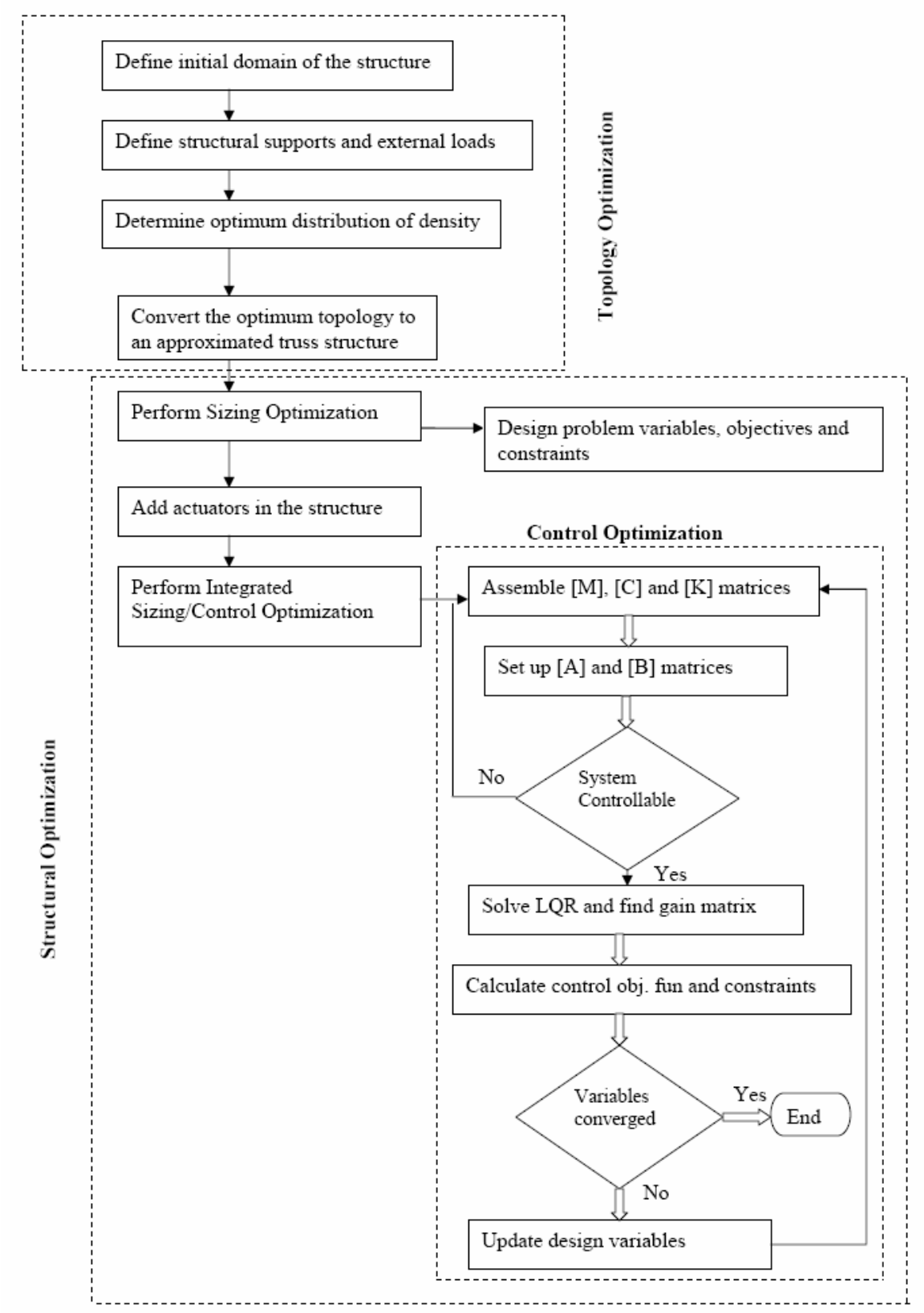

Figure 6.1 Steps for solving the Integrated Topology and Control Optimization Problem

