Dynamic Moving Load Identification Using Optimal Sensor Placement and Model Reduction

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DYNAMIC MOVING LOAD IDENTIFICATION USING OPTIMAL SENSOR PLACEMENT AND MODEL REDUCTION

by

Paul Augustine

A Thesis Submitted in Partial Fulfilment of the Requirements for the Degree of

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at

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ABSTRACT

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by

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Under the Supervision of Professor Anoop K. Dhingra

A structure in service can be subjected to static, dynamic or moving loads. Several situations in practice involve estimation of moving loads which induce vibrations in the structure on which they are applied. An accurate estimation of these loads will ensure product quality and reliability of the final design, and mitigate the cost of structural health monitoring systems. The moving nature of dynamic loads increases the computational difficulty of the problem. One of the types of Inverse Problems involves estimation of the applied load from measured structural response such as strain or accelerations.

Measuring response at a limited number of locations causes unavailability of the full structural response, which can lead to inaccurate results. The unavailability of full structural response is mainly due to three reasons - (i) financial constraints limiting the number of sensors that can be used, (ii) inaccessibility of loading locations to place sensors, and (iii) sensor influence on structural response. The load recovered from limited structural response data will be prone to errors. Ill-conditioning of the inverse problem can be eliminated by choosing optimum sensor locations on the structure, which leads to
precise load estimates. No studies could be found which consider optimum sensor placement while recovering dynamic moving loads acting on a structure.

In this thesis, the recovery of the dynamic moving loads through measurement of structural response at a finite number of optimally selected locations is investigated. Optimum sensor locations are identified using the D-optimal design algorithm. Separate algorithms are developed for dynamic moving load recovery using strain measurements and acceleration measurements. The developed algorithms are successfully implemented using ANSYS APDL and MATLAB programming environment. Compared to conventional algorithms for estimating moving loads, the developed methods make the dynamic moving load recovery procedure accurate and relatively easy to implement.
# TABLE OF CONTENTS

**Chapter 1 Introduction** .................................................................................................................. 1
   1.1 Problem Statement .................................................................................................................. 1
   1.2 Limitation of Load Cells ........................................................................................................ 1
   1.3 Using Structure as a Load Transducer .................................................................................. 2
   1.4 Limitations of Inverse Load Identification Method .............................................................. 2
   1.5 Practical Application .............................................................................................................. 4
   1.6 Organization of the Material ................................................................................................. 5

**Chapter 2 Literature Review** ......................................................................................................... 7
   2.1 Moving Load Identification History ...................................................................................... 7
   2.2 Time Domain and Frequency-Time Domain Methods ............................................................ 8
   2.3 Finite Element Methods ........................................................................................................ 9
   2.4 Other Algorithms in Moving Load Identification ................................................................. 10
   2.5 Regularization Techniques .................................................................................................... 11
   2.6 Optimum Location of Sensors ............................................................................................. 13
   2.7 Summary ............................................................................................................................... 14

**Chapter 3 Recovery of Quasi Static and Dynamic Moving Loads using Strain Measurements** ............................................................................................................................................. 16
   3.1 Theoretical Development ........................................................................................................ 16
   3.2 Generation of the Candidate Set ............................................................................................ 20
   3.3 Determination of the Number of Strain Gages ...................................................................... 23
   3.4 Determination of the D-Optimal Design ............................................................................... 23
   3.5 Numerical Examples ............................................................................................................... 26
      3.5.1 Quasi-Static Load Recovery ......................................................................................... 27
      3.5.2 Dynamic Moving Load Recovery: Orthogonal Loads .................................................... 29
      3.5.3 Dynamic Moving Load Recovery: Parallel Loads ......................................................... 32
      3.5.4 Recovery of Noisy Moving Loads with Uneven Cardinal Degrees of Freedom .............. 33
   3.6 Summary ............................................................................................................................... 34

**Chapter 4 Dynamic Moving Load Recovery Using Acceleration Measurements and Model Reduction** ............................................................................................................................................. 54
4.1 Matrix Representation of Structural Dynamics of a System ........................................54
  4.1.1 Physical Coordinate Representation .........................................................54
  4.1.2 Modal Coordinate Representation ............................................................55
  4.1.3 State Space Representation ........................................................................57

4.2 Model Order Reduction ....................................................................................58
  4.2.1 Static condensation (Guyan Reduction) .......................................................58
  4.2.2 Component mode synthesis ........................................................................61
    4.2.2.1 Normal Modes of a Structure ...............................................................61
    4.2.2.2 Static Modes of a Structure .................................................................61
    4.2.2.3 CMS Substructuring Method ...............................................................62

4.3 Load Recovery Using Acceleration Measurements ...........................................64
  4.3.1 Dynamic Non-Moving Load Recovery without Reduction .........................65
  4.3.2 Example: Dynamic Load Recovery without Reduction ..............................66
  4.3.3 Dynamic Non-Moving Load Recovery using D-Optimal Design and Model
    Reduction ........................................................................................................68
    4.3.3.1 Candidate Set for Accelerometers .......................................................69
    4.3.3.2 D-optimal Design for Accelerometers .................................................70
    4.3.3.3 Solution Procedure using D-optimal Design and Model Order Reduction
      ......................................................................................................................70
  4.3.4 Load Recovery using D-Optimal Design and Reduced Modal Model ..........73
  4.3.5 Moving Load Recovery using D-Optimal Design and Reduced Modal Model
    .........................................................................................................................75

4.4 Summary .........................................................................................................76

Chapter 5 Conclusions and Future Work ............................................................88

BIBLIOGRAPHY ...................................................................................................91
LIST OF FIGURES

Figure 2.1: A Simply Supported Beam Subjected to Moving Load $F(t)$ ...................... 15
Figure 3.1: Graphical Representation of Recovery of a Dynamic Moving Load .......... 42
Figure 3.3: Dimensions of 3D Bent Cantilever Beam ............................................. 43
Figure 3.4: Solid Elements of 3D Bent Cantilever Beam ......................................... 43
Figure 3.5: ANSYS SOLID45 Element (Ref. [28]) .................................................. 44
Figure 3.6: ANSYS SHELL41 Element (Ref. [28]) .................................................. 44
Figure 3.7: Shell Elements of 3D Bent Cantilever Beam .......................................... 45
Figure 3.8: Loads applied on node number 561 ......................................................... 45
Figure 3.9: Optimum Gage Locations and Orientations for Bent Cantilever Beam ...... 46
Figure 3.10: Recovery of Sine Wave Load ................................................................. 46
Figure 3.11: Recovery of Square Wave Load ............................................................. 47
Figure 3.12: Recovery of Random Load ..................................................................... 47
Figure 3.13: Dynamic Moving Load Passing through a 3D Simply Supported Beam:
Orthogonal Loads ....................................................................................................... 48
Figure 3.14: Nodes along Load Path and Cardinal Degrees of Freedom on 3D Simply
Supported Beam (SSB): Orthogonal Loads ............................................................... 48
Figure 3.15: Optimum Locations of 3D SSB under Dynamic-Moving Load: Orthogonal
Loads ............................................................................................................................. 49
Figure 3.16: Recovery of Dynamic Moving Load Orthogonal Loads: Load 1 .......... 49
Figure 3.17: Recovery of Dynamic Moving Load Orthogonal Loads: Load 2 .......... 50
Figure 3.18: Load Passing Nodes and Cardinal Degrees of Freedom of Dynamic Moving
Load: Parallel Loads ..................................................................................................... 50
Figure 3.19: Optimum Locations of 3D SSB under Dynamic-Moving Load: Parallel
Loads ............................................................................................................................. 51
Figure 3.20: Recovery of Dynamic Moving Load: Parallel Loads: Load 1 .......... 51
Figure 3.21: Recovery of Dynamic Moving Load: Parallel Loads: Load 2 .......... 52
Figure 3.22: Recovery of Dynamic Moving Load: Parallel Loads with Noise and Uneven
Selection of Cardinal Degrees of Freedom: Load 1 ................................................ 52
Figure 4.1: 15 Degrees of Freedom Spring-Mass System ........................................... 82
Figure 4.2: Load Recovery-No Reduction with 2 Modes ............................................ 82
Figure 4.3: Load Recovery-No Reduction with 5 Modes................................. 83
Figure 4.4: Load Recovery-No Reduction with All Modes (15 Modes).............. 83
Figure 4.5: ANSYS Plot of Optimum Sensor Locations for Spring Mass System ...... 84
Figure 4.6: Dynamic Load Recovered using Guyan Reduction and CB Reduction...... 84
Figure 4.7: Dynamic Non-moving Load Recovered using Reduced Modal Model for
Spring Mass System ......................................................................................... 85
Figure 4.8: Finite Element Model of Cantilever Beam and Optimum Accelerometer
Locations (Top View) ....................................................................................... 85
Figure 4.9: Finite Element Model of Cantilever Beam and Optimum Accelerometer
Locations (Bottom View) ................................................................................ 86
Figure 4.10: Non-moving Dynamic Load Recovery using Reduced Modal Model for a
3D Cantilever Beam ......................................................................................... 86
Figure 4.11: Path of Dynamic Moving Load Acting on a 3D Cantilever Beam ............ 87
Figure 4.12: Dynamic Moving Load Recovery using Reduced Modal Model for a 3D
Cantilever Beam ............................................................................................... 87
LIST OF TABLES

Table 3.1: Material Property of Bent Cantilever Beam .............................................. 36
Table 3.2: Optimum Gage Location and Orientation for Bent Cantilever Beam .......... 36
Table 3.3: Selected Cardinal Degrees of Freedom: Orthogonal Loads ..................... 37
Table 3.4: Optimum Gage Locations for Dynamic Moving Load: Orthogonal Loads .... 38
Table 3.5: Selected Cardinal Degrees of Freedom: Parallel Loads .......................... 39
Table 3.6: Optimum Gage Locations for Dynamic Moving Load: Parallel Loads ...... 40
Table 3.7: Selected Cardinal Degrees of Freedom: Parallel Loads with Noise and Uneven Selection of Cardinal Degrees of Freedom .......................................................... 41
Table 4.1: Optimum Sensor Locations for 15 DOF Spring Mass Problem .................... 78
Table 4.2: Input Data for 3D Cantilever Beam for Model Order Reduction ................... 78
Table 4.3: Optimum Locations for a 3D Cantilever Beam Under Dynamic Moving Load ............................................................................................................................................. 79
Table 4.4: Input Data for time step 1 for 3D Cantilever Beam Under Dynamic Moving Load ............................................................................................................................................. 79
Table 4.5: Input Data for time step 5 for 3D Cantilever Beam Under Dynamic Moving Load ............................................................................................................................................. 80
Table 4.6: Input Data for time step 16 for 3D Cantilever Beam Under Dynamic Moving Load ............................................................................................................................................. 80
Table 4.7: Input Data for time step 18 for 3D Cantilever Beam Under Dynamic Moving Load ............................................................................................................................................. 81
Table 4.8: Input Data for time step 19 for 3D Cantilever Beam Under Dynamic Moving Load ............................................................................................................................................. 81
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Chapter 1 Introduction

1.1 Problem Statement

Loads which change in magnitude and position with respect to time are generally known as dynamic moving loads. Moving loads can induce dynamic stresses in bodies and structures, and cause them to vibrate intensively, especially at high velocities. The intensive vibration induced by the moving load can severely affect the integrity and safety of the structure. In order to avoid damage from dynamic moving loads, it is important to have precise information on the true value of the moving load during the design stage itself. This notion of identifying the true value of a dynamic moving load is known as “dynamic moving load identification.” Prior information about the true value and position of the moving load will facilitate a reliable and cost effective design of the structure and thereby a reduction in structural health monitoring cost can be achieved.

1.2 Limitation of Load Cells

Dynamic moving load may be identified by placing load transducers (load cells) between the load causing body and the structure. In many applications, this direct method of load identification using load cell is not recommended due to certain limitations. Firstly, the introduction of a load transducer can affect the dynamic characteristics of the system, and thereby the system response may differ from the original system. Therefore, the whole purpose of introducing a load transducer becomes futile. Secondly, it is not feasible to place load cells for certain types of loads, imposed on the structure such as aerodynamic loads, moving fluids, seismic loads etc. Lastly, the loads which are not in direct contact with the structure may not be accurately measured using load cells. A
temperature source moving at a distance is an example of such a situation. Fourthly, the inaccessibility of load transferring locations may restrict the user from introducing a load transducer, which makes the direct method less flexible.

1.3 Using Structure as a Load Transducer

The limitations of direct load measurement are overcome by the inverse load identification method. In the indirect method, the response from the structure under dynamic moving load such as strain, acceleration, bending moment etc can be utilized to identify the load imposed on the structure. The response from the structure can be tapped by transforming the structure itself into self transducer by placing sensors on it. This indirect method is more feasible than the direct method because it overcomes the limitations of the direct method mentioned above.