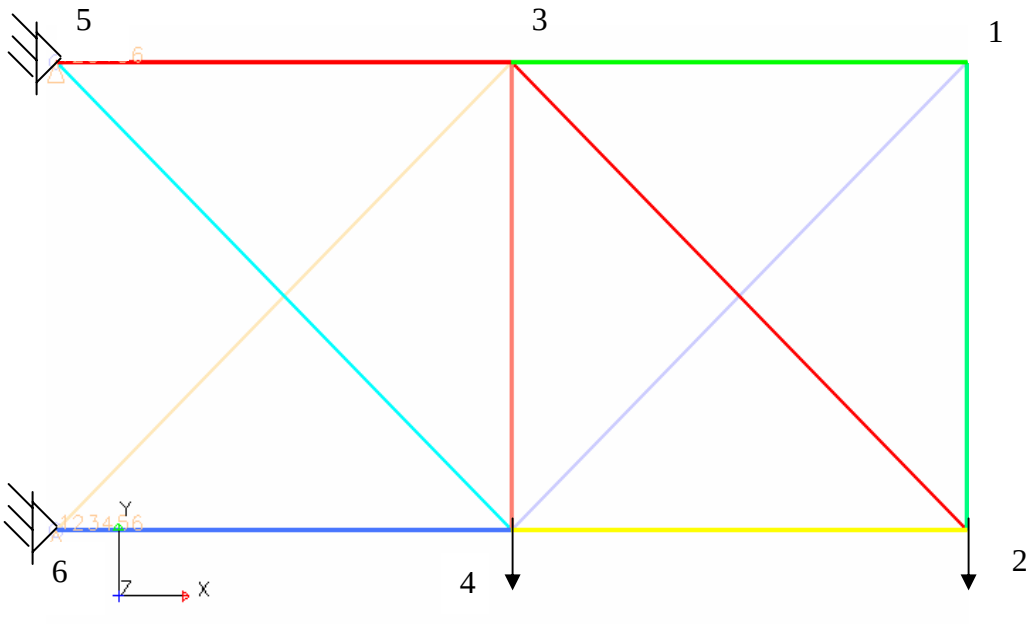

Figure 6.2 Ten bar truss with two applied loads

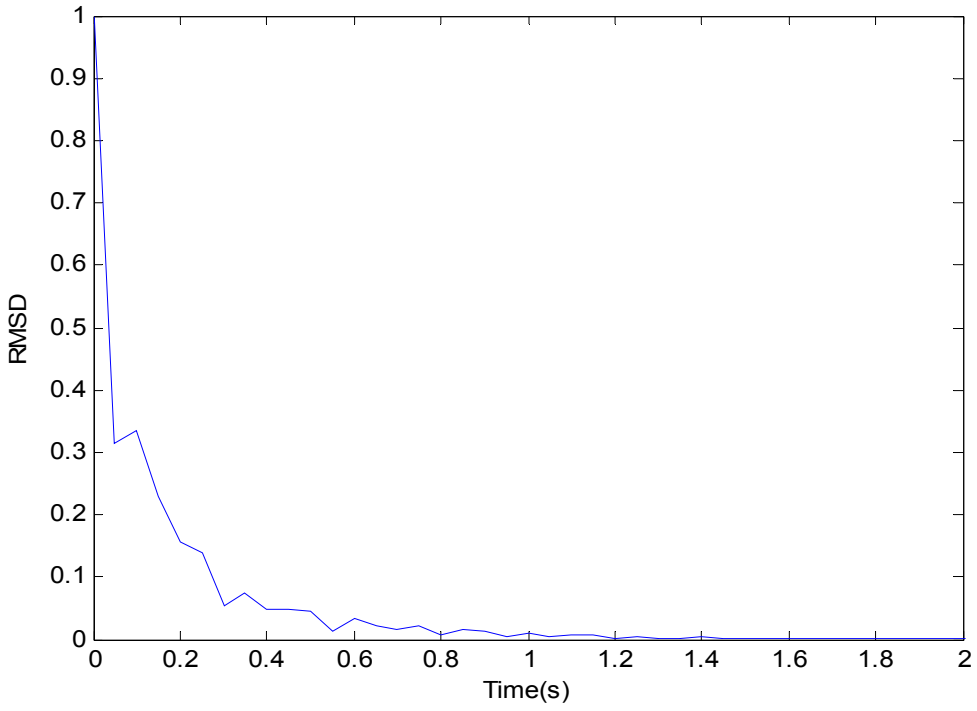

Figure 6.3 Transient response of 10 bar structure at optimum design

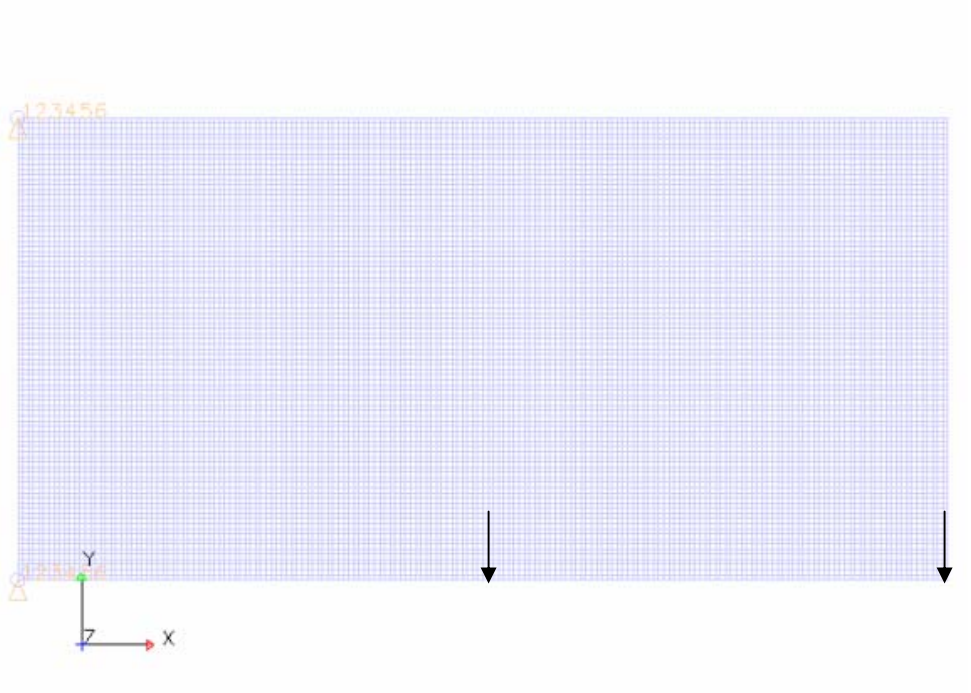

Figure 6.4 Problem domain for example 1 showing support and points of load application

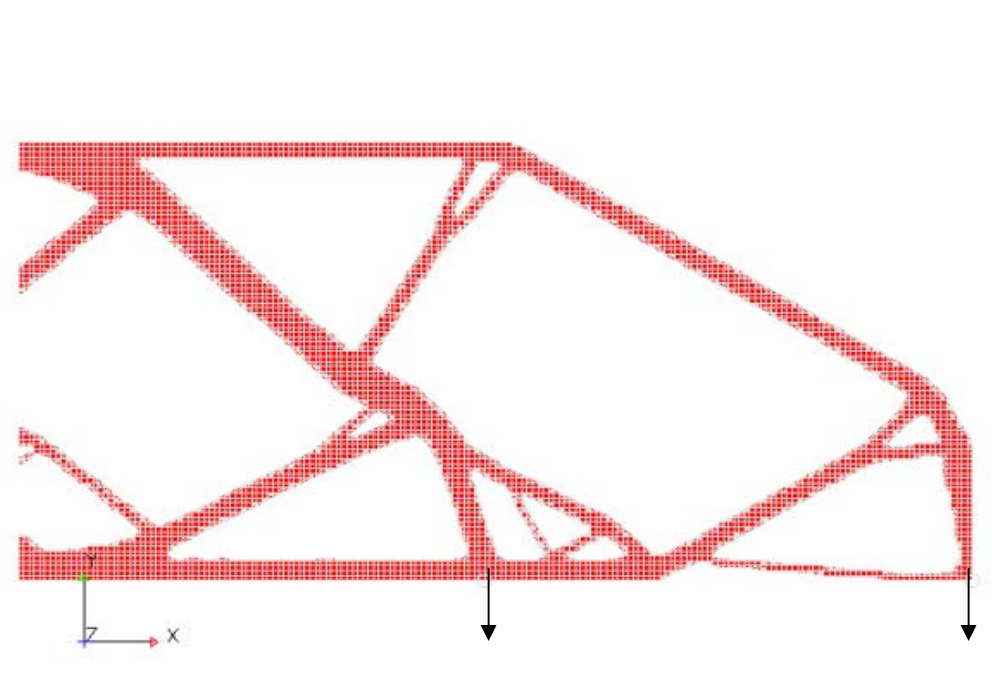

Figure 6.5 Optimum topology for example 1 with 20% volume constraint

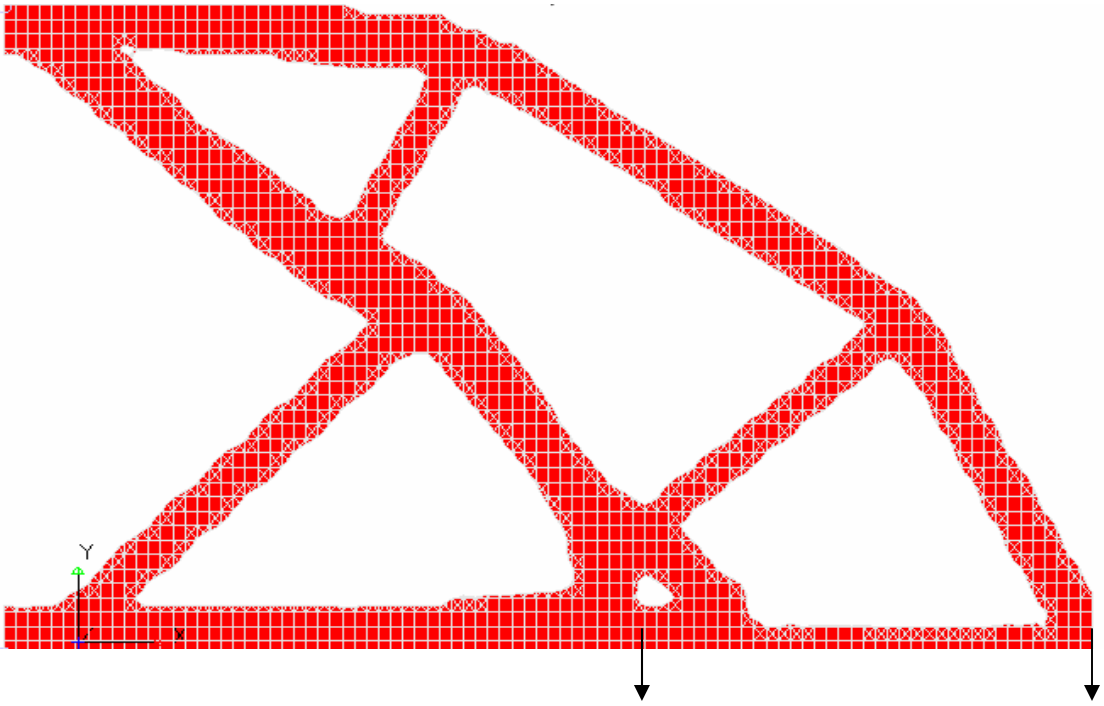

Figure 6.6 Optimum topology for example 1 with 25% volume constraint

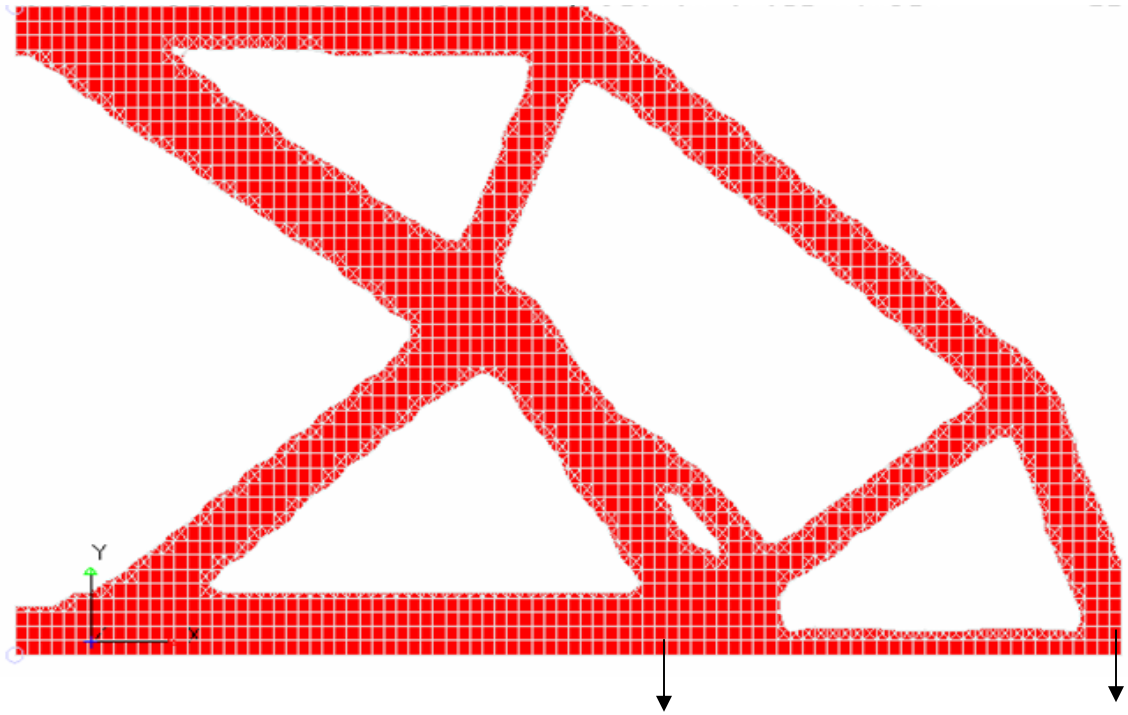

Figure 6.7 Optimum topology for example 1 with 30% volume constraint

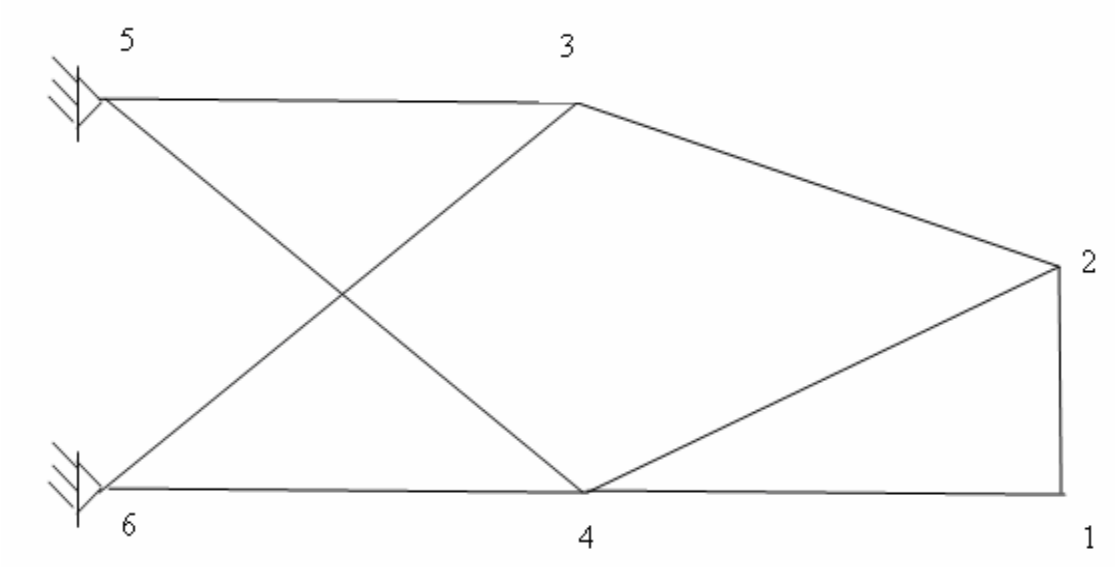