1.4 Limitations of Inverse Load Identification Method

The inverse problem described above appears to be straightforward and easily solvable. But this is misleading because the inverse problem tends to be highly ill-conditioned. Ill-conditioned matrices are generally incapable of solving a system of linear equations accurately and are prone to amplifying numerical errors. The condition number of these matrices will be high mainly because the columns within the matrices will be dependent upon each other. Ill-conditioning exists in inverse load identification problems because it is impossible to measure the response at all locations of a structure; instead random locations or manually selected locations are chosen to place sensors to retrieve the structural response. This limited response measured at finite number of locations causes the unavailability of full structural response and is mainly due to three reasons: (i)
financial constraints limiting the number of sensors that can be used, (ii) inaccessibility of loading locations to place sensors, and (iii) sensor influence on overall structural response.

Financial constraints are the most important reason behind the use of a limited number of sensors. Even after deciding the number of sensors to use, not all locations will be available to place sensors on a structure. The points of load application together with some other inaccessible locations will be unavailable for placement of sensors. Also, in some cases where the total mass of the system is comparatively close to the mass of the sensors, more number of sensors will affect the structural response of the system. This is because of the added mass of placed sensors. Moreover the use of maximum number of sensors is generally not accepted in any technical environment due to financial constraints. Due to all of the above discussed reasons, majority of important response information remain unmeasured and the load recovered from such insufficient structural response data will be prone to errors.

Ill-conditioning also occurs due to other reasons. A vehicle-bridge interaction system, which is a typical moving load problem, has several unaccounted structural and environmental parameters that can cause serious noise in structural response. This noise can cause ill-conditioning and will result in inaccurate solutions. Researchers have used several techniques to deal with ill-conditioning in moving load problems.

A detailed review of current methods in moving load identification and techniques to avoid ill-conditioning is provided in chapter 2. The existing methods to avoid ill-conditioning are computationally challenging and may not always lead a precise
solution. To overcome this issue, this thesis presents a new algorithm for dynamic moving load identification which is computationally easy to implement and provides accurate load estimates.

### 1.5 Practical Application

The methods developed in this thesis can contribute significantly to the industrial design applications which have non-stationary loading such as overhead cranes, mobile cranes, side-lifter cranes, tower cranes, elevators, escalators among other heavy lifting equipments. Apart from that, railroads and civil structures (bridges) can also be benefit from using the methods developed in this thesis.

The proposed methods can be implemented during the development phase of a product. After developing the prototype of a product, the engineers would take the prototype to a proving ground in order to understand the performance of the product under real working environment. The intention behind this testing process is to understand the loads and the behaviour of the product against loads under actual conditions. If the product failed during the testing process or if it presents an inefficient performance during the testing, engineers would be able to understand the direction, location and nature of the load during the test event. The loads estimated in the test environment are often significantly different than the theoretical loads used during the initial design stage. The estimated loads can be used to enhance the performance of the product under development. If the engineers failed to quantify the incoming loads, the testing process will continue and the whole product development cycle will become more expensive.
This situation can easily be handled by the methods developed in this thesis by turning the structures into load cells. Engineers can quantify the incoming load during the first testing process itself by employing the methods developed in this thesis and this will directly mitigate the expense related to the product development cycle. Also by designing a product for the real incoming loads, industries can develop products which are highly reliable and safe for public use.

1.6 Organization of the Material

Chapter 1 explains the importance of this thesis and answers several significant questions that come to a reader’s mind while reading this thesis. Also, it briefly overviews the challenges that might arise during the course of this thesis.

Chapter 2 details previous methods and algorithms presented in the field moving load identification along with a theoretical formulation. All algorithms have advantages and disadvantages of their own and the need for a new algorithm for moving load identification is clearly explained in this chapter.

Chapter 3 explains the optimization algorithm used in this thesis in detail and uses strain data to recover a dynamic moving load. Prior to the application of developed method to a moving load problem, the method is applied to a non-moving, quasi-static, bent cantilever beam problem and the results are discussed. Three specific problems of dynamic moving load recovery are discussed and the results are shown.

Chapter 4 uses accelerometer data to recover dynamic moving loads. Along with the algorithm used in chapter 3, chapter 4 uses the concept of model reduction in its recovery process. Basic model reduction techniques and an advanced technique used in
this thesis are explained in this chapter. The concepts are applied to a discrete as well as to a continuous system.

Chapter 5 provides concluding remarks for this thesis along with a scope of future work. Major results from previous chapters are discussed in this chapter. Some potential areas of future work are discussed based the results of fourth chapter.
Chapter 2 Literature Review

Research and development in identifying the load profile of a moving load began in the mid twentieth century. It was believed that the collapse of Stephenson’s bridge across river Dee at Chester in England in 1847 triggered the research of moving load problems. Recently, several techniques have been developed; each of them used for a specific application, each with unique advantages and disadvantages. All of these methods utilize measured structural response such as strain, displacement, acceleration, and bending moment, to estimate the acting load. The accuracy of the recovered load depends upon several factors including the algorithm in use, inclusion of static and dynamic properties of structure, and location of sensors on the structure. Some of the prominent techniques in the field of moving load identification are discussed in this chapter.

2.1 Moving Load Identification History

Theoretical formulations were first developed to solve identification of moving loads before practical approaches were developed. Krylov (1905) and Timoshenko (1922) solved the classical simply supported beam problem which is acted upon by a constant load moving at a uniform speed. Fryba (1972) compiles the theoretical response of a structure under various types of moving loads with significant structural parameters. Assuming beam behaviour governed by Bernoulli-Euler’s differential equation, for a beam with a constant cross section and a constant mass per unit length (Fig. (2.1)), the response to a moving load can be described as given below:
where $F(t)$ is the applied load, $E$ is the Young’s modulus, $J$ is the constant moment of inertia of the beam cross section, $x$ is the length coordinate from the left hand end of the beam, $t$ is the time coordinate with $t = 0$, the instant of the force arriving upon the beam. In Eq. (2.1), $v(x, t)$ is the beam deflection at the point $x$ and time $t$, $\mu$ is the constant mass per unit length, $\omega_b$ is circular frequency of the beam, $l$ is the span length, $c$ is the constant speed of load motion, and $\delta(x)$ is the Dirac delta function.

A bridge-vehicle interaction system is a typical moving load problem and the study by Fryba on vehicle axle loads significantly contributed towards the field of bridge design. Most of the traditional Bridge Weigh-in-Motion (B-WIM) systems could measure only static axle loads of vehicle, and they were very expensive and subject to bias. The bias can be reduced by weighing the vehicle for a longer period of time but this approach may not be a practical solution (Moses, 1979). Conventional B-WIM systems were replaced by techniques developed using the theoretical framework provided by Fryba. His contribution towards the field of moving load identification formed the basis for several time-domain and frequency-time domain moving load identification techniques.

### 2.2 Time Domain and Frequency-Time Domain Methods

Law et al. (1997) developed a time domain method from Eqn. (2.1) to estimate the dynamic moving vehicle load by utilizing bending moment and acceleration response of the structure. As shown in Eqn. (2.1), the vehicle axle load was modeled as a point load which moves at a constant velocity. One major disadvantage of this method is that the agreement between predicted loads and actual loads were not same throughout the
system. The accuracy was more at the centre of the beam than at the boundaries. Also, computational cost of this method is directly proportional to the number of axles on the vehicle.

Law et al. (1999) further developed a frequency-time domain method in which the recovered load is expressed as a mathematical representation of frequency. Fourier transformation was utilized in this technique. The number of structural modes involved in the computation is relevant in both time domain method and frequency-time domain method. As more normal modes are involved in the estimation, the accuracy of the estimated loads improves. However, as a practical matter, it is impossible to observe and measure the full modes of a structure. Consistency of both time domain and frequency-time domain methods varies with respect to the frequency range.

Yu and Chan (2003) utilized singular value decomposition to make frequency-time-domain method acceptable in all range of frequencies, and thus they were able to estimate the load history of moving load precisely. The time domain method is found to be better than the frequency-time domain method in solving ill-posed problems.

2.3 Finite Element Methods

The use of finite element method in recovery of moving loads started to gain wide acceptance about ten years ago. Law et al. (2004) replaced conventional computational approach by finite element method to furnish a methodology that identifies the dynamic moving load acting on a bridge deck. Hermitian cubic interpolation shape function was used to develop the response of each finite element of beam model in global coordinate system. The accuracy of the developed technique was tested against road roughness
factor to demonstrate the reliability of the developed technique. Also, the methodology is comparatively not sensitive to sampling frequency, vehicle velocity, noise level of measurement, and road roughness factor. Rowley (2007) modeled the bridge under investigation as a finite element model, in which each element was designed as an orthotropic rectangular plate. The inclusion of finite element method in the field of moving load identification has reduced theoretical complication in calculating structural response.

2.4 Other Algorithms in Moving Load Identification

Several techniques in the field of applied numerical methods were adapted in order to obtain accurate estimates of loads acting on the structure. Time-domain and frequency-time-domain methods have already been explained in the previous section. Some other algorithms which are significant in the area of load estimation are explained in this section.

Several researchers used Dynamic Programming, originally developed by Trujillo (1975), to estimate the dynamic load history. Dynamic programming is computationally expensive and the accuracy of load estimation depends upon the optimal values for unknown initial conditions. Noh and Lee (2012) applied coupled genetic algorithm to estimate the dynamic moving load parameters using finite element methods. The major advantage of genetic algorithm is that even for complex problem, a global solution can be achieved without providing considerable amount of data in advance.

O’Connor and Chan et al. (1988) developed Interpretive Method I, which utilizes the response from inertial and damping forces to compute the dynamic vehicle-bridge
interaction forces. The bridge deck is modeled as an assembly of lumped masses interconnected by massless elastic beam elements. Chan et al. (1999) later developed Interpretive Method II which is similar to Interpretive Method I, but utilized Euler’s equation of beams to model the bridge deck. Interpretive method I is independent of vibration modes but Interpretive method II needs at least the first three modes to accurately identify more than one moving load. Both Interpretive methods I and II are less accurate in identifying moving loads than time domain and frequency-time domain methods. All the above discussed methods can be ill-conditioned due to insufficient structural response measurements and regularization techniques are generally utilized to overcome this difficulty.

2.5 Regularization Techniques

While in many physical problems there are infinite number of locations where to place sensors on a structure, financial and physical constrains limit the measurement only from finite number of locations. This becomes the main cause of error because the number of data points in hand is very less compared to the total data points available. A use of regularization techniques helps to reduce the difference between theoretical and experimental measurements thereby minimizing the error. Law et al. (2001) proposed regularization techniques while using time domain method and frequency-time domain method. Most of the researchers used Tikhonov regularization method to reduce computational error. Identification of optimal regularization parameter is a major difficulty in performing Tikhonov regularization with time domain and frequency domain methods. The L curve method by Hansen (1992) and GCV methods by Zhu (2002) are
used by the researchers for the identification of optimal regularization parameter based on experience and prior information.

Pinkaew (2006) proposed regularization using updated static component technique in order to estimate the dynamic effects of vehicle axle loads. The technique extracts the static components of the identified axle loads and leaves only the associated dynamic components to be identified. Iterations are then performed to improve the accuracy of identified results. The errors associated with the least square estimate and conventional regularization methods are subdued by this technique. The technique promises higher accuracy at lower velocities but produces unacceptable results at higher velocities.

Singular value decomposition is also used as a solution approach in inverse load calculation. It can also be utilized to identify the rank (or column dependency) of a matrix. This dependency is very crucial in quantifying the ill-conditioning of a matrix. The approach by using singular value decomposition to avoid ill-conditioning can be computationally expensive. This can be mitigated by utilizing a partial singular value decomposition which is originally developed by Vogel et al. (1994). Rank Revealing QR factorization developed by Bazan et al. (1996) can also be utilized instead of full singular value decomposition.

As mentioned before, ill-conditioning in matrices occurs because of insufficient input data and errors in measured response. One potential approach to address this problem is to measure structural response at locations where it can produce the best results in load recovery.
2.6 Optimum Location of Sensors

Several researchers studied the ill-conditioning of inverse problems and developed several techniques to avoid that. An excellent study on ill-conditioning while solving an inverse problem can be seen in the study by Stevens (1987). Busby and Trujillo (1986) cast the inverse problem as a minimization problem in which the difference between predicted model response and measured structural response is minimized. Masroor and Zachary (1991) conducted statistical analysis to study the relation between load recovery and measured strain components at a finite number of locations. Their study shows that the location of sensor placement has significant effect on the accuracy of recovered load, and the placement of sensor at a low sensitivity location may result in ill-conditioning. They defined a statistical parameter which directly relates the variance of load estimates and sensor locations. The minimization of this parameter leads to the minimization of variance of load estimates. Masroor and Zachary expected the user to manually select the sensor locations while estimating the loads. The locations selected by the user may not be the combination of sensors which produces least variance in load estimates, and thus they might not be the optimal sensor locations.

Recently, Gupta (2013) further developed this technique to identify optimum strain gage locations to identify both static and non-moving dynamic loads, based on the D-optimal criteria developed by Mitchell (1974), Galil (1980) and Johnson et al. (1983). D-optimal (Determinant-optimal) methods utilize k-exchange algorithm to select optimum sensor locations. By using this algorithm for location selection, the best sensor locations are identified from all available locations. Along with the optimum location, the
optimum orientation of a strain gage is also identified using the D-optimal design method.

### 2.7 Summary

Indirect load identification is a typical example of an inverse problem and a small measurement error will result in an inaccurate load estimate. Majority of moving load identification techniques are indirect in nature and computationally expensive. Developments in the field of moving load identification clearly lack the domain of optimum sensor location identification. Most of the researchers placed their sensors based either on ease of installation or on certain prior knowledge about the problem. Due to this, the measured responses are prone to noise and may also be insufficient to recover the load accurately. It is assumed that, if the response data is measured at optimum sensor locations, the accuracy of recovering a moving load is higher and the utilization of regularization techniques can be eliminated. The solution approach proposed in this thesis is based on the above statement. The developed method and its application in dynamic moving load identification is explained in the following chapters.
Figure 2.1: A Simply Supported Beam Subjected to Moving Load $F(t)$
Chapter 3 Recovery of Quasi Static and Dynamic Moving Loads using Strain Measurements

Using strain gages for recovery of imposed loads has been tried for several years. In this chapter, the strain gage is used as a sensor and a new methodology to recover quasi-static non moving loads and dynamic moving loads is explained in detail. Identification of optimum sensor location for accurate estimation of imposed loads is the key process in this thesis and is explained in Secs. 3.1 to 3.4.

3.1 Theoretical Development

As mentioned in the previous chapter, by measuring structural response at optimum sensor locations, the computational cost of moving load recovery procedure can be reduced with an increase in accuracy. D-optimal design algorithms are used to identify a set of optimum locations and are explained later in this section. An important assumption made here is that the stress induced on the structure will never go beyond the elastic limit and the displacements are small enough so that the principle of superposition holds. Also, the material of the structure under investigation in this thesis is isotropic in nature.

Eqn. (3.1) has been developed for a structure under quasi-static load (Masroor and Zachary (1991)) and it outlines a linear relationship between applied quasi-static load and strain. It is assumed that the same equation can be approximated for a structure under dynamic moving load. For dynamic moving load acting on a structure, the linear relationship between strain and applied force can be written as shown below:
\[
\varepsilon(t) = [A][F(t)]
\]  (3.1)

where \(\varepsilon(t) \in \mathbb{R}^{g \times t}\) is a matrix of strain measurements at \(g\) locations, and \(t\) is the number of time-steps. Each column vector represents time-step and each row vector represents each strain gage location. \([A] \in \mathbb{R}^{g \times m}\) is called the system matrix, which holds the linear relation between applied load and measured strain. \([F(t)] \in \mathbb{R}^{m \times t}\) is the matrix that contains the information about dynamic moving load. Similar to the strain matrix, each column represents a separate time-step and each row vector contains the load applied at specific location. The superscript \(m\) stands for the number of load applied locations (cardinal degrees of freedom). The concept of cardinal degrees of freedom is explained later in this section.

By using left-pseudo inverse method of least square estimates, the applied load can be calculated inversely. Assuming, both system matrix and measured strain are known, Eqn. (3.2) shows the calculation of dynamic moving load:

\[
\begin{align*}
[F(t)] &= \left( [A]^{T}[A] \right)^{-1}[A]^{T}\varepsilon(t) \\
\end{align*}
\]  (3.2)

where \([F(t)]\) is a matrix of estimated load value and \(\varepsilon(t)\) is a matrix of measured strain. The variance of estimated force can be used here to determine the accuracy of estimation. Assuming the errors produced are distributed independently and identically, the variance-covariance matrix for load estimates can be calculated using Eqn. (3.3):

\[
\text{var}(F) = \sigma^2 \left( [A]^{T}A \right)^{-1}
\]  (3.3)
where \( \sigma \) and \( \sigma^2 \) are the standard deviation and variance of strain measurements respectively. The matrix \( [A^T A]^{-1} \) is called the sensitivity of \([A]\), and the minimization of this sensitivity matrix will lead to increased precision in load recovery. This notion forms the basis for D-optimal design algorithm. The minimum of sensitivity matrix is formed by a optimum combination of number of strain gages, the angular orientation of strain gages and most importantly the location of strain gages. To minimize the sensitivity of \([A]\), computational techniques are needed such that the optimum combination of the above mentioned factors can be identified effectively. The sensitivity of \([A]\) can be reduced by maximizing the determinant of its sensitivity matrix \([A^T A]\).

Certain assumptions must hold before using the method presented in this section. Along with the direction of load in Cartesian coordinates, the path of load is known to the user as prior information. Since the moving load passes from one node to another in the finite element model at each time step, it is advisable to identify the loads only at some specific locations. The principal notion is that the full space or total structure is divided into a finite number of equally separated sub-spaces and the load is recovered at these sub-spaces separately for different time steps. Each sub-space is represented by at least one degree of freedom in which the load is passing, and is called cardinal degree of freedom. Hence each moving load for a full structure with \( s \) sub-spaces will have at least \( s \) cardinal degrees of freedom.

Even though the load will pass through the full path of action, the points of interest are cardinal degrees of freedom only. Treating each cardinal degrees of freedom as a separate load case and utilizing the technique which is going to be explained in the
next section, it is possible to recover the dynamic moving load. Since the load is moving, it is necessary to recover the full history of the moving load. By utilizing interpolation techniques, the method is capable of recovering the full profile of a moving load as shown in Fig. 3.1.

As mentioned before, it may be helpful to plot the load history of recovered dynamic moving load. Since the loads are recovered at the cardinal points only, interpolation methods can be used to estimate the loads at rest of the locations. For static-moving loads, linear interpolation may be sufficient but for dynamic-moving loads, interpolation techniques of higher order must be used. ‘Spline’ technique in MATLAB programming environment provides a better solution for problems with harmonic load. Spline technique employs a third order cubic interpolation technique to compute the load history at discrete point intervals (de Boor, 1978).

Prior to the application of interpolation techniques it is essential to determine the loads at cardinal degrees of freedom. In order to estimate the load at these locations, a set of optimum sensor locations needs to be identified. Thus the initial step in this method becomes the identification of optimum sensor locations. By following a procedure systematically, one can identify the optimum location and optimum orientation angle for \( n \) number of strain gages. The procedure is as follows:

- Generation of the candidate set,
- Determination of the number of strain gages to be used, and
- Determination of D-optimal design.
3.2 Generation of the Candidate Set

Using the finite element method, the full structure can be meshed into numerous finite elements. The meshing should be done such that each element size is similar to an available strain gage size. Initially all elements have equal potential to become an optimum location. Based on certain criteria, the designer needs to identify the possible locations where the strain gages can be mounted. Firstly, all inaccessible locations are eliminated from the total because there are certain locations where it is impossible to mount strain gages and record measurements. Secondly, assuming the load application locations are known, it is sensible to eliminate those locations where the loads are applied. Damage to the strain gage and related equipment can thus be avoided. The remaining sets of locations combined with its angular orientations are called a candidate set for optimum sensor placement. The following section will detail the procedure to construct $[A]_{\text{candidate}}$ matrix.

As mentioned earlier, the optimum sensor locations $[A]_{\text{optimum}} \in \mathbb{R}^{g \times m}$ are a set of strain measurement location and orientations for all possible gages that provide the most precise estimates of the applied loads. $[A]_{\text{optimum}}$ is a subset of the candidate set $[A]_{\text{candidate}}$. The number of rows $g$ of matrix $[A]_{\text{optimum}}$ represents the number of required strain gages to be mounted on the structure and the number of columns $m$ represents the number of locations at which moving load will be recovered or number of cardinal degrees of freedom.
A finite element model of any structure is three dimensional in nature and thus each element after meshing will also be in three dimension. Considering practical aspects, it is important to retrieve surface strains from these elements because in reality it is only possible to measure strains at the surface. This problem can be solved by two methods: (i) develop a 2D model of the structure and mesh it using shell elements, (ii) even though the structure is modeled in three dimension, one can coat the surface with shell elements to retrieve strain data for these shell elements alone. The second method has better acceptability because model conversion is not viable in all cases and a 2D model may not give results as accurate as a 3D model.

It is important to know the reason behind the selection of elements instead of nodes for obtaining strain data. The nodal strain is the average of the adjacent elements strain and the error associated will also be averaged. This averaging can be avoided by directly utilizing the element strain data. Also, the orientation of the strain gage is calculated with respect to the element coordinate system located at the centroid of the element. But, if the nodal data is considered, orientation measurement will become more complex since element associated with a particular node has its own orientation. Another straight and simple answer is related to the physical considerations. Considering all of these reasons, it is recommended to use elemental strain data instead of nodal data for strain measurements.

As mentioned earlier, in moving load problem, it is sensible to identify the loads only at some specific locations or cardinal degrees of freedom. Hence, the primary step of this algorithm is to select some cardinal degrees of freedom where the user needs to identify the magnitude of a dynamic moving load. Then using a finite element software, a
moving load of unit magnitude is passed through the same path as original load. After this, strain tensors are obtained for all the candidate locations, only when the load is at the pre-selected cardinal degrees of freedom. For different cardinal degrees of freedom, the strain tensor for all the candidate sensor locations is saved separately.

It has been noticed that the strain tensor will vary for a change in angular orientation of the strain gage. By using rotation matrices, it is possible to rotate the strain tensor from one coordinate system to another, and thus the strain tensor at another orientation is obtained. The strain tensors can be transformed from the \( xyz \) coordinate to \( x'y'z' \) coordinate by using the following equation.

\[
[e]_{x'y'z'} = [T] [e]_{xyz} [T]^T
\]  

(3.4)

where \([T]\) is the transformation matrix, also called the rotation matrix, that contains the direction cosines for the \( x'y'z' \) coordinate system with respect to \( xyz \) coordinate system. For this operation, one coordinate axis needs to be fixed and the other two can rotate. The shell element’s local coordinate system used in this procedure has its \( z \) direction orthogonal to the plane of the element and hence the strain transformation involves rotation about the \( z \)-axis. The transformation matrix is shown in Eqn. (3.5).

\[
[T] = \begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  

(3.5)

The third row and column doesn’t have any rotation terms, the numerical values at that particular direction \( z \) will be preserved. For each element, 18 possible directions have been chosen in which strain gage can be oriented, from 0 to 170 degree with an increment of 10 degree. Strain gages are mostly sensitive in their axial direction, and thus
the candidate set will consists only of $x'^{x'}$ direction strain components after rotation and all other estimates will be eliminated. Compiling together, each column of the final candidate set of matrix $[A]_{\text{candidate}}$ will represent each cardinal degrees of freedom and each column will have strain of all the candidate locations in all 18 directions.

### 3.3 Determination of the Number of Strain Gages

The accuracy of recovered load will improve by including more strain gages. Adding more gages offsets the cost effectiveness of the proposed procedure. Since the algorithm uses left pseudo inverse to recover the dynamic moving load as shown in Eqn. (3.2), the general condition is that the number of gages should be greater than or equal to the number of loads to be identified. In dynamic moving load identification, the number of loads is referred to the number of cardinal degrees of freedom. Hence, the number of strain gages must be greater than the number of cardinal degrees of freedom.

### 3.4 Determination of the D-Optimal Design

The identification of optimum locations is a process of identifying a set of $g$ gage locations along with their orientations that together provides the least variance in load estimate. Based on the required number of optimum gages, an algorithm should select the optimum $g$ gages from $[A]_{\text{candidate}}$ which satisfy the condition stated above. The notion of using trial and error method is extremely time consuming and no guarantee is provided for a correct solution. For instance, let matrix $[A] \in R^{g \times m}$ be a random set of $g$ strain gages which is a subset of $[A]_{\text{candidate}}$. 
Several statisticians (Stevens (1987), Masroor and Zachary (1991)) have done research to improve the algorithm, which reduces the variance of a matrix $[A]$. A suitable approach to determine $[A]_{optimum} \in R^{g \times m}$ is to find $[A]$, which has the maximum value for the determinant of $[A]^T [A]$. The design that maximizes $| [A]^T [A] |$ is called D-optimal design. Mitchell (1974) presented a D-optimal algorithm, where D denotes determinant of the matrix. D-optimal designs guarantee low variance among parameters and low correlation between parameters. The major difficulty is the existence of local maxima, which can only be handled by an efficient algorithm.

D-optimal designs are usually constructed by algorithms that sequentially add and delete points from a potential design by using a candidate set of points spaced over the region of interest. Galil (1980) and Johnson et al. (1983) developed algorithms which generate with D-optimal designs, using sequential exchange algorithm and k-exchange algorithm respectively. The general outline of these algorithms is as follows.