Figure 6.8 Approximated optimum topology for example 1

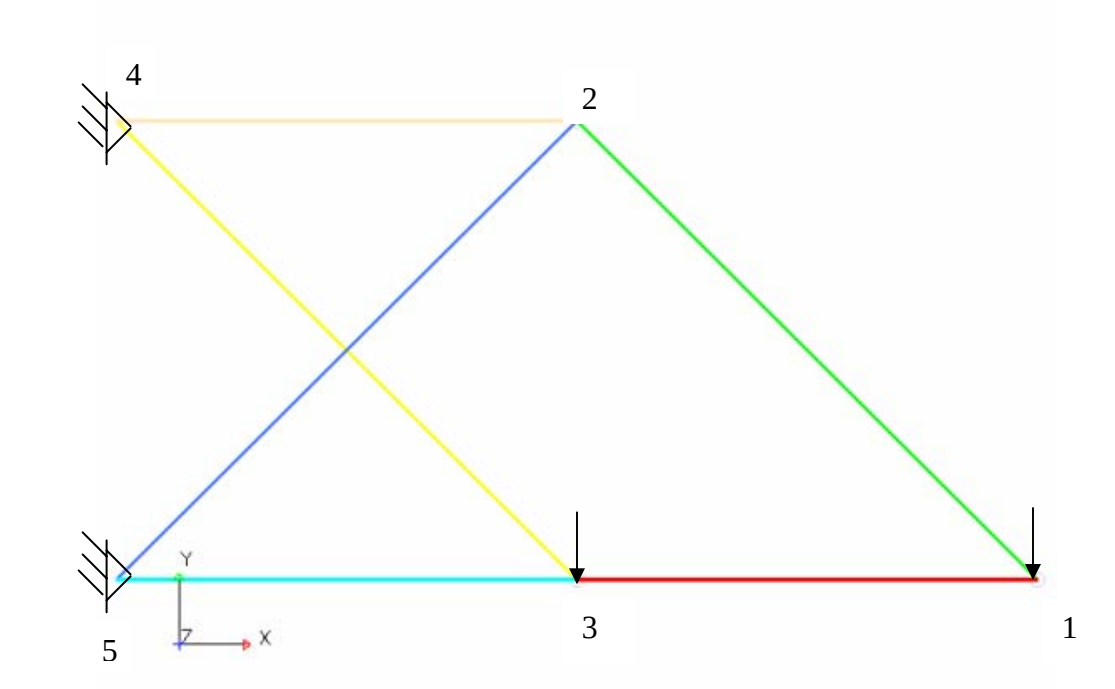

Figure 6.9 Another Approximated optimum topology for example 1

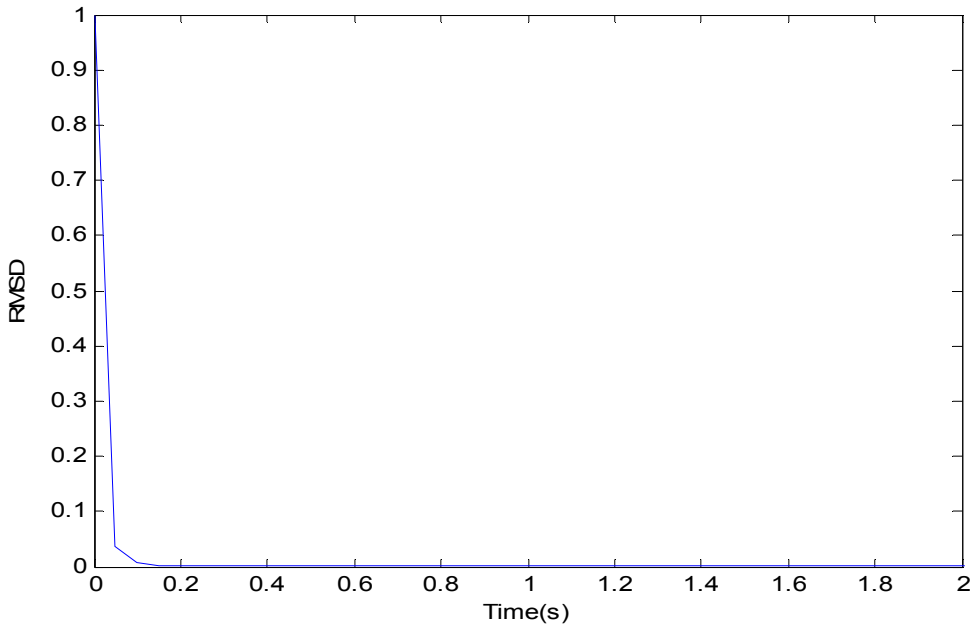

Figure 6.10 Transient response of 6 bar truss at optimum design

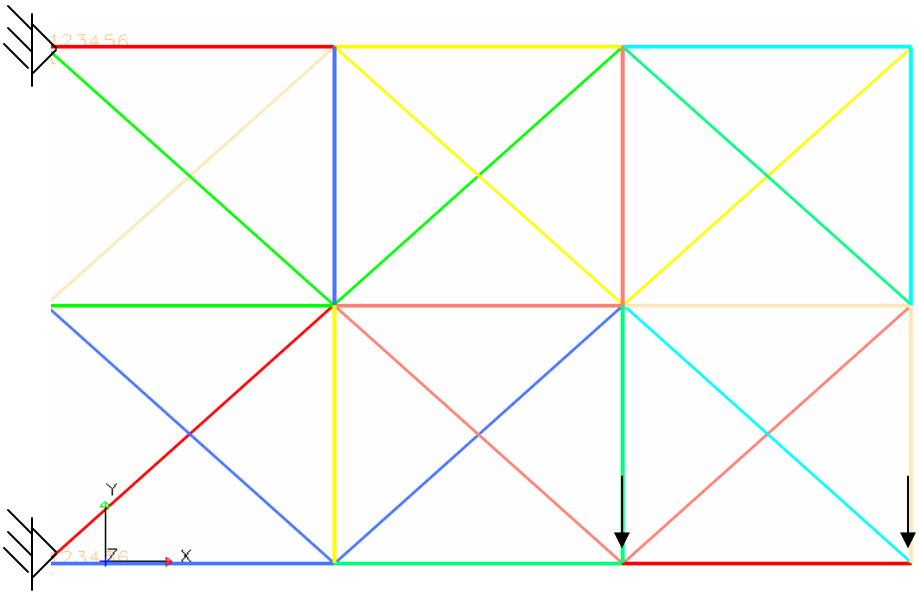

Figure 6.11 3x2 plane grid of Ohsaki and Katoh (2005)