The objective of the algorithm is to determine a set of gages that provide the least variance, which means $g$-rows in $[A]_{optimum}$ matrix must have the maximum possible prediction variance. To select $g$-rows, augmentation and reduction of $[A]$ matrix is required. With optimal augmentation, the candidate gage with maximum prediction variance is added as a row to the matrix $[A]$. Similarly, optimal reduction of the augmented design is achieved by eliminating the candidate gage of the matrix having minimum prediction variance. This procedure of addition and deletion of candidate points in a sequential manner continues until no further improvement can be made in the objective function.
Explaining the sequential exchange algorithm in more detail, the first step is to develop a matrix, \( [A] \) which has randomly selected \( g \) strain gages as rows and the number of applied loads \( m \) as columns. If \( n \) candidate points are there in the candidate matrix, the remaining \((n-g)\) gages are still in the candidate set. Out of the remaining \((n-g)\) gages in the candidate set, a candidate point is then selected and the corresponding row is augmented to the matrix \([A]\) to form \([A]_+\) such as the determinant of \([A]_+^T[A]_+\) is maximum. After this, out of the \( g+1 \) rows in matrix \([A]_+\), a row is deleted to construct a matrix \([A]_-\) such that the determinant of \([A]_-^T[A]_-\) is maximum. This process of augmenting and deleting rows continues until there is no further improvement in the answer for the determinant of \([A]_-^T[A]_-\). The final D-optimal design, \([A]_{optimum}\) is the \([A]\) matrix, which will provide the least variance for \( g \) gages. It is very expensive to compute the determinant at each step by using \( M = |[A]_+^T[A]_+| \). An alternate formula (Gupta, 2013) for computing the determinant \(|[A]_+^T[A]_+|\) from \( M \) when the row \( y^T \) is augmented to the matrix \([A]_+\) is:

\[
|M_+| = |M| \left( 1 + y^T M^{-1} y \right) \tag{3.6}
\]

where \([+]\) denotes addition and is replaced by subtraction in the case of deleting a row \( y^T \) from \([A]_+\). In order to be able to use Eqn. (3.6), \( M^{-1} \) can be maintained and updated as the row \( y^T \) is augmented to the matrix \([A]_+\) by:
\[ |M|-1 = |M|-1 [-] (M^{-1}y)(M^{-1}y)^T \]

\[ (1 + y^T M^{-1}y) \]

(3.7)

where \([-\]

\) denotes subtraction and is replaced by addition in the case of deleting a row \(y^T\) from \([A]_+\). Once the optimum strain gage locations and orientations, \([A]_{optimum}\) are known, place the strain gages at these optimum locations before the application of the unknown loads. Strains are then measured at these optimum locations, \([\varepsilon(t)]_{optimum} \in R^{n \times t}\), only when the load is at the cardinal points. This forms the strain tensor for dynamic moving load and by using Eqn. (3.8), the unknown moving loads \([F(t)]\) can be estimated.

\[ [F(t)]_{estimate} = \left( [A]_{optimum}^T [A]_{optimum} \right)^{-1} [A]_{optimum}^T [\varepsilon(t)]_{optimum} \]

(3.8)

A flowchart of the above described sequential programming algorithm is provided in Fig. (3.2). This algorithm was implemented in MATLAB. The finite element models of the system under consideration were constructed in ANSYS.

3.5 Numerical Examples

The dynamic load estimation technique discussed above is illustrated with the help of four examples. The first example is recovery of a non-moving quasi-static loads on a bent cantilever beam. The remaining three examples show the recovery of dynamic moving load on a simply supported beam. All four examples illustrate that the proposed procedure can be used to estimate the imposed loads fairly accurately.
3.5.1 Quasi-Static Load Recovery

This section will explain the recovery of three quasi static loads acting on a 3-dimensional bent cantilever beam, based on the concepts explained in Sec. 3.1. Before designing an experiment for recovery of a moving load, it is reasonable to test the algorithm on a non-moving load. For this experiment, since the load is non-moving, no cardinal degrees of freedom are required.

Quasi-static loads work similar to static loads, and it can be differentiated based on the time-step. In static load case, the load is applied only at one particular time-step and therefore structural response is studied only for that particular time-step. In quasi-static load case, the loads are acting at different time-steps and the responses need to be treated separately. Since the imposed loads are independent of the load history, we can solve the problem separately at each time-step, by treating the input force as separate static loads at distinct times. Thus the objective of this example is to test the capability of the algorithm to identify optimum strain gage locations based on strain data and recover the static loads in time domain.

In order to perform the experiment, ANSYS-APDL software is employed to design the cantilever beam and then to extract the strain data. The material used was steel with material properties listed in Table. 3.1.

The thickness of the beam, 0.03 m is constant throughout the length of 1.83 m. The beam height is 0.45 m, and is considered isotropic in nature, i.e. the material has uniform properties in all the three coordinate directions. The beam dimensions are shown in Fig. (3.3). The structure shown in Fig. (3.4) is map meshed with SOLID45 element in
ANSYS where each element has eight nodes (see Fig. (3.5)). The total number of elements after meshing is 600 and the beam has 1368 nodes. Each node has three degrees of freedom and the design has a total of 4104 degrees of freedom. Concatenation is performed in order to do map meshing on this structure.

Practically, it is not possible to place strain gages at all locations of a structure. In this design, only the top and side faces are considered to be the potential gage placement locations. In that case, it is necessary to mesh those surfaces with a shell element such that surface strains can be retrieved accurately. This process of meshing a shell element on the top of a solid element to extract surface responses is called coating. The shell elements were given near zero values for the modulus of elasticity and the thickness so that they do not change the elastic characteristics of the problem. SHELL41 element of ANSYS is used for this purpose (see Fig. (3.6)). Since SHELL41 has only 4 nodes per element, it has better compatibility with SOLID45 than any other shell elements. The number of elements thus becomes 1544 and the nodes remain the same. The bent beam with shell coated elements is shown in Fig. (3.7).

For this example, the location of load application is assumed to be known as prior information. The loads to be identified are applied at node 561 in three different directions, namely x, y and z. As discussed in Sec. 3.2, the strain data is generated by applying unit loads, one after the other, in all three of the above mentioned directions. Strain tensors were obtained at the centroid of each shell element for each load case separately. Remember in Sec. 3.2, the details are explained in line with moving load but in this problem since the loads are non-moving, unit loads are applied at one particular location (node 561) where the loads are acting.
To generate the candidate set, strain tensors were transformed using Eqn. (3.5) for angular orientations ranging from 0 to 170 degree with an increment of 10 degree. In this problem, the total number of load cases is 3 because three loads need to be identified at one time-step. Hence it is necessary to select at least 3 strain gages for the reason explained in Sec. 3.4. A total of 4 gages are used in this example to estimate imposed loads. Optimum gage locations and orientations were identified using the algorithm explained in Sec. 3.4. Optimum gage locations and angular orientations are shown in Fig. (3.9) and listed in Table 3.2.

Next, three time varying quasi static loads were applied at the same time at node number 561. As mentioned before, each load will be acting in a different direction. The loads applied are as follows:

- Sine wave of amplitude 1.0 and frequency 2.0 in x direction
- Square wave of amplitude 3.0 and frequency 2.0 in y direction
- Random load in the limit (0,1) in z direction

Strain tensors were extracted at optimum gages and by using transformation matrix, strain tensors at optimum orientation was calculated. For each time-step, the computation was performed as a separate static analysis. Applied loads were recovered exactly in time-domain using Eqn. (3.8) and are depicted in Figs. (3.10) to Fig. (3.12).

3.5.2 Dynamic Moving Load Recovery: Orthogonal Loads

In this example, the task is to recover dynamic moving loads passing through the structure shown in Fig. (3.13). The structure under investigation is a simply supported
beam of length 9.0 m, width 2.4 m and thickness 0.3 m. The material used is steel which has the Young’s modulus $E = 209$ GPa and Poisson’s ratio equal to 0.29.

The beam is meshed with SOLID45 elements, and has 240 elements. The beam is coated with SHELL41 elements so that surface strain information can be extracted. After surface coating, the number of elements becomes 796. The total number of nodes and degrees of freedom of the beam are 558 and 1674 respectively.

The dynamic moving load is programmed using ANSYS APDL software. Using transient solution phase for each time-step, the load is designed to move from one node to another. In this problem, the load is moving from node number 359 to 562 with an increment of 7 nodes. There are a total of 31 nodes along the beam length. Avoiding the nodes at the boundary, 29 nodes remain in the path. The load will pass through all these 29 nodes, resulting in 29 time-steps for this problem.

Two loads are under investigation for this example. Both act at the same time at the same node, but in orthogonal directions, and move at a constant velocity of 3m/s. If two or more loads were acting in a particular direction at the same time-step, then the load recovered at that time-step will be the sum of all applied loads.

As mentioned earlier, the objective is to recover the load at certain time-steps or certain cardinal degrees of freedom and then use interpolation methods to recover the full load history. The selection of cardinal degrees of freedom becomes the first step in this procedure. If the selected cardinal degrees of freedom are spaced equally, the interpolation becomes easier. Also, test for unevenly spaced cardinal degrees of freedom was also done and is discussed later. For the current example, there are 29 time-steps, and
the number of subspaces selected is 5. The cardinal degrees of freedom selected are given in Table 3.3 and shown in Fig. (3.14). These are the degrees of freedom which are kept as a reference.

The solution procedure focuses on these particular cardinal degrees of freedom. As described earlier, the input to the D-optimal algorithm is generated by moving unit loads at the same velocity of the actual load through the load path by using any finite element software. Since both loads are acting in different directions, separate unit loads are moved for both cases. In total, there are 10 load cases, 2 loads moving through 5 cardinal degrees of freedom. Strain tensors will then be measured for all candidate locations only when the unit load is at these cardinal degrees of freedom. This strain data is treated as the input for D-optimal algorithm.

By using Eqn. (3.5), the strain tensor at different directions was estimated for each possible gage location. This data will form the \( [A]_{candidate} \) matrix. Since the number of loads to be estimated was 10, the number of strain gages to be used must be \( \geq 10 \); therefore, for this problem 10 gages were used. The D-optimality criterion, as discussed earlier, is used to find the optimum gage locations and angular orientations for the given number of strain gages to form \( [A]_{optimum} \). The optimum gage locations and angular orientations are listed in Table 3.4, and the elements corresponding to the optimum gage locations are depicted in Fig. (3.15).

Next, \( \left[ \varepsilon(t) \right]_{optimum} \) is obtained by placing strain gages at these optimum locations in optimum orientations. Strain tensors were extracted from all these gages only when the
actual load reaches the predefined cardinal degrees of freedom. To recover the load, Eqn. (3.8) is then used. For each time-step, based on the number of load cases, the applied loads can be recovered. It is noticed that the load recovered is approximately zero at all other locations other than the location at which the load is actually acting for a particular time-step. Until now, loads at only 10 cardinal degrees of freedom were estimated. Using higher order interpolation techniques in MATLAB, a complete load history of the dynamic moving load can be estimated precisely. Both applied loads are recovered accurately as shown in Fig. (3.16) and Fig. (3.17).

3.5.3 Dynamic Moving Load Recovery: Parallel Loads

In the previous example, both loads were passing through the same nodes. In this example, the two loads are passing through different nodes but parallel to each other at a constant velocity of 3m/s. This example is representative of loads acting on the axle of a vehicle. Axle loads are approximated as point loads. The left axle load, load 1, will move through the left side and the right axle load, load 2, will move through the right side of the beam. Cardinal degrees of freedom are selected as listed in Table 3.5. Since there are two loads passing through different nodes, separate nodes are selected for cardinal degrees of freedom rather than separate degrees of freedom of same node as before. The length of the axle is 1.8 m and load 1 is acting from node number 356 to 552 with an increment of 7 for 29 time-steps and load 2 is acting from node number 362 to 558 with an increment of 7 for the same time-step. The cardinal degrees of freedom are listed in Table 3.5 and are depicted in Fig. (3.18) along with the load path.

After deciding the cardinal degrees of freedom, by following the same method described in orthogonal moving load recovery, the optimum strain gage locations were
identified. Since there are 10 load cases, 10 optimum gage locations were demanded. Optimum gage locations are depicted in Fig. (3.19). All optimum orientations are in relation with the x coordinate axis. Table 3.6 lists the optimum gage locations along with their orientations.

Placing gages at these optimum locations and extracting the strain tensors, dynamic moving load acting on the axle can be recovered using Eqn. (3.8). The recovered loads are shown in Fig. (3.20) and Fig. (3.21). It can be seen that the applied loads are recovered quite accurately.

3.5.4 Recovery of Noisy Moving Loads with Uneven Cardinal Degrees of Freedom

Even though for the last two examples, the cardinal degrees of freedom were evenly spaced (taken at equal intervals of time), practical limitations may deny the flexibility of selecting evenly spaced cardinal degrees of freedom. Considering this fact, a test was performed to evaluate the reliability of the proposed method for unevenly spaced cardinal degrees of freedom. For this, unevenly separated subspaces are selected, which results in the selection of cardinal points that are not equally spaced.

Also, as mentioned before, the strain data is prone to experimental noise and this might cause errors in load prediction. In order to validate the algorithm in the presence of noise in response measurements, a 5% randomly generated noise signal was added to the strain data before loads were recovered. The structure under investigation and path of action of the loads remains the same as in the third example.
As per the algorithm, it is necessary to select the cardinal degrees of freedom. For this particular problem, selection of cardinal degrees of freedom was made uneven as mentioned above. Table 3.7 list the cardinal degrees of freedom selected for this problem. Since the load is moving at a constant velocity, the time gap between each cardinal degrees of freedom is found to be uneven. Also through this problem, the assumption of loads not moving at a constant velocity is also being tested. If the load is moving at a non uniform velocity, it might reach the cardinal degrees of freedom which are equally spaced at unequal time gaps. Thus an additional test for non uniform velocity is not required. Loads recovered at cardinal degrees of freedom were used to develop the moving load history by using higher order interpolation. Recovery of dynamic moving loads for this problem is depicted in Fig. (3.22) and Fig. (3.23). Once again, it can be seen that the applied loads are determined accurately even when noise is present in strain measurements.