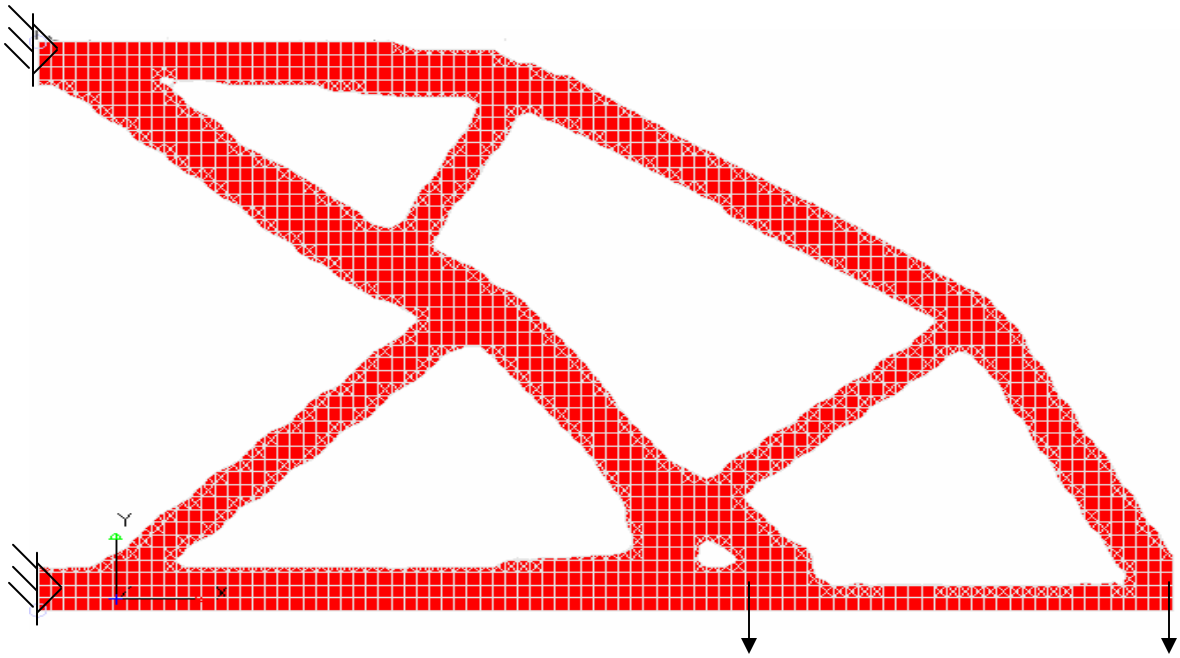

Figure 6.12 Optimum topology for example 2

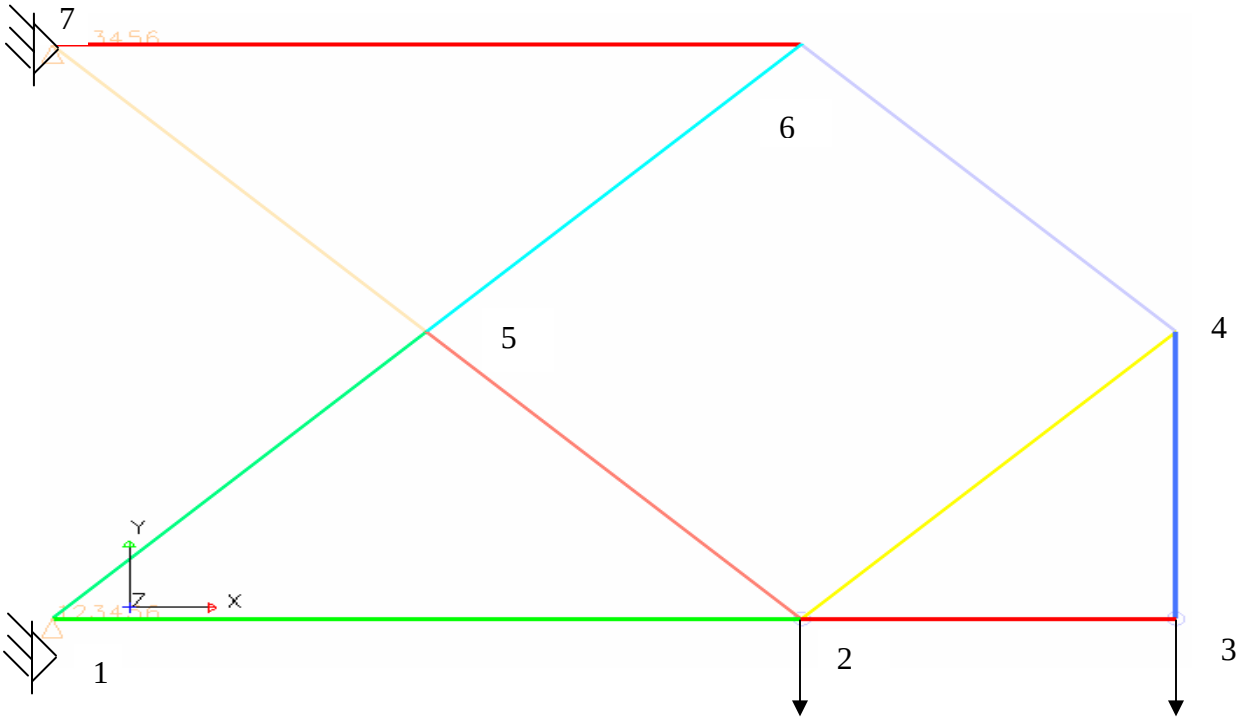

Figure 6.13 Approximated Optimum topology of 3x2 plane grid (with node stretching)