**3.6 Summary**

A new computational method is presented to recover dynamic moving load(s) using strain measurements at optimum strain gage locations. The chapter explains the concept of cardinal degrees of freedom and considering each cardinal degrees of freedom as separate load cases. As more strain gages are used in load recovery, the accuracy of recovered load improves. It is seen that the accuracy of recovered moving loads is quite high even with limited number of gages. Dynamic moving loads affect the integrity of a structure and are in general difficult to recover, but at the cost of more cardinal degrees of freedom, even this task can be achieved by implementing the procedure proposed in this chapter.
The developed method produces similar quality results even when the moving loads move at a non-uniform velocity which assures that the accuracy of the proposed method is independent of the velocity of the moving load. Since measurement noise within measured strain data is natural in a real environment, the method is tested in the presence of random noise present in the strain data. Even in the presence of noise, the load estimates are obtained with a high degree of accuracy which proves the reliability of the developed method.
Table 3.1: Material Property of Bent Cantilever Beam

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value (SI Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>201 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.29</td>
</tr>
<tr>
<td>Density</td>
<td>7635 kg/m³</td>
</tr>
</tbody>
</table>

Table 3.2: Optimum Gage Location and Orientation for Bent Cantilever Beam

<table>
<thead>
<tr>
<th>Gage Number</th>
<th>Optimum Gage Location (Element Number)</th>
<th>Orientation (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>601</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>780</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1063</td>
<td>170</td>
</tr>
<tr>
<td>4</td>
<td>1214</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 3.3: Selected Cardinal Degrees of Freedom: Orthogonal Loads

<table>
<thead>
<tr>
<th>No.</th>
<th>Time-step in Seconds</th>
<th>Cardinal DoF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(For Load 1)</td>
</tr>
<tr>
<td>1</td>
<td>0.10</td>
<td>359 – y dof</td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
<td>408 – y dof</td>
</tr>
<tr>
<td>3</td>
<td>1.50</td>
<td>457 – y dof</td>
</tr>
<tr>
<td>4</td>
<td>2.20</td>
<td>506 – y dof</td>
</tr>
<tr>
<td>5</td>
<td>2.90</td>
<td>555 – y dof</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(For Load 2)</td>
</tr>
<tr>
<td>6</td>
<td>0.10</td>
<td>359 – x dof</td>
</tr>
<tr>
<td>7</td>
<td>0.80</td>
<td>408 – x dof</td>
</tr>
<tr>
<td>8</td>
<td>1.50</td>
<td>457 – x dof</td>
</tr>
<tr>
<td>9</td>
<td>2.20</td>
<td>506 – x dof</td>
</tr>
<tr>
<td>10</td>
<td>2.90</td>
<td>555 – x dof</td>
</tr>
</tbody>
</table>
Table 3.4: Optimum Gage Locations for Dynamic Moving Load: Orthogonal Loads

<table>
<thead>
<tr>
<th>Gage Number</th>
<th>Optimum Gage Location (Element Number)</th>
<th>Orientation (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>391</td>
<td>160</td>
</tr>
<tr>
<td>2</td>
<td>449</td>
<td>50</td>
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<tr>
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</tr>
<tr>
<td>5</td>
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</tr>
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</tr>
<tr>
<td>10</td>
<td>690</td>
<td>160</td>
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</table>
**Table 3.5:** Selected Cardinal Degrees of Freedom: Parallel Loads

<table>
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<th>Cardinal DoF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( For Load 1)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.10</td>
<td>356 – y dof</td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
<td>405 – y dof</td>
</tr>
<tr>
<td>3</td>
<td>1.50</td>
<td>454– y dof</td>
</tr>
<tr>
<td>4</td>
<td>2.20</td>
<td>503– y dof</td>
</tr>
<tr>
<td>5</td>
<td>2.90</td>
<td>552– y dof</td>
</tr>
<tr>
<td></td>
<td>( For Load 2)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.10</td>
<td>362– y dof</td>
</tr>
<tr>
<td>7</td>
<td>0.80</td>
<td>411– y dof</td>
</tr>
<tr>
<td>8</td>
<td>1.50</td>
<td>460– y dof</td>
</tr>
<tr>
<td>9</td>
<td>2.20</td>
<td>509– y dof</td>
</tr>
<tr>
<td>10</td>
<td>2.90</td>
<td>558– y dof</td>
</tr>
</tbody>
</table>
Table 3.6: Optimum Gage Locations for Dynamic Moving Load: Parallel Loads

<table>
<thead>
<tr>
<th>Gage Number</th>
<th>Optimum Gage Location (Element Number)</th>
<th>Orientation (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>302</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>308</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>315</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>329</td>
<td>170</td>
</tr>
<tr>
<td>5</td>
<td>512</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>518</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>525</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>539</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>570</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>780</td>
<td>0</td>
</tr>
</tbody>
</table>
### Table 3.7: Selected Cardinal Degrees of Freedom: Parallel Loads with Noise and Uneven Selection of Cardinal Degrees of Freedom

<table>
<thead>
<tr>
<th>No.</th>
<th>Time-Step in Seconds</th>
<th>Cardinal DoF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(For Load 1)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.10</td>
<td>356– $y$ dof</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>384– $y$ dof</td>
</tr>
<tr>
<td>3</td>
<td>1.20</td>
<td>433– $y$ dof</td>
</tr>
<tr>
<td>4</td>
<td>1.70</td>
<td>468– $y$ dof</td>
</tr>
<tr>
<td>5</td>
<td>2.70</td>
<td>538– $y$ dof</td>
</tr>
<tr>
<td></td>
<td>(For Load 2)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.10</td>
<td>362– $y$ dof</td>
</tr>
<tr>
<td>7</td>
<td>0.50</td>
<td>390– $y$ dof</td>
</tr>
<tr>
<td>8</td>
<td>1.20</td>
<td>439– $y$ dof</td>
</tr>
<tr>
<td>9</td>
<td>1.70</td>
<td>474– $y$ dof</td>
</tr>
<tr>
<td>10</td>
<td>2.70</td>
<td>544– $y$ dof</td>
</tr>
</tbody>
</table>
**Figure 3.1:** Graphical Representation of Recovery of a Dynamic Moving Load

**Figure 3.2:** Flowchart of the Sequential Exchange Algorithm (Gupta, 2013)
Figure 3.3: Dimensions of 3D Bent Cantilever Beam

Figure 3.4: Solid Elements of 3D Bent Cantilever Beam
Figure 3.5: ANSYS SOLID45 Element (Ref. [28])

Figure 3.6: ANSYS SHELL41 Element (Ref. [28])
Figure 3.7: Shell Elements of 3D Bent Cantilever Beam

Figure 3.8: Loads applied on node number 561
Figure 3.9: Optimum Gage Locations and Orientations for Bent Cantilever Beam

Figure 3.10: Recovery of Sine Wave Load
Figure 3.11: Recovery of Square Wave Load

Figure 3.12: Recovery of Random Load
Figure 3.13: Dynamic Moving Load Passing through a 3D Simply Supported Beam: Orthogonal Loads

Figure 3.14: Nodes along Load Path and Cardinal Degrees of Freedom on 3D Simply Supported Beam (SSB): Orthogonal Loads
Figure 3.15: Optimum Locations of 3D SSB under Dynamic-Moving Load: Orthogonal Loads

Figure 3.16: Recovery of Dynamic Moving Load Orthogonal Loads: Load 1
Figure 3.17: Recovery of Dynamic Moving Load Orthogonal Loads: Load 2

Figure 3.18: Load Passing Nodes and Cardinal Degrees of Freedom of Dynamic Moving Load: Parallel Loads
Figure 3.19: Optimum Locations of 3D SSB under Dynamic-Moving Load: Parallel Loads

Figure 3.20: Recovery of Dynamic Moving Load: Parallel Loads: Load 1
Figure 3.21: Recovery of Dynamic Moving Load: Parallel Loads: Load 2

Figure 3.22: Recovery of Dynamic Moving Load: Parallel Loads with Noise and Uneven Selection of Cardinal Degrees of Freedom: Load 1
Figure 3.23: Recovery of Dynamic Moving Load: Parallel Loads with Noise and Uneven Selection of Cardinal Degrees of Freedom: Load 2
Chapter 4 Dynamic Moving Load Recovery Using Acceleration Measurements and Model Reduction

This chapter details another methodology to recover dynamic moving loads using acceleration response of the structure. The acceleration response can be measured by using accelerometers. The basic theories and equations used in this chapter are explained in Secs. 4.1 and 4.2. Throughout this chapter, an assumption is made that the structure under investigation is linear in nature.

4.1 Matrix Representation of Structural Dynamics of a System

There are many ways in which the response of a structure under load(s) can be represented. In order to represent the linear relationship between signals and parameters, mathematicians use linear algebra. This development, which is otherwise called the matrix representation, was adapted by scientists and engineers to perform efficient computations in structural analysis. Matrix representations and its inverse calculations are used in this chapter extensively. The following sections will explain some of the representations used in this chapter.

4.1.1 Physical Coordinate Representation

For simple systems, the representation of a structure under dynamic load can be expressed using partial differential equation. If the system is complex in nature and and/or has complex boundary conditions, finite element method can be used instead of analytical approaches. In the finite element method, the structure is divided into finite elements of definite shape and size.
The linear second order ordinary differential equations of a structure can be represented in physical coordinates by the following equation

\[ [M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \tag{4.1} \]

where \([M], [C],\) and \([K]\) are the mass, damping and stiffness matrices of the structure respectively. These system matrices can be formed using finite element equations for simple structure or by using finite element software for complex structures. The structural displacement vector is \(x(t)\) and the force vector is \(F(t)\). The acceleration response is \(\ddot{x}(t)\). Note that vectors are specified using curly bracket, \{\} and matrices are specified using square bracket, \[\].

4.1.2 Modal Coordinate Representation

It is possible to represent the structure in modal coordinates, also called the generalized coordinates. A harmonic solution for Eqn. (4.1) by using Eqn. (4.2) is assumed for this representation:

\[ \{x(t)\} = \{A\} \sin(\omega t) \tag{4.2} \]

where \(\{A\}\) is a vector of constants, \(\omega\) is the natural frequency in (rad/sec) and \(t\) is time in seconds. To solve the problem in modal coordinates, it is necessary to solve the eigenvalue problem. The eigenvalue and eigenvector solution of Eqn. (4.3), which formed from Eqn. (4.1) gives the natural frequency, \(\omega\) and normal mode shapes, \([\Phi]\) of the system respectively.

\[ ([\omega^2][M] - [K][\Phi])[\Phi] = [0] \tag{4.3} \]
Using the expansion theorem, the physical coordinates can be represented using a linear combination of normal modes as shown in Eqn. (4.4), and the relation between generalized coordinate and physical coordinate can be represented by using Eqn. (4.5).

\[
x(t) = q_1(t)\Phi^1 + q_2(t)\Phi^2 + \ldots + q_n(t)\Phi^n
\]  
(4.4)

\[
[x(t)]_{nxt} = [\Phi]_{nxt}[q(t)]_{nxt}
\]  
(4.5)

where \( n \) is the total degrees of freedom of the structure and \( q(t) \) is called the mode participation factor which represents the involvement of each mode shape in the structural response. Mode participation factors are time dependent but mode shapes are independent of time. The system in Eqn. (4.1) can be decoupled using the modal coordinates if the modes are normalized with respect to the mass matrix. Thus a set of decoupled system of equations can be formed using normalized matrices \([K]\) and \([M]\) given in Eqn. (4.6) and (4.7) below.

\[
[\Phi]^T[M][\Phi] = [I]
\]  
(4.6)

\[
[\Phi]^T[K][\Phi] = [\omega^2]
\]  
(4.7)

Here, \([I]\) is the identity matrix and

\[
[\omega^2] = \begin{bmatrix}
\omega_1^2 & 0 & \ldots & 0 \\
0 & \omega_2^2 & \ldots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & \ldots & \omega_n^2
\end{bmatrix}
\]

Using mode participation factor, \( \{q(t)\} \) and mode shapes, \([\Phi]\), Eqn. (4.1) can be represented in modal coordinates using the following equation
\[ [M][\Phi] \{q(t)\} + [C][\Phi] \{q(t)\} + [K][\Phi] \{q(t)\} = \{F(t)\} \quad (4.8) \]

### 4.1.3 State Space Representation

State space in general is the representation of physical system in mathematical model as a combination input-output signals and system parameters in first order differential equations. This representation is essential in this chapter, and is used in MATLAB programming environment to compute the system response. The continuous time-invariant state space equation can be represented as:

\[ \{\dot{u}(t)\} = [A_c]\{u(t)\} + [B_c]\{f(t)\} \quad (4.9) \]

where, \([A_c]\) and \([B_c]\) represents the system matrix and input matrix respectively, and the subscript \(c\) stands for continuous system. The state variables are represented by the vector \(u(t)\). System matrices \([A_c]\) and \([B_c]\) are given as follows:

\[
[A_c] = \begin{bmatrix}
0 & I \\
-M^{-1}[K] & -M^{-1}[C]
\end{bmatrix} \quad (4.10)
\]

\[
[B_c] = \begin{bmatrix}
0 \\
M^{-1}
\end{bmatrix} \quad (4.11)
\]

Even though most real world problems are continuous nature, it is impractical to get input-output signal in continuous form. To avoid this difficulty, the time which is continuous in nature is discretized into finite time steps. Signals are measured at these time steps and we are still be able to use the state space representation. For this, it is necessary to transform the state-space representation from the continuous to discrete space. The transformation is done as follows:
\[
\{u\}_{n+1} = [A_d]\{u\}_n + [B_d]\{f\}_n
\]  
(4.12)

\[
[A_d] = e^{A_d}\Delta t
\]  
(4.13)

\[
[B_d] = [A_d]^{-1}([A_d] - I)[B_d]
\]  
(4.14)

where, subscript \(t\) stands for current time-step and \(d\) stands for discrete system. The time increment is represented by the superscript \(\Delta t\). Eqn. (4.12) represents the state space representation of discrete time-invariant system.

### 4.2 Model Order Reduction

The accuracy from the developed model depends upon the number of degrees of freedom of the system. As meshing or discretization in finite element method becomes finer, the number of elements generated will be more and more and thus the number of degrees of freedom will increase. It is impractical to measure the response at each and every degree of freedom, instead certain methods can be utilized to condense the whole system into a reduced system, with relatively less degrees of freedom but posses the dynamic nature of the full model. This reduction (condensation) technique is called model order reduction. There are several condensation techniques in practice for years. Some of these techniques will be employed in this chapter and thus it is important to discuss them in this section.

#### 4.2.1 Static condensation (Guyan Reduction)

Static condensation, originally developed by Guyan (1965) is the basis for all reduction techniques. The dynamic (inertia) effect is ignored in this technique, and therefore it is static condensation. Despite being fifty years old, it is still one among the
most popular condensation methods used. This approach has been included in several structural analyses softwares. A detailed derivation is given below.

Neglecting the inertial and damping effects acting on the structure, Eqn. (4.1) becomes Eqn. (4.15)

\[
\begin{bmatrix}
K \\
\end{bmatrix} \begin{bmatrix}
x(t) \\
\end{bmatrix} = \begin{bmatrix}
F(t) \\
\end{bmatrix}
\]

(4.15)

The equation above contains full degrees of freedom of the system. While performing condensation techniques, some degrees of freedom are retained and others are ignored. The selection of retained degrees of freedom completely lies with the user. The total degrees of freedom of a structural system can be divided into

a) Master degrees of freedom

b) Slave degrees of freedom

In static condensation, master degrees of freedom are the degrees of freedom where the external forces are applied, all other degrees of freedom are to be considered as slaves. Assuming, slave coordinates do not possess considerable amount of information, a full model becomes a reduced one by eliminating the slave coordinates. After determining the master and slave degrees of freedom, Eqn. (4.15) can be rearranged as shown below.

\[
\begin{bmatrix}
K_{mm} & K_{ms} \\
K_{sm} & K_{ss} \\
\end{bmatrix} \begin{bmatrix}
x_m(t) \\
x_s(t) \\
\end{bmatrix} = \begin{bmatrix}
F_m(t) \\
F_s(t) \\
\end{bmatrix}
\]

(4.16)
where, $[K_{mm}]R^{mm}$, $[K_{ms}]R^{mx}$, $[K_{sm}]R^{sx}$, and $[K_{ss}]R^{ss}$. The superscripts ‘$m$’ and ‘$s$’ represent the total number of master and slave degrees of freedom respectively. Since no force is applied on slave degrees of freedom, the second part of Eqn. (4.16) becomes:

$$[K_{sm}][x_m(t)] + [K_{ss}][x_s(t)] = [F_s(t)] = 0$$  \hspace{1cm} (4.17)

The slave coordinates can be related to the master coordinates. The fundamental purpose of static reduction is to express the full system in terms of reduced model and the essential key is given by Eqn. (4.17). A combination of master and slave degrees of freedom can be expressed as shown below.

$$\{x(t)\} = \begin{bmatrix} x_m(t) \\ x_s(t) \end{bmatrix} = \begin{bmatrix} I_{mm} \\ -K_{ss}^{-1}K_{sm} \end{bmatrix} \{x_m(t)\}$$  \hspace{1cm} (4.18)

where $I$ is an identity matrix and Guyan transformation matrix, $T$ is given by Eqn. (4.19)

$$[T_{\text{guyan}}] = \begin{bmatrix} I_{mm} \\ -K_{ss}^{-1}K_{sm} \end{bmatrix}$$  \hspace{1cm} (4.19)

By using the transformation matrix, $T_{\text{guyan}}$, the mass, stiffness and force matrix of full model can be represented in reduced form as shown below

$$[M_{\text{reduced}}] = [T_{\text{guyan}}^T][M_{\text{full}}][T_{\text{guyan}}]$$  \hspace{1cm} (4.20)

$$[K_{\text{reduced}}] = [T_{\text{guyan}}^T][K_{\text{full}}][T_{\text{guyan}}]$$  \hspace{1cm} (4.21)

$$[F_{\text{reduced}}] = [T_{\text{guyan}}^T][F_{\text{full}}]$$  \hspace{1cm} (4.22)

Eqns. (4.19) to (4.22) forms the basis for static reduction.
4.2.2 Component mode synthesis

Component mode synthesis (CMS) is one of the popular model reduction techniques for large structural models. CMS was originally developed by Hurty in 1965 and modified into its present form by Craig and Bampton in 1968, thus CMS method is also known as Craig-Bampton/CB method. The notion of substructuring is introduced in this method where the complete structure divided into finite number of small substructures. Discretization is performed on each substructure either by using finite element method or by any other means. The total degrees of freedom of each substructure can then be divided into two sets:

a) Boundary degrees of freedom, $b$

b) Internal degrees of freedom, $i$

Boundary degrees of freedom are those which share between substructures and internal degrees of freedom belongs to only the relevant substructure. In this dynamic condensation method, both normal modes and static modes will be used.

4.2.2.1 Normal Modes of a Structure

Normal modes of a substructure are defined by the motion of interior coordinates, relative with all boundaries fixed and no force acts on the substructure. The constrained normal modes of any substructure can be computed by solving the eigenvalue problem defined by Eqn. (4.25).

4.2.2.2 Static Modes of a Structure

Static modes of a structure are due to successive unit displacement of each boundary degrees of freedom with all other boundary degrees of freedom fixed and
keeping all internal degrees of freedom free. The static modes can be solved by Eqn. (4.24).

### 4.2.2.3 CMS Substructuring Method

As mentioned earlier, a structural system is first divided into many substructures. For example, a 15 degree of freedom system can be divided into two substructures A and B with 7 degrees of freedom and 8 degrees of freedom respectively. The 7th coordinate and 8th coordinate are called boundary degree of freedom and the remaining coordinates are called internal degrees of freedom. This division becomes complex in nodes with higher degree of freedom. Taking nth substructure and considering the inertia terms, the equilibrium equation of any undamped structural system can be written as Eqn. (4.23).

\[
\begin{bmatrix}
M_{bb} & M_{bi} \\
M_{ib} & M_{ii}
\end{bmatrix}
\begin{bmatrix}
\ddot{x}(t)_{bb} \\
\ddot{x}(t)_{ii}
\end{bmatrix}
+
\begin{bmatrix}
K_{bb} & K_{bi} \\
K_{ib} & K_{ii}
\end{bmatrix}
\begin{bmatrix}
x(t)_{b} \\
x(t)_{i}
\end{bmatrix}
= \begin{bmatrix}
F_{b}(t) \\
F_{i}(t)
\end{bmatrix}
\tag{4.23}
\]

where subscripts ‘b’ and ‘i’ represent the boundary degrees of freedom and internal degrees of freedom respectively. For convenience, considering the stiffness term alone in Eqn. (4.23), and assuming zero force acting on the internal degrees of freedom, second part of the same gives the following equation which relates the boundary degrees of freedom and static modes of the internal degrees of freedom.

\[
\{x(t)\}_i^s = -[K]_{ii}^{-1} [K]_{ib} \{x(t)\}_b
\tag{4.24}
\]

The displacement of the internal degrees of freedom is the sum of static modes and normal modes. The solution to the eigenvalue problem of Eqn. (4.25) forms the constrained modal matrix \([\Phi_c]\), and constrained normal modes, \(\{x(t)\}_i^s\) are given by Eqn. (4.26).
\[-[\omega^2][M_u] + [K_u] = \{0\}\]
\[\{x(t)\}_i^u = \left\{\Phi_c \right\}_p \{q(t)\}_p\]  

(4.25)

(4.26)

where \(p\) is the number of constrained Craig-Bampton normal modes. Generally, the number of constrained normal modes will be very less compared to the internal degrees of freedom. The full displacement vector \(\{x(t)\}\) can then be expressed as:

\[
\{x(t)\} = \begin{cases} 
\{x(t)_b\} \\
\{x(t)_i\}
\end{cases} = \begin{cases} 
\{x(t)_b\} \\
-\{K\}_i^{-1}\{K\}_b \{x(t)\}_b + \left\{\Phi_c \right\}_p \{q(t)\}_p
\end{cases}
\]

(4.27)

\[
= [\Psi]_{CB} \begin{cases} 
\{x(t)_b\} \\
\{q(t)\}_p
\end{cases}
\]

where \([\Psi]_{CB}\) denotes the transformation matrix that transforms from the full model to the CB reduced model. The transformation matrix function for the \(n^{th}\) substructure is given by the following equation.

\[
[\Psi]_{CB} = \begin{bmatrix} 
[J] & [0] \\
-\{K\}_i^{-1}\{K\}_b & [\Phi]_c
\end{bmatrix}
\]

(4.28)

The reduced mass, stiffness and damping matrices can then be formed by using the following equations respectively:

\[
[M]_{CB} = [\Psi]_{CB}^T[M]_{full}[\Psi]_{CB}
\]

(4.29)

\[
[C]_{CB} = [\Psi]_{CB}^T[C]_{full}[\Psi]_{CB}
\]

(4.30)

\[
[K]_{CB} = [\Psi]_{CB}^T[K]_{full}[\Psi]_{CB}
\]

(4.31)

\[
\{x(t)\}_i^u = -\{K\}_i^{-1}\{K\}_b \{x(t)\}_b
\]

(4.32)
Studies show that, Guyan reduction is a good approximation for small eigenvalues of a finite element model, but for problems where higher frequency effects are significant, a rejection of inertial forces will produce inaccurate results. The CMS method developed by Craig and Bampton builds upon Guyan reduction to enhance the accuracy in the model reduction process. Even though, both reduction methods discussed above are available in commercial FE analysis software, for research purposes, both methods were coded in MATLAB programming environment in this thesis. Presented next is the load recovery based on the theories presented in the previous section. The load will be reconstructed using acceleration measurements.

4.3 Load Recovery Using Acceleration Measurements

In the previous chapter, we presented a methodology to recover dynamic-moving loads using strain measurements. Even though uneven subspaces and cardinal points do not impose a limitation on the proposed strain based method, this chapter presents an alternate approach to load recovery using acceleration measurements. Acceleration can be measured using accelerometers. The measured acceleration data is then used to recover the dynamic-moving load in time-domain.

Since at higher frequencies, it is evident that normal modes and static modes have more information about the system response, these mode shapes has been used to recover the dynamic load. D-optimal design algorithm used in the previous chapter is also used in this chapter. Model reduction methods are also engaged to reduce the computational effort and enhance the accuracy of the load recovered. Specific problems are solved numerically to prove the efficiency of the proposed algorithm.
4.3.1 Dynamic Non-Moving Load Recovery without Reduction

As explained in Sec. 4.2, the total degrees of freedom of any system can be reduced to a finite number of degrees of freedom, which have the ability to reasonably capture the dynamic nature of the full system. Before using the reduction techniques, this section will explain the load recovery of a non-moving dynamic load in modal coordinates without reduction of the system matrices. The second order equation of motion of a structure with n degrees of freedom can be written in a matrix form as shown below:

\[
[M]_{nn}[\ddot{x}(t)]_{nt} + [C]_{nn}[\dot{x}(t)]_{nt} + [K]_{nn}[x(t)]_{nt} = [F(t)]_{nt}
\]  

(4.33)

Here, t stands for the number of time-steps in transient analysis. Mass, Damping and Stiffness matrices can be obtained either by writing the equation of motion or by using the finite element method. Normally, for simple 1-Dimensional problems, it is possible to develop the system matrices manually, but finite element software can be used for complex systems. Acceleration data \([\ddot{x}(t)]\) can be obtained either by experiments or using FE software; velocity and displacement matrices can then be obtained by numerical integration. If the full response of the structure is available, load recovery is accurately possible by using the above equation. Eqn. (4.1) shows the equation of motion in physical coordinates; as explained in Sec. 4.1, the system can also be represented in modal coordinates. The relation between physical coordinate and modal coordinates is as follows:

\[
[\ddot{x}(t)]_{nt} = [\Phi]_{nn}[\ddot{q}(t)]_{nt}
\]

(4.34)
where $[\Phi]_{n\times n}$ is the modal matrix, obtained by solving the eigenvalue problem of Eqn. (4.3), and $[\ddot{q}(t)]_{m\times 1}$ is the second derivative of the mode participation factor $q(t)$. The modal matrix is time-invariant and the mode participation factor (mpf) is time-variant. For each transient time-step, mpf will contain the modal mass for each mode.