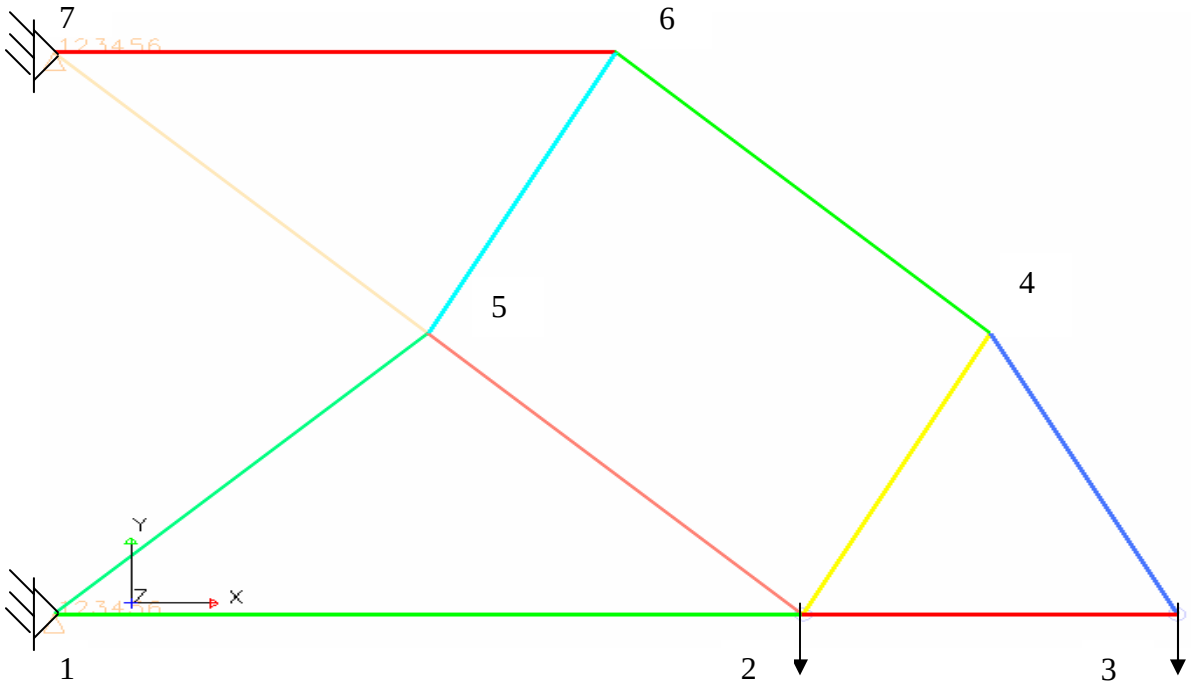

Figure 6.14 Approximated Optimum topology of 3x2 plane grid (without node stretching)

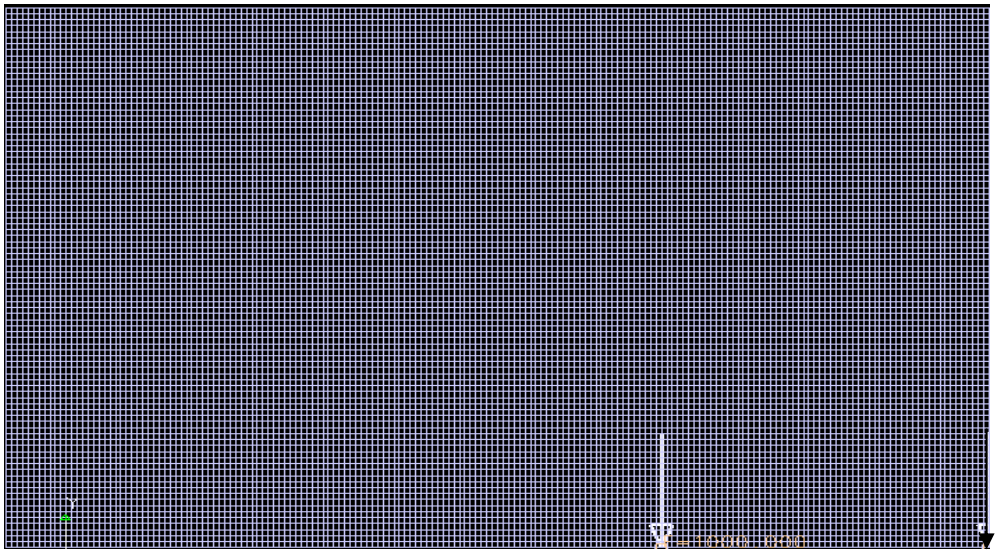

Figure 6.15 Problem domain with supports and points of load application for example 3