Mode participation factor can be obtained either from the finite element method or by using least square estimates. For a fixed number of retained modes $m \leq n$, where $n$ denotes the degrees of freedom in the full model, the least square estimate of mpf is obtained by the following equations:

$$
[q(t)]_{m\times 1} = ([\Phi]_{n\times n}^T[\Phi]_{n\times n})^{-1}[\Phi]_{n\times n}^T[x(t)]_{n\times 1} \tag{4.35}
$$

$$
[\dot{q}(t)]_{m\times 1} = ([\Phi]_{n\times n}^T[\Phi]_{n\times n})^{-1}[\Phi]_{n\times n}^T[\dot{x}(t)]_{n\times 1} \tag{4.36}
$$

$$
[q(t)]_{m\times 1} = ([\Phi]_{n\times n}^T[\Phi]_{n\times n})^{-1}[\Phi]_{n\times n}^T[x(t)]_{n\times 1} \tag{4.37}
$$

The dynamic moving load can then be recovered by using the following equation.

$$
[F(t)]_{m\times 1} = [M]_{n\times n}[\Phi]_{n\times m}[\ddot{q}(t)]_{m\times 1} + [C]_{n\times n}[\Phi]_{n\times m}[\dot{q}(t)]_{m\times 1} + [K]_{n\times n}[\Phi]_{n\times m}[q(t)]_{m\times 1} \tag{4.38}
$$

It is evident that as the number of modes increases, the accuracy of the recovered load is also increased.

### 4.3.2 Example: Dynamic Load Recovery without Reduction

For numerical illustration of the above described method, a 1-dimensional spring-mass system is chosen as an example as depicted in Fig. (4.1). The system is considered to be undamped and has a total 15 masses and 16 springs. The first and last masses are attached to fixed boundary conditions and no other boundary conditions were prescribed in the system. The mass is incremented from the first to the last by 10 kg, where the first
mass is 20 kg and the last mass is 160 kg. Similarly, the stiffness values were incremented by $0.5 \times 10^6$ N/m, from $0.5 \times 10^6$ to $7.5 \times 10^6$ N/m.

A load of $F_6 = 500(\sin 10\pi t) + 250(\cos 5\pi t)$ is applied at mass 6. The system can be simulated either using MATLAB environment or by using ANSYS APDL language. While programming in MATLAB, ode45 tool is used to calculate the system response. If using ANSYS, COMBIN14 element type is used to model the spring and MASS21 element type is used to model the mass. No meshing is required since elements are created directly from nodes in this problem. The mass and stiffness matrices can be assembled either by writing the equations of motion, or by using the HBMAT method in ANSYS APDL.

The modal matrix $[\Phi]$ is obtained either by solving the eigenvalue problem in MATLAB or outputting directly from ANSYS. In this example, where no reduction is applied to the mass and stiffness matrices, accelerometers are assumed to be placed at all the 15 masses. Mode participation factors for the first two, five and full (fifteen) modes are developed using the least square solution from Eqns. (4.35) to (4.37). Then by using Eqn. (4.38), the dynamic non-moving load is recovered. Figs. (4.2), (4.3) and (4.4) shows the recovery of a non-moving dynamic load for the above described problem. From the recovered loads, it is evident that as the number of retained modes increases, the accuracy of recovered load will also increase.

Fig. (4.4) shows that if all modes are available, the load recovery can be done accurately. However, practically, it is impossible to observe all the modes and also to place sensors at all the locations. Some alternative methods are needed in which with
limited number of non-collocated sensor locations and mode shapes, load recovery is done precisely. Discussed next are some methods, which have been developed based on this objective.

4.3.3 Dynamic Non-Moving Load Recovery using D-Optimal Design and Model Reduction

This section presents a time-domain technique to recover a non-moving dynamic load from acceleration measurements, at finite number of optimally placed sensors. The attractiveness of this technique is that the sensors are not collocated with the node at which the load is applied. For this, reduction methods explained in Sec. 4.2 will be employed. At first, Guyan reduction methodology can be engaged to recover a dynamic load. The concept behind reduction remains unchanged, but the selection of master degrees of freedom will be based on some specific locations. There are large numbers of locations on a structure where the accelerometers can be placed. It is evident that all locations will not give similar response and the accuracy of predicted load depends significantly on the location of the sensor. The optimum sensor locations can be identified by using the D-optimal design explained in chapter 3. The major difference between D-optimal design based on strain measurements and accelerometer measurements is that in strain measurements, strain from an element is used and optimum locations are elements whereas in acceleration measurements, they are measured at each degree of freedom and hence the optimum locations are in terms of degrees of freedom. The following section will explain the development of a candidate set for D-optimal design and using these designs for identifying acceleration measurement locations.
4.3.3.1 Candidate Set for Accelerometers

The maximum number of modes available in any finite element model is equal to the maximum number of degrees of freedom. It is impossible to place sensors at all degrees of freedom, and hence only a finite number of modes will be involved in the computation of the response. Past research had shown that sufficient number of modes must be retained such that at least 90% of the modal mass is included in the reduced model.

Also, another important consideration is the number of accelerometers. As the number of sensors increases, the prediction of mode participation factor \([\dot{q}(t)]\) becomes more precise. But this approach is not cost effective and practical. If the number of modes is more than the number of accelerometers, the number of sensors is less than the number of unknowns to be estimated, the estimates cannot be determined accurately. Hence, the number of accelerometers must always be greater than or equal to the number of retained modes.

Instead of strain data which works with element number, mode shapes, \([\Phi]\) with appropriate degree of freedom will act as the input data to D-optimal design. Initially all unaccessible locations and the locations where the loads are applied were eliminated from the candidate set. Thus the number of degrees of freedom in the candidate set can be fixed.

\[
[\Phi]_{full} \rightarrow [\Phi]_{candidate\_set}
\]  

(4.39)
4.3.3.2 D-optimal Design for Accelerometers

The number of demanded optimum sensor locations must be greater than or equal to the number of mode shapes in the candidate set. By using the D-optimal design concept explained in chapter 3, optimum sensor locations can be identified. During the D-optimal design process, many subsets are generated can be represented as \( \Phi \). In terms of the subset of candidate set \( \Phi \), the approximate solution of Eqn. (4.35) becomes;

\[
[\ddot{q}(t)] \cong [\ddot{\tilde{q}}(t)] = [\Phi^T \Phi]^{-1} [\tilde{\Phi}^T][\tilde{x}(t)]
\]  

(4.40)

where, \( \tilde{\ddot{q}}(t) \) is an approximation of \( \ddot{q}(t) \), and \( \tilde{x}(t) \) is a subset of \( \ddot{x}(t) \). Since \( \ddot{x}(t) \), is prone to measurement errors, \( \ddot{\tilde{q}}(t) \) will also have calculation errors based on the inverse problem defined in Eqn. (4.35).

4.3.3.3 Solution Procedure using D-optimal Design and Model Order Reduction

After producing the optimum set \( [\Phi_{\text{optimum}}] \in R^{a \times m} \) from D-optimal design, ‘a’ accelerometers can be physically placed at those particular optimum locations. Acceleration measurements at optimum locations \( [\ddot{x}(t)]_{\text{optimum}} \) are recorded and corresponding numerical integration can produce \( [\ddot{x}(t)]_{\text{optimum}} \) and \( [x(t)]_{\text{optimum}} \) respectively. \( [\ddot{\tilde{q}}(t)] \), which is an approximation is of \( [\ddot{q}(t)] \) can be obtained by the following Eqn. (4.41):

\[
[\ddot{\tilde{q}}(t)] = [\Phi_{\text{optimum}}^T \Phi_{\text{optimum}}]^{-1} [\Phi_{\text{optimum}}^T][\ddot{x}(t)]_{\text{optimum}}
\]  

(4.41)
Comparing Eqn. (4.41) with Eqn. (3.8), it is important to notice that $\Phi_{\text{optimum}}$ has the same role as $A_{\text{optimum}}$. When using static condensation, it is important to divide the degrees of freedom into master degrees of freedom and slave degrees of freedom. The master degrees of freedom can be chosen as the optimum locations and the locations where the loads are applied. Corresponding reduced stiffness matrix $[K]_{\text{Guyan}}$ can be produced either by following the Sec. 3.2.1 or directly using ANSYS substructuring method. It is evident that both reduced matrices will produce similar accuracy in load recovery. Then by following the Eqn. (4.42), dynamic loads can be recovered:

$$[F(t)] = [K]_{\text{Guyan}} [\Phi]_{\text{optimum}} [\tilde{q}(t)]$$  \hspace{1cm} (4.42)

The load recovered using Guyan reduction process will be less accurate because inertia terms are neglected. Craig-Bampton method, also known as fixed interface CMS method, can be used to recover loads with more accuracy. When comparing to static reduction, a significant improvement in load recovery was observed when using CB reduction with D-optimal Design.

After computing the acceleration at optimum locations and corresponding velocity and displacements, Eqn. (4.27) is used to obtain $[\dot{x}(t)_b]$ and its derivatives. Eqn. (4.1) can be thus be rewritten as follows:

$$[M]_{\text{CB}} [\ddot{x}(t)_b] + [C]_{\text{CB}} [\dot{x}(t)_b] + [K]_{\text{CB}} [x(t)_b] = \begin{bmatrix} F_{b}(t) \\ F_{f}(t) \end{bmatrix}$$  \hspace{1cm} (4.43)
where all the reduced system matrices can be obtained from Eqns. (4.29) to (4.31). It is important to note that, the boundary degrees of freedom are a combination of the degrees of freedom where the loads are acting and some randomly selected locations.

The same example problem from the previous section is used in this section to test the efficacy of load recovery using Guyan as well as Craig-Bampton reduction. A load of $F(t) = 500\sin(30nt) + 350\cos(20nt)$ is applied at the 3rd mass of the spring mass system. The modal matrix, otherwise called the eigenvector matrix, is formed by solving the Eqn. (4.4).

A total of 5 accelerometers were used and the number of retained mode shapes is 4. The number of Craig-Bampton constrained normal modes was 2. Optimum locations were identified using D-optimal design and are listed below. The degrees of freedom at which the loads are applied and the optimum locations will form a set of boundary degrees of freedom and more detailed theory behind the reduction process is explained in Sec. 4.2.2. Dynamic load at 6th mass can be recovered using Eqn. (4.42) for Guyan reduction and Eqn. (4.43) for CB reduction. Fig. (4.6) presents the recovered dynamic load at 6th degree of freedom using Guyan reduction and CB reduction.

The load recovered shown in Fig. (4.6) was found to be unacceptable, especially using Guyan Reduction. Also in general, Guyan reduction is not recommended at higher frequencies where inertia effects have to be accounted. The Craig-Bampton method needs to be improved in such a way that the load recovered will be highly accurate.
4.3.4 Load Recovery using D-Optimal Design and Reduced Modal Model

The previous section details the use of D-optimization with Guyan and CB reduction in dynamic non-moving load identification. This section deals with the recovery of non-moving dynamic load acting on a structure using improved Craig-Bampton method, also called Reduced Modal Model, which was originally developed by Gupta (2013).

Consider the Craig-Bampton reduced model Eqn. (4.43) which is in its matrix form. Treating the equation as an eigenvalue problem, the eigenvector solution will yield reduced Craig-Bampton modal matrix, \([\Phi]_{cb}\). Since both Craig-Bampton reduced mass and stiffness matrix has the capability to capture the dynamic nature of the full model, it is assumed that \([\Phi]_{cb}\) also captures the modal information of the full model.

The reduced CB modal matrix can be transformed to the modal coordinates by using the following transformation:

\[
\begin{bmatrix}
\ddot{x}(t)

q(t)
\end{bmatrix} = [\Phi]_{cb} [\ddot{q}(t)]_{cb}
\]

where \([q(t)]_{cb}\) are the reduced Craig-Bampton normal modes. Pre-multiplying the above equation by Craig Bampton transformation matrix \([\Psi]_{cb}\) yields:

\[
[\Psi]_{cb} \begin{bmatrix}
\ddot{x}(t)

q(t)
\end{bmatrix} = [\Psi]_{cb} [\Phi]_{cb} [\ddot{q}(t)]_{cb}
\]

Using Eqn. (4.27) and substituting \([\Phi]_{u} = [\Psi]_{cb} [\Phi]_{cb}\), gives the following equation:
\[ [x(t)] = [\Phi]_u [\ddot{q}(t)]_{CB} \quad (4.46) \]

where the subscript \( u \) stands for updated matrix. Comparing with Eqn. (4.5), Eqn. (4.46) has more ability to capture the dynamic nature of the full system as no modes are truncated here. Hence, the recovery of acceleration using Eq. (4.46) should be more accurate than by using Eq. (4.5). Still, \( \ddot{q}(t)_{CB} \) need to be identified in Eqn. (4.46), which can be solved similar to Eqn. (4.41) i.e. by identifying the optimum locations \( [\Phi]_{u_{optimum}} \) from \( [\Phi]_u \). After retrieving the accelerations at these optimum locations \( [q(t)]_{CB} \), can be identified by using Eq. (4.47)

\[ [\dddot{q}(t)] = [\Phi]^T_{u_{optimum}} [\Phi]_{u_{optimum}}^{-1} [\Phi]^T_{u_{optimum}} [\dddot{x}(t)]_{optimum} \quad (4.47) \]

The full displacement and acceleration can then be identified using the Eqn. (4.46). Again, \( \left\{ \dddot{x}(t) \right\} \) still need to be identified using Eqn. (4.27). Then similar to conventional Craig-Bampton method, using Eqn. (4.43), the applied dynamic load can be recovered accurately.

The spring mass system used in the previous section is used again and dynamic loads were recovered accurately. Fig. (4.7) depicts the capability of the described method. The spring mass system used is 1-dimensional in nature and to further prove the capability of the proposed method it is necessary to apply the method to a 3-dimensional system. A 3-dimensional cantilever beam is modeled and a non-moving dynamic load is applied and recovered.

A load of magnitude \( F(t) = 500 \sin(30\pi t) + 350 \cos(20\pi t) \), is applied at the free end of a cantilever beam. The beam is 25 inch long, 5 inch wide and 0.5 inch thick. The finite
element model of the beam is made in ANSYS software and was meshed using SOLID45 element type which has three degrees of freedom at each node. The system has 200 nodes and 600 degrees of freedom. The system matrices were generated from ANSYS by using Harwell-Boeing file format. The assumed inputs for the whole algorithm are provided in Table 4.2.

Based on the defined boundary degrees of freedom, Craig Bampton reduced system matrices were produced using Eqns. (4.28) till (4.31) and an updated modal matrix is then formed by using the relation \[ \Phi_U = \Psi_{CB} \Phi_{CB} \]. After eliminating degrees of freedom at which loads are applied from \( \Phi_U \), D-optimal design algorithm was used to identify optimum accelerometer locations from \( \Phi_{U,\text{candidate}} \), and is shown in Fig. (4.8) and (4.9). By using Eqn. (4.47), the mode participation factor of the retained modes can be calculated from acceleration measurements at optimum locations. Using Eqn. (4.46), the system response for the whole structure can be identified and then transformed to match the CB reduced system matrices by using Eqn. (4.27). Applied load was then recovered by using Eqn. (4.43). The recovered load is shown in Fig. (4.10). It is clear from the output that the reduced modal model algorithm is far better than conventional Craig Bampton method in dynamic load recovery procedure.

4.3.5 Moving Load Recovery using D-Optimal Design and Reduced Modal Model

The same structure used in the previous section is used with a dynamic moving load of magnitude \( F(t) = 500\sin(30\pi t) + 350\cos(20\pi t) \). The load was moving from the left most end to the right most end of the beam and is shown in Fig. (4.11). A total of 19 load passing locations were formed in the problem and the intention was to identify the
dynamic moving load at all these locations together as a one single solution. The same procedure used in the previous section is used to recover the dynamic moving load. The moving load is recovered at time steps 1, 5, 16, 18 and 19. Interpolation is done between these time steps to obtain a complete history of the moving load.

Along with the degrees of freedom at which the load is applied, some boundary degrees of freedom must also be specified to recover the dynamic load in reduced modal model. It has been observed that the best set of these boundary degrees of freedom will change for each load applied location for an accurate solution. A global set for these boundary locations could not be identified and hence the solution procedure was done separately for each load passing location.

Thirty optimum locations were identified and are listed in Table. 4.3. The loads were identified at specific load steps and higher order interpolation was used to recover the dynamic moving load. Loads at time step 1, 5, 16, 18 and 19 were identified and the input assumptions are given from Table 4.4 through Table 4.8. The recovered load is given in Fig. (4.12).

4.4 Summary

A new computational method is presented to recover a dynamic moving load using acceleration measurements at optimum sensor locations. From numerical simulations, it is clear that as more and more modes are involved in the load recovery procedure, the accuracy of recovered load improves. Unfortunately more number of retained modes will demand more accelerometers to be placed and hence the approach soon ceases to become practically viable. An acceptable solution for the problem is to
reduce the size of the system matrices by using model order reduction methods. Conventional model reduction algorithms are explained and are used to recover the load along with D-optimal algorithm. From the results it is clear that the reduction methods need to be developed further for more accurate solutions. The chapter explains a recently developed reduced modal model algorithm which can be used to recover dynamic loads accurately.

Reduced modal model algorithm preserves the dynamic nature of the structure more than any other reduction methods explained in this chapter. Even though the algorithm is capable of accurately recovering applied loads, it suffers from a drawback in the selection of suitable boundary degrees of freedom. For the moving load problem, due to this shortcoming, the loads were recovered separately and then interpolated together to get the final output. By identifying a global set of optimum boundary degrees of freedom, reduced modal model method has the capability to recover a dynamic moving load without interpolation.
**Table 4.1:** Optimum Sensor Locations for 15 DOF Spring Mass Problem

<table>
<thead>
<tr>
<th>Sensor No.</th>
<th>Degree of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

**Table 4.2:** Input Data for 3D Cantilever Beam for Model Order Reduction

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Degrees of Freedom</td>
<td>600</td>
<td>Candidate Set</td>
<td>597</td>
<td>Optimum accelerometers</td>
<td>8</td>
</tr>
<tr>
<td>Modes Involved</td>
<td>7</td>
<td>Boundary Degrees of Freedom</td>
<td>213, 280,372, 425</td>
<td>CB Constrained Modes</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 4.3: Optimum Locations for a 3D Cantilever Beam Under Dynamic Moving Load

<table>
<thead>
<tr>
<th>Optimum Degrees of Freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,20,32,44,68,71,95,98,101,113,</td>
</tr>
<tr>
<td>122,128,131,146,158,164,185,194,197,203,</td>
</tr>
<tr>
<td>215,221,230,242,248,374,380,413,500,599</td>
</tr>
</tbody>
</table>

Table 4.4: Input Data for time step 1 for 3D Cantilever Beam Under Dynamic Moving Load

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Degrees of Freedom</td>
<td>600</td>
<td>Candidate Set</td>
<td>543</td>
<td>Optimum accelerometers</td>
<td>30</td>
</tr>
<tr>
<td>Modes Involved</td>
<td>29</td>
<td>Boundary Degrees of Freedom</td>
<td>210,278,372,434</td>
<td>CB Constrained Modes</td>
<td>10</td>
</tr>
</tbody>
</table>
### Table 4.5: Input Data for time step 5 for 3D Cantilever Beam Under Dynamic Moving Load

<table>
<thead>
<tr>
<th>Variable</th>
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<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Degrees of Freedom</td>
<td>600</td>
<td>Candidate Set</td>
<td>543</td>
<td>Optimum accelerometers</td>
<td>30</td>
</tr>
<tr>
<td>Modes Involved</td>
<td>29</td>
<td>Boundary Degrees of Freedom</td>
<td>12,280,372,470</td>
<td>CB Constrained Modes</td>
<td>10</td>
</tr>
</tbody>
</table>

### Table 4.6: Input Data for time step 16 for 3D Cantilever Beam Under Dynamic Moving Load

<table>
<thead>
<tr>
<th>Variable</th>
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<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Degrees of Freedom</td>
<td>600</td>
<td>Candidate Set</td>
<td>543</td>
<td>Optimum accelerometers</td>
<td>30</td>
</tr>
<tr>
<td>Modes Involved</td>
<td>29</td>
<td>Boundary Degrees of Freedom</td>
<td>212,278,371,569</td>
<td>CB Constrained Modes</td>
<td>10</td>
</tr>
</tbody>
</table>
### Table 4.7: Input Data for time step 18 for 3D Cantilever Beam Under Dynamic Moving Load

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Degrees of Freedom</td>
<td>600</td>
<td>Candidate Set</td>
<td>543</td>
<td>Optimum accelerometers</td>
<td>30</td>
</tr>
<tr>
<td>Modes Involved</td>
<td>29</td>
<td>Boundary Degrees of Freedom</td>
<td>213, 280, 372, 587</td>
<td>CB Constrained Modes</td>
<td>10</td>
</tr>
</tbody>
</table>

### Table 4.8: Input Data for time step 19 for 3D Cantilever Beam Under Dynamic Moving Load

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Degrees of Freedom</td>
<td>600</td>
<td>Candidate Set</td>
<td>543</td>
<td>Optimum accelerometers</td>
<td>30</td>
</tr>
<tr>
<td>Modes Involved</td>
<td>29</td>
<td>Boundary Degrees of Freedom</td>
<td>213, 280, 372, 596</td>
<td>CB Constrained Modes</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 4.1: 15 Degrees of Freedom Spring-Mass System

Figure 4.2: Load Recovery-No Reduction with 2 Modes
Figure 4.3: Load Recovery-No Reduction with 5 Modes

Figure 4.4: Load Recovery-No Reduction with All Modes (15 Modes)
Figure 4.5: ANSYS Plot of Optimum Sensor Locations for Spring Mass System

Figure 4.6: Dynamic Load Recovered using Guyan Reduction and CB Reduction
Figure 4.7: Dynamic Non-moving Load Recovered using Reduced Modal Model for Spring Mass System

Figure 4.8: Finite Element Model of Cantilever Beam and Optimum Accelerometer Locations (Top View)
Figure 4.9: Finite Element Model of Cantilever Beam and Optimum Accelerometer Locations (Bottom View)

Figure 4.10: Non-moving Dynamic Load Recovery using Reduced Modal Model for a 3D Cantilever Beam
Figure 4.11: Path of Dynamic Moving Load Acting on a 3D Cantilever Beam

Figure 4.12: Dynamic Moving Load Recovery using Reduced Modal Model for a 3D Cantilever Beam
Chapter 5 Conclusions and Future Work

The thesis details algorithms to recover dynamic moving load(s) using structural response measured at optimum locations. Chapter 1 explains the problem statement and the necessity of this thesis in detail. Identification of the true value of a dynamic moving load is significant for an optimized design solution. Structural components constructed by considering an assumed design load may fail. Direct methods such as placing a load cell to recover loads have several disadvantages and are explained in chapter 1. The disadvantages of direct method are solved indirect method, in which a structure itself is converted into a transducer or called “self transducer.” Sensor location is one among the major factor in placing a sensor on a structure and is framed as an optimization problem in this thesis. In order to predict the load accurately by avoiding the ill-conditioning, the sensor must be placed at an optimum location where the structural response will give an accurate estimate of load acting.

No studies were found during the literature review where an optimum sensor location is used to identify the true value of a dynamic moving load. A discussion of several former methods is provided in chapter 2. Since former methods neglect the effect of optimum sensor location, most of them used a regularization technique to reduce the error in the solution. These regularization techniques will increase the computational difficulty of the problem with a non-agreeable accuracy.

Chapter 3 explains the algorithm used to recover a dynamic moving load using strain gages. The concept of identifying optimum sensor location using D-optimal design is explained under this chapter. After placing the strain gages at optimum locations, strain
data is recorded only for some degrees of freedom and are called cardinal degrees of freedom. Dynamic moving loads are estimated only for these cardinal degrees of freedom. A complete load profile can then estimated by using suitable interpolation techniques. Three specific problems have been discussed in chapter 3 which proves the capability and reliability of the developed method. The algorithm is also tested with noise present in the input strain response data. Even in the presence of random noise present in the strain data, the load estimates are obtained with a high degree of accuracy.

Chapter 4 details the recovery of a dynamic moving load using accelerometers. In this approach, mode shapes have a great significance and their influence is explained in Sec. 4.1. As more modes are involved in the estimation of the system response, the more will be the accuracy of recovered load. Since there exists a condition in the employed algorithm that the number of accelerometers included in the recovery procedure must be greater than the number of retained mode shapes, more and more number of modes cannot be simply included in the procedure. This problem is solved by using model reduction techniques, in which only a limited number of modes are used in the entire recovery procedure. Former model reduction methods are explained in chapter 4 including the advanced model reduction method used in this thesis. A specific problem is solved by using the advanced model reduction method and D-optimal algorithm.

Simple structures such as simply supported and cantilever beams are chosen to test the developed methods. This selection was made because they can easily represent most of the structures under dynamic moving load in the real world. Interpolation techniques are still used in the recovery procedure using accelerometers to recover the full load profile of a dynamic moving load. The utilization of interpolation methods is
due to the lack of an optimum set of boundary degrees of freedom that could be used during the reduction process. Optimum boundary degrees of freedom are particular set of degrees of freedom that can provide an accurate estimate of load when combined with the degrees of freedom on which loads are acting during model reduction and load recovery procedure. As an area of future research, any investigation that identify the optimum boundary degrees of freedom with load bearing degrees of freedom will enhance the usability of the algorithm developed for accelerometers.
BIBLIOGRAPHY