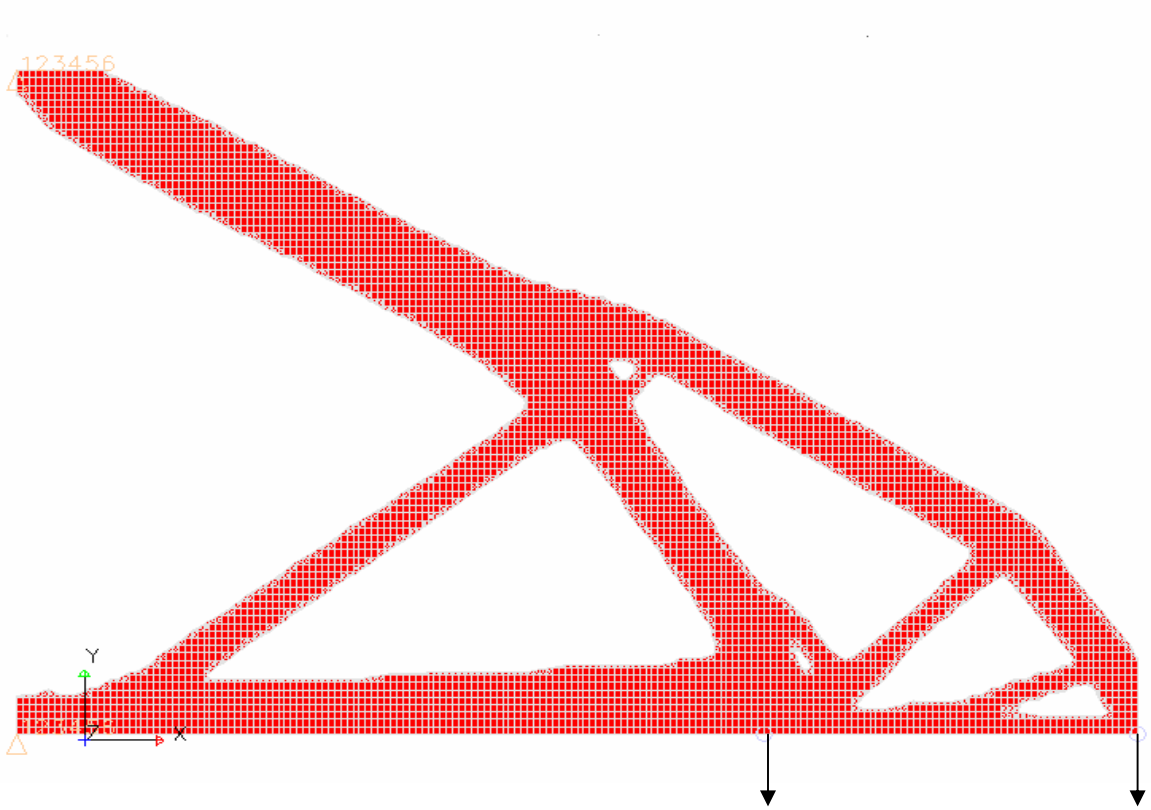

Figure 6.16 Optimum topology for example 3 with 25% volume constraint

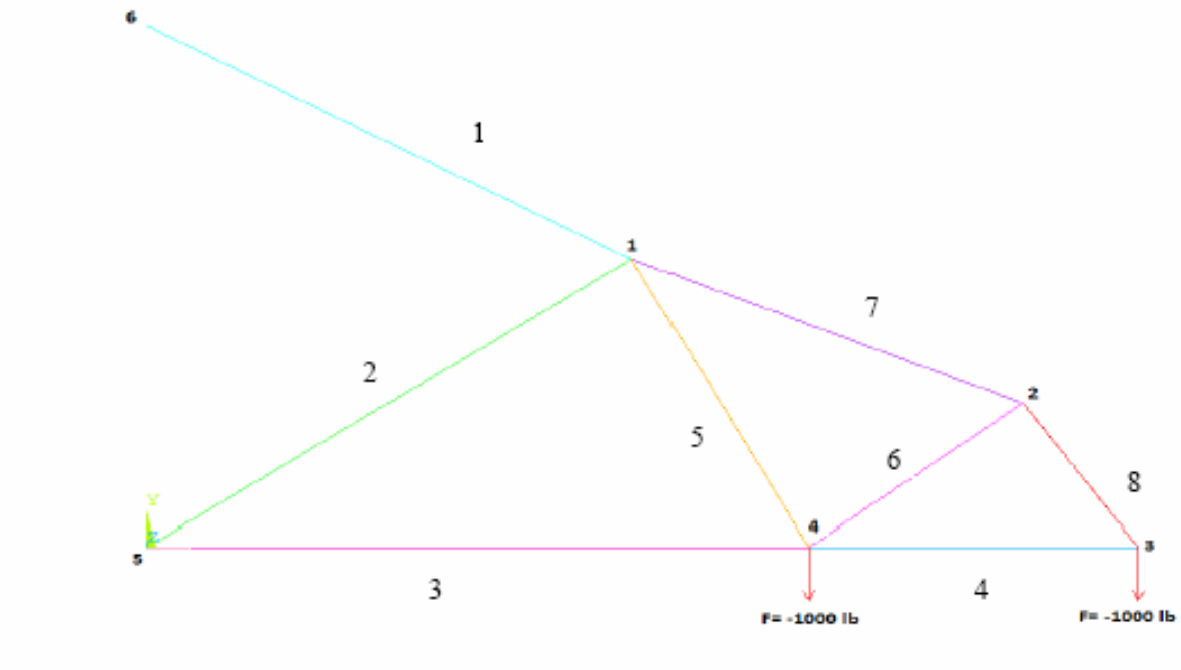

Figure 6.17 Approximated topology for example 3

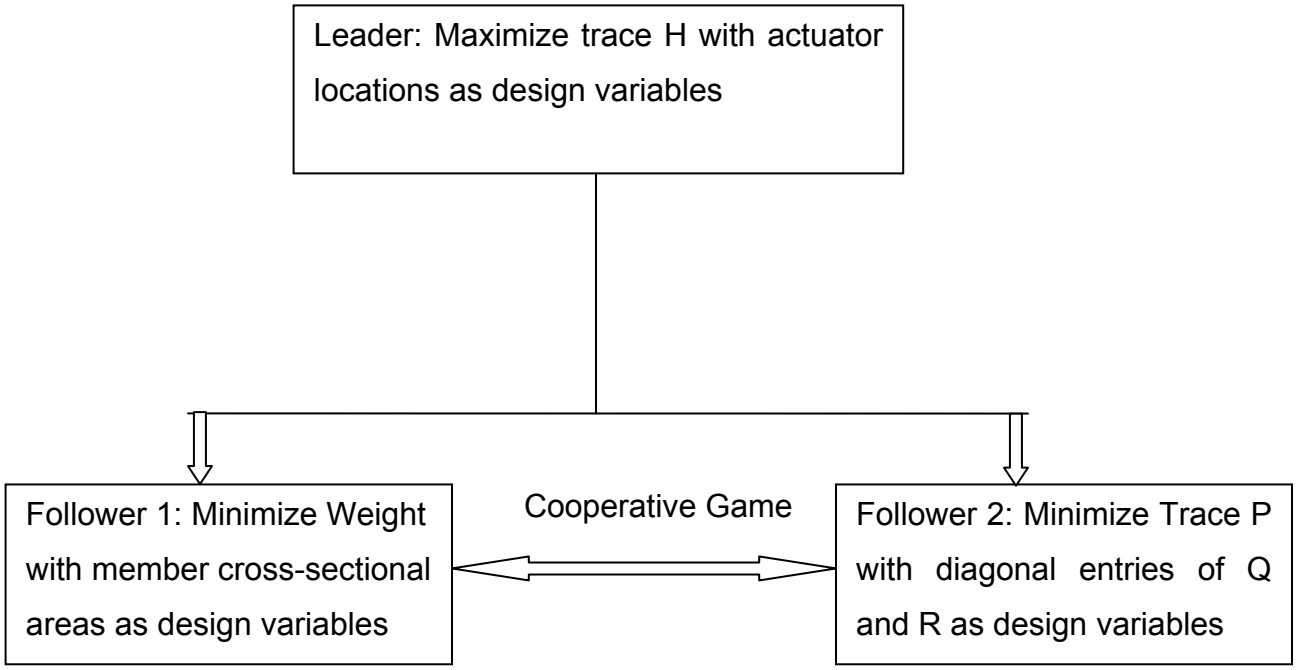

Figure 6.18 Two level Stackelberg and Cooperative Game

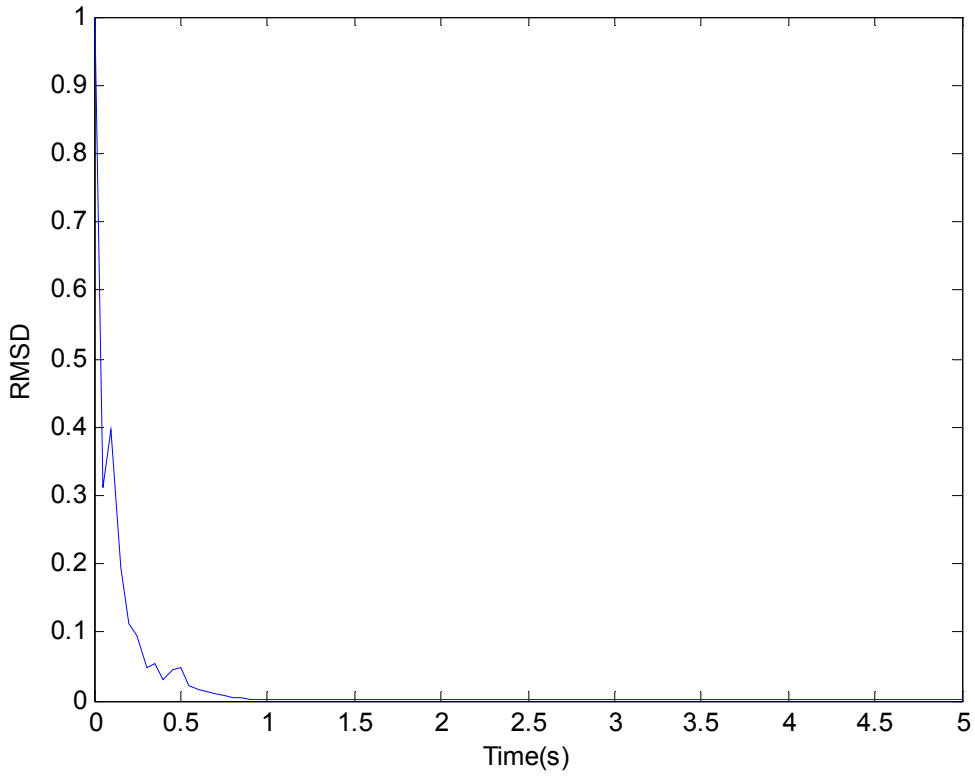

Figure 6.19 Transient response at optimum design - example 3

Chapter 7

Conclusions and Future Work

This thesis presented some solution techniques using a multi-objective formulation, for a comprehensive treatment of the structural and topological considerations in the design of actively controlled structures. The objectives of this dissertation can be divided into the three broad areas: (1) A simultaneous structure and control design of actively controlled structures with mixed discrete and continuous design variables representing actuator locations and member cross-sectional areas, (2) Multi-objective formulation and solution of structure and control design problem using game theory approaches, and (3) Comprehensive treatment of topological considerations in sizing and control optimization of actively controlled structures.

7.1 Actuator Placement in Structural-Control Design

This thesis presented an approach for finding the optimum number and optimum location of actuators in the design of actively controlled structures such that the structure satisfies the requirement on weight, control effort and performance. The member cross-sectional areas are also determined while solving the optimization problem. The structure and control designs are linked through constraints on structural and control performance. Since the locations of actuators are discrete (0-1) variables whereas the cross-sectional areas are continuous, this mixed discrete-continuous variable problem is solved using a genetic algorithm based approach. The constrained optimization problem is

converted to an equivalent unconstrained problem by using penalty function concept so that genetic algorithms can be used to obtain an optimum solution. The numerical results presented for an example problem show that the proposed approach can successfully design an optimum controller to minimize the weight of the structure and maximize the energy dissipated by the controller to bring the structure to its equilibrium position when subjected to an external disturbance.

7.2 Multi-level/Multi-Objective Optimization

In case of large complex structures where the number of design variables is large, the problem size becomes unmanageable and requires more computational time and effort. By dividing the whole problem into smaller sub problems (sub levels) makes the problem easy to solve. In this case each level has its own objective function and design variables, and an exchange of information is done between different levels. In this thesis, for the multi-objective problem considered, the design variables are cross-sectional areas of the members, locations of the actuators and the diagonal entries of Q and R matrices. The problem is divided into two levels. Two game theory approaches are used to solve the multi-objective structural control optimization problem. In the first approach, the two objectives considered are at the same level and a bargaining function between them is constructed and maximized using cooperative game theory. In the second approach, Stackelberg game theoretic formulation is used when the two objectives considered are not on the same level. In this method, the two objectives are treated at two levels with one level as the leader and the other as the follower. The discrete and continuous variables

are also separated into two levels with each level having its own objective function. Member cross-sectional areas are the design variables with the objective function corresponding to minimizing the weight of the structure in one level; the actuator locations are design variables with the objective of maximizing the energy dissipated by the actuators in the other level. The solution approach includes a blend of genetic algorithms and sequential quadratic programming techniques. A computational procedure based on variable updating using response surface methods is developed for exchanging information between the two levels.

7.3 Topology Optimization

This thesis also considers the simultaneous structural and control design of actively controlled structures with optimized topology. The available literature on simultaneous structural and control optimization primarily deals with structures with known topologies. It has been recognized that the performance of a controlled structure can be significantly improved by optimization of topology. The approach presented in this thesis involves first performing the topology optimization followed by a structural and control system optimization of the optimum topology.

Two approaches are considered in this work. The first is a sizing optimization of a structure with known topology, and the second is a determination of optimum topology followed by sizing and control optimization of the optimized topology. The approach to topology optimization involves defining a domain for the structure, points of applied loads and supports. Topology

optimization is performed by creating design variables associated with the Young's Modulus and density of each element in the design space. The design variable value ranges between 0 and 1 where 0 indicates the element has no stiffness or mass and 1 indicates the element has its normal stiffness and mass. The objective function for the topology optimization is a minimization of strain energy. Based on the results of the numerical examples, it is seen that irrespective of the fact that only structural optimization is performed or an integrated structural and control optimization problem is solved, the optimum topology formulation always results in a better structural and control designs compared to fixed topology formulation. The solution approach presented for optimizing the topology and structure and control system design is not intense because control system optimization is performed once optimum topology is determined.

7.4 Scope for Future Work

The techniques proposed in this thesis use the linear quadratic regulator (LQR) theory for the control system design. Though the LQR theory is efficient and a popular method for control design, it suffers from a major limitation that all states must be measured exactly when specifying the control law. In case of higher order systems, measuring all states can be very expensive. Another limitation of this controller is that uncertainties/disturbances in the system cannot be considered by using LQR design. Since many real world problems may preclude exact measurement of all state variables, further research in this area

should consider the influence of uncertainties on the controller performance and stability.

The DOE-RSM method used in the thesis for capturing the change in follower's variable as a function of leader's variable involves approximating the RRS for the follower's problem. Based on the DOE set up used for the follower's problem, a fixed function results as an approximation for RRS. This RRS does not get updated while iterations continue in the leader's problem. An updating of the RRS as more data becomes available can improve the efficiency of this method. Moving least squares method or Kriging techniques can both be used to update the RRS of the follower as iterations continue for the leader problem. This aspect of model updating will be explored in the near future.

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- Ali, A. and Dhingra, A.K., (2013) “Integrated Topology and Sizing Optimization of Actively Controlled Structures,” accepted to *Engineering Optimization Journal*

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